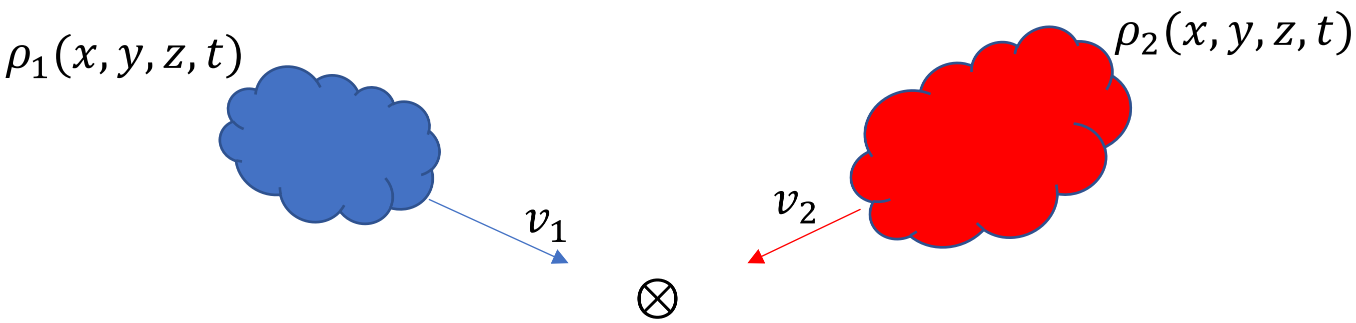
ILUMI4d(*) (Integrated LUMInosity4d based on REACT4d)

Algorithm to compute the single bunch luminosity and space-time structure of luminosity at a collider

- Luminosity and space time structure of luminosity at a collider
- Analytical formula
- ILUMI4d
- Comparison of ILUMI4D results with analytical formula and standard numerical integrator



General problem: moving 3d objects with defined density distribution collide in space
The reactions are created where the two objects volumes overlap during the crossing
-> how many reactions were created during the crossing ?
-> where did the reactions happen (3d coordinate of reaction points and time of reaction)?

(*) ILUMI4d and REACT4d author: Joachim Baechler

Luminosity and space-time structure of luminosity

-> single bunch crossing at a collider

(1) $N_i = \int d^3\, \boldsymbol{x} \rho_i(\boldsymbol{x}, t)$, $i = 1{,}2$ -> normalisation

(2) $\rho_i = \rho_i(\boldsymbol{x}, t)$, $i = 1{,}2$ -> density of particles in bunch

that move with velocities v_i in the Lab frame.

Furman, M.A.
The Moller luminosity factor
LBNL-53553,CBPNote-543

$$L_{sc} = \int dt\, d^3\boldsymbol{x} \rho_1 \rho_2 \sqrt{(\boldsymbol{v_1} - \boldsymbol{v_2})^2 - \frac{(\boldsymbol{v_1} \times \boldsymbol{v_2})^2}{c^2}} \quad \left[\frac{1}{area}\right] \quad ; \quad K = \sqrt{(\boldsymbol{v_1} - \boldsymbol{v_2})^2 - \frac{(\boldsymbol{v_1} \times \boldsymbol{v_2})^2}{c^2}}$$

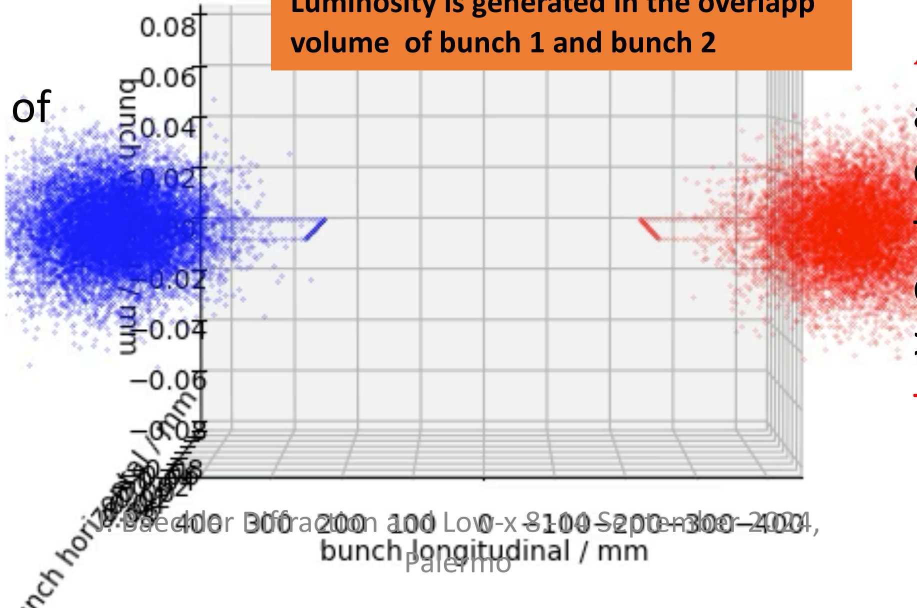
$$\mu_{sc} = L_{sc} \cdot \sigma_{proc} \qquad \mu_{sc} = \textit{average pileup generated during the single bunch crossing}$$

analytical formula and standard numerical integration algorithm allow to compute L_{sc} , μ_{sc} and $L_{\Delta t_i}(x, y, z, \Delta t_i)$

ILUMI4d allows to compute for gaussian and non gaussian L_{sc} and μ_{sc} , $L_{\Delta t_i}(x, y, z, \Delta t_i), V^{vertex}_{\Delta t_i}(x, y, z, \Delta t_i)$

L_{sc} and μ are essential for the operation of experiments and the accelerator

Luminosity is generated in the overlapp volume of bunch 1 and bunch 2



$L_{\Delta t_i}(x, y, z, \Delta t_i)$ *and* $V^{vertex}_{\Delta t_i}(x, y, z, \Delta t_i)$ are key for:

optimal machine settings

trigger

data analyis

>lumi is expressed as volumetric object for each time step Δt_i (time frame)

Analytical formula to compute the luminosity (**):

(gaussian bunch distribution)

(5) $L = K \cdot f \cdot N_b \cdot N_1 \cdot N_2 \cdot \iiint \int_{-\infty}^{+\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0)\ \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0)dx\,dy\,ds\,ds_0$

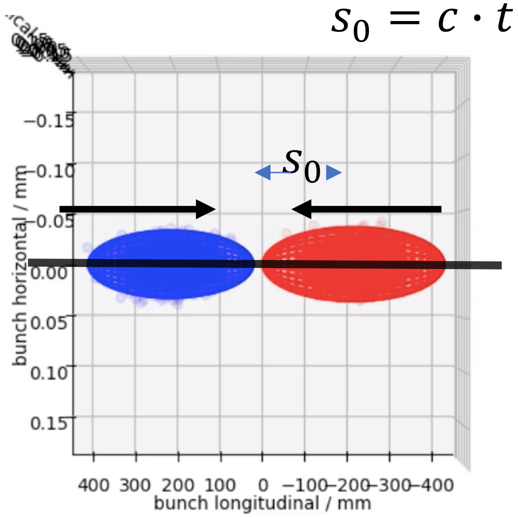
(6) $\rho_{iz}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{(z)^2}{2\sigma_z^2}\right)$ where i=1,2, $z = x, y$

(7) $\rho_s(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s\pm s_0)^2}{2\sigma_s^2}\right)$

(**) W. Herr, 'concept of luminosity' https://cds.cern.ch/record/941318/files/p361.pdf

Luminosity is generated within the overlap volume

$s_0 = c \cdot t$



With $\sigma_{1x} = \sigma_{2x}$, $\sigma_{1y} = \sigma_{2y}$, $\sigma_{1z} = \sigma_{2z}$ and head on colission $\theta_{c1} = \theta_{c1} = 0$

(8) $L = \frac{2\,N_1 N_2 \cdot f \cdot n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \iiint \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx\,dy\,ds\,ds_0$

(9) $L = \frac{N_1\,N_2\,f\,N_b}{4\pi\sigma_x\sigma_y}$ $\sigma_{x,y} = transverse\ beam\ size\ at\ ip$

(10) $L_{sc} = \frac{N_1\,N_2}{4\pi\sigma_x\sigma_y}$

Single bunch luminosity

(11) $\sigma_{xy}^{trans} = \frac{\sigma_x}{\sqrt{2}}$

(12) $\sigma_z^{long} = \frac{\sigma_z}{\sqrt{2}}$

Beam spot size

output of analytical formula

integral over bunch crossing time ->

No information on space-time structure of luminosity

Two selected publications with reference to LUMI at a collider

- **Concept of luminosity** *Werner Herr and Bruno Muratori*

CERN, Geneva, Switzerland
now at Daresbury Laboratory, United Kingdom
https://cds.cern.ch/record/941318/files/p361.pdf. (2006)

….To evaluate this integral one should know all distributions. An analytical calculation is not al- ways possible and a numerical integration may be required. However in many cases the beams follow ”reasonable” profiles and we can obtain closed solutions.

….7.2 Luminous region and space structure of luminosity

In addition to the number of events, the space structure is important for the design and running of a particle physics experiment. The questions we asked are therefore ….

- **Luminosity of a Collider with Asymmetric Beams**

I. N. Meshkov *a*, *b*, *
a Joint Institute for Nuclear Research, Dubna, Russia b St. Petersburg University, St. Petersburg, Russia

ISSN 1547-4771, Physics of Particles and Nuclei Letters, 2018, Vol. 15, No. 5, pp. 506–509. © Pleiades Publishing, Ltd., 2018.

The problem of calculating the luminosity of a collider is known since the appearance of the first accelerators with colliding beams.

Nevertheless, a fairly compact formula describing the luminosity of the collider in the general case of collision of two beams with arbitrary parameters, required to perform analytical or numerical calculations, has not yet been proposed. Such a formula is helpful in selection of collider parameters …..

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ILUMI4D (*)

- algorithm to compute the luminosity and the space-time structure of luminosity at a collider with bunched beams and coasting beams for arbitrary beam parameter and bunch shapes (gaussian and non gaussian)

➢ is based on vectorised point clouds (stochastically distributed), moving independently in relation to each other on individual trajectories

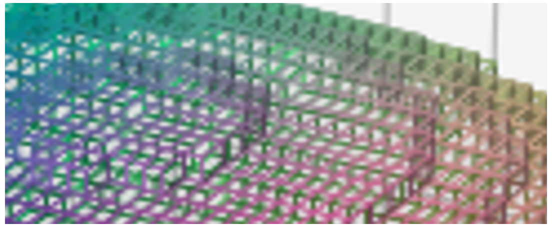
- allows to compute the luminosity for any distribution of particles in the overlap volume of the colliding bunches

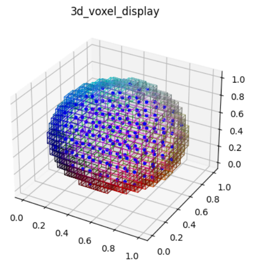
➢ in this presentation a 3d gaussian distribution for the point cloud (10k elements) is used as example. This allows to compare the results (luminosity, beam spot size) of ilumi4d with the analytical formula and with a standard numerical integrator

- the bunch collision process is split up in space-time frames of equal distance in space

➢ for every space time frame the bunch overlap volume is voxelized (experiment coordinate system) and the luminosity and the reaction probability based on the cross section for each voxel: geometric voxel -> lumi voxel -> reaction voxel

A voxel is a three-dimensional counterpart to a pixel. It represents a value on a regular grid in a three-dimensional space. Voxels are frequently used in the visualization and analysis of medical and scientific data. [Wikipedia]



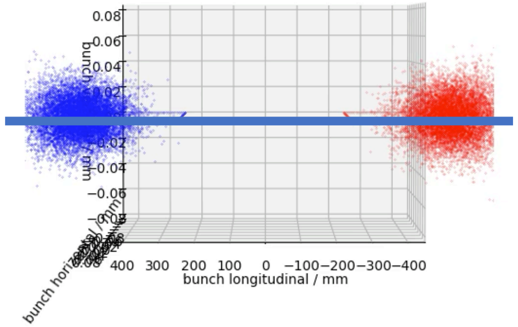


reflects the 2-dimensional target charge density
→ voxel acts as local proton-flow meter

REMARK(*):
ILUMI4d should be seen as mean to compute the luminosity and space time structure of luminosity with precision of 1% relative to analytical formula and standard numerical intergrator

4 main steps of ILUMI4d algorithm

gaussian bunch distribution



- bunch1 and bunch2 trajectories with

  -> n equidistant space points at distance $\Delta t * c$ (time frames)
- point cloud generated according to 3d-gaussian distribution at every spacepoint with (k elements representing each of the two bunches with N_1, N_2 protons)
- For each space point the overlap region of bunch 1 and bunch 2 is voxelized with cubic voxels and the **luminosity for each voxel** (lumi voxel) is computed for the real bunch population of N_1, N_2 protons
- The vertex distribution is generated for every time frame according to the lumi voxel distribution and the process cross section

General formula for single bunch luminosity in ILUMI4d

-> general formula for single bunch luminosity computation

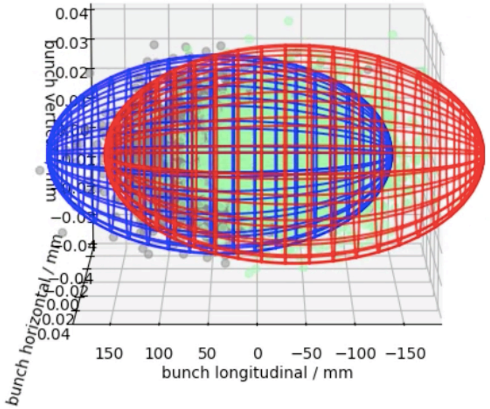
- $\boldsymbol{L_{sc}} = \int dt\, d^3\boldsymbol{x}\rho_1\rho_2\sqrt{(\,\boldsymbol{v_1} - \ \boldsymbol{v_2})^2 \ - \frac{(\boldsymbol{v_1}\times\boldsymbol{v_2})^2}{\boldsymbol{c^2}}}$

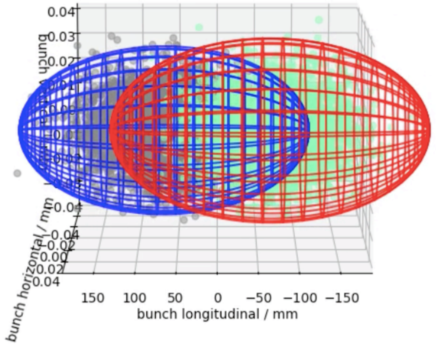
-> ILUMI4D

$$L_{sc}^{4d} = \sum_{i=1}^{nfr}(\sum_{l,m,n}^{nvox} D^{-1}\cdot\left(V_{l,m,n,i}^{exp} \circ \ \rho_{l,m,n,i}^{1\,exp} \circ \rho_{l,m,n,i}^{2\,exp}\right)\cdot\sqrt{(\,\vec{v}_1 - \vec{v}_2\ )^2 \ - \frac{(\vec{v}_1\times\vec{v}_2)^2}{c^2}} \ \cdot\Delta t\,\Big)$$

$\boldsymbol{l, m, n}$: voxel index
i : time frame index
nfr=number of time frames
$\mathbf{nfr}\cdot\triangle\,\boldsymbol{t} \geqq$ bunch crossing time

- $\boldsymbol{\rho_{l,m,n,i}^{1\,exp}}$ = density bunch 1 particle
- $\boldsymbol{\rho_{l,m,n,i}^{2\,exp}}$ = density bunch 2 particle
- $\boldsymbol{V_{l,m,n,i}^{exp}}$ = voxel volume
- $\boldsymbol{D^{-1}}$ = scaling factor - normalisation
- $\boldsymbol{\Delta t}$ = $\boldsymbol{time\ interval\ of\ time\ frame}$





Goal:

Compute probability that an interaction vertex is generated in a voxel for each time frame

-> the lumi_voxels stores the luminosity

-> multiplication of lumi_voxels with σ_{prod} -> vertex_voxel

L_{sc}^{4d} = single bunch crossing luminosity

$$L_{sc}^{4d} = \sum_{i=1}^{nfr} \left(\sum_{l,m,n} D^{-1} \cdot \left(V_{l,m,n,i}^{exp} \circ \rho_{l,m,n,i}^{1\,exp} \circ \rho_{l,m,n,i}^{2\,exp} \right) \cdot \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{(\vec{v}_1 \times \vec{v}_2)^2}{c^2}} \cdot \Delta t \right)$$

$LUMI_{l,m,n,i}^{exp}$ = voxel luminosity in a single time frame

$$LUMI_{l,m,n,i}^{exp} = D^{-1} \cdot \left(V_{l,m,n,i}^{exp} \circ \rho_{l,m,n,i}^{1\,exp} \circ \rho_{l,m,n,i}^{2\,exp} \right) \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \frac{(\vec{v}_1 \times \vec{v}_2)^2}{c^2}} \cdot \Delta t.$$

μ_i = number of vertices produced within a single time frame

$$\mu_i = \sum_{l,m,n} LUMI_{l,m,n,i}^{exp} \cdot \sigma_{prod}$$

- $vertex_{l,m,n,i}^{exp}$ = probabiity of vertex generation in a lumi voxel

$$vertex_{l,m,n,i}^{exp} = LUMI_{l,m,n,i}^{exp} \cdot \sigma_{prod}$$

The ILUMI4d algorithm is based on statistical distributions which are convoluted

-> results will be compared to analytical formula and general numerical integrator

-> only relative precision can be quoted

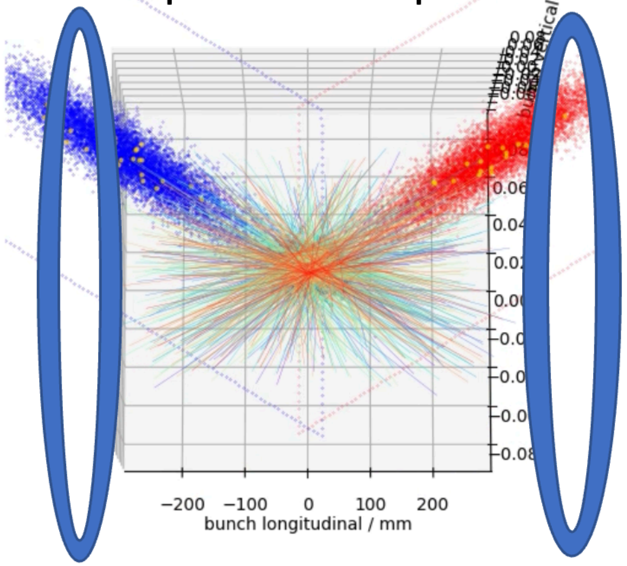
Change of luminiosity by lumi levelling (videos produced with ILUMI4d)

->the computation of luminosity with the hour glass effect needs, requires numerical integration !!

- crossing angle

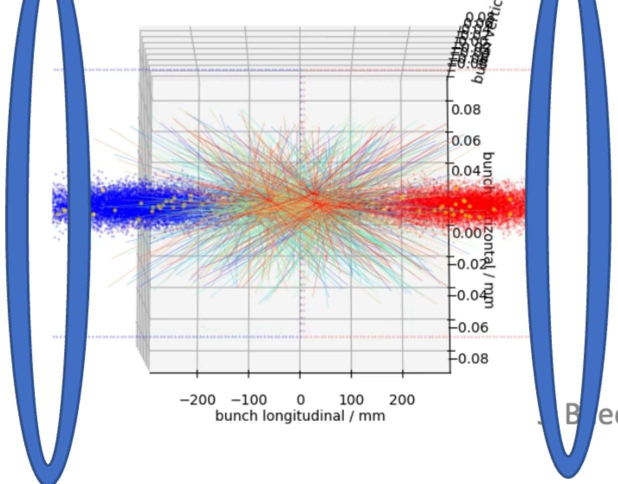
Top view on experiment

Color code of tracks corresponds
to creation time:
blue: start of bunch crossing
red: end of bunch crossing
color increment about 30 ps

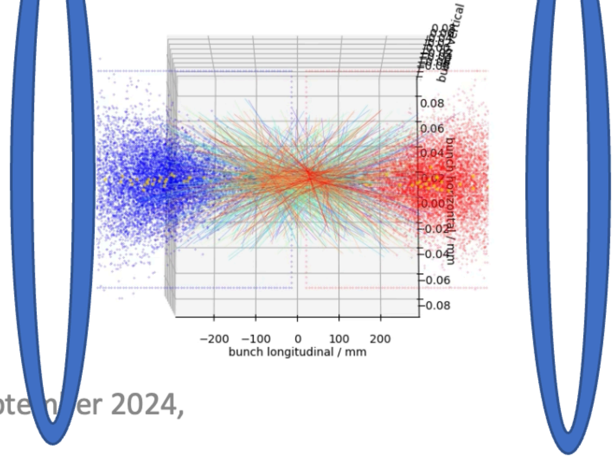


- β^*- levelling $\beta(z) = \beta^*(1 + (\frac{z}{\beta^*})^2)$ $\quad$ *example with no crossing angle*

Hourglass effect off
can be computed with analytical formula

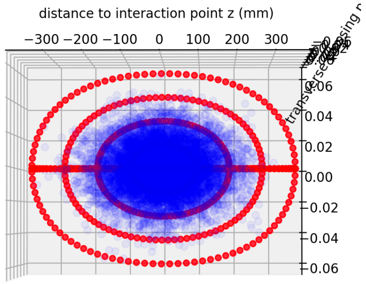


hourglass effect on
-> needs numerical integration

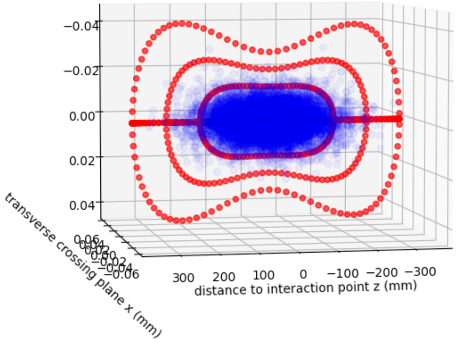


Change of bunch size due to hour glass effect
β-functions have their minimum in the interaction point

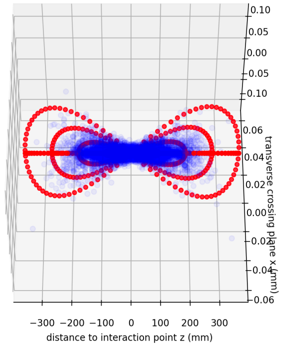
- $\beta(z) = \beta^*(1 + (\frac{z}{\beta^*})^2)$
- $\sigma = \sqrt{\beta(z) \cdot \varepsilon}$
- $\sigma = \sqrt{\beta^*(1 + (\frac{z}{\beta^*})^2) \cdot \varepsilon}$



$\beta^* = 0.6m$



$\beta^* = 0.15m$



$\beta^* = 0.015m$

Benchmarking of ILUMI4d with analytical formula and numerical integrator with standard settings

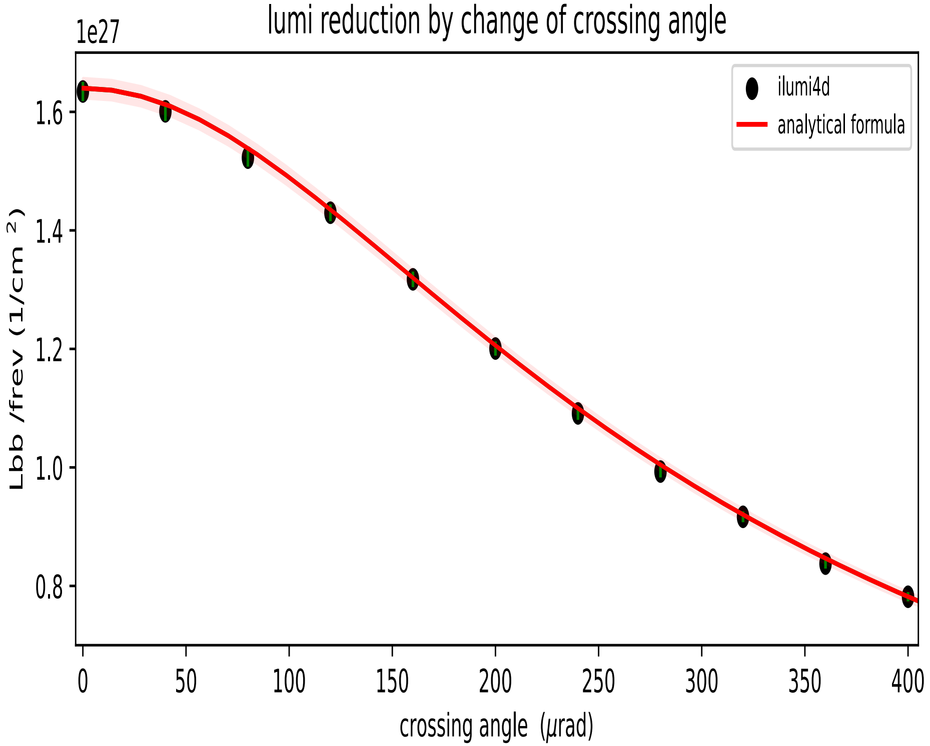
Analytical formula: (https://lpc.web.cern.ch/lumiCalc.html) - standard settings when opening the link

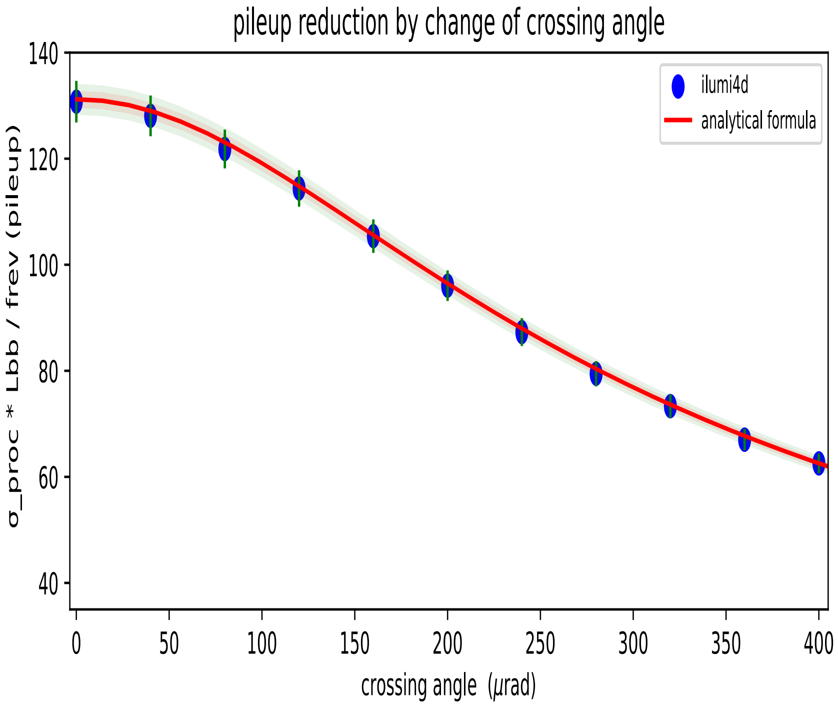
Numerical integrator: https://gitlab.cern.ch/xbuffat/PyLumi

Comparison of ILUMI4d with analytical formula
single bunch lumi and beam spot size (no hourglass effect)

Comparison of ILUMI4d with numerical integrator
single bunch lumi (hourglass effect)

Lumi and pileup reduction by change of crossing angle ---> no hourglass effect

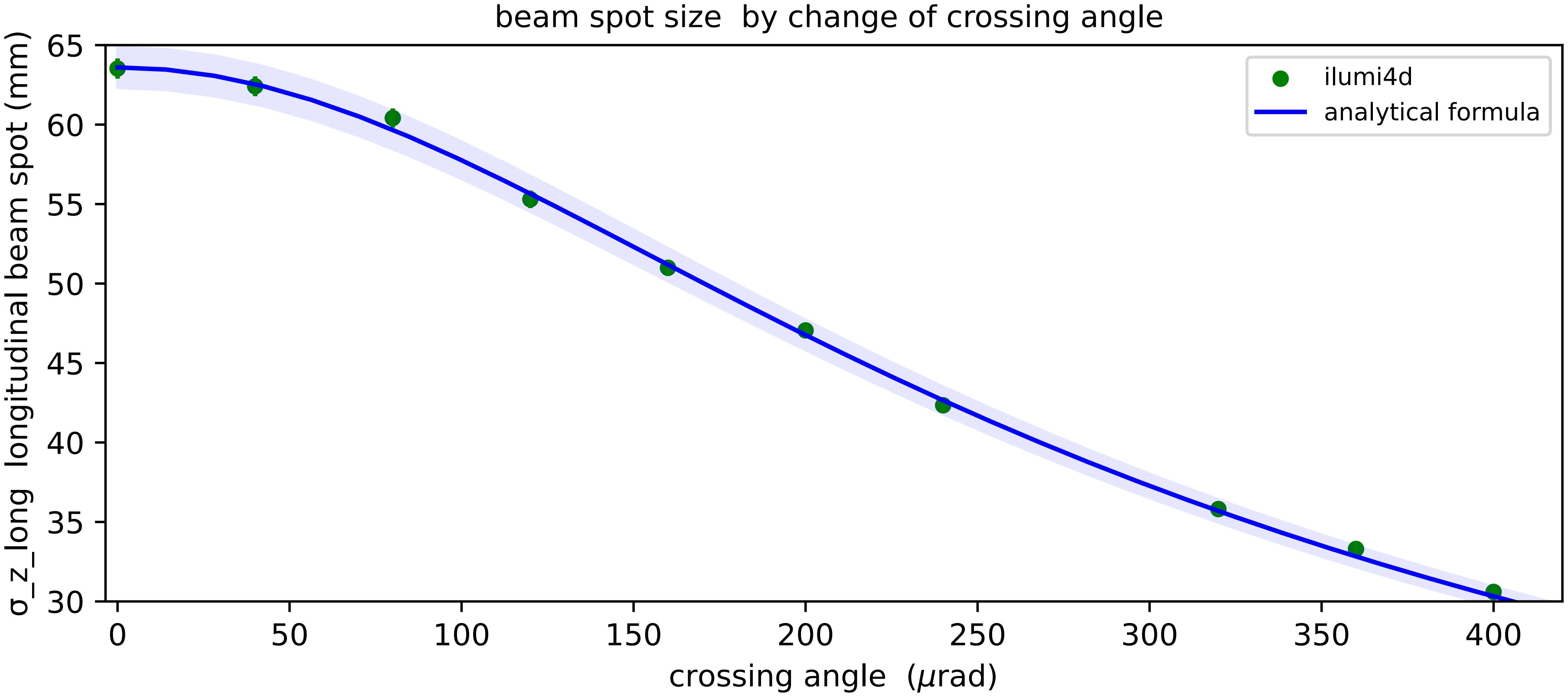




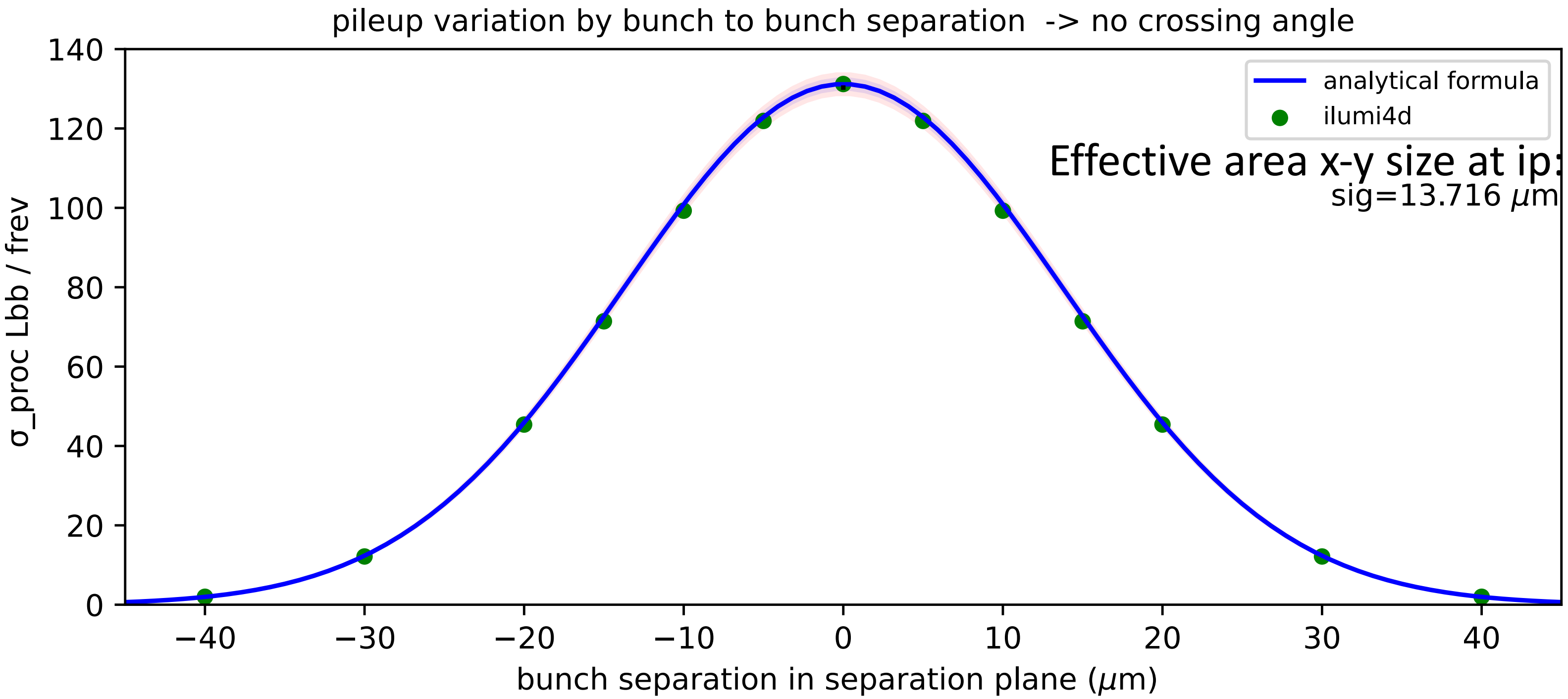
$$L_{sb} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} \cdot \frac{1}{\sqrt{1+(\frac{\sigma_z}{\sigma_y}\ \frac{\phi}{2})^2}}$$

formula: https://cds.cern.ch/record/691967/files/project-note-301.pdf

Beam spot size longitudinal as function of crossing angle
-> no hourglass effect



Pileup variation by bunch separation
(no crossing angle)



Effective area x-y size at ip with analytical formula: 13.8 μm

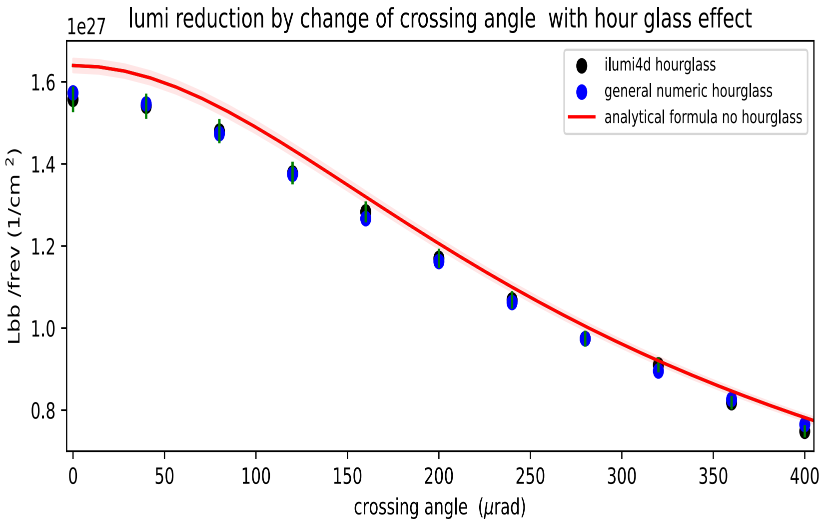
$$\mathcal{L} = \frac{f_{\mathrm{rev}} N_1 N_2}{2\pi \Sigma_x \Sigma_y} \mathcal{S}$$

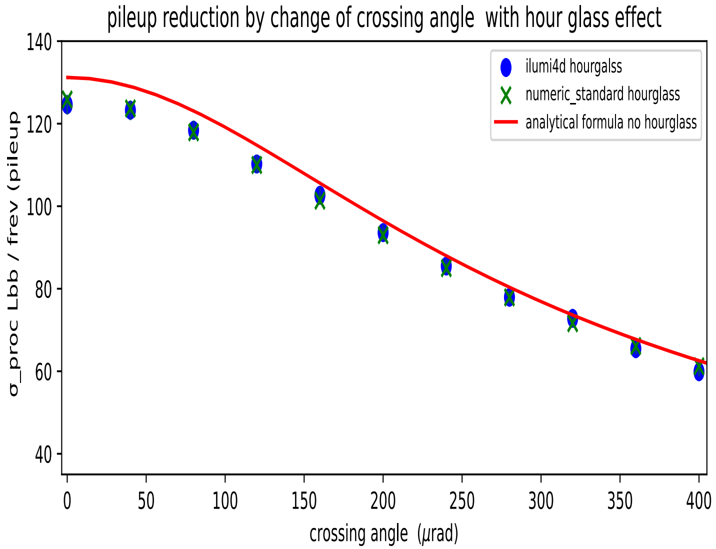
$$\mathcal{S} = \exp\left(\frac{-d^2}{2\Sigma_d^2}\right)$$

https://journals.aps.org/prab/pdf/10.1103/PhysRevAccelBeams.21.102801

Lumi and pileup reduction crossing angle

hour glass effect on - comparison to numerical integrator





analytical formula (needs numerical integration):
https://cds.cern.ch/record/691967/files/project-note-301.pdf

$$\sigma_z = \sigma_z^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2}$$

z = x,y

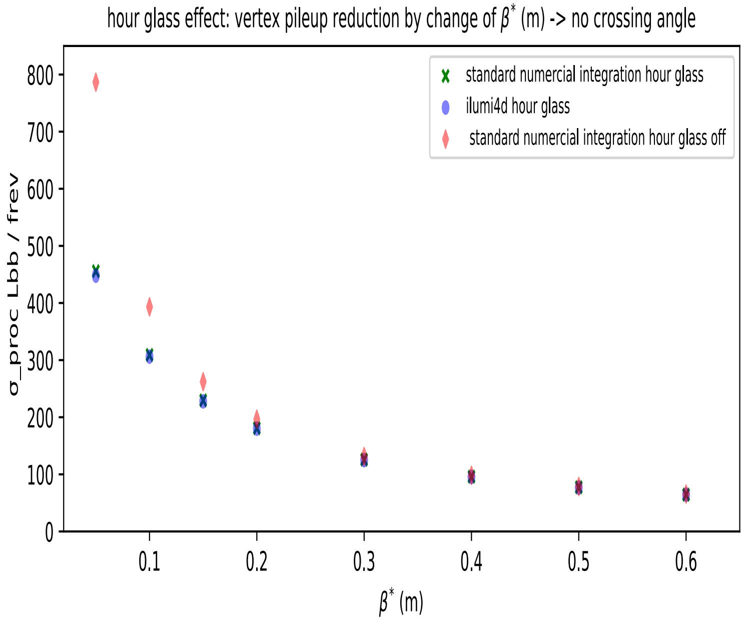
where

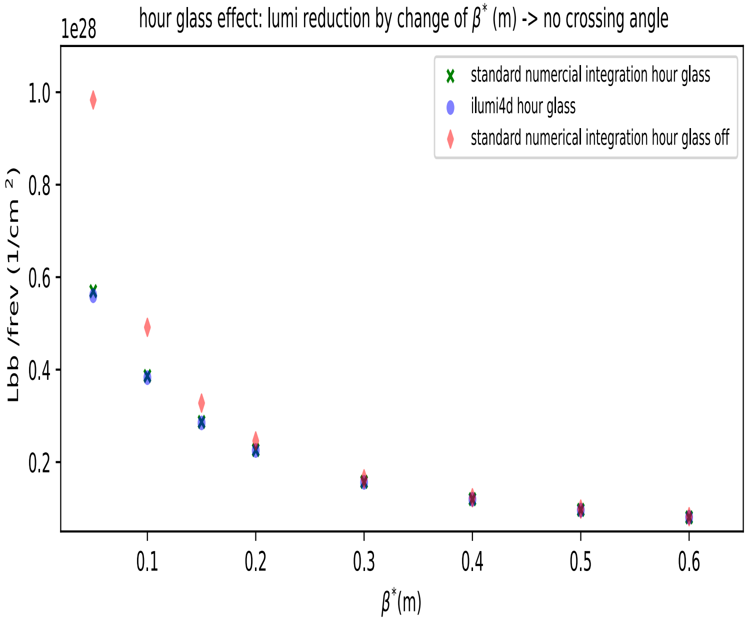
$$\mathcal{L}_{HG} = \left(\frac{N_1 N_2 f B}{4\pi\sigma_x^*\sigma_y^*}\right) \frac{\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} \frac{e^{-s^2 A}}{1+(\frac{s}{\beta^*})^2} ds,$$

$$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x)^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} = \frac{\sigma_s^2 \sin^2\frac{\phi}{2} + (\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\cos^2\frac{\phi}{2}}{(\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\sigma_s^2}.$$

Lumi and pile up variation with change of β^*

hour glass effect on , no crossing angle





Space time structure and lumi region
results of single bunch collision simulation with ILUMI4d.
-> no benchmarking possible

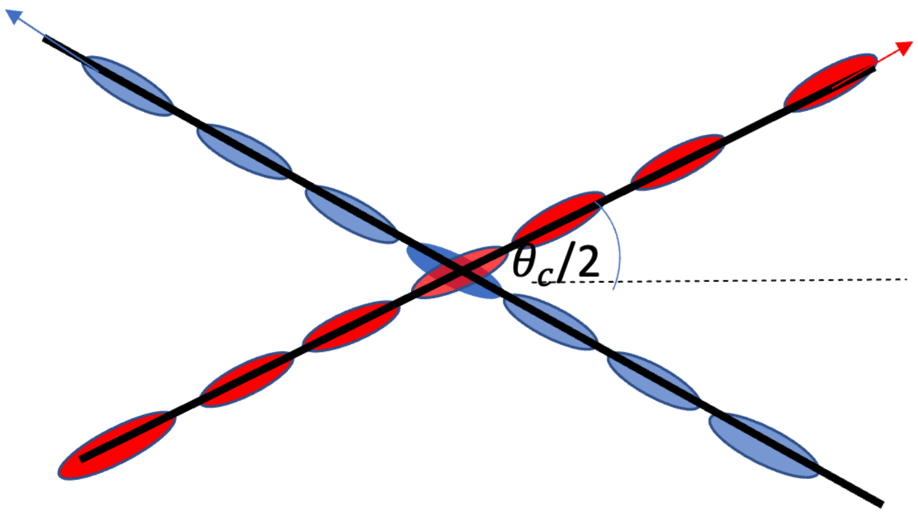
- vertex footprints in the collision region

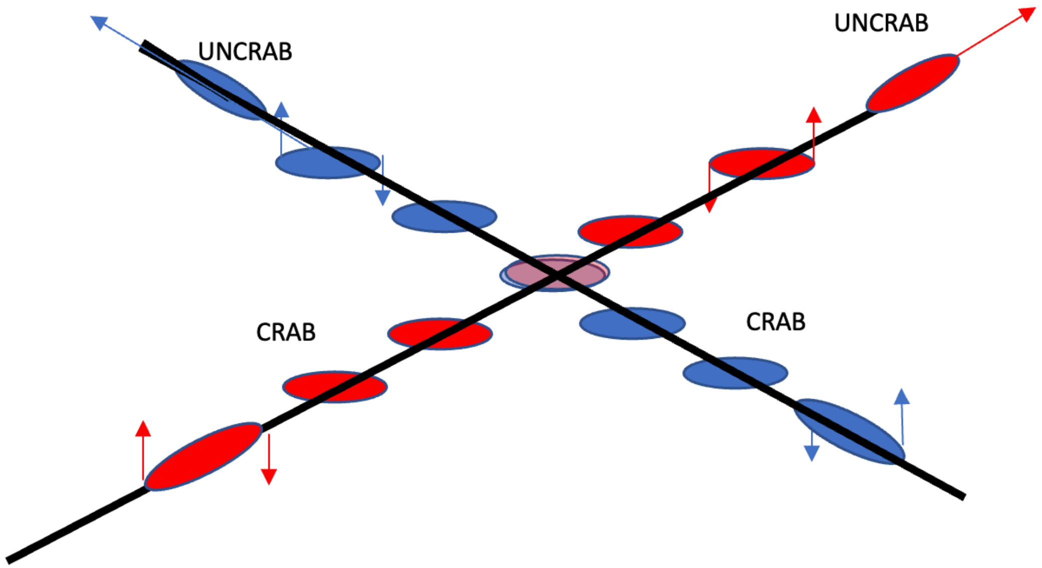
  standard crossing angle
  crab angle

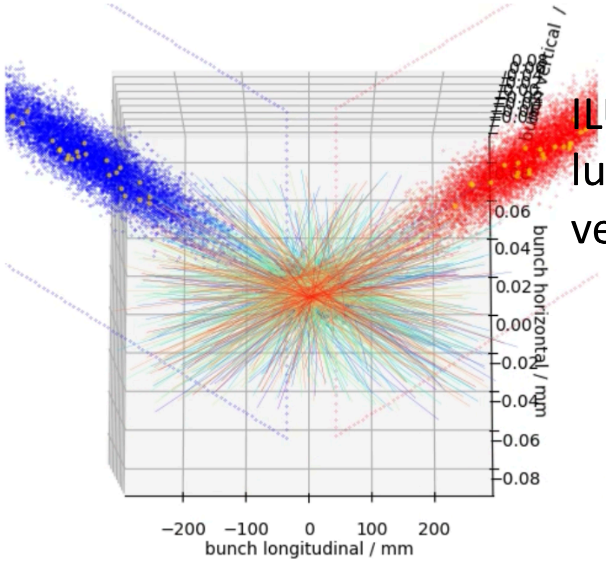
- forward physics proton time distribution

  time structure of protons in Roman Pot detector

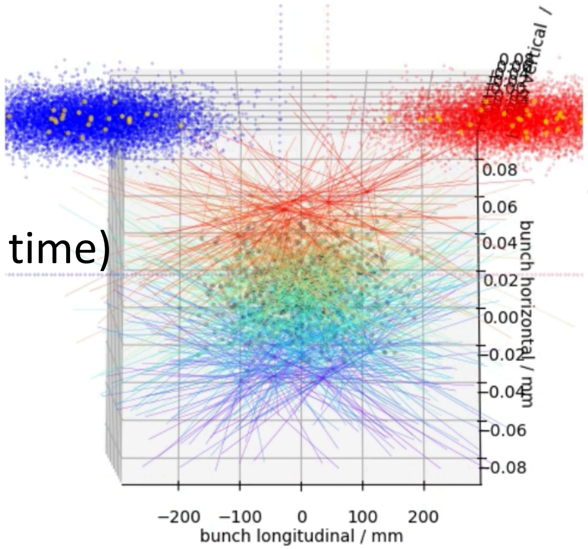
Crab cavity effect on transverse lumi distribution at interaction region





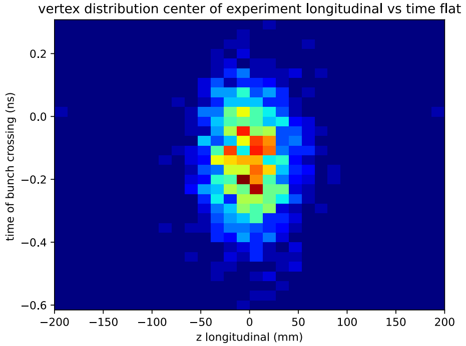


ILUMI4d
lumi region computation
vertex footprint (space and time)

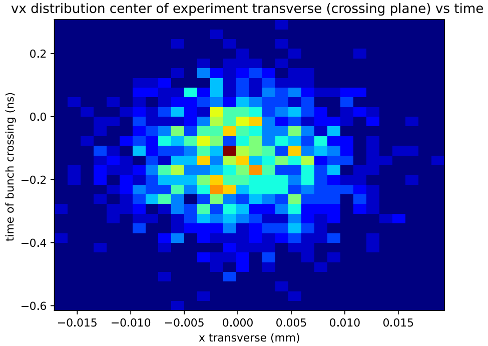


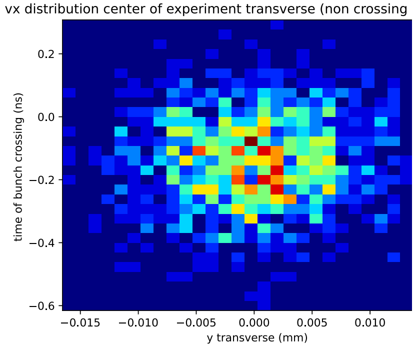
Color code of tracks corresponds to the creation time
blue: start of bunch crossing
red: end of bunch crossing

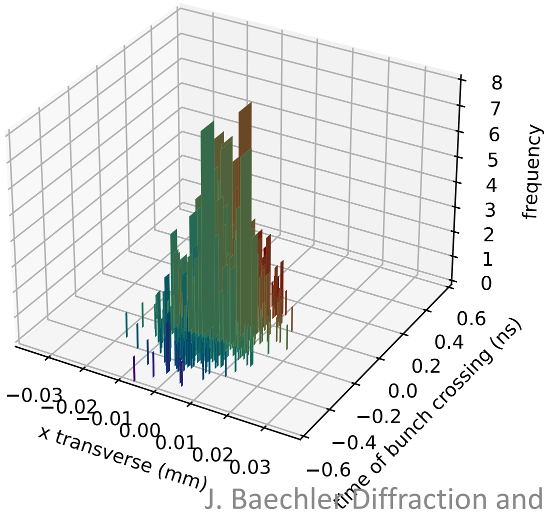
Crossing angle no crab

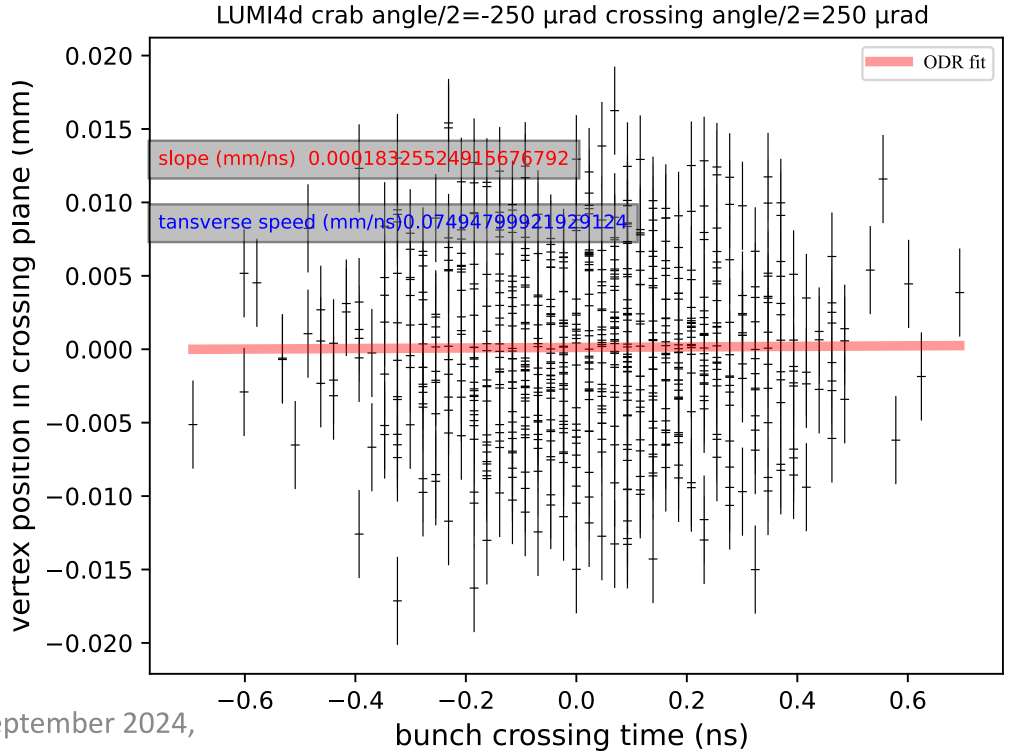


time

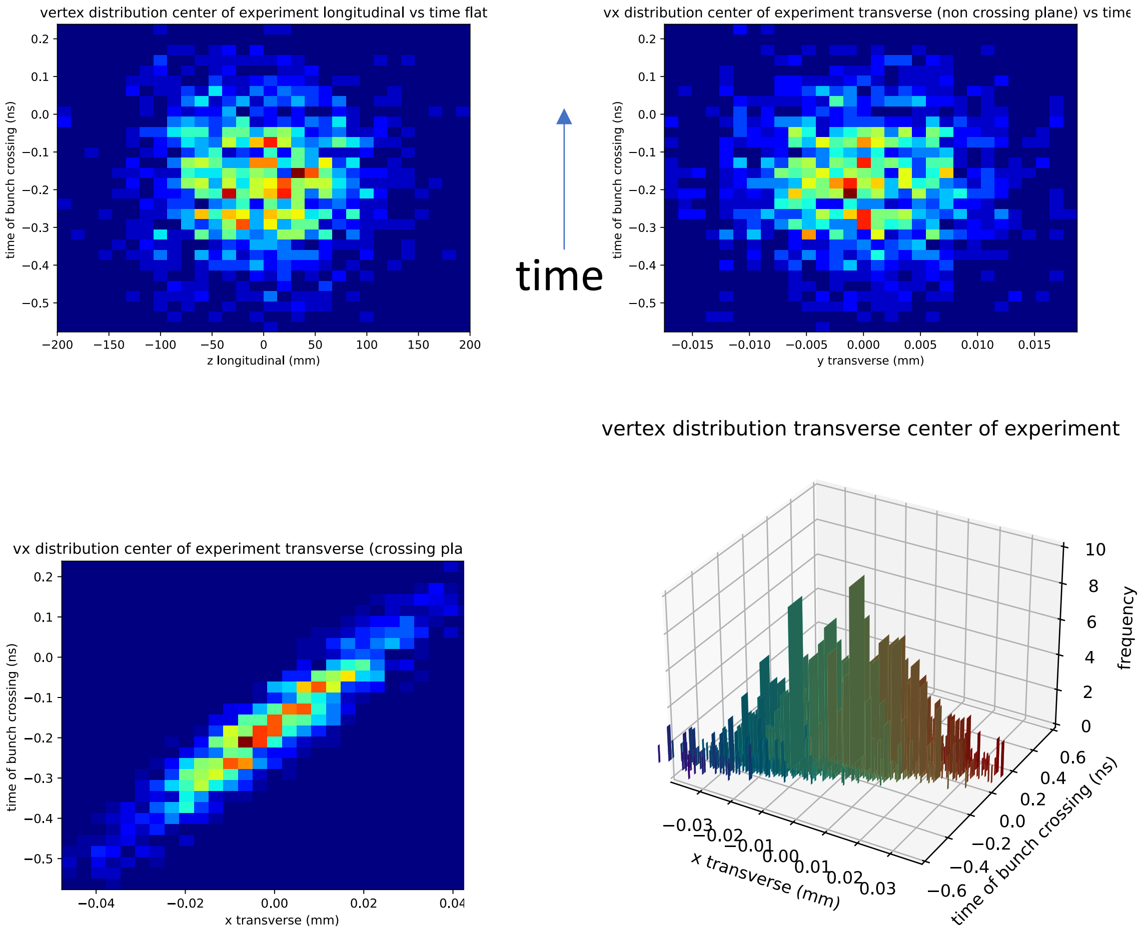






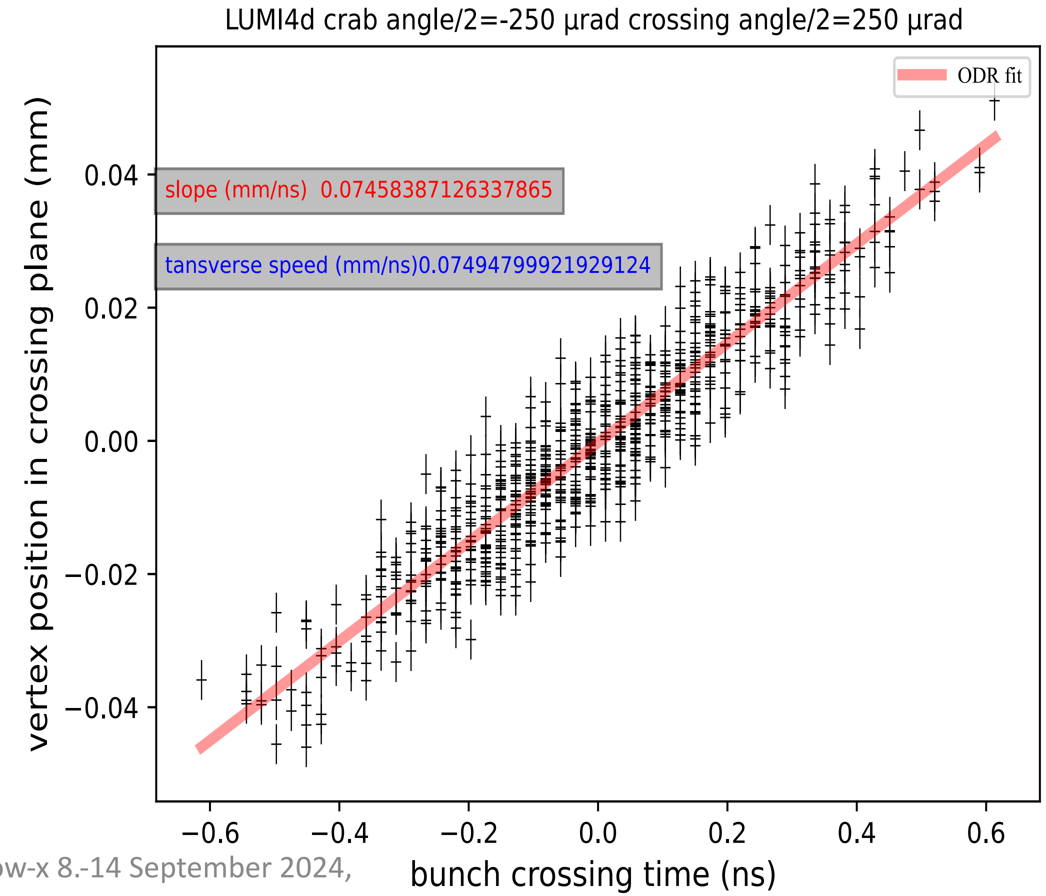


Impact of crab cavity on transverse vertex distribution



vertex distribution at interaction point

transverse vertex positions as function of
bunch crossing time -> crab space time clock

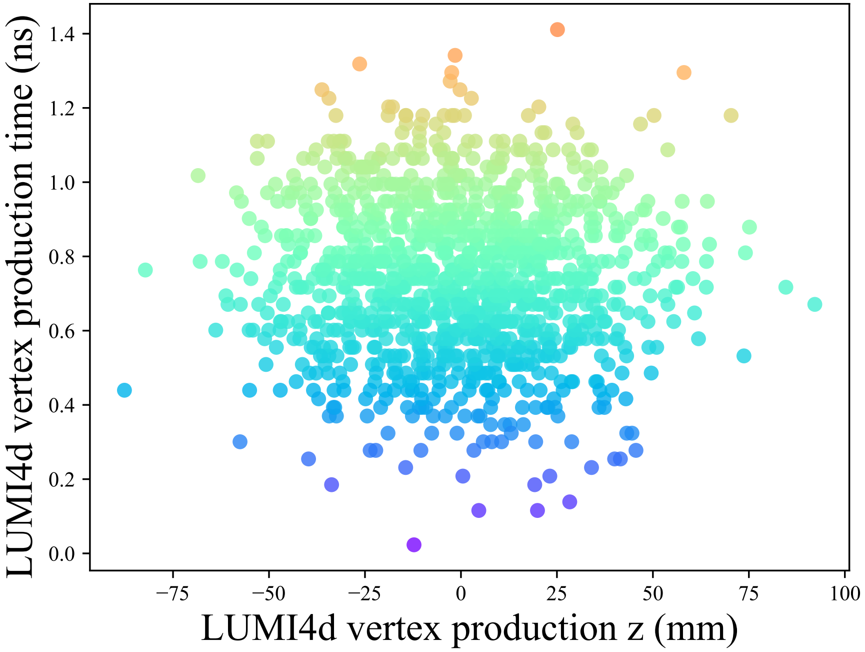


ILUMI4D: crab cavity

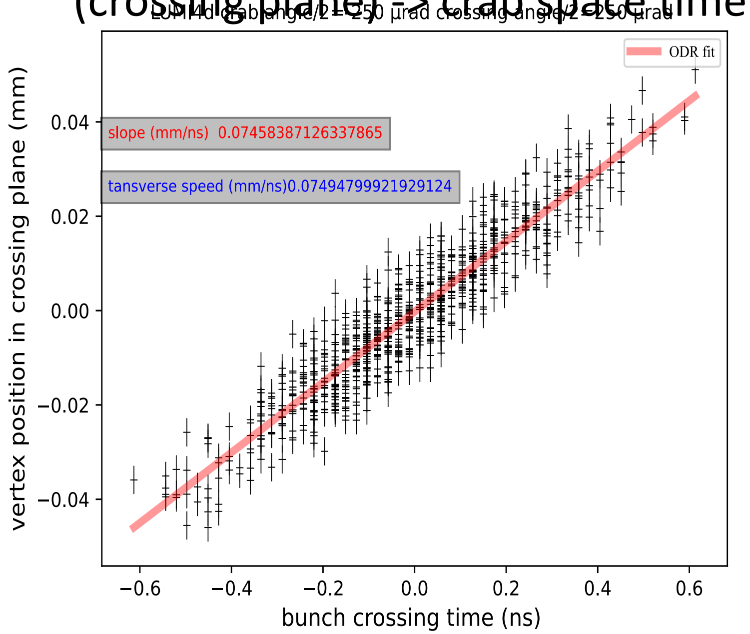
Color code of vertex points corresponds
to creation time blue: start of bunch crossing
red: end of bunch crossing

Longitudinal vertex distribution

time



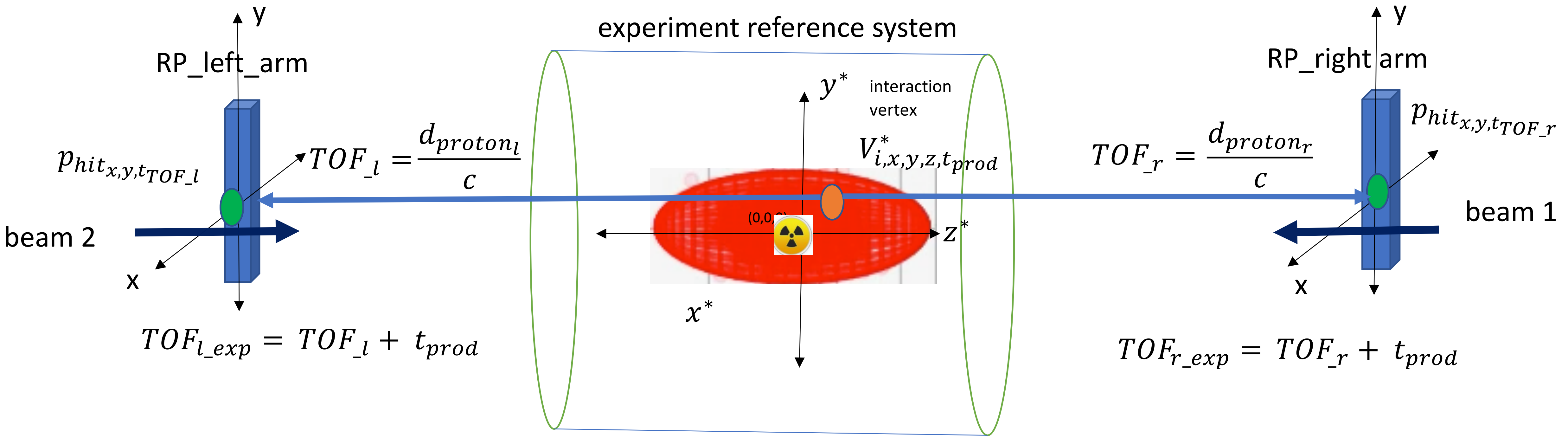
Transverse vertex distribution (crossing plane) -> crab space time clock



time

Timing with RP at a collider

double pommeron colllision: p p emission -> RP detectors: proton-proton emission tomograph   TOF p-pet



beam spot

z^*

$V^*{}_{i,x,y,z,t_{prod}}. = vertex\ position\ (x^*, y^*, z^*)\ \ at\ bunch\_crossing\ time\ (t_{prod}\ )$

During bunch crossing process:

the beam spot
- changes size
-changes position (crab-cavity)

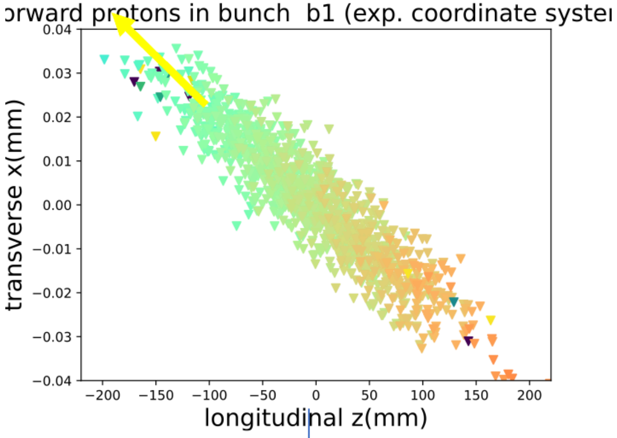
$(TOF_{l\_exp} + TOF_{r\_exp}) = t_{prod} + const$

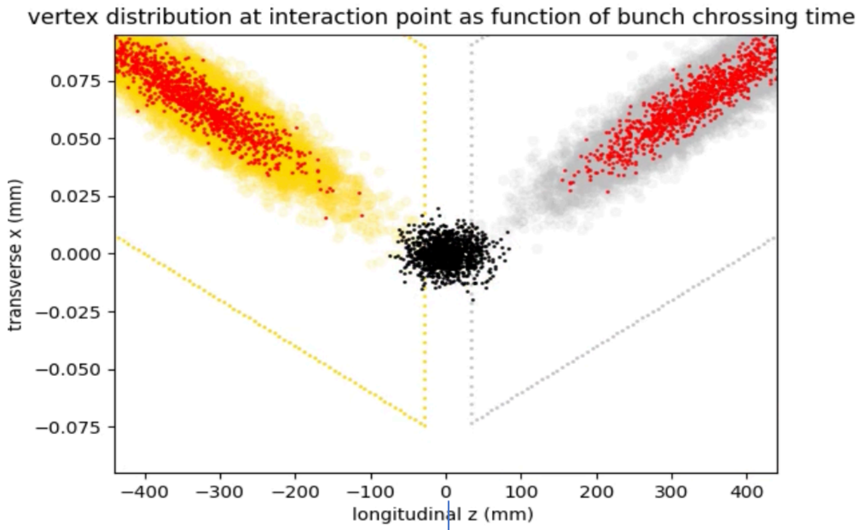
$(TOF_{l\_exp} - TOF_{r\_exp}) * \frac{c}{2} = V_{x,y,z,t_{prod}}$

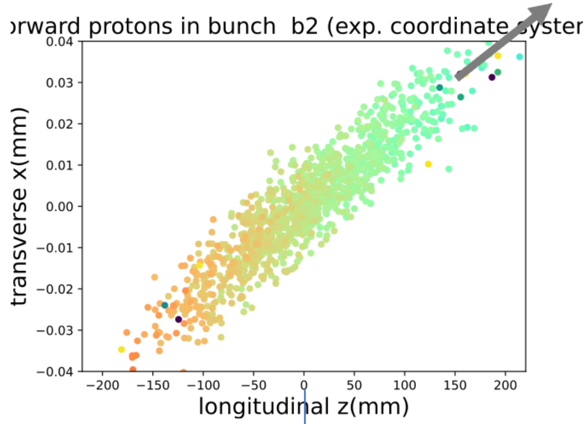
Forward physics:

beam spot size at interaction point and arrival time distribution of forward protons detected by RP detector

-> ILUMI4d allows to compute the p12=B1⊗ $b2$ and p21=B2⊗ $b1$ convolution factors

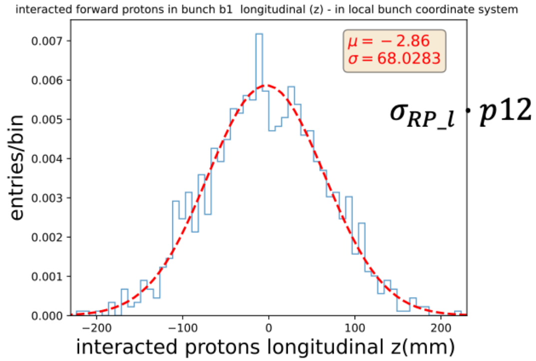




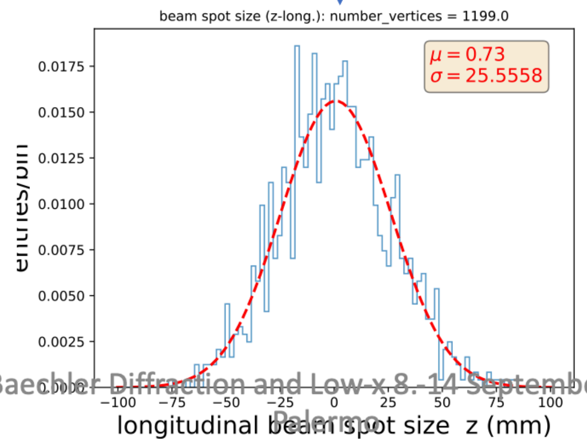


This distribution moves with speed c towards RP_left

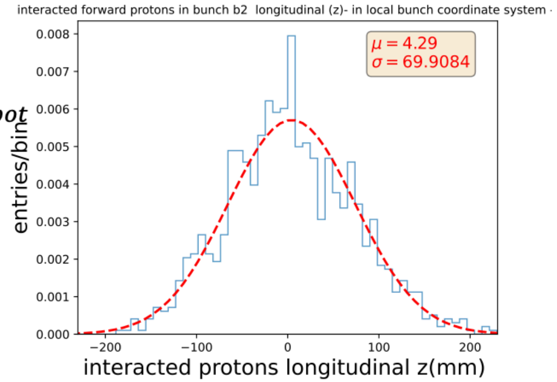
This distribution moves with speed c towards RP_right



$$\sigma_{RP\_l} \cdot p12 = \sigma_{beam\_spot}$$



$$\sigma_{RP\_r} \cdot p21 = \sigma_{beam\_spot}$$



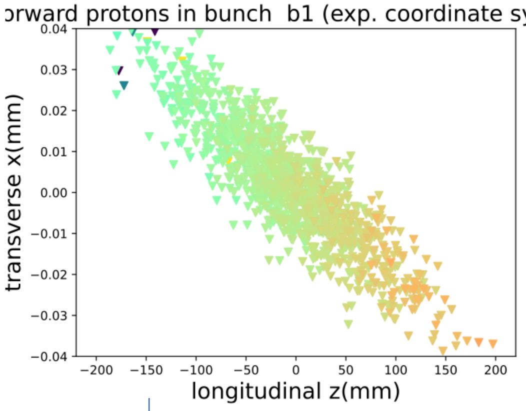
Corresponds
to arrival time distribution at RP_left

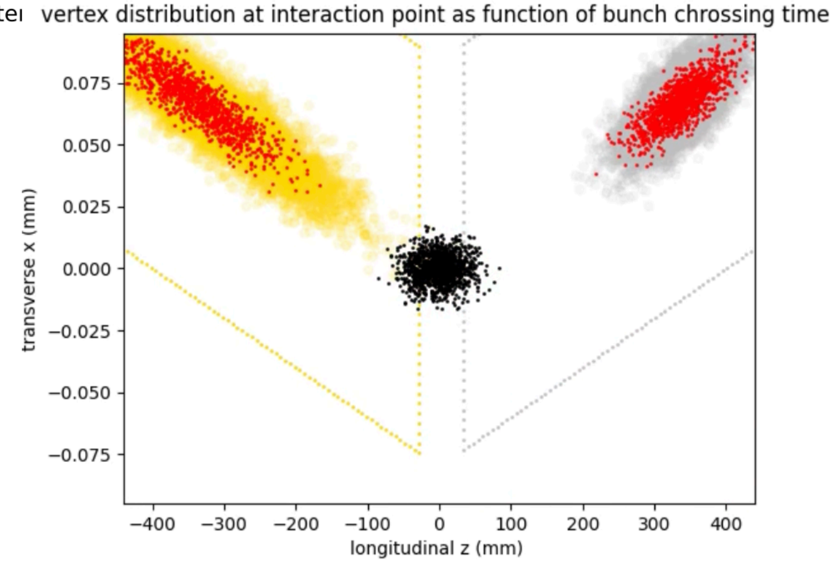
Corresponds
to arrival time distribution at RP_right

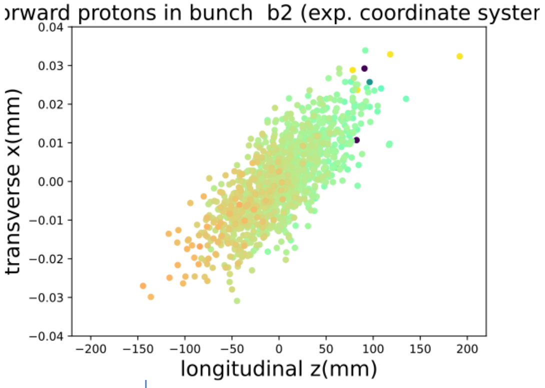
Forward physics:

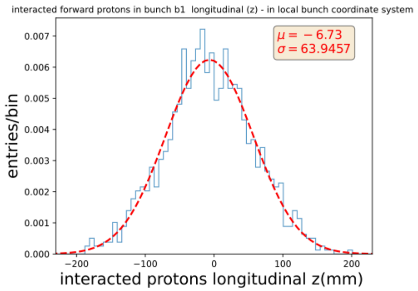
beam spot size at interaction point and arrival time distribution of forward protons detected by RP detector

-> ILUMI4d allows to compute the p12=B1$\otimes$ $b2$ and p21=B2$\otimes$ $b1$ convolution factors (asymetric beams)

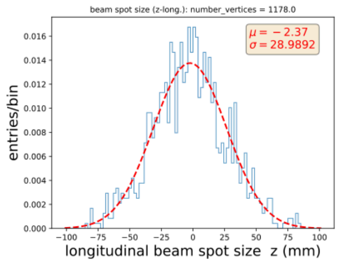




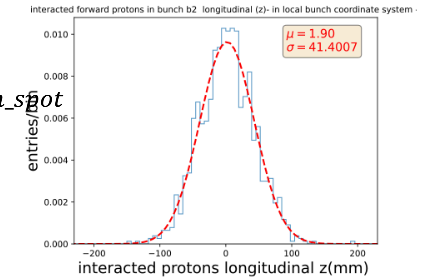




$$\sigma_{RP\_l} \cdot p12 = \sigma_{beam\_spot}$$

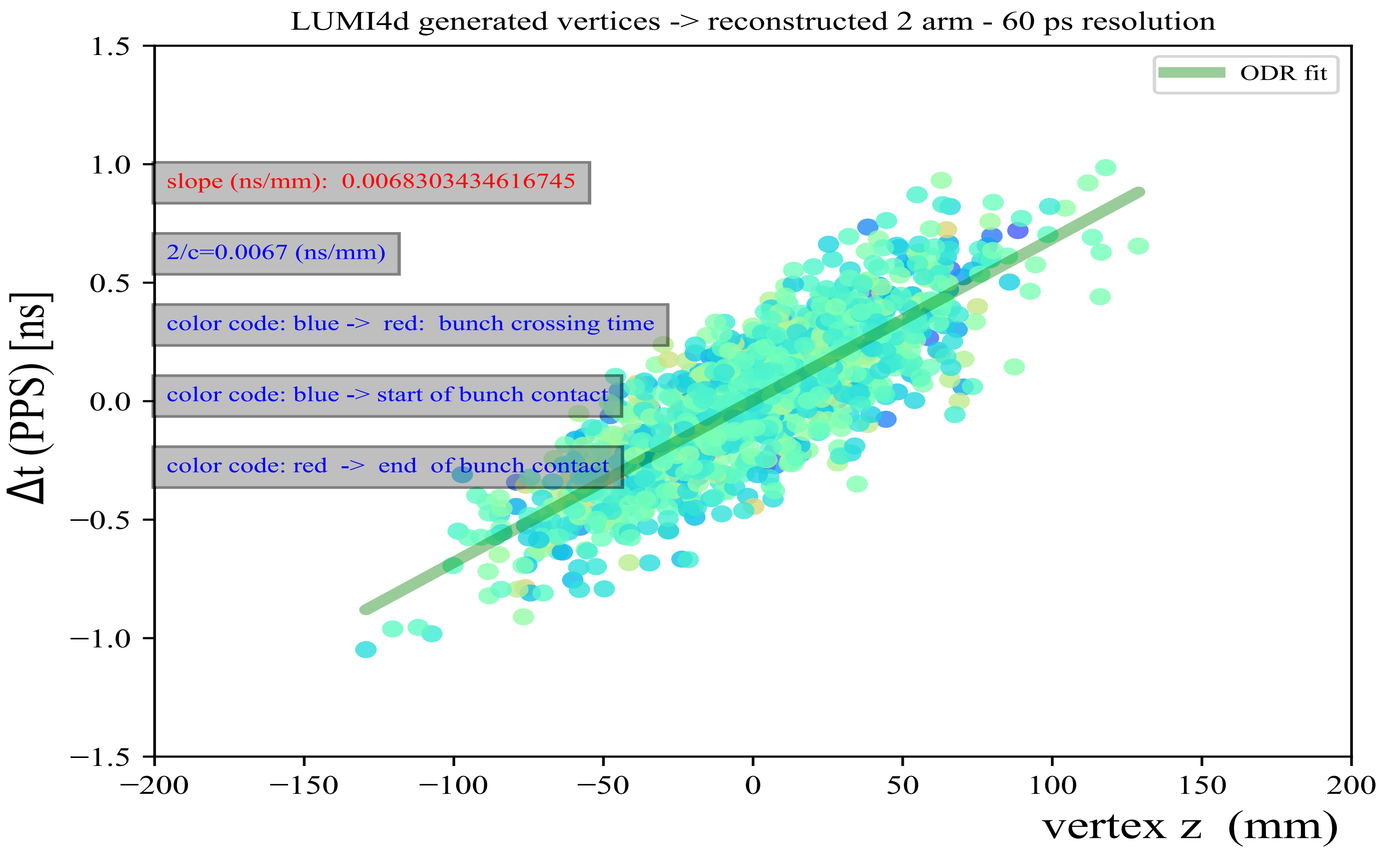


$$\sigma_{RP\_r} \cdot p21 = \sigma_{beam\_spot}$$



ILUMI4D reconstructed vertices forward detector

$$(TOF_{l\_exp} - TOF_{r\_exp}) * \frac{c}{2} = V_{x,y,z,t_{prod}}$$



Goal of accelerator operation:

keep the luminosity and the pileup constant during a fill, when the bunch intensity declines as function of time

To keep the luminostiy constant during a fill at HL-LHC, the following parameter relevant for the luminosity are changed

$$\varepsilon_x, \varepsilon_y, \beta^*$$

-> the beam size, effective area and the beam spot size change with the parameter change during a fill -> **hour glass effect**

-> **the bunch length, crossing angle and crab angle are kept constant**

-> the crab cavity introduces a transverse speed in the crossing plane

The space time structure of the luminosity changes during a fill

HL-LHC TDR

https://cds.cern.ch/record/2749422/files/127-117-PB.pdf

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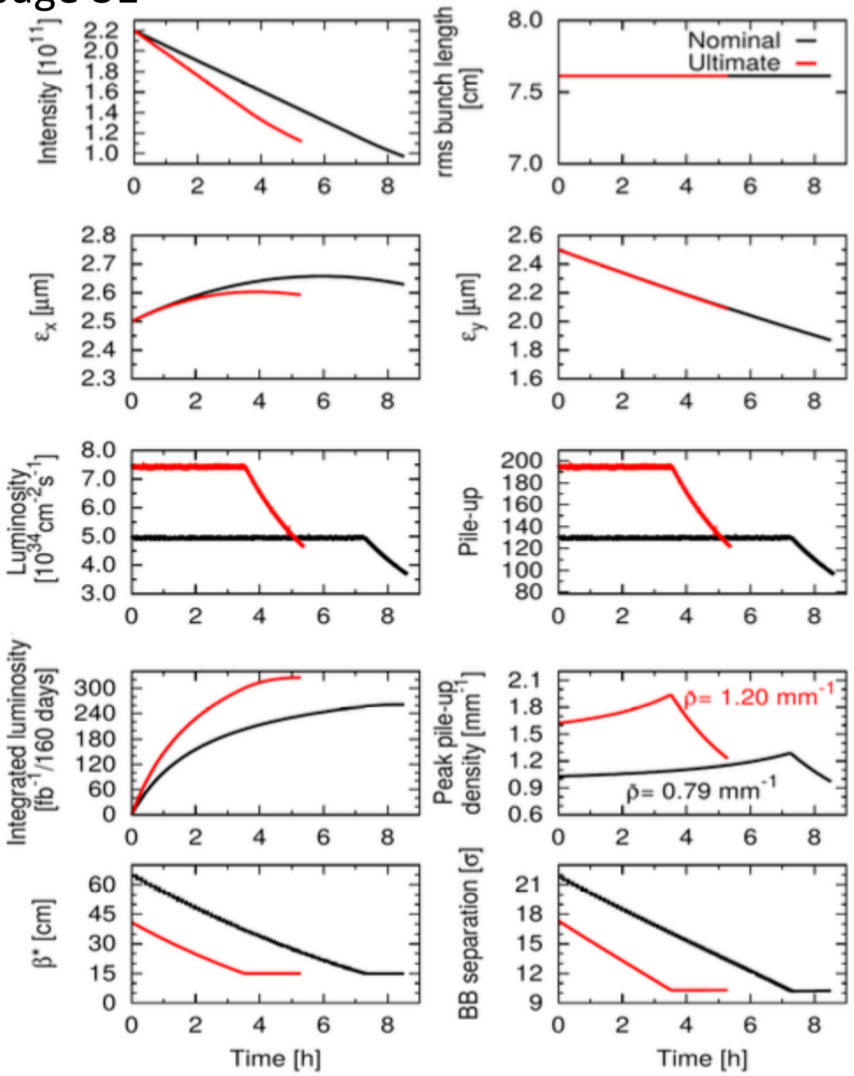

Figure 2-3: Evolution of the main beam and machine parameters for the nominal and ultimate scenarios.

Summary and Conclusions (1)

ILUMI4d allows to compute the single bunch luminosity and space time structure of luminosity for arbitrary bunch parameters and operation scenario (including fixed target and moving targets – important for medical applications EBRT)

- the crossing angle, crab-angle, hour glass effect (can be switched on&off), bunch cogging and bunch separation are taken into account

- the 3d vertex footprint, the longitudinal and transverse beam spot size is computed as function of the bunch crossing time (time frames) on 20 picosecond scale (super bunch with $\sim 4k$ vertices)

- the production time spectrum of forward protons and the longitudinal vertex distribution in the central detector can be computed (reconstruction of beam spot size)

- the results are stored in n-tuple for further analysis and the virtual collision process is visualised in mp3 videos

ILUMI4d ->**Virtual Collision Concept : compute lumi and lumi characterstics for the different bunch parameters**

Summary and Conclusions (2)

- The algorithm ILUMI4d is based on the replica of nature with unbiased statistical samples and implementation of deep reaction processes in the convolution process.
- This concept can be applied to many different processes in nature where convolution processes are involved.
- The main program is REACT4d
  several applications are under development (PET4d, MED4d, ….