Small-x phenomenology in collinear factorisation

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Diffraction & Low-x 2024: Tonnara Trabia, 9 September 2024

Small-x logarithms and collinear factorisation

Collinear factorisation:

$$\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(z, \alpha_s(Q^2)\right) f_i\left(\frac{x}{z}, Q^2\right)$$

DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}f_i\left(x, \mu^2\right)}{\mathrm{d}\mu^2} = \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(z, \alpha_s(\mu^2)\right) f_j\left(\frac{x}{z}, \mu^2\right)$$

Perturbative expasion of C_i, P_{ij} $\rightarrow$ **logarithmic enhancements**

Small-x logarithms: $\alpha_s^n \frac{1}{x} \log^k\left(\frac{1}{x}\right) \quad 0 \leq k \leq n-1$ (single logs)

$\alpha_s \log\left(\frac{1}{x}\right) \sim 1$ (fixed order) perturbation theory fails $\rightarrow$ **all-order resummation**

Collinear factorisation:

$$\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(z, \alpha_s(Q^2)\right) f_i\left(\frac{x}{z}, Q^2\right)$$

DGLAP evolution:

$$\mu^2 \frac{\mathrm{d} f_i\left(x, \mu^2\right)}{\mathrm{d}\mu^2} = \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(z, \alpha_s(\mu^2)\right) f_j\left(\frac{x}{z}, \mu^2\right)$$

k_t-factorisation: S. Catani, F. Hautmann [hep-ph/9405388]

$$\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}k_t^2\, \mathcal{C}_g\left(\frac{x}{z}, \alpha_s, Q^2, k_t^2\right) \mathcal{F}_g\left(z, Q^2, k_t^2\right) + \ldots$$

BFKL equation singlet sector, $t = \log(1/x)$:

$$\frac{\mathrm{d}}{\mathrm{d}t} f\left(t, q^2\right) = \int_0^\infty \frac{\mathrm{d}k^2}{k^2} K\left(\frac{q^2}{k^2}, \alpha_s\right) f\left(t, k^2\right)$$

G. Altarelli, S. Forte [hep-ph/9703417], M. Bonvini [1212.0480]

(Some) Recent results

Hybrid Factorisation Approach (HyF) $\rightarrow$ `JETHAD` code [arXiv:2008.00501],[2103.07396]

D. Bolognino, F.G. Celiberto, L. Delle Rose, M. Fucilla,

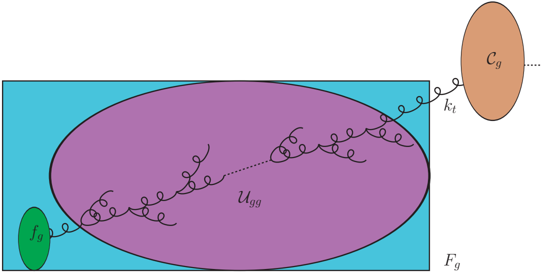
G. Gatto, D.Y. Ivanov, M.M.A. Mohammed, A. Papa

- full or partial NLL accuracy for many LHC processes:
  Mueller-Navelet jet emissions, Drell-Yan pair, light or heavy-light hadron, quarkonium
  Z/Higgs+jet at NLL/NLO^{-} [2408.08757],[2408.08731] + **Poster session tomorrow**

Lipatov EFT + High-energy factorisation

J.P. Lansberg, **M. Nefedov**, M. A. Ozcelik

- NLO impact factors [2112.06789], [2408.09440], [2408.06234]
  NLL+NLO matched predictions for η_c produciton $\rightarrow$ **later talk today**

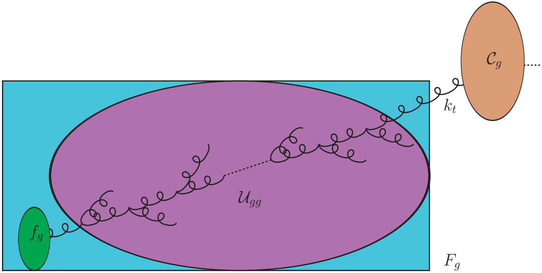


Collinear factorisation:

$$\sigma(N, Q^2) = \int_0^1 \mathrm{d}z\, z^N \sigma(z, Q^2) = C_i(N, \alpha_s, \mu_F^2) f_i(N, \mu_F^2)$$

k_t-factorisation: [Catani, Hautmann hep-ph/9405388]

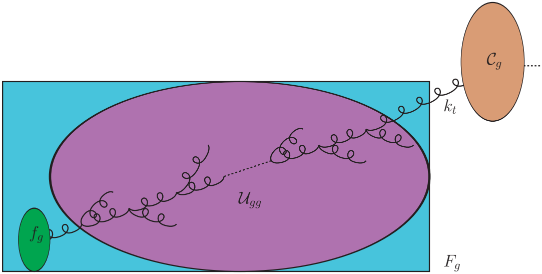
$$\sigma(N, Q^2) = \int_0^1 \mathrm{d}z\, z^N \sigma(z, Q^2) = \int \mathrm{d}k_t^2\, \mathcal{C}(N, \alpha_s, Q^2, k_t^2) \mathcal{F}(N, Q^2, k_t^2)$$



$$\mathcal{F}(N, k_t^2, Q^2) = U(N, k_t^2, Q^2)\, f(N, Q^2)$$

Mellin-space formalism, fixed coupling: G. Altarelli, R. D. Ball, S. Forte [0802.0032]...

$$U(N, k_t^2, Q^2) = \gamma(\alpha_s, N)\left(\frac{k_t^2}{Q^2}\right)^{\gamma(\alpha_s,N)} \xrightarrow[\mathrm{NLL}\gamma]{} \frac{\mathrm{d}}{\mathrm{d}k_t^2} \exp\left[\int_{Q^3}^{k_t^2} \frac{\mathrm{d}{q_T}^2}{{q_T}^2} \gamma_r(N, \alpha_s({q_T}^2))\right],$$



Return to direct space $\rightarrow$ HELL

$$C_g(z, \alpha_s, Q^2) = \int_z^1 \frac{\mathrm{d}z'}{z'} \int \mathrm{d}k_t^2 \, \mathcal{C}_g\left(\frac{z}{z'}, \alpha_s, Q^2, k_t^2\right) U(z', k_t^2, Q^2)$$

M. Bonvini, S. Marzani [1805.06460],[1708.07510],[1607.02153]

- Resummation of Deep Inelastic Scattering
- Determination of Collinear PDFs with NNLO+NLLx evolution

R. D. Ball, V. Bertone, M. Bonvini, S. Marzani, J. Rojo, L. Rottoli [1710.05935],

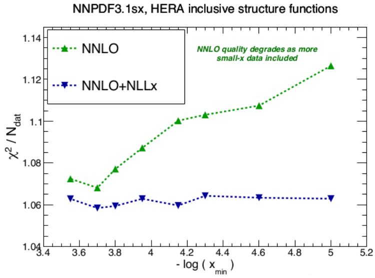
xFitterCollaboration [1802.00064]

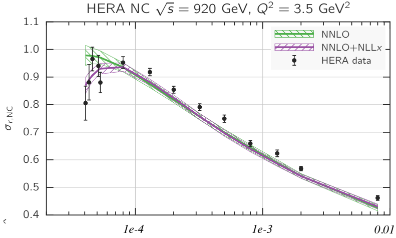
M. Bonvini, F. Giuli, FS [1906.06573][2311.08785]

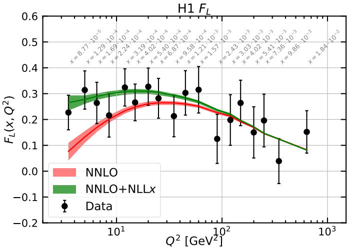
Successful description of this region when including small-x resummation!

- NNPDF3.1 framework [1710.05935]
- xFitter framework [1802.00064]

Turnover reproduced →



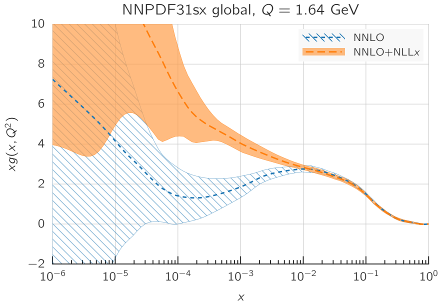


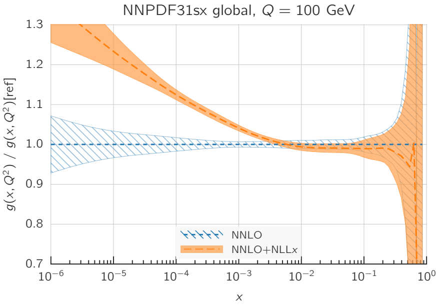


χ^2/N_{dat}	NNLO	NNLO+NLLx
xFitter	**1.23**	**1.17**
NNPDF3.1sx	**1.130**	**1.100**
		reduction!

No extra parameters →**refined theory only**

Impact on gluon PDF





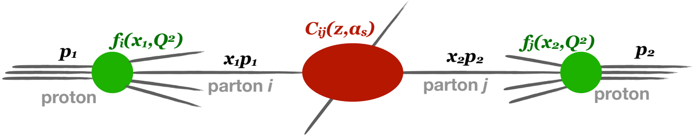
Dramatic effect of resummation on the gluon PDF at $x \lesssim 10^{-3}$

- Larger effect at higher energy scales →impact for LHC and FCC-hh
- Onset of saturation at some point

Differential cross section in collinear factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\ldots} = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}\hat{y}\, \mathscr{L}_{ij}\left(\frac{x}{z}, \hat{y}, Q^2\right) \frac{\mathrm{d}C_{ij}}{\mathrm{d}y\ldots}(z, Y-\hat{y}, \ldots, \alpha_s)$$

$$\mathscr{L}_{ij}(x, \hat{y}, Q^2) = f_i(\sqrt{x}e^{\hat{y}}, Q^2)\, f_j(\sqrt{x}e^{-\hat{y}}, Q^2)\, \vartheta(e^{-2|\hat{y}|} - x)$$



$\frac{Q^2}{s} = x < z$

note: typically $\sqrt{z}e^{\pm\hat{y}} \sim \sqrt{x}$

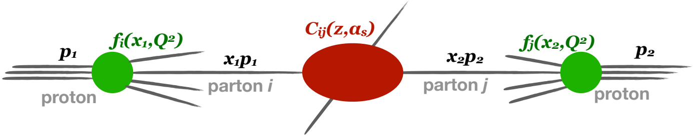
Processes considered so far in HELL:

- $gg \to H$ (inclusive cross section) [1802.07758], [1805.08785]
- $c\bar{c}$, $b\bar{b}$ pair production M. Bonvini, FS [2211.10142]
- $gg \to H$ (differential in Higgs phase space) M.Bonvini, B. Bernardini [TBA]
- Drell-Yan *(in progress)*

Resummation of LHC observables

Differential resummation at proton collider

$$\frac{\mathrm{d}C_{gg}}{\mathrm{d}y\ldots}(z,y,\ldots)=\int_0^\infty \mathrm{d}k_1^2\int_0^\infty \mathrm{d}k_2^2\int_z^1\frac{\mathrm{d}x}{x}\int\mathrm{d}\hat{y}\,\frac{\mathrm{d}\mathcal{C}_{gg}}{\mathrm{d}y\ldots}(x,y-\hat{y},k_1^2,k_2^2,\ldots,\alpha_s)$$
$$\times\frac{\mathrm{d}}{\mathrm{d}k_1^2}U\left(\sqrt{\frac{z}{x}}e^{\hat{y}},k_1^2,Q^2\right)\frac{\mathrm{d}}{\mathrm{d}k_2^2}U\left(\sqrt{\frac{z}{x}}e^{-\hat{y}},k_2^2,Q^2\right)\vartheta\left(e^{-2|\hat{y}|}-\frac{z}{x}\right)$$



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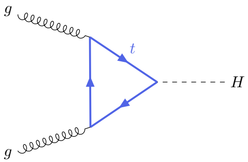
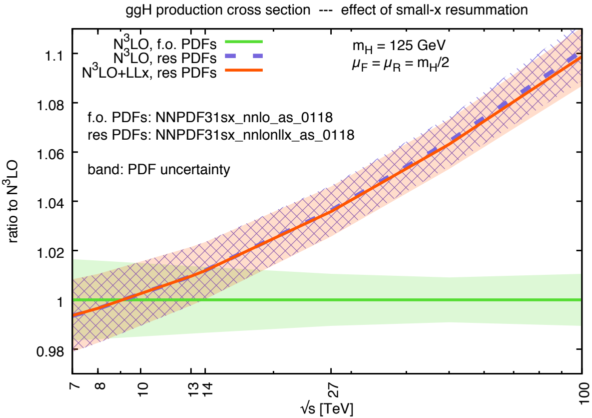
$gg \to H$ inclusive cross section

M. Bonvini, S. Marzani [1802.07758], [1805.08785]





ggH cross section at FCC-hh can be $\sim 10\%$ larger than expected with NNLO PDFs!

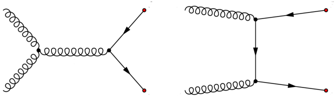
At LHC $+1\%$ effect$+$ expected enhancement at differential level

Progress toward full NLLx resummation with NLO impact factor/coefficient function

F.G. Celiberto et al. [2408.08731]

Fully differential heavy-quark pair production

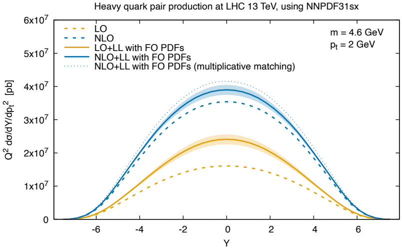
M. Bonvini, FS [2211.10142]



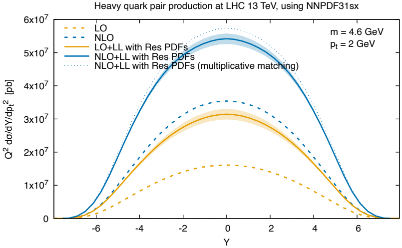
$$\frac{d\sigma}{dQ^2\, dY\, dq_t} \to \text{pair kinematics}$$

$$\frac{d\sigma}{dy\, dp_t} \to \text{single kinematics}$$

Small-x resummation crucial for
charm and bottom production



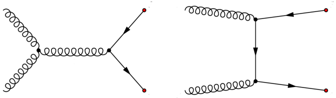
FO PDFs ↑ **↓ Resummed PDFs**



Charm (and bottom) pair production at LHC

Fully differential heavy-quark pair production

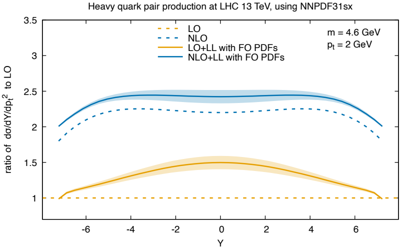
M. Bonvini, FS [2211.10142]



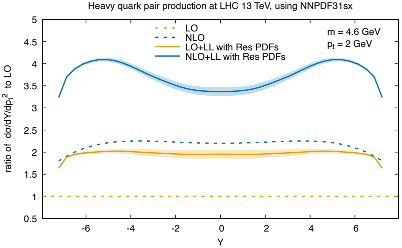
$\frac{d\sigma}{dQ^2\,dY\,dq_t} \rightarrow$ pair kinematics

$\frac{d\sigma}{dy\,dp_t} \rightarrow$ single kinematics

Small-x resummation crucial for **charm and bottom production**



FO PDFs ↑ ↓ **Resummed PDFs**



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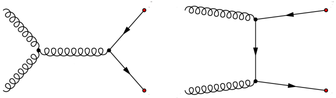
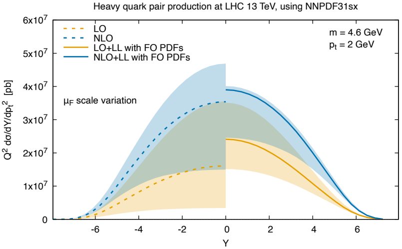


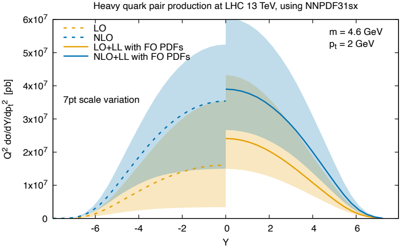
$\frac{d\sigma}{dQ^2\, dY\, dq_t} \to$ pair kinematics

$\frac{d\sigma}{dy\, dp_t} \to$ single kinematics

Small-x resummation crucial for **charm and bottom production**



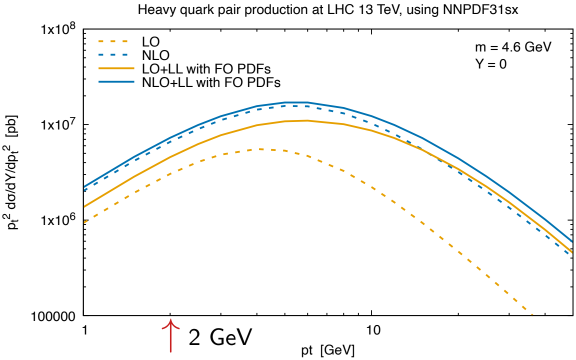
μ_F **variations** $\uparrow$ $\downarrow$ **7-points variations**



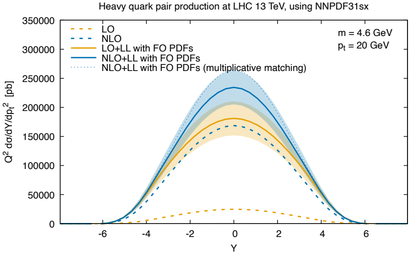
Heavy-quark pair production at LHC

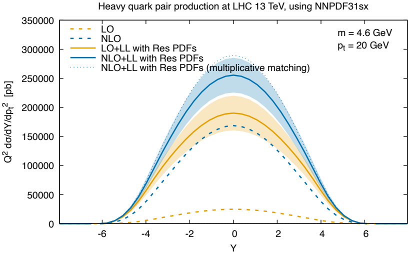
Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**

M. Bonvini, FS [2211.10142]



Results for $\frac{d\sigma}{dy\, dp_t}$ in **single quark kinematics** M. Bonvini, FS [2211.10142]





Conclusion and outlook

Key messages:

- Resummation is needed at small-x, especially $x \lesssim 10^{-3}$
- Significant impact expected at LHC at low invariant mass and large rapidity
- Future colliders will be sensitive to this effect

Outlook:

- Wide array of process resummed to NLL+NLO accuracy $\rightarrow$more NLO coefficient functions are needed
- Differential resummed results for LHC pheno $\rightarrow$potential to perform an updated NLLx+aN3LO global PDF fit

Backup

Small-x resummation of DGLAP evolution

Let's focus on the gluon PDF, which is appropriate at LL.

Mellin transf.: $$h(N) = \int_0^1 \mathrm{d}z\, z^N h(z)\,,$$

DGLAP: $$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} f(N,\mu^2) = \gamma(N,\alpha_s) f(N,\mu^2)$$

BFKL: $$x \frac{\mathrm{d}}{\mathrm{d}x} \mathcal{F}(x,M) = \chi(M,\alpha_s) \mathcal{F}(x,M)$$

Perturbatively computing

$$\chi(M,\alpha_s) = \alpha_s \chi_0(M) + \alpha_s^2 \chi_1(M) + \ldots$$

LO does not reproduce fixed Q^2, $x \to 0$ asymptotic behaviour (large χ_n).

Several solutions [Altarelli, Ball Forte][Ciafaloni, Colferai, Salam, Stasto][Thorne, White]
aim to combine resummation of collinear and high-energy logarithms

ABF method

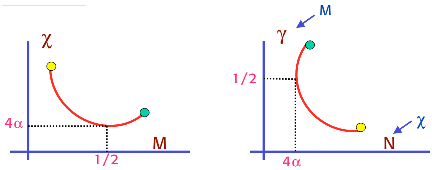
If the strong coupling is frozen [Altarelli,Ball,Forte 1995,...,2008]

$$N = \int_0^\infty \frac{dk'^2}{k'^2} \mathcal{K}\left(\frac{k^2}{k'^2}, \alpha_s\right) \left(\frac{k'^2}{k^2}\right)^{\gamma(N,\alpha_s)}$$

where $\chi(M, \alpha_s) = \int \frac{dk'^2}{k'^2} \left(\frac{k'^2}{k^2}\right)^M \mathcal{K}\left(\frac{k^2}{k'^2}, \alpha_s\right)$ is the BFKL kernel in M Mellin space

The duality relation **at fixed coupling** is an inverse-function relation in double-Mellin space

$$\chi(\gamma(N, \alpha_s), \alpha_s) = N \qquad \leftrightarrow \qquad \gamma(\chi(M, \alpha_s), \alpha_s) = M$$



ABF method

Expand the DGLAP anoumalous in logarithmic orders

$$\gamma(\chi, \alpha_s) = \gamma_s\left(\frac{\alpha_s}{N}\right) + \alpha_s \gamma_{ss}\left(\frac{\alpha_s}{N}\right) + \dots$$

on the other hand

$$\begin{aligned}\frac{N}{\alpha_s} &= \frac{\chi(\gamma(N, \alpha_s), \alpha_s)}{\alpha_s} \\ &= \chi_0\left(\gamma_s\left(\frac{\alpha_s}{N}\right)\right) + \alpha_s\left[\chi_0'\left(\gamma_s\left(\frac{\alpha_s}{N}\right)\right)\gamma_{ss}\left(\frac{\alpha_s}{N}\right) + \chi_1\left(\gamma_s\left(\frac{\alpha_s}{N}\right)\right)\right] + \mathcal{O}\left(\frac{\alpha_s^{k+2}}{N^k}\right)\end{aligned}$$

So we can infer the resummed anomalous dimension from the knowledge of the BFKL kernel

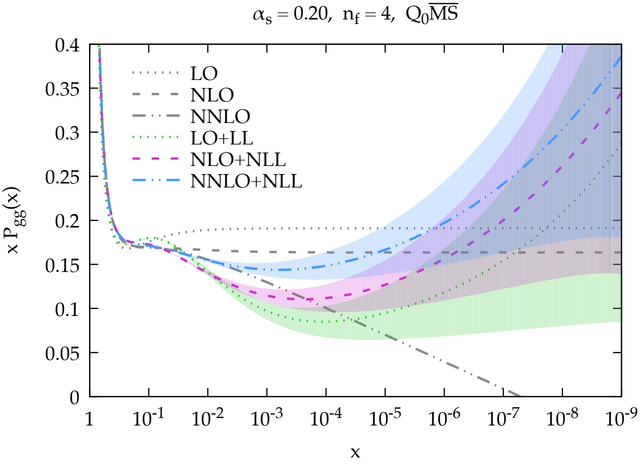
$$\gamma_s\left(\frac{\alpha_s}{N}\right) = \chi_0^{-1}\left(\frac{N}{\alpha_s}\right) \qquad \gamma_{ss}\left(\frac{\alpha_s}{N}\right) = -\chi_1\left(\gamma_s\left(\frac{\alpha_s}{N}\right)\right)\left(\chi_0'\left(\gamma_s\left(\frac{\alpha_s}{N}\right)\right)\right)^{-1}$$

Which can then be matched to the fixed order determination of γ itself

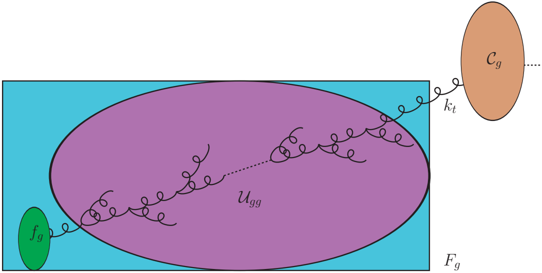
$$\begin{aligned}\gamma_{\text{doubleleading}}(\alpha_s, N) &= \left[\alpha_s \gamma_0(N) + \gamma_s\left(\frac{\alpha_s}{N}\right) - \text{ double counting}\right] \\ &= \left[\alpha_s^2 \gamma_1(N) + \alpha_s \gamma_{ss}\left(\frac{\alpha_s}{N}\right) - \text{ double counting}\right] + \dots\end{aligned}$$

Small-x resummation in the gluon-gluon splitting function

Resummation with `HELL` [Bonvini, Marzani, Peraro 1607.02153] [Bonvini, Marzani, Museli 1708.07510] [Bonvini, Marzani 1805.06460]



For $Q \sim 5$ GeV $\rightarrow$ **perturbative instability** from $x \sim 10^{-3}$ and below.

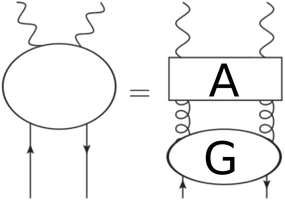


$\mathcal{C}(\cdots) \to$ off-shell continuation collinear Born counterpart.

- This is the only process dependent part of the computation.
- 2 Gluon Irreducible (2GI) in t channel $\leftarrow$ gauge-invariance at Born level.
- Off-shellness of the incoming gluon

$$k_{\text{collinear}} \to k_{\text{in}} = zp_1 + k_t, \quad \frac{1}{D-2}\sum_{\lambda} \varepsilon^{\mu}_{\lambda}(k_{\text{in}})\varepsilon^{*\nu}_{\lambda}(k_{\text{in}}) \to -\frac{k_t^{\mu} k_t^{\nu}}{k_t^2}$$

$$\left\langle -\frac{k_t^{\mu} k_t^{\nu}}{k_t^2} \right\rangle_{\hat{k}_t} \xrightarrow[|k_t| \to 0]{} \frac{1}{D-2} d^{\mu\nu}(z_1 p_1, n)$$



2GI decomposition [Catani, Ciafaloni, Hautmann Nucl.Phys.B 366 (1991) 135-188]

$$Q^2\sigma = \frac{Q^2}{2S}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\left[A_{0,\mu\nu}(k,q) + \frac{\alpha_s}{4\pi}A_{1,\mu\nu}(k,q,n)\right]d^{\mu\mu'}(k,n)d^{\nu\nu'}(k,n)G_{\mu'\nu'}(p_1,k)\,.$$

at LO

n = Light cone gauge axis

$$\frac{Q^2}{2S}\int \frac{\mathrm{d}^4k}{(2\pi)^4}[A_{0,\mu\nu}(k,q)]d^{\mu\mu'}(k,n)d^{\nu\nu'}(k,n)G_{\mu'\nu'}(p_1,k) =$$
$$\int_x^1 \frac{\mathrm{d}z}{z}\int \mathrm{d}k_t^2\,\mathcal{C}_g\left(\frac{x}{z},\alpha_s,Q^2,k_t^2\right)\mathcal{F}_g(z,Q^2,k_t^2)$$

ansatz for G at NLL ← from HELL evolutor + NLL resummation of an. dim

APFEL+HELL $\rightarrow$ make possible a PDF fit with small-x resummation

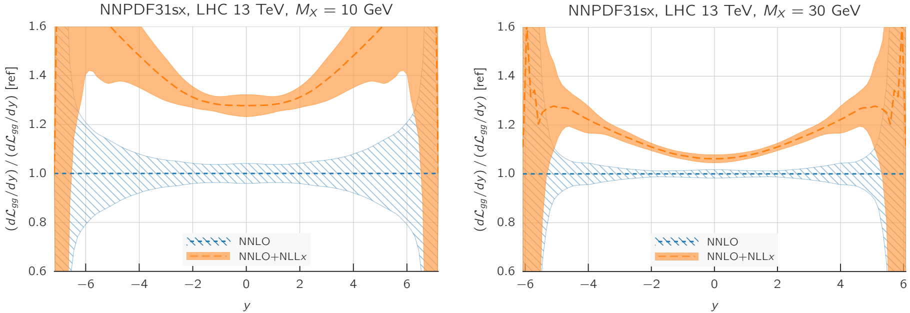
NNPDF3.1sx [1710.05935]	xFitter [1802.00064]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
VFNS: FONLL	VFNS: FONLL
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC (~ 4000 datapoints)	only HERA data (~ 1200 datapoints)
NLO, NLO+NLL, NNLO, NNLO+NLL	NNLO, NNLO+NLL

One interesting difference in the HERA data we include:

Lowest Q^2 HERA bins	NNPDF3.1/HERAPDF2.0	NNPDF3.1sx/xFitter
$Q^2 = 3.5$ GeV^2	included	included
$Q^2 = 2.7$ GeV^2	excluded	included
$Q^2 = 2.0$ GeV^2	excluded	excluded

Impact on parton luminosities

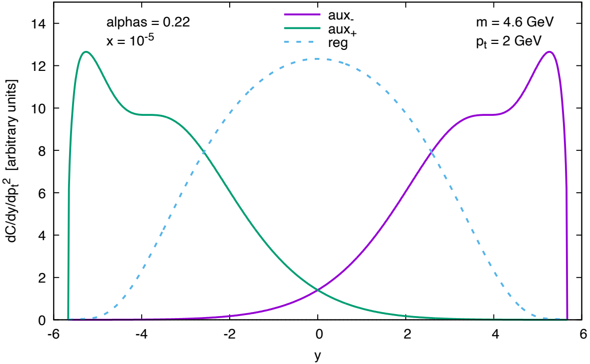
Gluon-gluon parton luminosities $\mathscr{L}_{gg}$



Significant impact for small invariant masses, especially at large rapidities

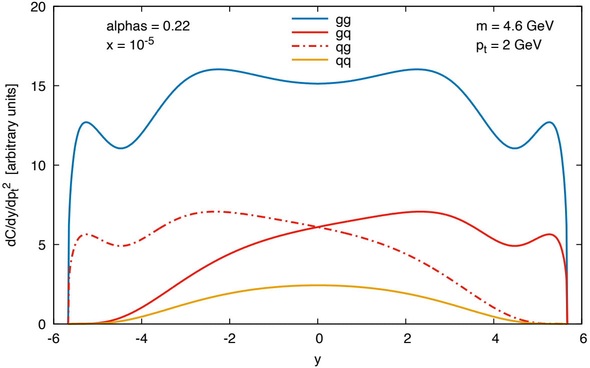
Heavy-quark pair production at LHC

Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**



- Pure resummed
- Open heavy quark final state → add fragmentation to compare with exp. data
- Auxilliary channels provide dominant contribution → **hybrid factorisation**

Results for $\frac{d\sigma}{dy\,dp_t}$ in **single quark kinematics**

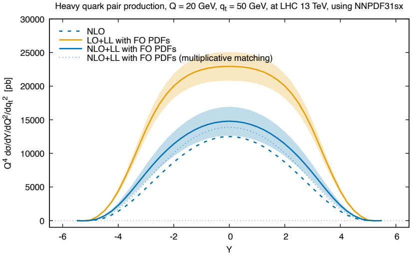


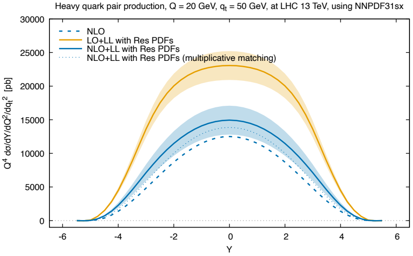
- Pure resummed
- Open heavy quark final state → add fragmentation to compare with exp. data

$$\mathcal{F}_g(N,\xi) = \left[U_{\rm reg}\left(N, Q^2\xi, \mu_{\rm F}^2\right) + \delta(\xi)\right] f_g(N,\mu_{\rm F}^2) + \frac{C_F}{C_A} U_{\rm reg}\left(N, Q^2\xi, \mu_{\rm F}^2\right) f_q(N,\mu_{\rm F}^2).$$

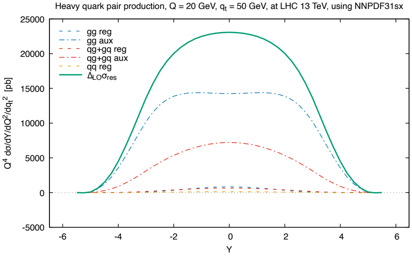
Heavy-quark pair production at LHC

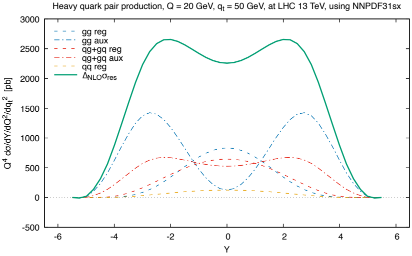
New results for $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\, dY\, \mathrm{d}q_t}$ in **pair kinematics**





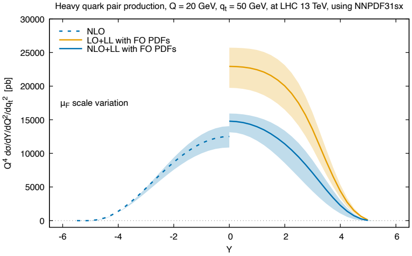
New results for $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\, dY\, \mathrm{d}q_t}$ in **pair kinematics**

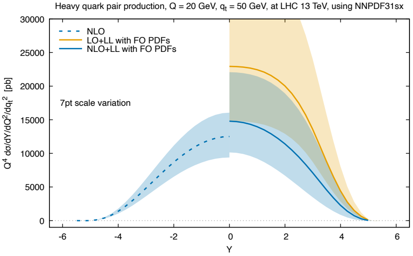




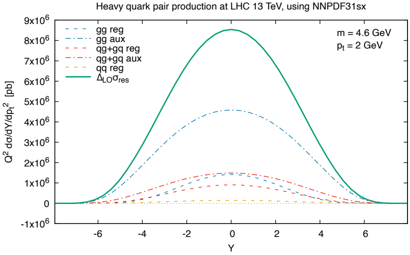
Heavy-quark pair production at LHC

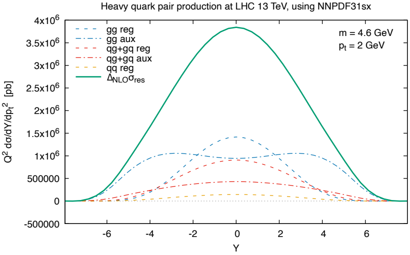
New results for $\frac{d\sigma}{dQ^2\, dY\, dq_t}$ in **pair kinematics**



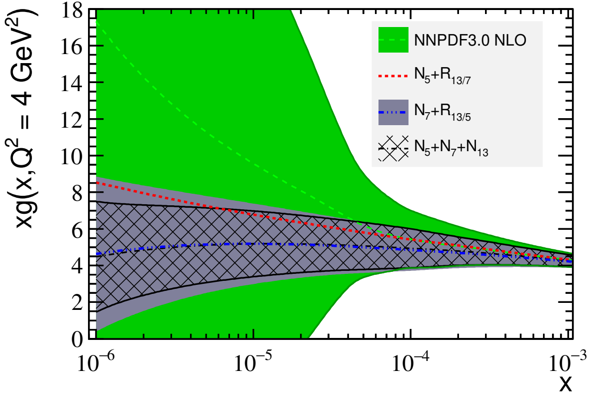


Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**





Impact of heavy meson data on gluon PDFGluon PDF uncertainty reduction from heavy quarks in LHC Run 2



[Gauld, Rojo 1610.09373]