RG improved JIMWLK Hamiltonian: running coupling and DGLAP resummation

Michael Lublinsky

Ben-Gurion University of the Negev, Israel

A. Kovner, M. Lublinsky, V. V. Skokov and Z. Zhao, arXiv:2308.15545 [hep-ph], in JHEP

T. Altinoluk, G. Beuf, M. Lublinsky and V. V. Skokov, arXiv:2310.10738 [hep-ph], in JHEP

High Energy Scattering

Target $(\rho^t = \rho^-)$ **Projectile** $(\rho^p = \rho^+)$

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle \qquad\qquad \rho^+ \sim \int \mathbf{dk^+ a^\dagger\, T\, a}$$

S-matrix:

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T} \,\langle\, \mathbf{P}|\;\; \hat{\mathbf{S}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle \qquad\qquad \mathbf{Y} \;\sim\; \ln(\mathbf{s})$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathrm{Y}} \;=\; \langle \mathbf{T} \,\langle\, \mathbf{P}|\;\; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

How do these averages change with increase in energy of the process?

$$\partial_{\mathrm{Y}} \langle \hat{\mathcal{O}} \rangle_{\mathrm{Y}} \;=\; -\mathcal{H}\, \langle \hat{\mathcal{O}} \rangle_{\mathrm{Y}} \qquad\qquad \mathcal{H} \;\rightarrow\; \text{the HE effective Hamiltonian}$$

$\mathcal{H}$ **defines the high energy limit of QCD and is universal**

Expansion in α_s

$$\mathcal{H} = \mathcal{H}_{\mathbf{LO}}(\alpha_{\mathrm{s}}) + \mathcal{H}_{\mathbf{NLO}}(\alpha_{\mathrm{s}}^{\mathbf{2}}) + \ldots; \qquad \mathcal{H} = \mathcal{H}[\rho^{\mathrm{t}}, \delta/\delta\rho^{\mathrm{t}}]$$

JIMWLK Hamiltonian is a limit of $\mathcal{H}$ for dilute partonic system ($\rho^{\mathrm{p}} \to \mathbf{0}$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{LO}}$ **(1997-2002),** $\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{NLO}}$ **with massless quarks (2007-2016),** $\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{NLO}}(\mathrm{m_q})$ **(2022)**

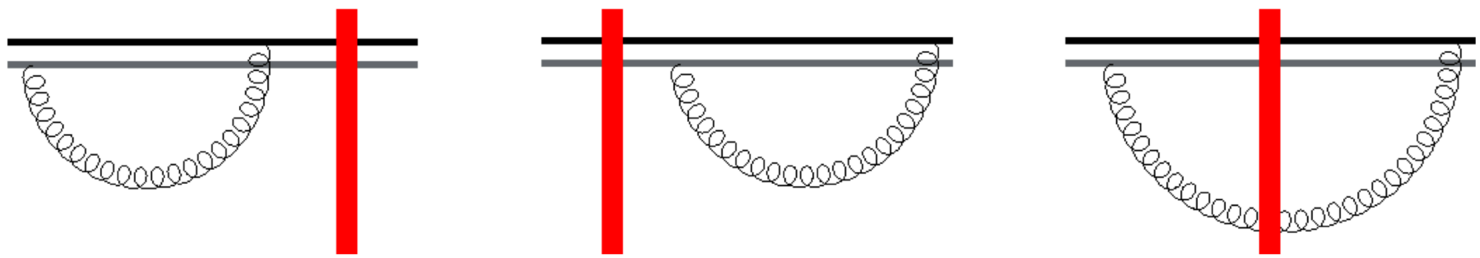
Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

LO JIMWLK Hamiltonian

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

$$\mathcal{H}^{\rm JIMWLK}_{\rm LO} = \int_{x,y,z} K_{\rm LO}\,\left\{J^a_L(x)J^a_L(y) + J^a_R(x)J^a_R(y) - 2J^a_L(x)S^{ab}_A(z)J^b_R(y)\right\}$$

$$K_{\rm LO}(x,y,z) = \frac{\alpha_s}{2\,\pi^2}\,\frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2}$$



$$S_A{}^{cd}(z) = \mathcal{P}\exp\left\{i\int dx^+\,T^a\,\alpha^a_t(z,x^+)\right\}^{cd}. \qquad \Delta\text{''}\,\alpha_t = \rho_t \quad ({\rm YM})$$

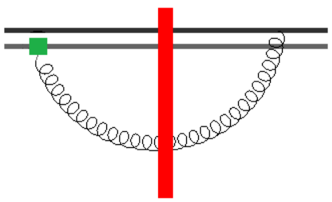
Here $\rho^p \to J_L$ and $\hat{S}\rho^p \to J_R$ are left and right SU(N) generators:

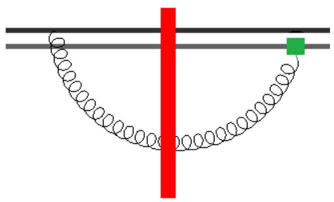
$$J^a_L(x)S^{ij}_A(z) = (T^a S_A(z))^{ij}\,\delta^2(x-z) \qquad J^a_R(x)S^{ij}_A(z) = (S_A(z)T^a)^{ij}\,\delta^2(x-z)$$

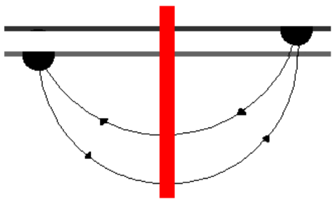
$\mathcal{H}^{\rm JIMWLK}$ contains all the LO BFKL / BKP / TPV physics

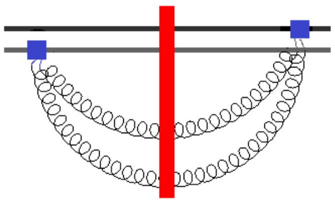
JIMWLK Hamiltonian @ NLO

Kovner, ML & Mulian (2013) based on Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)







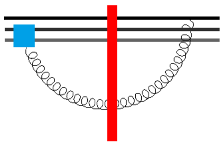


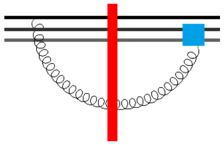
$$\mathcal{H}^{NLO\ JIMWLK} = \int_{x,y,z} {\color{blue}K_{JSJ}(x,y;z)} \left[J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) S_A^{ab}(z) J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} {\color{blue}K_{JSSJ}(x,y;z,z')} \left[f^{abc} f^{def} J_L^a(x) S_A^{be}(z) S_A^{cf}(z') J_R^d(y) - N_c J_L^a(x) S_A^{ab}(z) J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} {\color{blue}K_{q\bar{q}}(x,y;z,z')} \left[2\, J_L^a(x)\, tr[S_F^\dagger(z)\, t^a\, S_F(z') t^b]\, J_R^b(y) \;-\; J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z') f^{acb} \left[J_L^d(x)\, J_L^e(y)\, S_A^{dc}(z)\, S_A^{eb}(z')\, J_R^a(w) - J_L^a(w)\, S_A^{cd}(z)\, S_A^{be}(z')\, J_R^d(x)\, J_R^e(y)\right]$$
$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\, f^{bde} \left[J_L^d(x)\, J_L^e(y)\, S_A^{ba}(z)\, J_R^a(w) \;-\; J_L^a(w)\, S_A^{ab}(z)\, J_R^d(x)\, J_R^e(y)\right]$$
$$+ \int_{w,x,y} K_{JJJ}(w;x,y) f^{deb} \left[J_L^d(x)\, J_L^e(y)\, J_L^b(w) \;-\; J_R^d(x)\, J_R^e(y)\, J_R^b(w)\right].$$





Motivation and Objectives

Precise saturation physics phenomenology at NLO is badly needed.

The JIMWLK Hamiltonian at NLO is known for some years, but there are problems there.

- No known recipe for numerical evaluation
- Large transverse logarithms emerge: $\mathcal{H} \sim \alpha_s(\# + \alpha_s(\# + \mathbf{Log}))$,
  If the Log is large, then $\alpha_s \mathbf{Log} \sim \mathbf{1}$ – not a small correction to LO
  There are various types of the large Logs there:
  running coupling effects, (Ioffe) time ordering, DGLAP logs.
  All have to be identified, clearly separated, and independently resummed.

LO JIMWLK kernel beyond LO

$$\mathcal{H} = \int_{x,y,z} K(x,y;z)\left[J^a_L(x)\,J^a_L(y) \;+\; J^a_R(x)J^a_R(y) \;-\; 2J^a_L(x)\,S^{ab}_A(z)\,J^b_R(y)\right]$$

An effective kernel $K \;=\; K_{LO} \;+\; K_{NLO} \;+\; \ldots. \;\sim\; \alpha_s(\# \;+\; \alpha_s\,(\# + Logs) \;+\; \cdots)$

Large transverse logarithms emerge at NLO. There are various types of large Logs - all have to be identified, clearly separated, and independently resummed.
Proper resummation requires understanding of physics beyond NLO!

- Running coupling effects (UV divergent) – rcJIMWLK:

$$K_{LO} \;=\; \frac{\alpha_s}{2\pi^2}\frac{XY}{X^2Y^2} \;\rightarrow\; K_{rc} \;=\; \frac{\alpha_s[running]}{2\pi^2}\frac{XY}{X^2Y^2}$$

- DGLAP logs: Large transverse logs of the $\log(Q^T_s/Q^P_s)$ type (dilute-on-dense).

NLO Kernels (Large UV Logs only)

$$\mathbf{X} = \mathbf{x} - \mathbf{z}$$
$$Y = y - z$$

$$\mathbf{K_{JSJ}}(\mathrm{b\,terms}) = \frac{\alpha_s^2}{16\pi^3}\left\{-\mathbf{b}\frac{(\mathbf{x}-\mathbf{y})^2}{\mathbf{X}^2\mathbf{Y}^2}\ln(\mathbf{x}-\mathbf{y})^2\mu^2 + \frac{\mathbf{b}}{\mathbf{X}^2}\ln\mathbf{Y}^2\mu^2 + \frac{\mathbf{b}}{\mathbf{Y}^2}\ln\mathbf{X}^2\mu^2\right\} + \cdots$$

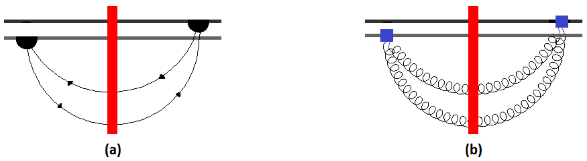
Here μ is the normalization point, $\mathbf{b} = \frac{11}{3}\mathbf{N_c} - \frac{2}{3}\mathbf{n_f}$ **,** $\mathbf{b}\ \ln \mathbf{Q}^2/\mu^2 \rightarrow \alpha_s(\mathbf{Q}^2)$

Huge ambiguity in identifying Q

Resum large Logs into an effective kernel $\mathbf{K} = \mathbf{K_{LO}} + \mathbf{K_{JSJ}} + \ldots.$

$$\int_{\mathbf{x\,y\,z,z'}} \mathbf{K_{JSSJ}}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{z}')\,\mathbf{J_L^a}(\mathbf{x})\mathbf{J_R^b}(\mathbf{y})\,\Big[\mathbf{D^{ab}}(\mathbf{z},\mathbf{z}')\Big] \quad \sim\ \mathbf{b}\ \times\ (\mathrm{UV\ divergent\ Log})$$

$$\mathbf{D^{ab}}(\mathbf{z},\mathbf{z}') \equiv \mathbf{Tr}[\mathbf{T^a S_A}(\mathbf{z})\mathbf{T^b S_A^+}(\mathbf{z}')]$$



The UV divergence in $JSSJ$ is trivial: when the two gluons are too close to each other ($z \sim z'$), they cannot be resolved by the target and hence should be counted as a single gluon scattering. We are thus prompted to introduce a "resolution scale" Q

Dressed Wilson line

Within the finite resolution Q, bare gluons $\to$ dressed gluons,
bare Wilson lines $\to$ *dressed Wilson lines*, $\mathbf{S} \to \mathbf{S_Q}$

$$\mathbf{S_Q^{ab}(z) = S_A^{ab}(z) + \frac{\alpha_s}{2\pi^2}\int_0^1 d\xi\, \sigma(\xi) \int^{Q^{-1}} \frac{d^2Z}{Z^2}\Big(D^{ab}(z + (1-\xi)Z, z - \xi Z) \;-\; N_c\, S_A^{ab}(z)\Big)}$$

ξ is the fraction of longitudinal momentum carried by one of the gluons.

$$\sigma(\xi) = \left[\frac{1}{\xi(1-\xi)}\left(\xi^2 + (1-\xi)^2 + \xi^2(1-\xi)^2\right)\right]_+ ; \quad 2N_c \int_0^1 d\xi \sigma(\xi) \;=\; -\frac{11 N_c}{3} \to -\, b$$

This is the P_{gg} splitting function except that we introduce the "+" prescription both for $\xi = 1$ and $\xi = 0$ poles The "+" prescription emerges from the $1/\xi$ subtraction absorbed into (LO)^2 part of the evolution.

The sign is negative – correcting for the over-subtraction in the LO.

We go beyond the usual DGLAP: we allow simultaneous scattering of all gluons.

For $\mathbf{Q > Q_s^T}$, $\mathbf{S_Q \simeq S_A}$ - the target does not resolve gluon splitting
at distances smaller than $1/\mathbf{Q_s^T}$.

Resolution scale and the running coupling

Express S in terms of $\mathrm{S_Q}$ and substitute it into the LO+NLO JIMWLK Hamiltonian. $\mathcal{H}[\mathrm{S}] \to \mathcal{H}[\mathrm{S_Q}]$. The Hamiltonian will feature $\ln \mathrm{Q}^2$ terms such as $\ln(\mathrm{Q}^2\mathrm{X}^2)$.

$$\mathbf{K} \;=\; \mathbf{K_{LO}} \left(\mathbf{1} + \frac{\alpha_s}{\mathbf{4}\pi}\mathbf{b}\,(\ln \mathbf{X}^2\mu^2 + \ln \mathbf{Y}^2\mu^2 - \ln \mathbf{Q}^{-2}\mu^2)\right) \;+\; \text{other}\,\mathrm{O}(\alpha_s^2)\,\text{terms}$$

We assume existence of a typical scale $\mathrm{Q_s^P} \ll \mathrm{Q_s^T}$ associated with the projectile, such that $\ln(\mathrm{Q_s^P}\mathrm{X}^2)$ are small. The UV finite parts of the Hamiltonian proportional to b do not have any large Logs

$$\mathbf{K_{in}} \;=\; \mathbf{K}(\mathbf{Q} = \mathbf{Q_s^P}) \;=\; \frac{\sqrt{\alpha_s(\mathbf{X})\,\alpha_s(\mathbf{Y})}}{\mathbf{2}\pi^{\mathbf{2}}}\,\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}\left[\mathbf{1} + \frac{\alpha_s}{\mathbf{8}\pi}\mathbf{b}\,(\text{small logs})\right]$$

However, at $Q = Q_s^P$, $\mathbf{S_Q}$ is very different from $\mathbf{S_A}$, $\mathbf{S_Q} \sim \mathbf{S_A}\,[\mathbf{1} + \alpha_s\,\#\,\mathbf{Log}(\mathbf{Q^2}/\mathbf{Q_s^T})]$. This large Log has to be resummed via inclusion of multiple consecutive DGLAP splittings:

$$\frac{\partial \mathbf{S_Q}(\mathbf{z})}{\partial \ln \mathbf{Q}} = -\frac{\alpha_s}{\mathbf{2}\pi^2} \int_\xi \sigma(\xi) \int_{\phi_\mathbf{Q}} [\mathbf{D_Q}(\mathbf{z}) \;-\; \mathbf{N_c}\, \mathbf{S_Q}(\mathbf{z})]$$

$$\mathbf{D_Q}(\mathbf{z_1}, \mathbf{z_2}) \equiv \mathbf{Tr}[\mathbf{T^a} \mathbf{S_Q}(\mathbf{z_1}) \mathbf{T^b} \mathbf{S_Q^+}(\mathbf{z_2})]$$

If we were to take $\mathbf{Q} = \mathbf{Q_s^T}$ then $\mathbf{S_Q} \simeq \mathbf{S_A}$ but the $\ln \mathbf{Q^2}$ terms in the Hamiltonian would be large and have to be resummed.

Either way, we have to resum large logs of the order $\log Q_s^T/Q_s^P$.

Functional RG

The resummed Hamiltonian should be Q-independent:

$$\frac{\mathbf{d}\mathcal{H}}{\mathbf{d}\ln\mathbf{Q}} = \frac{\partial\mathcal{H}}{\partial\ln\mathbf{Q}} + \int_{\mathbf{u}}\left[\frac{\delta\mathcal{H}}{\delta\mathbf{S_Q}(\mathbf{u})}\frac{\partial\mathbf{S_Q}(\mathbf{u})}{\partial\ln\mathbf{Q}}\right] = \mathbf{0}$$

DGLAP-like evolution for the Hamiltonian (evolution in the space of Hamiltonians):

$$\mathcal{H}[\mathbf{Q_s^P}] \;=\; \mathbf{Exp}\left[\int_{\mathbf{Q_s^P}}^{\mathbf{Q_s^T}}\frac{\mathbf{dQ}}{\mathbf{Q}}\mathbf{H_{DGLAP}}\right]\mathcal{H}_{\mathbf{in}}$$

$$\mathbf{H_{DGLAP}} \;=\; \frac{\alpha_{\mathbf{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{u}}\int_{\xi}\sigma(\xi)\int_{\phi_{\mathbf{Q}}}\mathbf{Tr}\left(\left[\mathbf{D_Q}(\mathbf{u})\;-\;\mathbf{N_c}\,\mathbf{S_Q}(\mathbf{u})\right]\frac{\delta}{\delta\mathbf{S_Q}(\mathbf{u})}\right)$$

$\mathbf{Q_s^P} = \mathbf{Q_s^P}(\eta)$ **– Q_s^P is dynamical (rapidity dependent);**
hence the resummed Hamiltonian is too.

Weak target field approximation – linearization

$$\mathbf{S^{ab}_Q} = \delta^{\mathbf{ab}} + \mathbf{f^{abc}}\alpha^{\mathbf{c}}_{\mathbf{Q}}\,; \qquad \mathbf{D^{ab}_Q(z_1, z_2)} = \mathbf{N_c}\left(\delta^{\mathbf{ab}} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{f^{abc}}\left[\alpha^{\mathbf{c}}_{\mathbf{Q}}(\mathbf{z_1}) + (\alpha^{\mathbf{c}}_{\mathbf{Q}}(\mathbf{z_2}))^*\right]\right)$$

Expand the Hamiltonian (BFKL-like)

$$\mathbf{H_{DGLAP}} \sim \alpha_{\mathbf{Q}}\,\frac{\delta}{\delta\alpha_{\mathbf{Q}}}$$

$\mathbf{H_{DGLAP}}$ **is homogeneous and hence solvable**

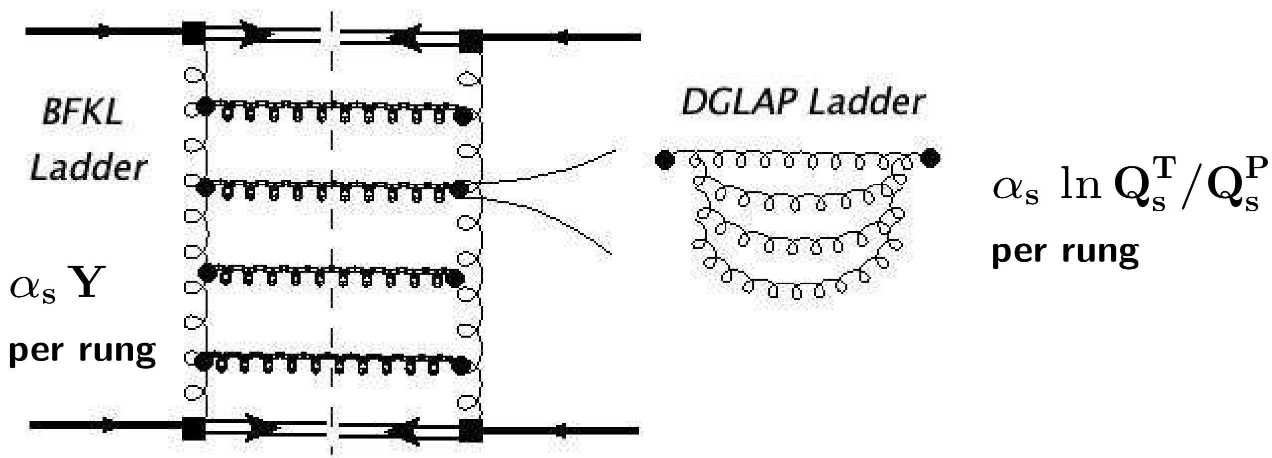
Saturation region

$$\mathbf{H_{DGLAP}} = \frac{\alpha_{\mathbf{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{u}}\int_{\xi}\sigma(\xi)\int_{\phi_{\mathbf{Q}}}\mathbf{Tr}\left([\mathbf{Tr}[\mathbf{T^a S_Q(z_1) T^b S^+_Q(z_2)}]_{\mathbf{u}} - \mathbf{N_c\,S_Q(u)}]\,\frac{\delta}{\delta\mathbf{S_Q(u)}}\right)$$

Since $|\mathbf{z_1} - \mathbf{z_2}| = \mathbf{1/Q} > \mathbf{1/Q^T_s}$, the two gluons are well separated and outside the correlation region in the target (in a sense of averaging over the target). Neglect the first term. H_{DGLAP} is again homogeneous

Summary/Outlook

- **DGLAP-like resummation inside the JIMWLK Hamiltonian has been performed. These DGLAP corrections are large whenever there is a large disparity between the correlation lengths (or saturation momenta) in the projectile and the target.**



- **rcJIMWLK emerges with the scale choice for the running coupling:**
  $\mathbf{K} \sim \sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}$