Lévy α-stable generalization of the ReBB model of elastic pp and p$\bar{\text{p}}$ scattering

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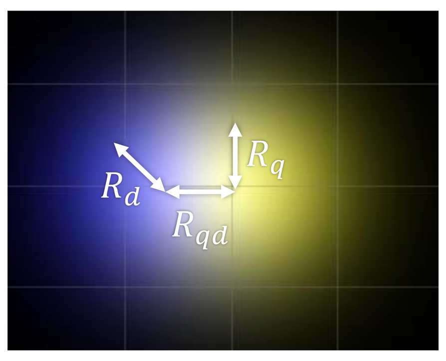
Introduction and preliminaries

Bialas-Bzdak (BB) models

A. Bialas and A. Bzdak, in 2007, published models for elastic proton-proton (pp) scattering [1, 2], the BB models for short. In these models the proton is described as a bound state of constituent quarks, and the probability of inelastic pp scattering is constructed based on R. J. Glauber's diffractive multiple scattering theory: all possible single and multiple binary inelastic collisions of the constituents is considered in a way that constituent back scattering is prohibited; as a result, the collision of two protons is inelastic if at least one constituent-constituent collision is inelastic. The elastic scattering amplitude is then calculated based on the unitarity relation neglecting the sub-dominant real part of the scattering amplitude.

The p = (q,d) Real extended Bialas-Bzdak (ReBB) model

In 2015, the BB model was extended: a real part of the scattering amplitude was added in a unitary manner [3] leading to the Real extended Bialas-Bzdak model, the ReBB model for short. It was also found that the p=(q,d) version of the BB model is consistent with the experimentally observed features of elastic pp scattering. The p = (q,d) version of the model describes the proton as a bound state of a constituent quark and a constituent diquark in a way that the diquark is treated as a single entity not as a bound state of two quarks. Two basic ingredients of the BB model are the inelastic scattering probabilities of two constituents as a function of their relative transverse position and the quark-diquark distribution inside the proton. The constituent-constituent inelastic scattering probabilities have Gaussian shapes that follow from the Gaussian-shaped parton distributions of the constituents characterized by the scale parameters R_q and R_d. The quark-diquark distribution inside the proton has also a Gaussian shape with scale parameter R_{qd} that characterizes the separation between the quark and the diquark constituents inside the proton.



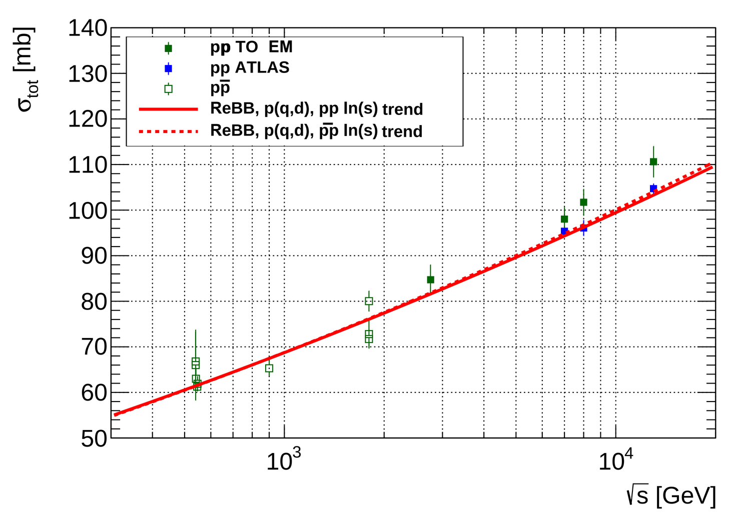
ReBB model versus data

It was found in studies published in 2021 and 2022 [3, 4] that the ReBB model describes all the available data not only on elastic pp scattering but also on elastic proton-antiproton (p$\bar{\text{p}}$) scattering in a statistically acceptable manner i.e. with a confidence level (CL) ≥ 0.1% in the kinematic range:

$$0.38\ \text{GeV}^2 \le |t| \le 1.2\ \text{GeV}^2,$$
$$546\ \text{GeV} \le \sqrt{s} \le 8\ \text{TeV},$$

where t is the squared four-momentum transfer and s is the squared center of mass energy. Based on the ReBB model analysis of elastic pp and p$\bar{\text{p}}$ scattering, statistically significant signals of the t-channel odderon exchange were observed [4, 5].

Need for an improvement of the ReBB model at low-|t|

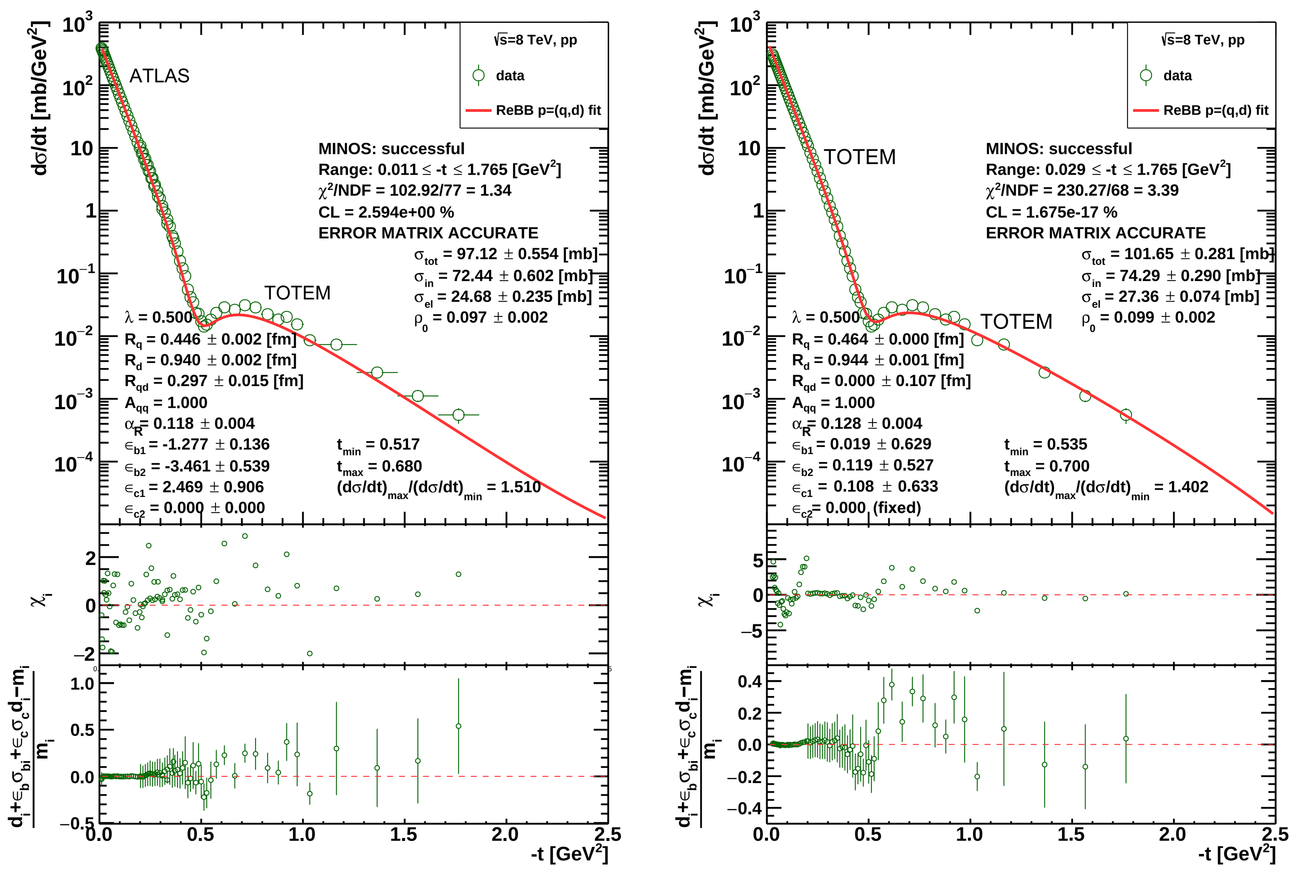


The ReBB model calibrated to the SPS UA4 p$\bar{\text{p}}$, Tevatron D0 p$\bar{\text{p}}$, and LHC TOTEM pp elastic $d\sigma/dt$ data in the kinematic range,

$$0.38\ \text{GeV}^2 \le |t| \le 1.2\ \text{GeV}^2,$$
$$546\ \text{GeV} \le \sqrt{s} \le 7\ \text{TeV},$$

perfectly describes the pp σ_{tot} data as measured by the LHC ATLAS experiment being systematically below the pp σ_{tot} data as measured by the LHC TOTEM experiment. Theoretically, $\sigma_{tot}(s) = 2\text{Im}T_{el}(s,t=0)$. Further studies may be important within a model that describes the elastic pp data both at low-$|t|$ and high-$|t|$ with CL ≥ 0.1%.

At $\sqrt{s}$ = 8 TeV, the ReBB model describes the ATLAS low-$|t|$ and the TOTEM high-$|t|$ data simultaneously with CL = 2.6% (the used χ^2 formula is the one derived by the PHENIX Collaboration [6]). At $\sqrt{s}$ = 8 TeV, the ReBB model fails to describe the TOTEM low-$|t|$ and the TOTEM high-$|t|$ data simultaneously with CL ≥ 0.1%. The TOTEM low-$|t|$ data shows a strong non-exponential behavior with a statistical significance grater than 7σ [7] which is not reproduced by the ReBB model containing Gaussian-shaped distributions.



Gaussian shape versus Levy α-stable shapes

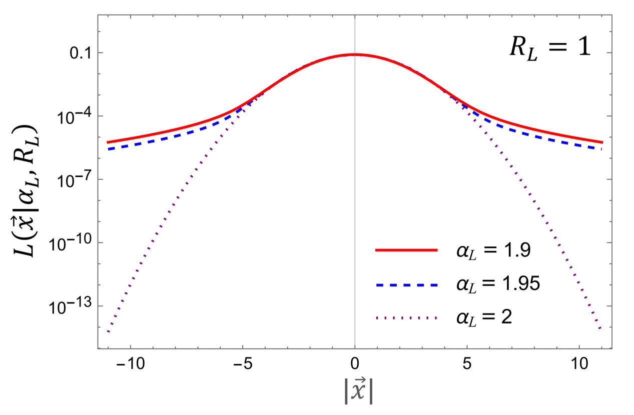
The use of Gaussian distributions are motivated by the central limit theorem. Generalized central limit theorems motivate the use of Levy α-stable distributions. The bivariate Gaussian and symmetric Levy α-stable distributions centered at 0 are:

$$G(\vec{x}|R_G) = \frac{1}{2\pi R_G^2} e^{-\frac{\vec{x}^2}{2R_G^2}},$$

$$L(\vec{x}|\alpha_L, R_L) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\vec{q}\cdot\vec{x}} e^{-|\vec{q}^2 R_L^2|^{\alpha_L/2}},$$

where $0 < \alpha_L \le 2$. For $\alpha_L = 2$ the Lévy α-stable distribution is the Gaussian distribution:

$$L(\vec{x}|\alpha_L = 2, R_L = R_G/\sqrt{2}) \equiv G(\vec{x}|R_G).$$



Lévy α-stable distributions with $\alpha_L < 2$ have tails behaving asymptotically as a power law (infinite variance): for large x and $\alpha_L < 2$, $L(x|\alpha_L, R_L) \sim |x|^{-(1+\alpha_L)}$.

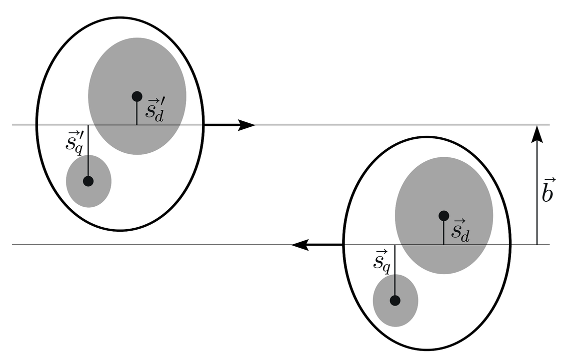
A simple model of elastic scattering with a Gaussian impact parameter amplitude yields a purely exponential t-distribution, while a simple model with a Levy α-stable impact parameter amplitude and $\alpha_L < 2$ yields a non-exponential t-distribution [9].

$$\tilde{T}_{el}(s,b) = \frac{i+\rho_0(s)}{2}\sigma_{tot}(s) G\left(b\middle|\sqrt{B_0(s)}\right) \;\Rightarrow\; \frac{d\sigma}{dt}(s,-t) = a(s)e^{-tB_0(s)}$$

$$\tilde{T}_{el}(s,b) = \frac{i+\rho_0(s)}{2}\sigma_{tot}(s) L\left(b\middle|\alpha_L, \sqrt{B_L(s)}\right) \;\Rightarrow\; \frac{d\sigma}{dt}(s,-t) = a(s)e^{-|tB_L(s)|^{\alpha_L(s)/2}}$$

Levy α-stable generalized Bialas-Bzdak (LBB) model

In the p=(q,d) BB model the proton is a bound state of a constituent quark and a constituent diquark; the inelastic scattering probability of two protons at a fixed impact parameter vector ($\vec{b}$) and at fixed constituent transverse position vectors ($\vec{s}_q, \vec{s}_d, \vec{s}_q', \vec{s}_d'$) is given by a Glauber expansion:



$$\sigma(\vec{s}_q, \vec{s}_d; \vec{s}_q', \vec{s}_d'; \vec{b}) = 1 - \prod_{a\in\{q,d\}}\prod_{b\in\{q,d\}}\left[1 - \sigma_{ab}(\vec{b} + \vec{s}_b' - \vec{s}_a)\right].$$

The parton distributions of the constituent quark and the constituent diquark are now Levy α-stable distributions and the inelastic scattering probability for the collision of two constituents at a fixed relative transverse position $\vec{x}$ of the constituents:

$$\sigma_{ab}(\vec{x}) = A_{ab}\pi S_{ab}^2 \int d^2 r_a L(\vec{r}_a|\alpha_L, R_a/2) L(\vec{x} - \vec{r}_a|\alpha_L, R_b/2) \equiv A_{ab}\pi S_{ab}^2 L(\vec{x}|\alpha_L, S_{ab}/2),$$

$$S_{ab}^{\alpha_L} = R_a^{\alpha_L} + R_b^{\alpha_L}, \quad a, b \in \{q,d\}.$$

The distribution of the constituents inside the proton is now given in terms of a Levy α-stable distribution:

$$D(\vec{s}_q, \vec{s}_d) = (1+\lambda)^2 L(\vec{s}_q - \vec{s}_d|\alpha_L, R_{qd}/2)\delta^2(\vec{s}_q + \lambda\vec{s}_d),$$

$$\lambda = m_q/m_d, \quad \vec{s}_d = -\lambda\vec{s}_q, \quad \int d^2s_q d^2s_d D(\vec{s}_q, \vec{s}_d) = 1.$$

where m_q is the mass of the quark and m_d is the mass of the diquark.

The probability of inelastic scattering of protons at a fixed $\vec{b}$ is given by averaging over the constituent positions inside the protons:

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} d^2s_q d^2s_q' d^2s_d d^2s_d'\, D(\vec{s}_q, \vec{s}_d)\, D(\vec{s}_q', \vec{s}_d')\, \sigma(\vec{s}_q, \vec{s}_d; \vec{s}_q', \vec{s}_d'; \vec{b}).$$

The elastic scattering amplitude $\tilde{T}_{el}(s,b)$ is given via $\tilde{\sigma}_{in}(s,b)$ through the opacity function $\Omega(s,b)$:

$$\text{Re}\Omega(s,b) = -1/2\ln[1 - \tilde{\sigma}_{in}(s,b)], \quad \text{Im}\Omega(s,b) = -\alpha_R\tilde{\sigma}_{in}(s,b), \quad \tilde{T}_{el}(s,b) = i\left[1 - e^{-\Omega(s,b)}\right],$$

where $b = |\vec{b}|$ and the dependence on s follows from the s-dependence of the free parameters.

The $T_{el}(s,t)$ is obtained from $\tilde{T}_{el}(s,b)$: $\quad T_{el}(s,t) = \int d^2b\, e^{i\vec{q}\cdot\vec{b}}\, \tilde{T}_{el}(s,b), \quad |\vec{q}|^2 = -t.$

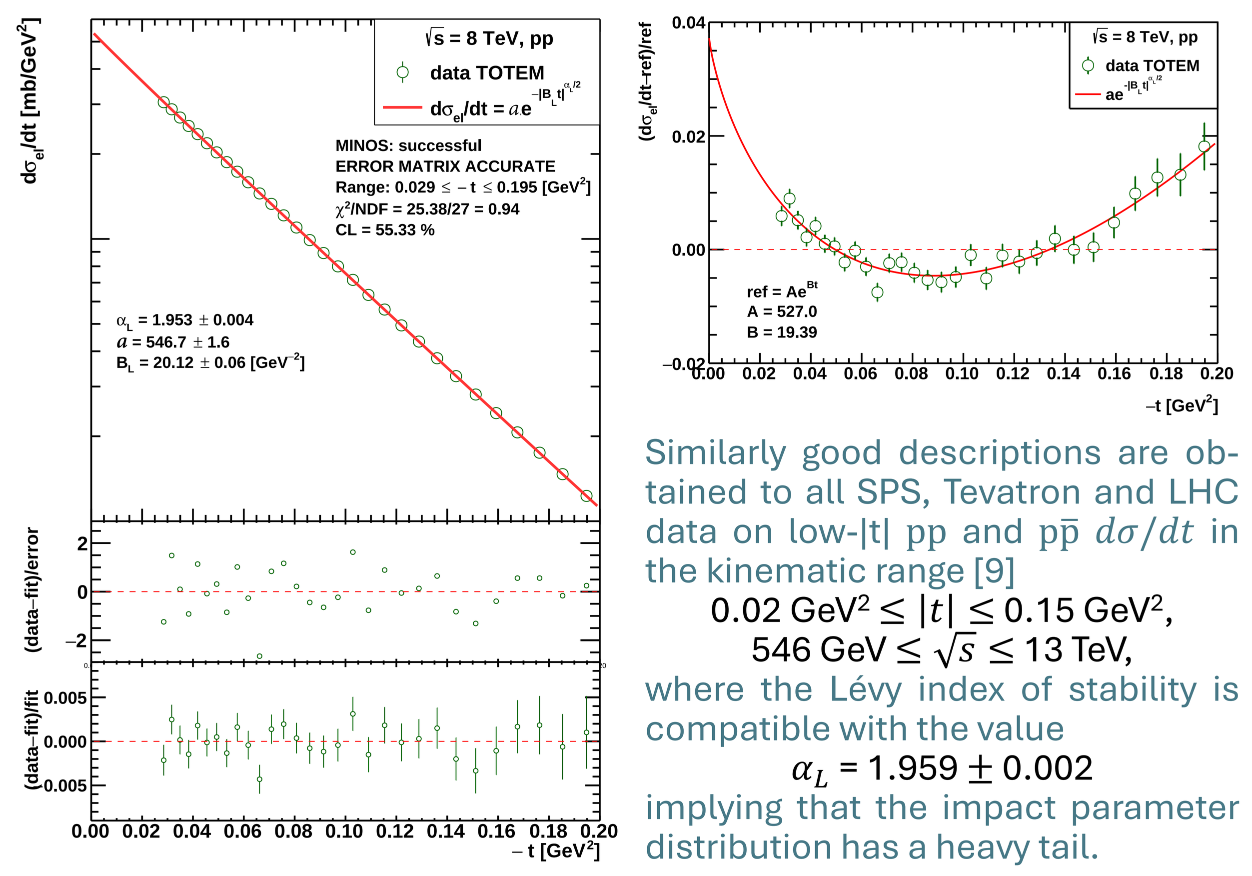
The new free parameter of the Levy α-stable generalized model is α_L, the Lévy index of stability; if $\alpha_L = 2$, the ReBB model with Gaussian distributions is recovered.

Illustration of the power of the Lévy α-stable generalization

Numerical calculations with the present form of the LBB model are time-consuming: a sequence of three integral calculations needs to be performed where the result of an integral is an integrand of the next integral. A relatively high computing capacity and/or an improved analytic insight is needed to proceed with the full LBB model. Interestingly, the LBB model reduces to the simple Lévy α-stable model of elastic scattering by approximating $\tilde{\sigma}_{in}(s,b)$ with a single Levy α-stable shape and applying approximations valid at low-$|t|$ [8]. The non-exponential Lévy α-stable model with

$$\alpha_L = 1.953 \pm 0.004$$

represents the LHC TOTEM $\sqrt{s}$ = 8 TeV low-$|t|$ differential cross section data with a confidence level of 55% [8].



Similarly good descriptions are obtained to all SPS, Tevatron and LHC data on low-|t| pp and p$\bar{\text{p}}$ $d\sigma/dt$ in the kinematic range [9]

$$0.02\ \text{GeV}^2 \le |t| \le 0.15\ \text{GeV}^2,$$
$$546\ \text{GeV} \le \sqrt{s} \le 13\ \text{TeV},$$

where the Lévy index of stability is compatible with the value

$$\alpha_L = 1.959 \pm 0.002$$

implying that the impact parameter distribution has a heavy tail.

Summary

The Lévy α-stable generalization of the Bialas-Bzdak model is done by generalizing from Gaussian shapes to Lévy α-stable shapes both (i) the inelastic scattering probabilities of two constituents and (ii) the quark-diquark distribution inside the proton. The LBB model is expected to describe simultaneously the low-$|t|$ and high-$|t|$ domains of elastic pp and p$\bar{\text{p}}$ $d\sigma/dt$ with a Lévy index of stability $\alpha_L < 2$. Thus the next step is to apply the full LBB model to describe the data. Given that the LBB model describes the data in a statistically satisfying manner i.e. with CL ≥ 0.1%, it can be used to study, e.g, (i) the discrepancy between ATLAS and TOTEM cross section measurements, and (ii) after considering the effects of the Coulomb-nuclear interference, the odderon contribution to parameter $\rho_0 = \text{Re}T_{el}(s,t)/\text{Im}T_{el}(s,t)|_{t\to 0}$ at $\sqrt{s}$ = 13 TeV.

Acknowledgements

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