Twist corrections to exclusive vector meson production in a saturation framework

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in collaboration with

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based on

R. Boussarie, M. F., L. Szymanowski, S. Wallon [arXiv:2407.18115]
R. Boussarie, M. F., L. Szymanowski, S. Wallon [arXiv:2407.18203]

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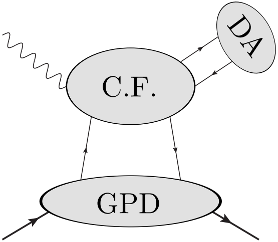
Deeply virtual meson production (DVMP)

- Exclusive vector meson leptoproduction

  $\gamma^{(*)}(p_\gamma) + P(p_0) \to \rho(p_\rho) + P(p_0')$

- Extensively studied at HERA



- NLO corrections to the production of a longitudinally polarized ρ-meson at small-x

  [Ivanov, Kotsky, Papa (2004)]

  [Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)]

  [Mäntysaari, Pentalla (2022)]

- Transversally polarized vector meson production start at the **twist-3**

  [Diehl, Gousset, Pire (1999)] **[Collins, Diehl (2000)]**

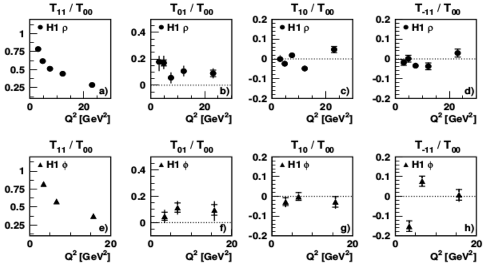
- Collinear treatment at the twist-3 leads to **end point singularities**

  [Mankiewicz, Piller (2000)] **[Anikin, Teryaev (2002)]**

Transversely polarized vector meson production

- HERA data for the ρ and ϕ meson **[F.D. Aaron et al. (2010)]**

$$\gamma^*(\lambda_\gamma)p \to V(\lambda_V)p \qquad \lambda_\gamma = 0, 1, -1 \quad \text{and} \quad \lambda_V = 0, 1, -1$$



- Exclusive vector meson production at the twist-3 in the dilute (BFKL) limit and forward case
  i. Restricted to s-channel helicity conserving (SCHC) amplitudes
  ii. Saturation effects only for the T_{00}

[Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]

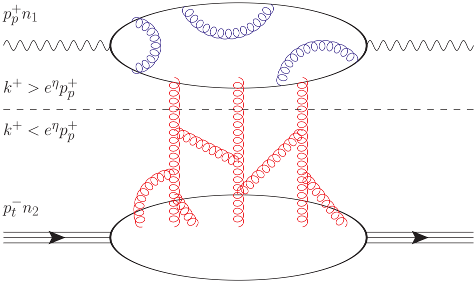
- Phenomenological studies at small-x

[Besse, Szymanowski, Wallon (2013)]

[Bolognino, Szczurek, Schäfer, Celiberto, Ivanov, Papa (2018-2021)]

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$n_1^2 = n_2^2 = 0 \quad n_1 \cdot n_2 = 1$$

- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

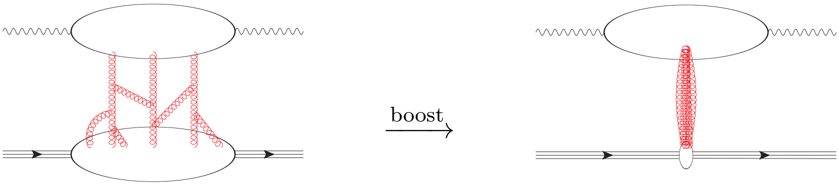
[McLerran and Venugopalan (1994)] [Balitsky (1996-2001)]

$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k}) \qquad e^\eta \ll 1$$

Shockwave approach

- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^{+}(x^{+}, x^{-}, \vec{x}) & = \Lambda^{-1} b_0^{+}(\Lambda x^{+}, \Lambda^{-1} x^{-}, \vec{x}) \\ b^{-}(x^{+}, x^{-}, \vec{x}) & = \Lambda b_0^{-}(\Lambda x^{+}, \Lambda^{-1} x^{-}, \vec{x}) \\ b^{i}(x^{+}, x^{-}, \vec{x}) & = b_0^{i}(\Lambda x^{+}, \Lambda^{-1} x^{-}, \vec{x}) \end{cases}$$



$b_0^{\mu}(x)$

$$b^{\mu}(x^{+}, x^{-}, \vec{x}) = \delta(x^{+}) \mathbf{B}(\vec{x}) n_2^{\mu} + \mathcal{O}\left(\Lambda^{-1}\right)$$

Shockwave approximation

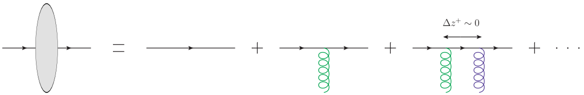
- Light-cone gauge $A \cdot n_2 = 0$

$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

Shockwave approach

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

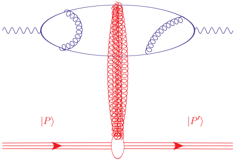
$$V_{\vec{z}}^{\eta} = 1 + ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) b_\eta^- \left(z_j^+, \vec{z}_i\right) \theta\left(z_{ij}^+\right) + \ldots$$



$$V_{\vec{z}}^{\eta} = \mathcal{P} \exp\left[ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}\right)\right]$$

- Factorization in the Shockwave approximation

$$\mathcal{M}^{\eta} = N_c \int d^d \boldsymbol{z}_1 d^d \boldsymbol{z}_2 \, \Phi^{\eta}(\boldsymbol{z}_1, \boldsymbol{z}_2) \langle P' \left| \mathcal{U}_{12}^{\eta}(\boldsymbol{z}_1, \boldsymbol{z}_2) \right| P \rangle$$



- **Dipole operator**

$$\mathcal{U}_{ij}^{\eta} = 1 - \frac{1}{N_c} \mathrm{Tr}\left(V_{\vec{z}_i}^{\eta} V_{\vec{z}_j}^{\eta\dagger}\right)$$

- **Balitsky-JIMWLK** evolution equations **[Balitsky (1995)]**

[Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

- **Effective background field operators**

$$[\psi_{\text{eff}}\ (z_0)]_{z_0^+<0} = \psi\,(z_0) - \int \mathrm{d}^D z_2 G_0\,(z_{02}) \left(V^\dagger_{\boldsymbol{z}_2} - 1\right) \gamma^+ \psi\,(z_2)\,\delta(z_2^+)$$

$$\left[\bar\psi_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \bar\psi\,(z_0) + \int \mathrm{d}^D z_1 \bar\psi\,(z_1)\,\gamma^+\,(V_{\boldsymbol{z}_1} - 1)\,G_0\,(z_{10})\,\delta(z_1^+)$$

$$\left[A^{\mu a}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = A^{\mu a}\,(z_0) + 2i\int \mathrm{d}^D z_3 \delta(z_3^+) F^b_{-\sigma}\,(z_3)\,G^{\mu\sigma_\perp}\,(z_{30}) \left(U^{ab}_{\boldsymbol{z}_3} - \delta^{ab}\right)$$

- Shockwave effective Feynman rules are easily reproduced

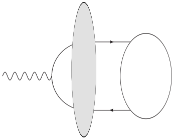
$$\left[v^{ij}_\alpha\,(p_{\bar q}, z_0)\right]_{z_0^+<0} \equiv \left[\psi^j_{\text{eff},\alpha}\,(z_0)\right]_{z_0^+<0} |i, p_{\bar q}\rangle = -\frac{(-i)^{d/2}}{2(2\pi)^{d/2}} \left(\frac{p^+_{\bar q}}{-z_0^+}\right)^{d/2} \theta(p^+_{\bar q})\theta(-z_0^+)$$

$$\times \int d^d z_2 V^{ij\dagger}_{\vec z_2} \frac{-z_0^+\gamma^- + \hat z_{20\perp}}{-z_0^+}\gamma^+ \frac{v(p_{\bar q})}{\sqrt{2p^+_{\bar q}}}\ \exp\left\{ip^+_{\bar q}\left(z_0^- - \frac{\vec z_{20}^{\,2}}{2z_0^+} + i0\right) - i\vec p_{\bar q}\cdot \vec z_{20}\right\}$$

$$G_{ij}\,(z_2, z_0)|_{z_2^+>0>z_0^+} \equiv \psi_i(z_2)\left[\bar\psi_{\text{eff},j}\,(z_0)\right]_{z_0^+<0}$$

$$= \frac{i\Gamma(d+1)}{4(2\pi)^{d+1}}\int d^d\vec z_1 V_{ij}\,(\vec z_1)\,\frac{\left(z_2^+\gamma^- + \hat z_{21\perp}\right)\gamma^+\left(-z_0^+\gamma^- + \hat z_{10\perp}\right)}{\left(-z_0^+ z_2^+\right)^{\frac{D}{2}}\left(-z_{20}^- + \frac{\vec z_{21}^2}{2z_2^+} - \frac{\vec z_{10}^2}{2z_0^+} + i\varepsilon\right)^{d+1}}\theta(z_2^+)\theta(-z_0^+)$$

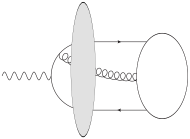
- **Two-body contribution**



i. Dependence of the leading Fock state wave function – with a minimal number of (valence) partons – on **transverse momentum**

$$\mathcal{A}_2 = -ie_f \int \mathrm{d}^D z_0 \theta(-z_0^+) \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\text{eff}}\left(z_0\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\text{eff}}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

- **Three-body contribution**

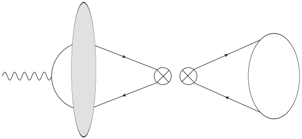


i. Distributions with a **non-minimal parton configuration**

$$\mathcal{A}_{3,q} = (-ie_q)\,(ig) \int \mathrm{d}^D z_4 \mathrm{d}^D z_0 \theta(-z_4^+)\theta(-z_0^+)$$
$$\times \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\text{eff}}\left(z_4\right) \gamma_\mu A^{\mu a}_{\text{eff}}(z_4) t^a G(z_{40}) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\text{eff}}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

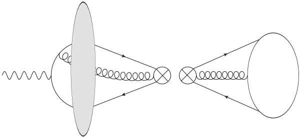
ρ-meson production: factorization

- **Two-body contribution**



$$\mathcal{A}_2 = ie_f \int \mathrm{d}^D z_0 \int \mathrm{d}^D z_1 \int \mathrm{d}^D z_2 \theta(-z_0^+)\delta(z_1^+)\delta(z_2^+) \left\langle M\left(p_M\right) \left| \overline{\psi}\left(z_1\right)\Gamma^\lambda \psi\left(z_2\right) \right| 0 \right\rangle$$
$$\times \left\langle P\left(p'\right) \left| 1 - \frac{1}{N_c} \mathrm{tr}\left(V_{\boldsymbol{z}_1} V_{\boldsymbol{z}_2}^\dagger\right) \right| P\left(p\right) \right\rangle \frac{1}{4} \mathrm{tr}_D \left[\gamma^+ G_0\left(z_{10}\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} G_0\left(z_{02}\right) \gamma^+ \Gamma_\lambda\right]$$

- **Three-body contribution**



$$\mathcal{A}_{q3} = -ie_q \int \mathrm{d}^D z_4 \mathrm{d}^D z_3 \mathrm{d}^D z_2 \mathrm{d}^D z_1 \mathrm{d}^D z_0 \theta(-z_4^+)\delta(z_3^+)\delta(z_2^+)\delta(z_1^+)\theta\left(-z_0^+\right) \mathrm{e}^{-i(q\cdot z_0)}$$
$$\times \left\langle P\left(p'\right) \left| \mathrm{tr}\left(V_{\boldsymbol{z}_1} t^a V_{\boldsymbol{z}_2}^\dagger t^b U_{\boldsymbol{z}_3}^{ab}\right) \right| P\left(p\right) \right\rangle \left\langle M\left(p_M\right) \left| \overline{\psi}\left(z_1\right)\Gamma^\lambda g F_{-\sigma}\left(z_3\right) \psi\left(z_2\right) \right| 0 \right\rangle$$
$$\times \frac{1}{N_c^2 - 1} \mathrm{tr}_D \left[\gamma^+ G_0\left(z_{14}\right) \gamma_\mu G^{\mu\sigma_\perp}\left(z_{34}\right) G_0\left(z_{40}\right) \hat{\varepsilon}_q G_0\left(z_{02}\right) \gamma^+ \Gamma_\lambda\right] - \text{n.i.}$$

Results: two-body contribution

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\Psi\left(x,\boldsymbol{r}\right) \int \mathrm{d}^d\boldsymbol{b}\ \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P\left(p'\right)\middle| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}}V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right)\middle| P\left(p\right)\right\rangle$$

- **Coordinate-space impact factor**

$$\Psi_2\left(x,\boldsymbol{r}\right) = e_q\delta\left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+}q_\mu\right)$$

$$\times\left[\phi_{\gamma^+}(x,\boldsymbol{r})\left(2x\bar{x}q^\mu - i(x-\bar{x})\frac{\partial}{\partial r_{\perp\mu}}\right) + \epsilon^{\mu\nu+-}\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r})\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Two-body vacuum to meson matrix elements

$$\phi_{\gamma^+}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-}\left\langle M\left(p_M\right)\middle|\overline{\psi}\left(r\right)\gamma^+\psi\left(0\right)\middle|0\right\rangle_{r^+=0}$$

$$\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-}\left\langle M\left(p_M\right)\middle|\overline{\psi}\left(r\right)\gamma^+\gamma^5\psi\left(0\right)\middle|0\right\rangle_{r^+=0}$$

Results: three-body contribution

- Three-body amplitude

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3}\int dx_i\theta(x_i)\right)\delta(1-x_1-x_2-x_3)\int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1+x_2\boldsymbol{z}_2+x_3\boldsymbol{z}_3)}$$

$$\times\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)\left\langle P\left(p'\right)\middle|\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}-\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}-\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}+\frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2}\middle|P\left(p\right)\right\rangle$$

- **Coordinate-space impact factor**

$$\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right) = \frac{e_q q^+}{2(4\pi)}\frac{N_c^2}{N_c^2-1}\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)$$

$$\times\left\{\chi_{\gamma^+\sigma}\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]\right.$$

$$\left.-\chi_{\gamma^+\gamma^5\sigma}\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)+(1\leftrightarrow 2)\right]\right\}$$

- Three-body vacuum to meson matrix elements

$$\chi_{\Gamma^\lambda,\sigma}\equiv\chi_{\Gamma^\lambda,\sigma}(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3) =$$

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}z_1^-}{2\pi}\frac{\mathrm{d}z_2^-}{2\pi}\frac{\mathrm{d}z_3^-}{2\pi}\mathrm{e}^{-ix_1q^+z_1^--ix_2q^+z_2^--ix_3q^+z_3^-}\left\langle M\left(p_M\right)\middle|\overline{\psi}\left(z_1\right)\Gamma^\lambda gF_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\middle|0\right\rangle_{z_{1,2,3}^+=0}$$

Covariant collinear factorization

- **Covariant collinear factorization**

  [Braun, Filyanov (1990)]

  [Ball, Braun, Koike, Tanaka (1998)]

  i. Minimal basis of *independent distributions* (twist-3 collinear DAs)

  ii. *Minimal numbers of parameters*

  iii. Easy to perform the calculation directly in coordinate space

- Idea: two- and three-body operators in gauge invariant form

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,gF^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\gamma^5\,[z,tz]\,g\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

where

$$[z,0]=\mathcal{P}_{\exp}\left[ig\int_0^1 dt A^\mu\,(tz)\,z_\mu\right]$$

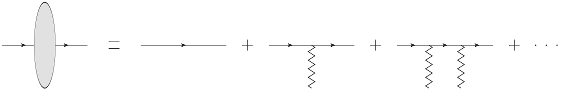
- The expansion for $z^2 \to 0$ should be taken
- Expansion of **string operators** in powers of deviation from the light-cone

  [Balitsky, Braun (1989)]

Dilute regime: two-body contribution

- **Reggeon** definition **[Caron-Huot (2013)]** $\quad R^a(\boldsymbol{z}) \equiv \dfrac{f^{abc}}{gC_A} \ln\left(U^{bc}_{\boldsymbol{z}}\right)$
- Expansion of the *Wilson line* in Reggeized gluons

$$V_{\boldsymbol{z}_1} = 1 + ig\boldsymbol{t}^a R^a(\boldsymbol{z}_1) - \frac{1}{2} g^2 \boldsymbol{t}^a \boldsymbol{t}^b R^a(\boldsymbol{z}_1) R^b(\boldsymbol{z}_1) + O\left(g^3\right)$$



- **BFKL k_T-factorization**

$$\mathcal{A}_2^{\text{dilute}} = \frac{g^2}{4N_c}(2\pi)^d \delta^d(\boldsymbol{q} - \boldsymbol{p}_M - \boldsymbol{\Delta}) \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathcal{U}(\ell) \int_0^1 \mathrm{d}x$$

$$\times \underbrace{\left[\Phi_2\left(x, \boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) + \Phi_2\left(x, -\boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) - \Phi_2(x, \bar{x}\boldsymbol{\Delta}) - \Phi_2(x, -x\boldsymbol{\Delta})\right]}_{\Phi_{2,\text{BFKL}}(x, \boldsymbol{l}, \boldsymbol{\Delta})}$$

- $\mathcal{U}(\boldsymbol{l}) \rightarrow k_T$-unintegrated gluon density (UGD) in the BFKL sense

$$\mathcal{U}(\boldsymbol{\ell}) \equiv \int \mathrm{d}^d \boldsymbol{v} \mathrm{e}^{-i(\boldsymbol{\ell}\cdot\boldsymbol{v})} \left\langle P\left(p'\right) \middle| R^a\left(\frac{\boldsymbol{v}}{2}\right) R^a\left(-\frac{\boldsymbol{v}}{2}\right) \middle| P(p) \right\rangle ,$$

- Φ_2 is the Fourier transform of Ψ_2

Explicit two-body term in the dilute and $\Delta = 0$ limit

- **BK impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) = 2\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{2,\text{f.}}(x) - \frac{x\bar{x}Q^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{2,\text{n.f.}}(x) \right]$$

- Helicity structures and DAs combinations

$$T_{\text{n.f.}} = \boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^* \qquad \phi_{2,\text{n.f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) + \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

$$T_{\text{f.}} = \frac{(\boldsymbol{\varepsilon}_q \cdot \boldsymbol{l})(\boldsymbol{\varepsilon}_M^* \cdot \boldsymbol{l})}{\boldsymbol{l}^2} - \frac{\boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^*}{2} \qquad \phi_{2,\text{f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) - \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

- **Forward limit matching**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 2\left(\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) - \Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{0})\right)$$

- **BFKL impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 4\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{\text{f.}}(x) + \frac{\boldsymbol{l}^2(\boldsymbol{l}^2 + 2x\bar{x}Q^2)}{x\bar{x}Q^2\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{\text{n.f.}}(x) \right]$$

Explicit three-body term in the dilute and $\Delta = 0$ limit

- The 3-body BFKL impact factor is a combination of 12 BK impact factors

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- Transverse to transverse transition in the **forward** and **dilute** limit

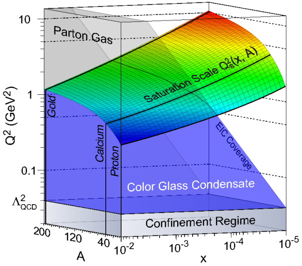
$$\mathcal{A}_{3T,\boldsymbol{\Delta}=\mathbf{0}}^{\text{dilute}} = e_q m_M \frac{g^2}{N_c}(2\pi)\delta\left(1-\frac{p_M^+}{q^+}\right)(2\pi)^2\delta^2\left(\boldsymbol{q}-\boldsymbol{p}_M\right)\int\frac{\mathrm{d}^d\boldsymbol{\ell}}{(2\pi)^d}\mathcal{U}(\boldsymbol{\ell})$$
$$\times\left(\prod_{i=1}^{3}\int_0^1\frac{dx_i}{x_i}\right)\frac{\delta(1-x_1-x_2-x_3)}{x_3}\frac{\boldsymbol{\ell}^2}{Q^2}\left\{T_{\text{f.}}\left[f_{3M}^V V\left(x_1,x_2\right)-f_{3M}^A A\left(x_1,x_2\right)\right]\right.$$
$$\times 2x_1\left(\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_2x_3}{x_2+x_3}Q^2}+\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_1x_3}{x_1+x_3}Q^2}-\frac{\bar{x}_3\left(1-c_f\right)}{\boldsymbol{\ell}^2+\frac{x_1x_2}{x_1+x_2}Q^2}+\frac{x_2-\bar{x}_1c_f}{\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2}+\frac{x_1-\bar{x}_2c_f}{\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2}\right)$$
$$-T_{\text{n.f.}}\left[f_{3M}^V V\left(x_1,x_2\right)+f_{3M}^A A\left(x_1,x_2\right)\right]$$
$$\left.\times\left(\frac{\left(1-c_f\right)x_1\bar{x}_3}{\bar{x}_3\boldsymbol{\ell}^2+x_1x_2Q^2}-\frac{c_fx_3^2}{\bar{x}_1\boldsymbol{\ell}^2+x_2x_3Q^2}-\frac{\left(x_2-\bar{x}_1c_f\right)x_1x_2}{\bar{x}_1\left(\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2\right)}-\frac{\left(x_1-\bar{x}_2c_f\right)\bar{x}_2}{\left(\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2\right)}\right)\right\}$$

- **The forward and dilute limit matches the previous result**
  [Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]
  BFKL approach + twist-expansion via light-cone collinear factorization

Summary

- **Transversally polarized light vector meson production**
  [Boussarie, M.F., Szymanowski, Wallon (arXiv:2407.18115)]
- DVMP in the **non-linear** regime in the transversely polarized case
- Both **forward** and **non-forward** results and **s-channel non-conserving helicity amplitudes**
- **Coordinate** and **momentum space** representations
- Reggeized gluon expansion **[Caron-Huot (2013)]** $\Longrightarrow$ **BFKL results**
- Complete description of HERA and future EIC data

- Higher-twist corrections are essential to describe medium energy data of exclusive processes
  [M. Defurne et al. (2016)]
- Method to deal with twist corrections at small-x including saturation



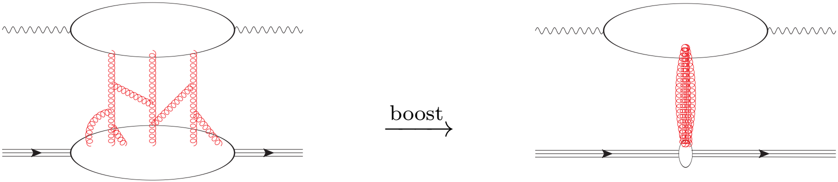
Thanks for your attention

Backup

Shockwave approach

- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+)\mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0$

$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

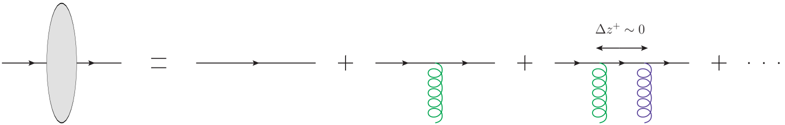
Shockwave approach

- Interactions with the simple shockwave field

  i. Independence from x^- $\Longrightarrow$ conservation of p^+ (**eikonal** approx.)

  ii. $\delta(x^+)$ $\Longrightarrow$ interactions at a **single transverse coordinate**.

- Quark line through the shockwave



$$V_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_i\right)\theta\left(z_{ij}^+\right) + \cdots$$

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

$$V_{\vec{z}}^{\eta} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}\right)\right]$$

Balitsky-JIMWLK evolution equations

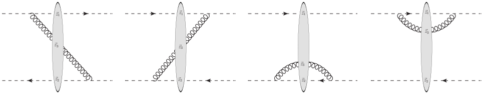
- ***Balitsky-JIMWLK evolution equations*** for the dipole
  [Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}} \right) \Big[\underbrace{\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12}}_{\text{BFKL}} - \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32} \Big]$$

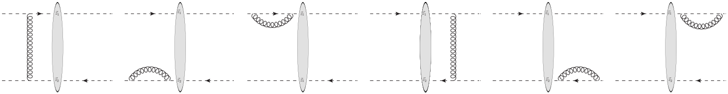
$$\frac{\partial \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32}}{\partial \eta} = \cdots \qquad \longleftarrow \qquad \textit{Balitsky hierarchy}$$

$$\vdots$$

- Double dipole contribution and Dipole contribution

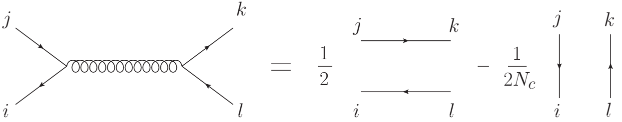


- Dipole contribution



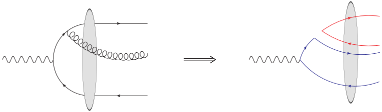
Balitsky-Kovchegov evolution equation

- Large-N_c limit **[G. 't Hooft (1974)]**



$$t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$

- Double dipole → Dipole × dipole



$$\langle \mathcal{U}^\eta_{13}\mathcal{U}^\eta_{32}\rangle \to \langle \mathcal{U}^\eta_{13}\rangle \, \langle \mathcal{U}^\eta_{32}\rangle$$

- Hierarchy of equations broken → closed non-linear **BK-equation**
  [I. I. Balitsky (1995)] [Y. V. Kovchegov (1999)]

$$\frac{\partial \langle \mathcal{U}^\eta_{12}\rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}}\right)\left[\langle \mathcal{U}^\eta_{13}\rangle + \langle \mathcal{U}^\eta_{32}\rangle - \langle \mathcal{U}^\eta_{12}\rangle - \langle \mathcal{U}^\eta_{13}\rangle\langle \mathcal{U}^\eta_{32}\rangle\right]$$

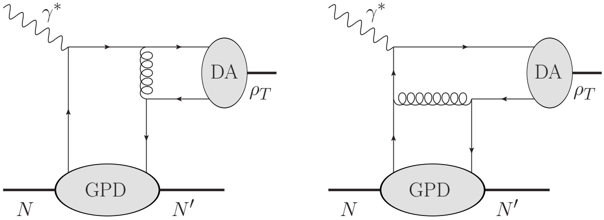
with $\langle \mathcal{U}^\eta_{12}\rangle \equiv \langle P'|\mathcal{U}^\eta_{12}|P\rangle$

Transversely polarized ρ-meson production

- The leading DA (twist 2) of ρ_T is **chiral odd** ($\sigma^{\mu\nu}$ coupling)
- The amplitude for $\gamma^* N \to \rho_T N'$ is zero to all order in perturbation theory at the leading twist

[Diehl, Gousset, Pire (1999)] [Collins, Diehl (2000)]

- Lowest order diagrammatic argument:

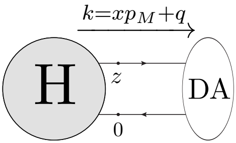


$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$$

Light-cone collinear factorization

- Most general two-body amplitude

$$\mathcal{A}_2 = \int \frac{d^4k}{(2\pi)^4} \int d^4 z e^{-ik\cdot z} \langle M(p_M)|\overline{\psi}^{\,i}_{\alpha}(z)\,\psi^{\,j}_{\beta}(0)\,|0\rangle H^{ij}_{2,\alpha\beta}$$



- The separation is choosen as $z^\mu = \lambda n_2^\mu \implies$ no Wilson line in the n_2 **light-cone gauge**
- Sudakov decomposition: $k = k^+ n_1 + q = x p_M + q$ and small q expansion
- Two-body amplitude after Fierz decomposition

$$\mathcal{A}_2 = \frac{1}{4N_c} p_M^+ \int \frac{\mathrm{d}x}{2\pi} \int \frac{\mathrm{d}q^-}{2\pi} \int \frac{\mathrm{d}^d \boldsymbol{q}}{(2\pi)^d} \int \mathrm{d}^D z\ \mathrm{e}^{-ixp_M^+ z^- - iq^- z^+ + i(\boldsymbol{q}\cdot\boldsymbol{z})}$$
$$\times \left\langle M\left(p_M\right)\middle|\overline{\psi}\left(z\right)\Gamma_\lambda \psi\left(0\right)\middle|0\right\rangle \mathrm{tr}\left[H_2\left(xp_M + q\right)\Gamma^\lambda\right]$$

Light-cone collinear factorization

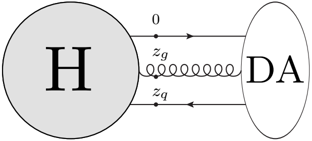
- **Taylor expansion** of the hard part

$$H_2\left(xp_M+q\right)=H_2\left(xp_M\right)+q_{\perp\mu}\left[\frac{\partial}{\partial q_{\perp\mu}}H_2\left(xp_M+q\right)\right]_{k=xp_M}+\text{h.t.}$$

- Two-body amplitude factorized form up to the **twist-3**

$$\mathcal{A}_2=\frac{1}{4N_c}\int\mathrm{d}x\,p_M^+\int\frac{\mathrm{d}z^-}{2\pi}\mathrm{e}^{-ixp_M^+z^-}\left\{\left\langle M\left(p\right)\left|\overline{\psi}\left(z^-\right)\Gamma_\lambda\psi\left(0\right)\right|0\right\rangle\mathrm{tr}\left[H_2\left(xp_M\right)\Gamma^\lambda\right]\right.$$
$$\left.+i\left\langle M\left(p_M\right)\left|\overline{\psi}\left(z^-\right)\overleftrightarrow{\partial}_{\perp\mu}\Gamma_\lambda\psi\left(0\right)\right|0\right\rangle\mathrm{tr}\left[\partial_\perp^\mu H_2\left(xp_M\right)\Gamma^\lambda\right]\right\}$$

- *Gauge invariance* is broken $\Longrightarrow$ need to include a **3-body contribution**



- Three-body contribution factorized

$$\mathcal{A}_3=\frac{1}{2(N_c^2-1)}\int\mathrm{d}x_q\mathrm{d}x_g\left(p_M^+\right)^2\int\frac{\mathrm{d}z_q^-}{2\pi}\frac{\mathrm{d}z_g^-}{2\pi}\mathrm{e}^{-ix_qp_M^+z_q^--ix_gp_M^+z_g^-}$$
$$\times\left\langle M\left(p\right)\left|\overline{\psi}\left(z_q^-\right)\Gamma_\lambda gA_\mu\left(z_g^-\right)\psi\left(0\right)\right|0\right\rangle\mathrm{tr}\left[t^bH_3^{\mu,b}\left(x_qp_M,x_gp_M\right)\Gamma^\lambda\right]$$

Light-cone collinear factorization

- *Light-cone collinear factorization*

  [Ellis, Furmanwski, Petronzio (1982)] [Anikin, Teryaev (2002)]
- Factorization around the dominant **light-cone direction** is naturally implemented in **momentum space**
- **Overcomplete set of distributions** must be reduced exploiting QCD equations of motion

$$\langle i(\hat{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad \langle i\psi_\alpha(0)(\bar{\psi}(z)\overleftarrow{\hat{D}}(z))_\beta\rangle = 0$$

- Invariance of the amplitude under **rotation on the light-cone**

  [Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]

  i. Independence of the amplitude from the choice of n

  ii. Given a "natural" choice n_0, we can define

$$n^\mu = \alpha p^\mu + \beta n_0^\mu + n_\perp^\mu$$

  iii. Imposing $p \cdot n = 1$ and $n^2 = 0 \longrightarrow \beta = 1\,, \alpha = -n_\perp^2/2$

  iiii. The freedom is parametrized in terms of the transverse component

$$\frac{\partial \mathcal{A}}{\partial n_\perp^\mu} = 0$$

Covariant collinear factorization

- Matrix elements without gauge links need to be related to the fully gauge invariant one within **twist-3** accuracy
- Three-body matrix element

$$\left\langle M(p)\left|\overline{\psi}\left(z_q\right)\gamma^{\lambda} gF^{\mu\nu}\left(z_g\right)\psi\left(z_{\overline{q}}\right)\right|0\right\rangle$$
$$\simeq \left\langle M(p)\left|\overline{\psi}(z_q)[z_q, z_g]\gamma^{\lambda} gF^{\mu\nu}(z_g)[z_g, z_{\overline{q}}]\psi(z_{\overline{q}})\right|0\right\rangle_{z_i^+=0}$$

- Two-body matrix element

$$\left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)[r,0]\Gamma^{\lambda}\psi\left(0\right)\right|0\right\rangle_{r^+=0} \longleftrightarrow \left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)\Gamma^{\lambda}\psi\left(0\right)\right|0\right\rangle_{r^+=0}$$

- Expansion of the gauge link

$$[z,0] = \mathcal{P}\exp\left[ig\int_0^1 dt A^{\mu}(tz)z_{\mu}\right] \simeq 1 + ig\int_0^1 dt A^{\mu}\left(tz\right)z_{\mu} + \text{h.t.}$$

- In a given n light-cone gauge

$$A^{\mu}(z) = \int_0^{\infty} \mathrm{d}\sigma\, \mathrm{e}^{-\epsilon\sigma} n_{\nu} F^{\mu\nu}(z+\sigma n)$$

- The difference between the two matrix elements is parameterized in terms of a three-body contribution

Dilute regime: three-body contribution

- Linearization in the three-body case → combination of more impact factors

$$\mathcal{A}_3^{\text{dilute}} = \left(\prod_{i=1}^{3} \int dx_i \theta(x_i)\right) \delta(1 - x_1 - x_2 - x_3) \frac{-g^2}{4N_c} \int \frac{d^d \boldsymbol{l}}{(2\pi)^d} \mathcal{U}(\ell)$$

$$\Big\{ \Phi_3' \left(\{x\}, \bar{x}_1 \boldsymbol{\Delta}, -x_2 \boldsymbol{\Delta}, -x_3 \boldsymbol{\Delta}\right) - \Phi_3' \left(\{x\}, \left(\frac{1-2x_1}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}, -x_2 \boldsymbol{\Delta}, \left(\frac{1-2x_3}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}\right)$$

$$-\Phi_3' \left(\{x\}, \left(\frac{1-2x_1}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}, -x_2 \boldsymbol{\Delta}, \left(\frac{1-2x_3}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}\right) + \Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, -x_2 \boldsymbol{\Delta}, \bar{x}_3 \boldsymbol{\Delta}\right)$$

$$+\Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, -x_2 \boldsymbol{\Delta}, \bar{x}_3 \boldsymbol{\Delta}\right) - \Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, \left(\frac{1-2x_2}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}, \left(\frac{1-2x_3}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}\right)$$

$$-\Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, \left(\frac{1-2x_2}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}, \left(\frac{1-2x_3}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}\right) + \Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, \bar{x}_2 \boldsymbol{\Delta}, -x_3 \boldsymbol{\Delta}\right)$$

$$-\frac{1}{N_c^2} \Big[\Phi_3' \left(\{x\}, \bar{x}_1 \boldsymbol{\Delta}, -x_2 \boldsymbol{\Delta}, -x_3 \boldsymbol{\Delta}\right) - \Phi_3' \left(\{x\}, \left(\frac{1-2x_1}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}, \left(\frac{1-2x_2}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}, -x_3 \boldsymbol{\Delta}\right)$$

$$-\Phi_3' \left(\{x\}, \left(\frac{1-2x_1}{2}\right) \boldsymbol{\Delta} + \boldsymbol{l}, \left(\frac{1-2x_2}{2}\right) \boldsymbol{\Delta} - \boldsymbol{l}, -x_3 \boldsymbol{\Delta}\right) + \Phi_3' \left(\{x\}, -x_1 \boldsymbol{\Delta}, \bar{x}_2 \boldsymbol{\Delta}, -x_3 \boldsymbol{\Delta}\right) \Big] \Big\}$$

where

$$\Phi_3'(\{x\}, \{\boldsymbol{p} + x \boldsymbol{p}_M\}) \equiv \Phi_3(\{x\}, \{\boldsymbol{p}\})$$

Dilute regime: three-body contribution

- Fourier transform

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- **Momentum space impact factor** (after twist expansion)

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \frac{e_q m_M}{4} c_f \left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+} q_\rho\right)\left(\varepsilon_M^{*\beta} - \frac{p_M^\beta}{p_M^+}\varepsilon_M^{*+}\right)\delta\left(1 - \frac{p_M^+}{q^+}\right)$$

$$\times\left(\prod_{j=1}^{3}\frac{\theta(1-x_j)\theta(x_j)}{x_j}\right)\frac{(2\pi)^3\delta^{(2)}\left(\sum_{i=1}^{3}\boldsymbol{p}_i + x_i\boldsymbol{p}_M\right)}{\left[Q^2 + \sum_{i=1}^{3}(\boldsymbol{p}_i + x_i\boldsymbol{p}_M)^2/x_i\right]}\left\{g_{\beta\sigma} f_{3M}^V V(x_1,x_2)\left(4g_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\right.\right.$$

$$\left.+\tilde{T}_1^{\sigma\rho\nu}(\{x\})\Big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2 + x_1(1-x_1)Q^2}\right) - \epsilon_{-+\sigma\beta} f_{3M}^A A(x_1,x_2)$$

$$\left.\times\left(4\frac{x_1x_2}{1-x_2}\epsilon^{\sigma\rho+-} + i\tilde{T}_2^{\sigma\rho\nu}(\{x\})\Big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2 + x_1(1-x_1)Q^2}\right)\right\}$$

$$+(1\leftrightarrow 2)$$

Higher-twist ρ-meson DAs

- Twist and chiral classification of the ρ-meson distribution amplitudes

[Ball, Braun, Koike, Tanaka (1998)]

- Two body distribution amplitudes

Twist	2	3	4
	$O(1)$	$O(1/Q)$	$O(1/Q^2)$
$e_\parallel$	$\phi_\parallel$	$\underline{h_\parallel^{(t)}}, \underline{h_\parallel^{(s)}}$	g_3
$e_\perp$	$\underline{\phi_\perp}$	$g_\perp^{(v)}, g_\perp^{(a)}$	$\underline{h_3}$

- $g_\perp^{(v)}$ and $g_\perp^{(a)}$ are the vector and axial two-body twist-3 DA

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle$$

- Vector and axial three-body twist-3 DA

$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,gF^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,g\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

Twist-expanded photon meson wavefunctions overlap

- 3-body twist-expanded photon meson wavefunctions overlap

$$\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right) = \frac{e_q m_M c_f}{8\pi}\delta\left(1-\frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)\left(\varepsilon_M^{*\mu}-\frac{p_M^\mu}{p_M^+}\varepsilon_M^{*+}\right)\left(\prod_{j=1}^{3}\theta(x_j)\theta(1-x_j)e^{-ix_j\boldsymbol{p}_M\boldsymbol{z}_j}\right)$$

$$\times\Bigg\{-if_{3M}^V g_{\sigma\mu}V(x_1,x_2)\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]$$

$$-\epsilon_{-+\sigma\beta}f_{3M}^A g_{\perp\perp\mu}^{\beta}A(x_1,x_2)\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)+(1\leftrightarrow 2)\right]\Bigg\}$$

- 2-body twist-expanded photon meson wavefunctions overlap

$$\Psi_2\left(x,\boldsymbol{r}\right) = e_q m_M f_M\delta\left(1-\frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu}-\frac{\varepsilon_q^+}{q^+}q_\mu\right)\left(\varepsilon_{M\alpha}^*-\frac{\varepsilon_M^{*+}}{p_M^+}p_{M\alpha}\right)$$

$$\times\left[-ir_\perp^\alpha(h(x)-\tilde{h}(x))\left(2x\bar{x}q^\mu+(x-\bar{x})\frac{-i\partial}{\partial r_{\perp\mu}}\right)+\epsilon^{\mu\nu+-}\epsilon^{+\alpha-\delta}r_{\perp\delta}\left(\frac{g_\perp^{(a)}(x)-\tilde{g}_\perp^{(a)}(x)}{4}\right)\frac{\partial}{\partial r_\perp^\nu}\right]$$

$$\times K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Genuine twist-3 related DAs

$$\widetilde{h}\left(x\right) = \frac{f_{3M}^V}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{V\left(x_q,x_{\overline{q}}\right)}{\left(1-x_q-x_{\overline{q}}\right)^2}\qquad \widetilde{g}_\perp^{(a)}\left(x\right) = 4\frac{f_{3M}^A}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{A\left(x_q,x_{\overline{q}}\right)}{(1-x_q-x_{\overline{q}}+i\epsilon)^2}$$