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Absorptive Corrections to the Electromagnetic Form Factor in High-Energy Elastic Proton-Proton Scattering

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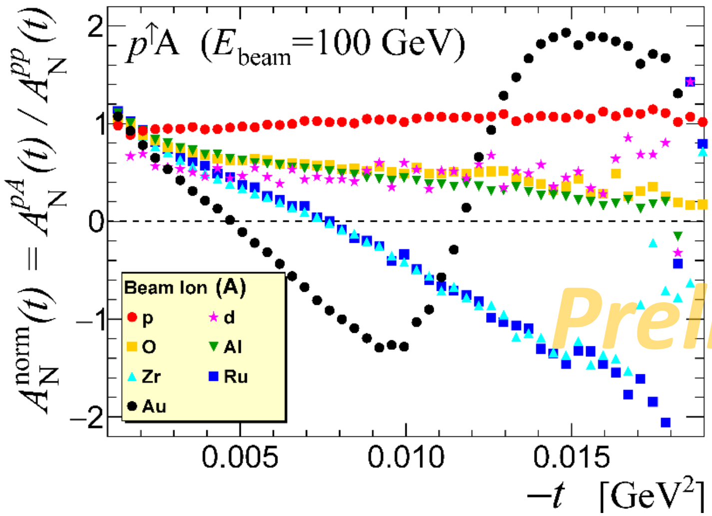
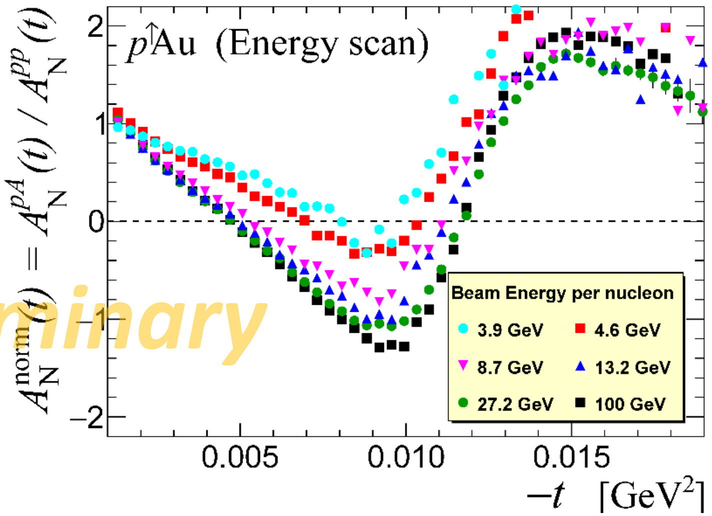
The of goal of this research

Numerous $p^{\uparrow}A$ analyzing powers have been measured using the RHIC Polarized Atomic Hydrogen Gas Jet Target (HJET).

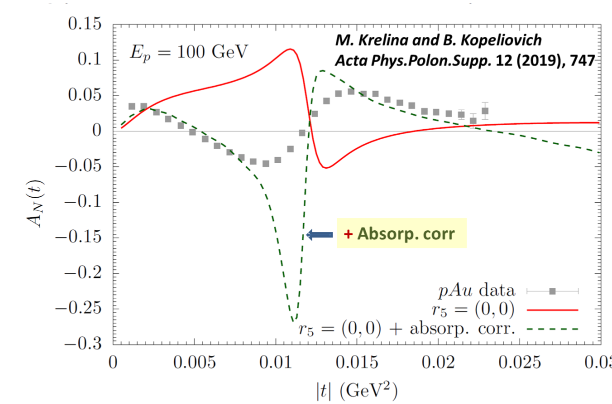




Here, $A_N^{pp}(t)$ represents the pure CNI (($r_5 = 0$) proton-proton analyzing power, calculated for a 100 GeV beam in the lab. system.

Boris Kopeliovich pointed out that absorptive corrections to the proton electromagnetic form factor may drastically alter theoretical interpretation of $p^{\uparrow}p$ and $p^{\uparrow}A$ analyzing power measurements.

B.Z. Kopeliovich et al., Phys. Lett. B **816**, 136262 (2021)
B.Z. Kopeliovich et al., Universe **10**, 63 (2024)



The primary goal of this research was to develop a parametrization for the absorptive correction to the non-flip electromagnetic form factor, which can be conveniently applied in HJET data analysis.
Absorptive corrections to the spin-flip amplitude are not considered here.

The Eikonal Phases in Elastic pp Scattering

The Coulomb amplitude:

$$f_C^{\mathrm{Born}}(t) = \frac{2\alpha}{t} e^{B_E t/2}$$

$$\chi_C(b) = \int_0^\infty q_T dq_T \frac{-2\alpha}{q_T^2+\lambda^2} e^{-B_E q_T^2/2} J_0(bq_T)$$

$$B_E = \frac{2\langle r_E^2 \rangle}{3} = 12.1\ \mathrm{GeV}^{-2}$$

$$q_T^2 \approx -t$$

$$-t_c = 8\pi\alpha/\sigma_{\mathrm{tot}} \approx 0.002\ \mathrm{GeV}^2$$

The hadronic amplitude:

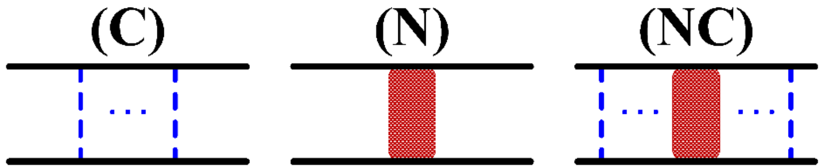
$$f_N^{\mathrm{Born}}(t) = (i+\rho)\frac{\sigma_{\mathrm{tot}}}{4\pi} e^{Bt/2}$$

$$= (i+\rho)\frac{2\alpha}{-t_c} e^{Bt/2}$$

$$\gamma_N(b) = \int_0^\infty q_T dq_T \frac{2\alpha}{-t_c} e^{-Bq_T^2/2} J_0(bq_T)$$

$$= \frac{2\alpha}{-Bt_c} e^{-b^2/2B}$$

Soft photon corrections to the amplitudes



$$f_C^\gamma(t) = \int_0^\infty bdbJ_0(bq_T)\; i\left[1 - e^{i\chi_C(b)}\right] = f_C^{\rm Born}(t)e^{i\Phi_C^\lambda}$$

$$f_{N+NC}^\gamma(t) = (i+\rho)\int_0^\infty bdbJ_0(bq_T)\, e^{i\chi_C(b)}\gamma_N(b) = f_N^{\rm Born}(t)e^{i\Phi_{NC}^\lambda}$$

$$f_N(t) = (i+\rho)\int_0^\infty bdbJ_0(bq_T)\,\gamma_N(b) = f_N^{\rm Born}(t)$$

In lowest order of α and $\lambda \to 0$, the phases Φ_C^λ and Φ_{NC}^λ were calculated analytically
[B.Z. Kopeliovich and A.V. Tarasov, Phys. Lett. B**497**, 44 (201)]

$$\Phi_C^\lambda/\alpha = \ln\frac{B_E\lambda^2}{2} + 2\gamma + \ln w - \mathrm{Ei}\left(\frac{w}{2}\right) + e^w\left[2E_1(w) - E_1\left(\frac{w}{2}\right)\right]$$

$$\Phi_{\rm NC}^\lambda/\alpha = \ln\frac{B_E\lambda^2}{2} + 2\gamma + \ln w + \ln\frac{B^2}{B_E^2} - \mathrm{Ei}(z) \approx \ln\frac{B_E\lambda^2}{2} + \gamma + \ln\frac{B+B_E}{B_E}$$

$\gamma = 0.5772\,...$

$w = -B_E t/2$

$z = -B^2 t/2(B+B_E)$

The phase difference is independent of the photon mass λ

$$\delta_C(t) = \Phi_C^\lambda - \Phi_{NC}^\lambda = -\alpha\left[\gamma + \ln\frac{(B+B_E)|t_c|}{2}\right] + \alpha\ln\frac{t_c}{t} = \alpha\left[\ln\frac{2}{|t|(B+8/\Lambda^2)} - \gamma\right]$$

The Absorptive Correction

- Typically, the proton electromagnetic form factor slope B_E used in high-energy pp data analysis is determined in ep scattering, which is free from initial/final state strong interactions between the colliding particles.
- In pp scattering, absorptive corrections, arising from the dependence of inelastic scattering probability on the impact parameter b, may significantly reduce the partial elastic amplitude at small impact parameters.
- To account for this correction, an absorptive factor can be applied:

$$i\left[1-e^{i\chi_C(b)}\right] \Rightarrow i\left[1-e^{i\chi_C(b)}\right] \times [1-\gamma_N(b)]$$
$$= i\left[1-e^{i\chi_C(b)}\right] - i\gamma_N(b) + i\gamma_N(b)\, e^{i\chi_C(b)}$$

using expressions on the previous page

$$f_C^{\gamma}(t) \Rightarrow f_C^{\gamma}(t) - i f_N(t) \times \left[1-e^{i\Phi_{NC}^{\lambda}}\right]$$

$$B_E \quad \Rightarrow \quad B_E^{\text{eff}} = B_E + 2\Phi_{NC}^{\lambda}/t_c$$

- However, the result explicitly depends on the photon mass λ.

Elimination of the Photon Mass Dependence

- It can be straightforwardly shown that adding a constant term to the eikonal phase

$$\chi_C(b) \to \chi_C(b) + \chi_0$$

results in corresponding alteration of the both Coulomb phases:

$$\Phi_C^\lambda \to \Phi_C^\lambda + \chi_0 \text{ and } \Phi_{NC}^\lambda \to \Phi_{NC}^\lambda + \chi_0$$

For example, $e^{i[\chi_C(b)+\chi_0]}\gamma_N(b) = e^{i\chi_0} \times e^{i\chi_C(b)}\gamma_N(b)$.

- Focusing on the infrared divergence, one finds,

$$\chi_C(b,\lambda) = -2\alpha \int_0^\infty \frac{q_T dq_T}{q_T^2 + \lambda^2} e^{-B_E q_T^2/2} J_0(bq_T) \Longrightarrow -\alpha \int_0^{q_T^{max}(b)} \frac{dq_T^2}{q_T^2 + \lambda^2} \Longrightarrow \chi_C(b,\boldsymbol{\lambda}_2) - \chi_C(b,\boldsymbol{\lambda}_1) \approx \alpha \ln \frac{\boldsymbol{\lambda}_2^2}{\boldsymbol{\lambda}_1^2}$$

$$\chi_C(b,\lambda) = \chi_C(b) + \boldsymbol{\alpha} \ln \frac{\boldsymbol{B_E \lambda^2}}{\boldsymbol{2}} + \mathbf{const}$$

- Thus, the dependence of Φ_{NC}^λ on the photon mass can be replaced by some (unknown) constant:

$$\Phi_{\rm NC}^\lambda \to \Phi_{\rm NC} \approx \alpha \left[C + \ln \frac{B + B_E}{2B_E} \right] \approx \alpha C$$

- Consequently, the electromagnetic form factor is modified as:

$$B_E^{\rm eff} = \frac{2\langle r_E^2 \rangle}{3} - \frac{\sigma_{\rm tot}}{4\pi} C$$

Absorptive Correction to the Differential Cross Section

- Ignoring the difference between the electromagnetic B_E and hadronic B slopes, the differential cross section is given by:

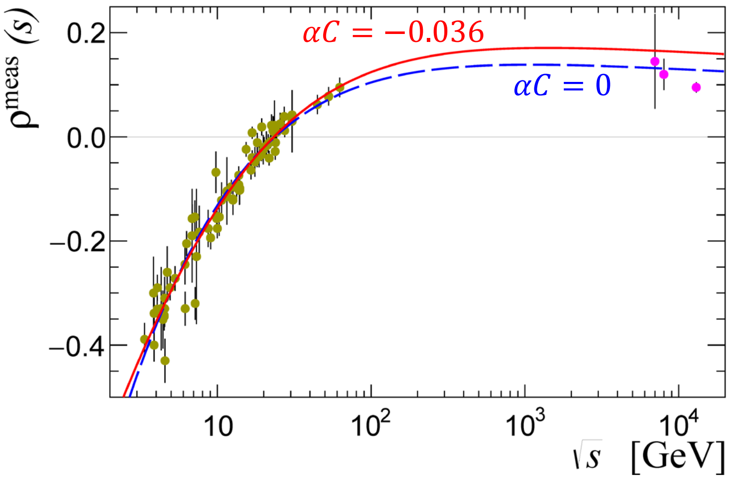
$$\frac{d\sigma}{dt} = \frac{\sigma_{\text{tot}}^2}{16\pi}\left[\left(\frac{t_c}{t}\right)^2 - 2(\rho + \delta_C - \alpha C)\frac{t_c}{t} + 1 + \rho^2\right] e^{Bt}$$

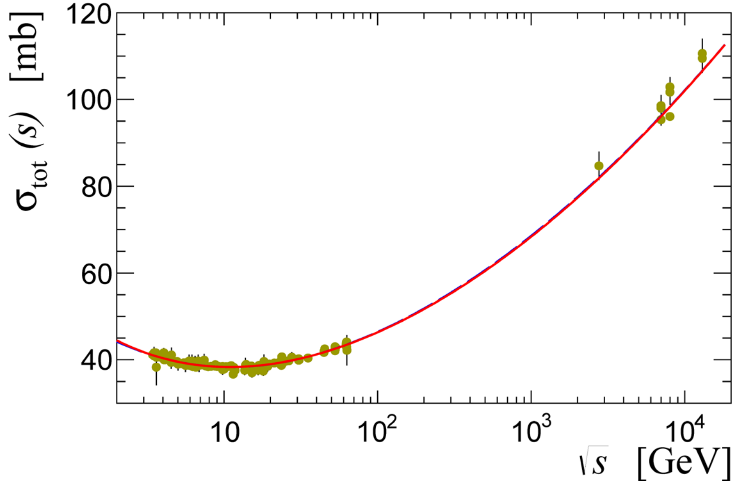
- Thus, if absorptive corrections are not accounted for in the experimental data analysis, the measured real-to-imaginary ratio ρ will be biased:

$$\rho^{\text{meas}} = \rho - \alpha C$$

- This bias is nearly independent of the center-of-mass energy squared s.
- If present, it can be evaluated in a Regge fit analysis of the available proton-proton experimental values of $\rho(s)$ and $\sigma_{\text{tot}}(s)$.

Regge Fit of Proton-Proton Measurements





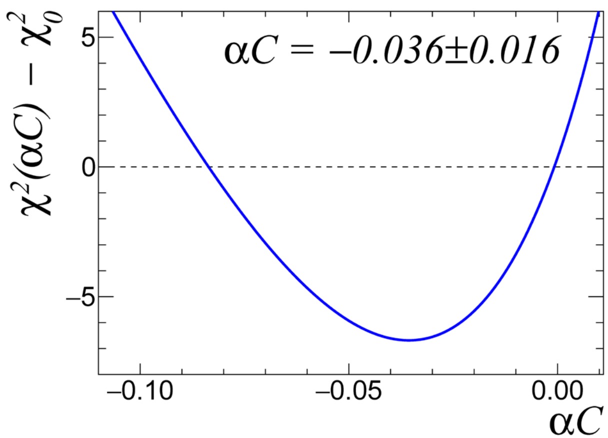
- The accelerator pp data for $p_{Lab} > 5\ \text{GeV}/c$ was taken from the PDG.
- The LHC measurements of ρ were not included in the fit

$$\sigma_{tot}(s)[i + \rho^{meas}(s) + \alpha C] = f_P P(s) + f_+ R^+(s) + f_- R^-(s)$$

$$P(s) \propto \pi f_F \ln\frac{s}{4m_p^2} + i\left(1 + f_F \ln^2\frac{s}{4m_p^2}\right)$$

$$R^{\pm}(s) \propto \left(1 \pm e^{-i\pi\alpha_{R^\pm}}\right)\left(\frac{s}{4m_p^2}\right)^{\alpha_{R^\pm}-1}$$

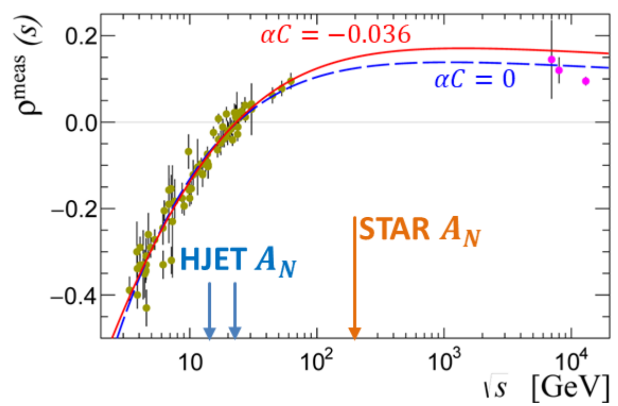
- In the fit, a non-zero value for the absorptive correction, $\alpha C = -0.036 \pm 0.016$, was found. The confidence level of non-zero correction is equivalent to 2.6 standard deviations.
- While this result cannot be considered conclusive proof, it suggests further attention.



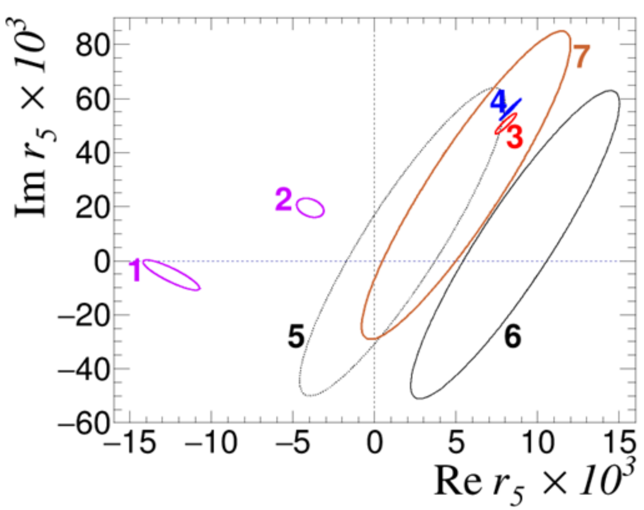
Absorptive Corrections to the Elastic $p^{\uparrow}p$ Analyzing Power $A_N(t)$

$$A_N(t) = \frac{\sqrt{-t}}{m_p} \times \frac{[\kappa_p - 2\,\mathrm{Im}\,r_5]\,t_c/t - 2\,\mathrm{Re}\,r_5}{\left(\frac{t_c}{t}\right)^2 - 2(\rho - \alpha C + \delta_C)\frac{t_c}{t} + 1 + \rho^2}$$

The hadronic spin-flip amplitude,
$\phi_{sf} = \mathrm{Im}\,\phi_{nf} \times r_5\,\sqrt{-t}/m_p$,
can be extracted from the measured analyzing power $A_N(t)$



- In the $A_N(t)$ measurements, ρ^{fit} from the Regge fit is used instead of $\rho - \alpha C$.
- In the HJET measurements at $\sqrt{s} = 13.76$ and 21.92 GeV, $\rho^{\mathrm{fit}} \approx \rho - \alpha C$, so the absorptive corrections do not alter the measured r_5.
- However, for STAR at $\sqrt{s} = 200$ GeV, applying absorptive correction $\alpha C = -0.036$ results in $\Delta\rho^{\mathrm{fit}} = 0.034$, leading to a biased measurement of r_5.



1-σ contours (stat+syst)

1. HJET, $\sqrt{s} = 13.76$ GeV
2. HJET, $\sqrt{s} = 21.92$ GeV
3. Extrapolation (Froissaron) to 200 GeV
4. Extrapolation (simple pole) to 200 GeV
5. STAR, $\sqrt{s} = 200$ GeV (as published)
6. STAR, + the $B_E - B$ and **spin-flip** absorptive corrections
7. **STAR, + non-flip absorptive correction $\alpha C = -0.036$**

For the combine fit (Regge+A_N), confidence of the bias in value of ρ is equivalent to 3.2 standard deviations.

Modified Calculation of the Absorptive Correction

To eliminate the dependence on photon mass in calculating the absorptive correction, a modified approach (though not yet proven or justified) can be considered:

If the electromagnetic phase is set to zero, $\Phi_C^\lambda(t) \equiv 0$, then the absorption can be incorporated by introducing an absorptive factor $1 - \gamma_N(b)$ in the impact parameter space.

In this scenario: $\Phi_{\rm NC}^\lambda \to -\delta_C(t)$, $\alpha C \to -\delta_C(t)$, and

$$\frac{d\sigma}{dt} = \frac{\sigma_{\rm tot}^2}{16\pi}\left[\left(\frac{t_c}{t}\right)^2 - 2(\rho + 2\delta_C)\frac{t_c}{t} + 1 + \rho^2\right]e^{Bt}$$

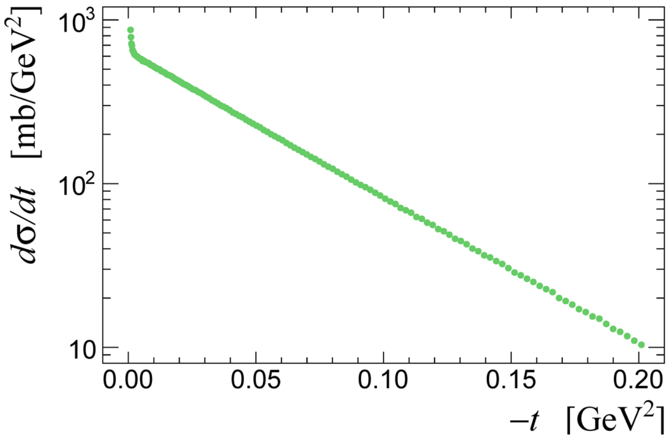
The average value of the absorptive correction,

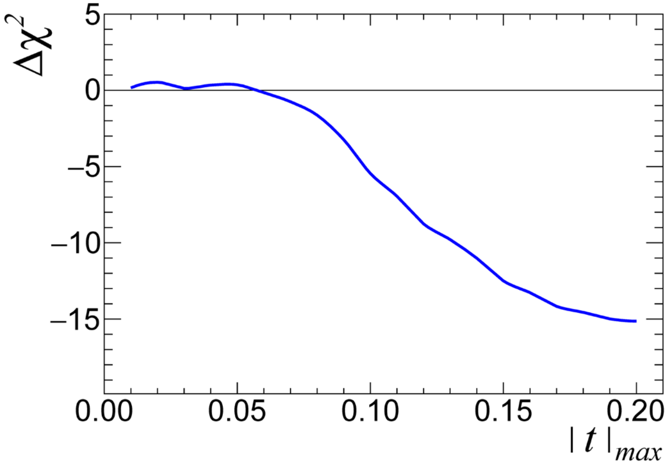
$$\langle -\delta_C(t)\rangle \approx -0.021,$$

agrees with the $\alpha C = -0.036 \pm 0.016$.

To estimate potential effect of the logarithmic term $\alpha \ln(t_c/t)$ in the absorptive correction on the measurement of ρ, a fit of the TOTEM measurement of $d\sigma/dt$ was performed. $\Delta\chi^2$ represents the chi-square difference between fits with and without the absorptive correction, depending on the t-range. The hadronic slope B does not depend on t.

TOTEM Collaboration, Eur. Phys. J. C **79**, 785 (2019)





The assumption that the absorptive correction can be approximated by the Coulomb phase $-\delta_C(t)$ is consistent with the experimental measurements considered.

Hadronic Slope Parametrization $B(t)$

- Significant improvement of $\Delta\chi^2$ for large $|t|_{\max}$ suggests considering the following parametrization of the slope

$$B(t) = \beta_0 \times [1 + \beta' \ln(t_c/t)]$$

- For the TOTEM measurement, this parametrization could exhaustively improve consistency with the measured $d\sigma/dt$ beyond the CNI region, as well as increase the value of ρ to align with the predictions from the Regge fit (before applying absorptive corrections).
- However, when absorptive correction are applied, the discrepancy between the two values of ρ comes back.
- The numerical value of $\beta' \approx 2.8\alpha$ suggests it might be due to a possible radiative corrections to the slope B.
- If this is the case, theoretically calculating the $\ln(t_c/t)$ corrections to the hadronic slope $B(t)$ could be critically important for experimental study of forward elastic proton-proton scattering.

Absorptive corrections were not considered in the results presented in the Table.

	$\|t\|_{\max} = 0.07\ \mathrm{GeV}^2$			$\|t\|_{\max} = 0.15\ \mathrm{GeV}^2$		
$B(t)$	χ^2/ndf	ρ	σ_{tot} [mb]	χ^2/ndf	ρ	σ_{tot} [mb]
TOTEM Collaboration, Eur. Phys. J. C 79, 785 (2019)						
β_0	0.9	0.09 ± 0.01	111.8 ± 3.1	2.1	–	–
$\beta_0+\beta_1 t$	0.9	0.10 ± 0.01	111.9 ± 3.1	1.0	0.09 ± 0.01	111.9 ± 3.1
$\beta_0+\beta_1 t+\beta_2 t^2$	0.9	0.09 ± 0.01	111.9 ± 3.0	0.9	0.10 ± 0.01	112.1 ± 3.1
This work						
β_0	67.2/76	$\mathbf{0.090 \pm 0.007}$	111.0 ± 2.4	**236.9/115**	0.070 ± 0.007	107.8 ± 2.3
$\beta_0\left(1 + 0.021 \ln\frac{t_c}{t}\right)$	67.6/76	$\mathbf{0.124 \pm 0.007}$	107.6 ± 2.2	**105.9/115**	0.119 ± 0.006	108.3 ± 2.2

Summary

- It has been demonstrated that alterations in the proton **non-flip** electromagnetic form factor due to absorptive corrections can introduce an effective bias in the measurement of the real-to-imaginary ratio ρ. This bias appears to be nearly independent of the center-of-mass energy s.
- The magnitude of this bias may be significant, comparable to the correction introduced by the Coulomb phase δ_C.
- However, the current methodology for evaluating absorptive corrections may need to be adjusted.
- The **non-flip** absorptive corrections do not alter the hadronic spin flip amplitude parameter r_5 determined at HJET but is for interpreting the $A_\mathrm{N}(t)$ measured at STAR.
- **Irrespective of the assumptions regarding absorptive corrections, it is important to note:**
  - The assumption that all prior experimental determinations of ρ are systematically biased improves the Regge fit for elastic pp measurements of $\rho(s)$ and $\sigma_{tot}(s)$.
  - However, this refit may increase the observed discrepancy in TOTEM between the measured value of ρ and the expected value from the fit.
  - In a combined analysis that includes the Regge fit and a comparison of HJET and STAR values of r_5, a bias in the value of ρ corresponds to a confidence level equivalent to 3.2 standard deviations.
  - Assumption that $B(t) = \beta_0[1 + \beta' \ln(t_c/t)]$ is well consistent with the TOTEM measurements of $d\sigma/dt$ and also eliminates the discrepancy between the measured and expected values of ρ.
  - If confirmed such a parametrization may require revisiting all previous measurements of ρ.

Backup

Elimination of the photon mass dependence

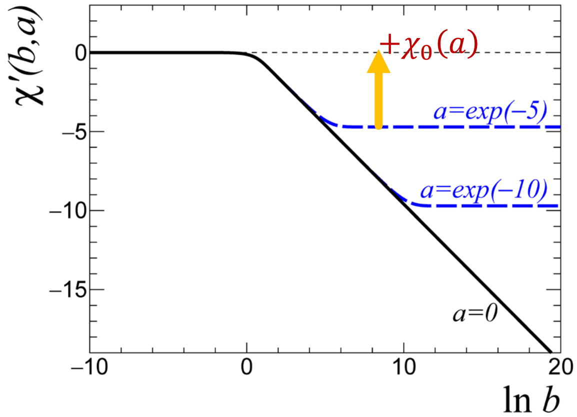
- It can be straightforwardly shown that adding a constant term to eikonal phase $\chi_C(b) \to \chi_C(b) + \chi_0$ results in corresponding alteration of the both Coulomb phases: $\Phi_C^\lambda \to \Phi_C^\lambda + \chi_0$ and $\Phi_{NC}^\lambda \to \Phi_{NC}^\lambda + \chi_0$.
- Introducing of the photon mass λ effectively suppress Coulomb amplitude $\chi_C(b) \to 0$ for large $b > 1/\lambda$.
- Technically, to achieve this adjustment, a λ-dependent constant χ_0^λ is added to $\chi_C(b)$:

$$q = q_T\sqrt{B_E/2}\,, \quad a = \lambda\sqrt{B_E/2}$$

$$\chi(b,a) = \int_0^\infty \frac{dq\, q e^{-q^2}}{q^2+a^2} J_0(bq) = \chi_0(a) + \chi'(b,a)$$

$$\chi_0(a) = \chi(0,a) = \frac{e^{-a^2}E_1(a^2)}{2} \approx -\ln a - \gamma/2$$

$$\chi'(b,a) = \int_0^\infty \frac{dq\, q e^{-q^2}}{q^2+a^2}\left[J_0(bq) - 1\right]$$



- Thus, the dependence of Φ_{NC}^λ on the photon mass can be eliminated in a straightforward manner:

$$\Phi_{\rm NC}^\lambda \to \Phi_{\rm NC} = \Phi_{\rm NC}^\lambda + \alpha e^{-B_E\lambda^2/2} E_1\left(B_E\lambda^2/2\right)$$

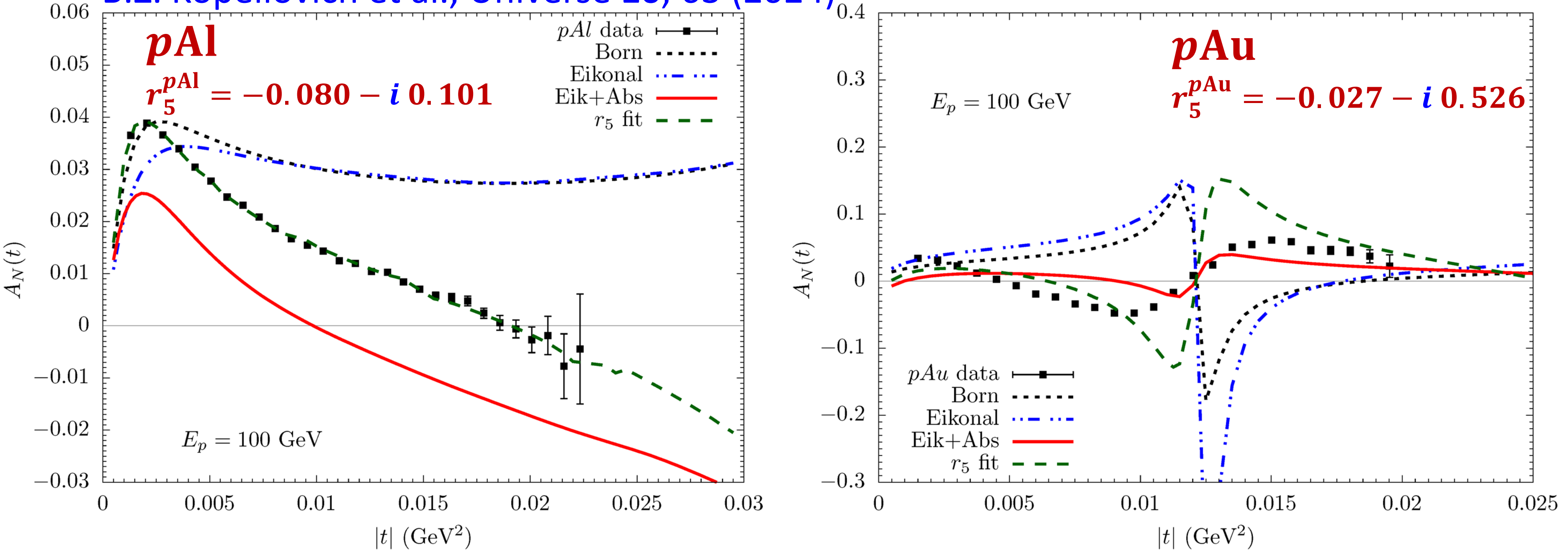
- However, in doing so, $\Phi_{\rm NC}$ is defined up to some photon mass-independent constant C:

$$\Phi_{\rm NC} \approx \alpha\left[C + \ln\frac{B + B_E}{2B_E}\right] \approx \alpha C$$

$$B_E^{\rm eff} = \frac{2\langle r_E^2\rangle}{3} - \frac{\sigma_{\rm tot}}{4\pi} C$$

Absorptive Corrections to the Elastic $p^{\uparrow}\mathrm{Au}$ *Analyzing Power* $A_N(t)$

B.Z. Kopeliovich et al., Universe **10**, 63 (2024)



- Values of r_5^{pAl} and r_5^{pAu} disagrees with $r_5^{pp} = -0.0125 - i\,0.0053$ (HJET, 100 GeV) if

  $r_5^{pA} = \frac{i+\rho^{pA}}{i+\rho^{pp}}\, r_5^{pp}$ [B.Z. Kopeliovich and T.L Trueman, Phys. Rev. D 64, 03404 (2001)]
- Probably, adjustment of the absorptive correction may improve the data fit.