# **Odderon contribution in light of the LHC low-***t* **data**

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> *Diffraction and Low-x Physics* Palermo, Italy, 2024

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# **Outline**

- Brief introduction to Odderon
- Formalism: Pomeron and Odderon inputs
- The tension between the TOTEM and ALFA/ATLAS measurements
- Results
- Conclusions

# **Introduction**

**E** The TOTEM publication of the measurements of the  $\sigma_{tot}^{pp}$  and the  $\rho^{pp}$ parameter at 13 TeV, prompted a renewal of interest in the potential existence of the high-energy *C*-odd (Odderon) contribution.

 $\Rightarrow$  The observed value  $\rho = (0.09 - 0.10) \pm 0.01$  turned out to be smaller than the predicted value ( $\rho = 0.13 - 0.14$ ) based on Disp. Rel.

 $\square$  The new ATLAS/ALFA data recently confirmed this value of  $\rho$ 

 $\Rightarrow$  However, the value of  $\sigma_{tot}^{pp}$  at 13 TeV reported by the ATLAS/ALFA team is approximately 5% lower than the average of values determined by TOTEM

# **Introduction**

**The relatively small value of**  $\rho$  can be explained by the admixture of the *C*-odd amplitude, which survives at high LHC energies

 $\Rightarrow$  Such amplitude with the intercept  $\alpha_{\text{Odd}}$  close to 1 was predicted by the perturbative QCD

 $\square$  Another indication in favor of the Odderon emerged when the  $d\sigma^{\bar{p}p}/dt$  data at  $\sqrt{s} = 1.96$  TeV was compared with the corresponding *pp* cross section (measured at 2.76 TeV but extrapolated to 1.96 TeV) in the diffractive dip region

 $\Rightarrow$  A clear difference was observed

 $\Rightarrow$  It is important to emphasize that this result depends on the specific extrapolation method between different energy levels

# **Introduction**

■ It should be noted that at very low *t* close to zero and in the diffractive dip region, we deal with different *C*-odd contributions

 $\Rightarrow$  To get a well-pronounced dip-bump structure near the dip in  $d\sigma^{pp}/dt|_{2.76 TeV}$  and rather flat behavior of  $d\sigma^{\bar{p}p}/dt|_{1.96 TeV}$ , the real part of the Odderon *pp* amplitude should be positive (in agreement with perturbative QCD expectation for the three gluon exchange)

 $\Rightarrow$  On the other hand, to explain the low value of  $\rho$  at  $t=0$ , we need a negative Odderon real part

 $\Rightarrow$  This negative real part could be induced by non-pert. effects

# **Formalism**

■ Our analysis is focused on differential cross-section data involving very small values of *t*

 $\Rightarrow$  It requires consideration of the Coulomb-nuclear interference (CNI) region:

 $\bm{F}^{C+N} = \bm{F}^N + \bm{e}^{i\alpha\phi(t)}\bm{F}^C$ 

□ We adopt

$$
\phi(t) = \kappa \left[ \gamma + \ln \left( \frac{\mathcal{B}|t|}{2} \right) + \ln \left( 1 + \frac{8}{B\Lambda^2} \right) + \frac{4|t|}{\Lambda^2} \ln \left( \frac{\Lambda^2}{4|t|} \right) - \frac{2|t|}{\Lambda^2} \right]
$$

where  $\kappa$  flips sign when going from *pp* ( $\kappa = -1$ ) to  $\bar{p}p$  ( $\kappa = +1$ )

# **Formalism**

 $\Rightarrow$   $\Lambda^2$  is fixed at 0.71 GeV<sup>2</sup>

⇒ *B* is the *t* slope of elastic *d*σ/*dt* ∝ *exp*(*Bt*) cross-section

 $\Rightarrow \gamma = 0.577...$  is the Euler-Mascheroni constant

 $\Rightarrow \alpha$  is the electromagnetic coupling:

$$
\alpha(\mathbf{q}^2) = \frac{\alpha(\mathbf{0})}{1 - \frac{\alpha(\mathbf{0})}{3\pi} \ln\left(\frac{\mathbf{q}^2 + m_{\mathbf{e}}^2}{m_{\mathbf{e}}^2}\right)}
$$

where  $\alpha(0) \approx 1/137$ 

# **Formalism**

# ■ The Coulomb amplitude is expressed as

$$
F^C = \kappa s \frac{2\alpha}{|t|} G^2(t)
$$

where *G*(*t*) is the electromagnetic form factor of the proton:

$$
G(t) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^2
$$

 $\square$  To account for the eikonalization, it is convenient to calculate the nuclear amplitude in terms of opacities:

$$
\Omega_i(\boldsymbol{s},b)=\frac{2}{s}\int_0^\infty q\,dq\,J_0(bq)\,F_i^N(\boldsymbol{s},t)
$$

where  $i = P$ ,  $\odot$  represent the Pomeron and Odderon exchanges

### **Pomeron input**

■ The single Pomeron contribution is given by

$$
\mathcal{F}_{\mathbb{P}}^{N}(\boldsymbol{s},t)=\beta_{\mathbb{P}}^{2}(t)\,\eta_{\mathbb{P}}(t)\left(\frac{\boldsymbol{s}}{\boldsymbol{s}_0}\right)^{\alpha_{\mathbb{P}}(t)}
$$

 $\Rightarrow \eta_\mathbb{P}(t) = -e^{-i\frac{\pi}{2}\alpha_\mathbb{P}(t)}$  is the even signature factor

 $\Rightarrow \beta_{\mathbb{P}}(t)$  is the elastic proton-Pomeron vertex

 $\Rightarrow \alpha_{\mathbb{P}}(t)$  is the Pomeron trajectory,

$$
\alpha_{\mathbb{P}}(t)=1+\epsilon+\alpha_{\mathbb{P}}'t+\frac{m_{\pi}^2\beta_{\pi}^2}{32\pi^3}\,h(\tau)
$$

where

$$
h(\tau) = -\frac{4}{\tau} F_{\pi}^{2}(t) \left[ 2\tau - (1+\tau)^{3/2} \ln \left( \frac{\sqrt{1+\tau}+1}{\sqrt{1+\tau}-1} \right) + \ln \left( \frac{m^{2}}{m_{\pi}^{2}} \right) \right]
$$

with  $\epsilon > 0$ ,  $\tau = 4 m_\pi^2/|t|$ ,  $m = 1$  GeV, and  $m_\pi = 139.6$  MeV

# **The Odderon input**

**E** From the standpoint of QCD (at the lowest order) the  $C = +1$ amplitude arises from the exchange of two gluons and the  $C = -1$ amplitude from the exchange of three gluons

■ Extensive theoretical studies have been directed towards uncovering corrections to these results, particularly in higher orders

 $\square$  In this scenario, the leading-log approximation allows for the summation of certain higher-order contributions to physical observables in high-energy particle scattering processes

 $\Rightarrow$  This approach was widely used in the study of the QCD-Pomeron through the BFKL equation

# **The Odderon input**

 $\Rightarrow$  In BFKL equation terms of the order  $(\alpha_s \ln(s))^n$  are systematically summed at high energy (large *s*) and small strong coupling α*<sup>s</sup>*

 $\Rightarrow$  The simplistic notion of bare two-gluon exchange gives way to the BFKL Pomeron, which, in an alternative representation, can be seen as the interaction of two reggeized gluons with one another

■ Beyond the BFKL Pomeron, the most elementary entity within perturbative QCD is the exchange involving three interacting reggeized gluons

 $\square$  The evolution of the three-gluon Odderon exchange as energy increases is governed by the BKP equation

 $\Rightarrow$  A bound state solution of this Odderon equation was obtained with the intercept  $\alpha_{\mathbb{O}}(0) = 1$ 

# **The Odderon input**

■ Based on these QCD findings, we adopt in this work the simplest conceivable form for the Odderon trajectory:

$$
\alpha_{\mathbb{O}}(t)=1
$$

# ■ The Odderon contribution is given by

$$
\mathit{F}^{N}_{\mathbb{O}}(s,t)=\beta^2_{\mathbb{O}}(t)\,\eta_{\mathbb{O}}(t)\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{O}}(t)}
$$

 $\Rightarrow \eta_{\mathbb{O}}(t)=-i e^{-i\frac{\pi}{2}\alpha_{\mathbb{O}}(t)}$  is the odd signature factor

 $\Rightarrow$   $\beta_{\Omega}(t)$  is the elastic proton-Odderon vertex

# *t* **dependence of the** β **vertices**

 $\blacksquare$   $\beta_{\mathbb{P}}(t)$  and  $\beta_{\mathbb{O}}(t)$  are parameterized accounting for the observed deviation from a pure exponential behavior of the low-|*t*| *d*σ/*dt* data at LHC energies

 $\Rightarrow$  Behavior identified by the TOTEM Collaboration

 $\square$  The TOTEM group has extended the pure exponential to a cumulant expansion,

$$
\frac{d\sigma}{dt}(t) = \left. \frac{d\sigma}{dt} \right|_{t=0} \exp \left( \sum_{n=1}^{N_b} b_n t^n \right)
$$

where the optimal fit was achieved for  $\textit{N}_{b} = 3$ , yielding  $\chi^{2}/\textit{DoF} = 1.22$ and a corresponding *p*-value of 8.0%

*t* **dependence of the** β **vertices**

■ Based on this result, we have written the Pomeron- and Odderon-proton vertices as

$$
\beta_{\mathbb{P}}(t)=\beta_{\mathbb{P}}(0)e^{(At+Bt^2+ Ct^3)/2}
$$

and

 $\beta_\mathbb{O}(t) = \beta_\mathbb{O}(0) e^{Dt/2}$ 

respectively

■ We allow for the low mass diffractive dissociation (Good-Walker formalism)

 $\Rightarrow$  The Pomeron and Odderon couplings to the two diffractive states  $|\phi_{\bf k}\rangle$  are

 $\beta_{i,k}(t) = (1 \pm \gamma)\beta_i(t)$ 

with  $i = P$  or  $\mathbb{O}$ , and  $\gamma = 0.55$ 

 $\square$  The eikonalized amplitude in  $(s, t)$ -space is then given by

$$
\mathcal{A}(s,t) = is \int_0^\infty b \, db \, J_0(bq) \left[ 1 - \frac{1}{4} \, e^{i(1+\gamma)^2 \Omega(s,b)/2} - \frac{1}{2} \, e^{i(1-\gamma)^2 \Omega(s,b)/2} - \frac{1}{4} \, e^{i(1-\gamma)^2 \Omega(s,b)/2} \right]
$$

where  $\Omega(s, b)$  is the total opacity

■ We consider two versions for the total opacity with different signs for the Odderon contribution

□ In the first version, referred to as 'Model **I**', we have

 $\Omega(s, b) = \Omega_{\mathbb{P}}(s, b) \mp \Omega_{\mathbb{O}}(s, b)$ 

□ In the second version, called 'Model **II**', we have

 $\Omega(s, b) = \Omega_{\mathbb{P}}(s, b) \pm \Omega_{\mathbb{O}}(s, b)$ 

 $\Rightarrow$  In both cases the upper sign is for *pp* and the lower sign is for  $\bar{p}p$ 

**The total cross section and the**  $\rho$  **parameter are expressed in terms** of the nuclear eikonalized amplitude  $A(s, t)$ :

$$
\sigma_{tot}(s) = \frac{4\pi}{s} \text{Im } A(s, t = 0)
$$

$$
\rho(\mathbf{s}) = \frac{\operatorname{Re} \mathcal{A}(\mathbf{s}, t=0)}{\operatorname{Im} \mathcal{A}(\mathbf{s}, t=0)}
$$

■ The full scattering amplitude is written as

$$
F^{C+N}(s,t) = A(s,t) + e^{i\alpha\phi(t)}F^{C}(s,t)
$$

# ■ Finally, the differential and the total elastic cross sections are given by

$$
\frac{d\sigma}{dt}(\mathbf{s},t)=\frac{\pi}{\mathbf{s}^2}\left|\mathcal{A}(\mathbf{s},t)+e^{i\alpha\phi}\mathcal{F}^C(\mathbf{s},t)\right|^2
$$

and

$$
\sigma_{el}(s) = \frac{\pi}{s^2} \int_{-\infty}^0 dt \, |\mathcal{A}(s,t)|^2
$$

■ The LHC has released exceptionally precise measurements of diffractive processes

 $\square$  These measurements, particularly the total and differential cross sections obtained from ATLAS and TOTEM Collaborations, enable us to determine the Pomeron and Odderon parameters accurately

 $\Rightarrow$  However, these experimental results unveil a noteworthy tension between the TOTEM and ATLAS measurements

 $\Rightarrow$  For instance, when comparing the TOTEM and the ATLAS result for  $\frac{p}{\sigma_{tot}}$  at  $\sqrt{s} = 8$  TeV, the discrepancy between the values corresponds to 2.6  $\sigma$ 

■ In order to systematically explore the tension between TOTEM and ATLAS results, we perform global fits to *pp* and  $\bar{p}p$  differential cross-section data while considering three distinct datasets:

**Ensemble A:** 
$$
\frac{d\sigma^{p\rho,pp}}{dt}\Big|_{\text{CERN-ISR}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{S}\bar{p}pS} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{Tevatron}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{ATLAS/ALFA}}
$$
  
\n**Ensemble T:**  $\frac{d\sigma^{p\rho,pp}}{dt}\Big|_{\text{CERN-ISR}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{S}\bar{p}pS} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{Tevatron}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{TOTEM}}$   
\n**Ensemble A**  $\oplus$  **T:**  $\frac{d\sigma^{p\rho,pp}}{dt}\Big|_{\text{CERN-ISR}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{S}\bar{p}pS} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{Tevatron}} + \frac{d\sigma^{p\rho}}{dt}\Big|_{\text{Tevatron}}$ 

 $\Rightarrow$  We carry out global fits to the two distinct ensembles using a  $\chi^2$ fitting procedure, where  $\chi^2_{\sf min}$  follows a  $\chi^2$  distribution with  $\nu$  DoF

 $\Rightarrow$  We adopt an interval  $\chi^2-\chi^2_{\sf min}$  corresponding to a 90% confidence level (CL).

■ Since the absolute values of cross sections measured at the same energy by different groups do not agree, we have introduced normalization factors *N<sup>i</sup>* for high-energy *d*σ/*dt* data

 $\Rightarrow$  *i* = 7[A], 8[A], and 13[A] for the ATLAS/ALFA data and *i* = 7[T], 8[*T*], and 13[*T*] for the TOTEM data (here the numbers within the indices *<sup>i</sup>* correspond to the values of <sup>√</sup> *s*)

 $\Rightarrow$  Analogous normalization factors are introduced for the Tevatron data with  $i=1.8[\bm{E}]$  and  $i=1.8[\bm{C}]$ , i.e.  $\bm{N_{1.8[\bm{E}]}}$  for the E710 data and  $\mathcal{N}_{1.8 [C]}$  for the CDF data

 $\Rightarrow$  Despite being the only data set measured at  $\sqrt{s} =$  546 GeV, we also included a normalization factor for  $\left. d\sigma^{\bar{p}p}/dt \right|_{\sqrt{\bar{s}}=546\,\mathrm{GeV}},$  namely *N*<sup>546</sup>

 $\blacksquare$  Furthermore, when dealing with the data sets incorporating normalization factors *N<sup>i</sup>* , we make use of the formula

$$
\chi^{2} = \sum_{ij} \frac{(N_{i} ds_{ij}^{th} - ds_{ij}^{exp})^{2}}{(\delta_{ij}^{rem})^{2}} + \sum_{i} \frac{(1 - N_{i})^{2}}{\delta_{i}^{2}}
$$

⇒ *i* denotes the particular set of data while *j* denotes the point *t<sup>j</sup>* in this set of data

⇒ *dsth* is the theoretically calculated *d*σ/*dt* cross section while *dsexp* is the value measured at the same *ij* point experimentally

 $\Rightarrow$   $\delta_i$  is the normalization uncertainty of the given (*i*) set of data and  $\delta_{ij}^{\prime em}$  is the remaining error at the point  $ij$  calculated as  $(\delta_{ij}^{rem})^2 = \delta_{tot,ij}^2 - \delta_i^2$ 

 $\Rightarrow$  As a rule the value of  $\delta^{\mathit{rem}}$  is dominantly the statistical error



FIG.1. Description of the *t* dependence of the elastic *pp*- and *pp*-cross sections measured at CERN-ISR. The dashed and solid curves depict the results obtained using Models I and II, respectively



FIG.2. Description of the *t* dependence of the elastic *pp*- and *pp*-cross sections measured at the *SppS*, the Tevatron and the LHC colliders. The dashed and solid curves depict the results obtained using Models I and II, respectively. The lower curves describe the ATLAS/ALFA (E710) data while the upper curves correspond to the TOTEM (CDF) data; in both cases, the normalization factors *N<sup>i</sup>* are accounted for



FIG.3. The same as Fig. 2 but for the CNI region where the Odderon contribution reveals itself



FIG.4. The same as Figs. 2 and 3 but in another scale to better see the quality of precise 13 TeV data description



FIG.5. Description of the total and elastic  $pp$  (•, ▲, ■) and  $\bar{p}p$  (◦) cross sections. The dotted and dashed-dotted curves represent the results for *pp* and  $\bar{p}p$  channels, respectively, obtained from the global fit to Ensemble A  $\oplus$  T using Model I. These curves are indistinguishable. The solid and dashed curves represent the results for *pp* and  $\bar{p}p$  channels, respectively, obtained from the global fit to Ensemble A ⊕ T using Model II



#### $\sqrt{s}$  (GeV)

FIG.6. *ρ* parameter for *pp* (▲, ■) and  $\bar{p}p$  (○) elastic amplitude. The dotted and dashed-dotted curves represent the results for *p<sup>pp</sup>* and *p<sup>pp</sup>*, respectively, obtained from the global fit to Ensemble A ⊕ T using the Model I. These curves are indistinguishable. The solid and dashed curves represent the results for p<sup>pp</sup> and p<sup>pp</sup>, respectively, obtained from the global fit to Ensemble A ⊕ T using the Model II

**Table:** Values of the parameters obtained in the global fits to Ensemble *A* ⊕ *T*.

<span id="page-28-0"></span>

# **Table:** Predictions for  $\sigma_{tot}^{\bar{p}p, pp}$ ,  $\sigma_{el}^{\bar{p}p, pp}$ , and  $\rho^{\bar{p}p, pp}$  using Models I and II. These results were derived for the scenario with  $D=A/2$







dashed (solid) curve represents the predicted p<sup>pp</sup> from the global fit using Ensemble T (Ensemble A)

# **Conclusions**

**■ The differential** *pp* and  $\bar{p}p$  cross sections  $d\sigma/dt$  at low  $|t| < 0.1$ GeV<sup>2</sup> and collider energies (from  $\sqrt{s}$  > 50 GeV to 13 TeV) are successfully described ( $\chi^2/\nu=1.11)$  within the two-channel eikonal model

 $\square$  To avoid the double counting we do not include in the fit the  $\sigma_{tot}$  and ρ data (which were obtained from the description of the same *d*σ/*dt* data points)

■ The model accounts for the screening of the Odderon contribution by the Pomerons including the *C*-even (Pomeron) and *C*-odd (Odderon) multiple exchanges

■ To resolve the discrepancy between the TOTEM and ATLAS/ALFA (CDF and E710 in the Tevatron case) data we introduce the normalization coefficients, *N<sup>i</sup>* writing the theoretical prediction as  $d\sigma^{exp}/dt = N_i d\sigma^{Th}/dt$ 

# **Conclusions**

■ We show that the presence of *C*-odd (Odderon) contribution essentially improves the fit; however it does not noticeably change the predicted value of  $\rho^{\rho\rho}$  at 13 TeV

■ The main lessons about the Odderon coming from this study are:

 $\Longrightarrow$  The description using the Odderon improves the fit (the  $\chi^2/\nu$  is the lowest one)

 $\implies$  The sign of the Odderon amplitude needed to describe the very low |*t*| data is opposite to that predicted by the perturbative QCD three-gluon exchange contribution

# **Conclusions**

 $\implies$  The quality of the description weakly depends on the Odderon *t*-slope, *D*, (leading to practically the same values of  $\sigma_{tot}$  and  $\rho$ ). However, for smaller *D* we need larger coupling  $\beta_0$  to compensate for a stronger absorption caused by the Pomeron screening at small impact parameters *b*

 $\Rightarrow$  The Odderon-proton coupling,  $\beta_0$ , is smaller than that for the Pomeron,  $\beta_{\mathbb{P}}$ . For  $D = A/2$  we get  $\beta_{\mathbb{O}}/\beta_{\mathbb{P}} = 0.40$ , however after accounting for screening by the Pomeron the final *C*-odd contribution to  $\rho$  at 13 TeV becomes quite small,  $\delta \rho = (\rho^{\bar{\rho} \rho} - \rho^{\rho \rho})/2 \leq$  0.004 (see Table [1\)](#page-28-0) and it will be challenging to enlarge it. Otherwise, we will get Table 1) and it will be challenging to emarge it. Otherwise, we<br>too large  $\rho^{\bar{\rho}\rho}$  at  $\sqrt{s}$  ∼ 541 GeV in disagreement with the data

# THANK YOU

# **Resummations in QCD**

■ Every physical observable can be written, in pQCD, as a power series in α*<sup>s</sup>*

 $\implies$  in these series the coupling constant is accompanied by large logarithms, which need to be resummed

 $\implies$  according to the type and to the powers of logarithms that are effectively resummed one gets different evolution equations

**The solution of the DGLAP equation sums over all orders in**  $\alpha_s$  **the** contributions from leading, single, collinear logarithms of the form  $\alpha_{\scriptscriptstyle S}$  ln  $\left(Q^2/Q_0^2\right)$ 

 $\implies$  it does not include leading, single, soft singularities of the form  $\alpha_s$  ln (1/*x*), which are treated instead by the BFKL equation

■ The BFKL equation describes the *x*-evolution of PDFs at fixed  $Q^2$ 

### **Resummations in QCD**

■ The phase space regions which contribute these logarithms enhancements are associated with configurations in which successive partons have strongly ordered transverse,  $k<sub>T</sub>$ , or longitudinal,  $k<sub>I</sub> \equiv x$ , momenta:

 $\Rightarrow$   $\alpha_s L_Q \sim$  1,  $\alpha_s L_X \ll$  1:  $Q^2 \gg k_{\mathcal{T},n}^2 \gg \cdots \gg k_{\mathcal{T},1}^2 \gg Q_0^2$ 

⇒ α*sL<sup>x</sup>* ∼ 1, α*sL<sup>Q</sup>* ≪ 1: *x* ≪ *x<sup>n</sup>* ≪ · · · ≪ *x*<sup>1</sup> ≪ *x*<sup>0</sup>

■ At small-*x* and slow  $Q^2$  (where gluons are dominant) we do not have strongly ordered *k<sup>T</sup>*

 $\Rightarrow$  we have to integrate over the full range of  $k<sub>T</sub>$ 

⇒ this leads us to work with the *unintegrated* gluon PDF  $\tilde{g}(x, k_T^2)$ :

$$
xg(x,Q^2) = \int^{Q^2} \frac{dk_T^2}{k_T^2} \tilde{g}(x,k_T^2)
$$

# **Positivity**

■ The phase factor is associated with the positivity property

 $\Rightarrow$  However, unlike Pomeron, the Odderon is not constrained by positivity requirements

 $\Rightarrow$  From a theoretical standpoint, this implies that it is not possible to determine the phase of the Odderon mathematically

 $\square$  This issue can be succinctly grasped: in the forward direction the physical amplitudes F *pp*  $\frac{p\rho}{\bar{p} \rho}(s)$  can be written as  $\mathcal{F}^{pp}_{\bar{p} \rho}$ *pp*¯ (*s*) = *F* <sup>+</sup>(*s*) ± *F* <sup>−</sup>(*s*)

 $\square$  Considering that the only relevant contributions are those arising from the Pomeron and the Odderon exchanges, we can write the symmetric and antisymmetric amplitudes as  $F^+(s) = R_{\mathbb{P}}(s) + i I_{\mathbb{P}}(s)$  $\mathsf{and}\; \bar{F}^{-}(s) = R_{\mathbb{O}}(s) + i \bar{I}_{\mathbb{O}}(s)$ 

 $\square$  From the optical theorem, we have  $s\sigma_{tot}^{pp,\bar{p}p}(s) = 4\pi$  Im  $\mathcal{F}_{\bar{p}p}^{pp}$  $\frac{p}{\bar{p}p}(s) > 0,$ which implies that

$$
\text{Im}\,\mathcal{F}^{pp}_{\bar{p}p}(s)=\mathit{I}_{\mathbb{P}}(s)\pm\mathit{I}_{\mathbb{O}}(s)>0
$$

and, in turn,

 $I_P(s) > |I_{\odot}(s)|$ 

As a consequence,

$$
\textit{l}_{\mathbb{P}}(s) = \frac{s}{2}\left[\sigma_{\textit{tot}}^{\textit{pp}}(s) + \sigma_{\textit{tot}}^{\textit{pp}}(s)\right] > 0
$$

while

$$
I_{\mathbb{O}}(s) = \frac{s}{2} \left[ \sigma_{tot}^{pp}(s) - \sigma_{tot}^{\bar{p}p}(s) \right]
$$

is not bound by the same positivity requirements