Lévy α-stable generalization of the ReBB model of elastic pp and $p\bar{p}$ scattering

based on Universe 2023, 9(8), 361 & Universe 2024, 10(3), 127

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Diffraction and Low-x 2024

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ELTE EÖTVÖS LORÁND UNIVERSITY

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MATE

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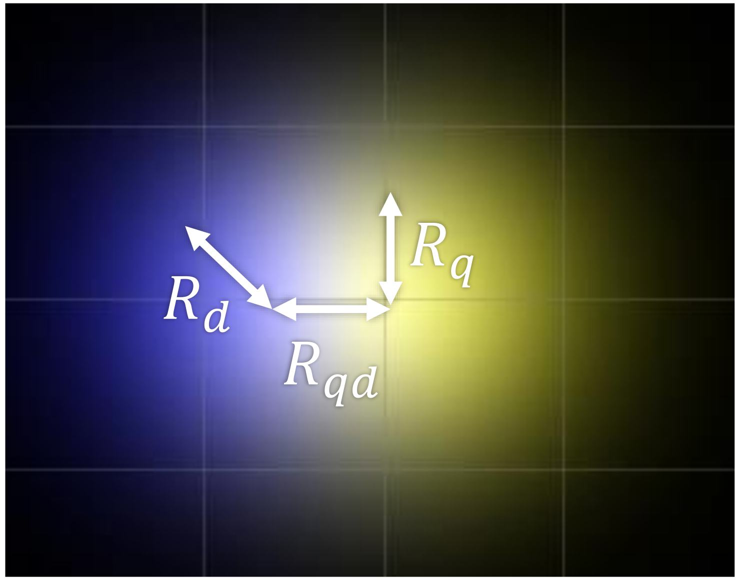

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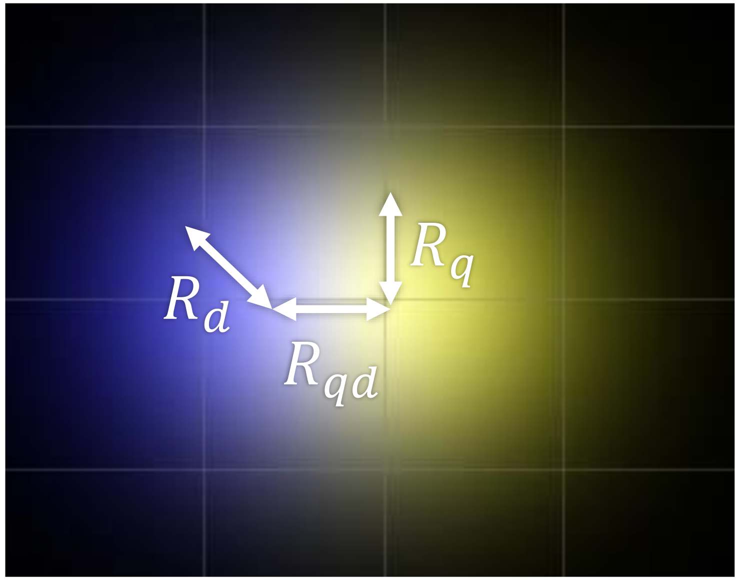

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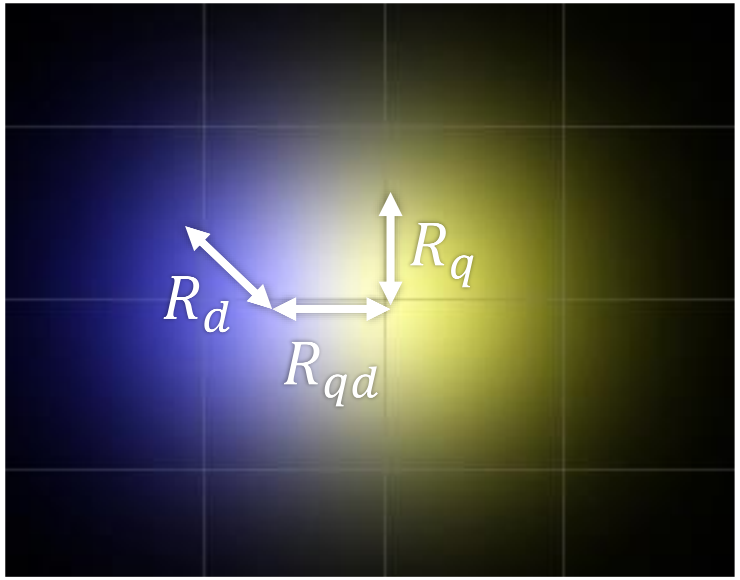

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$0.38\ \text{GeV}^2 \leq |t| \leq 1.2\ \text{GeV}^2$,

$546\ \text{GeV} \leq \sqrt{s} \leq 8\ \text{TeV}$,

(t is the squared four-momentum transfer, s is the squared center of mass energy)

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bivariate Gaussian and symmetric Levy α-stable distributions centered at $\vec{0}$:

$$G(\vec{x}|R_G) = \frac{1}{2\pi R_G^2} e^{-\frac{\vec{x}^2}{2R_G^2}}$$

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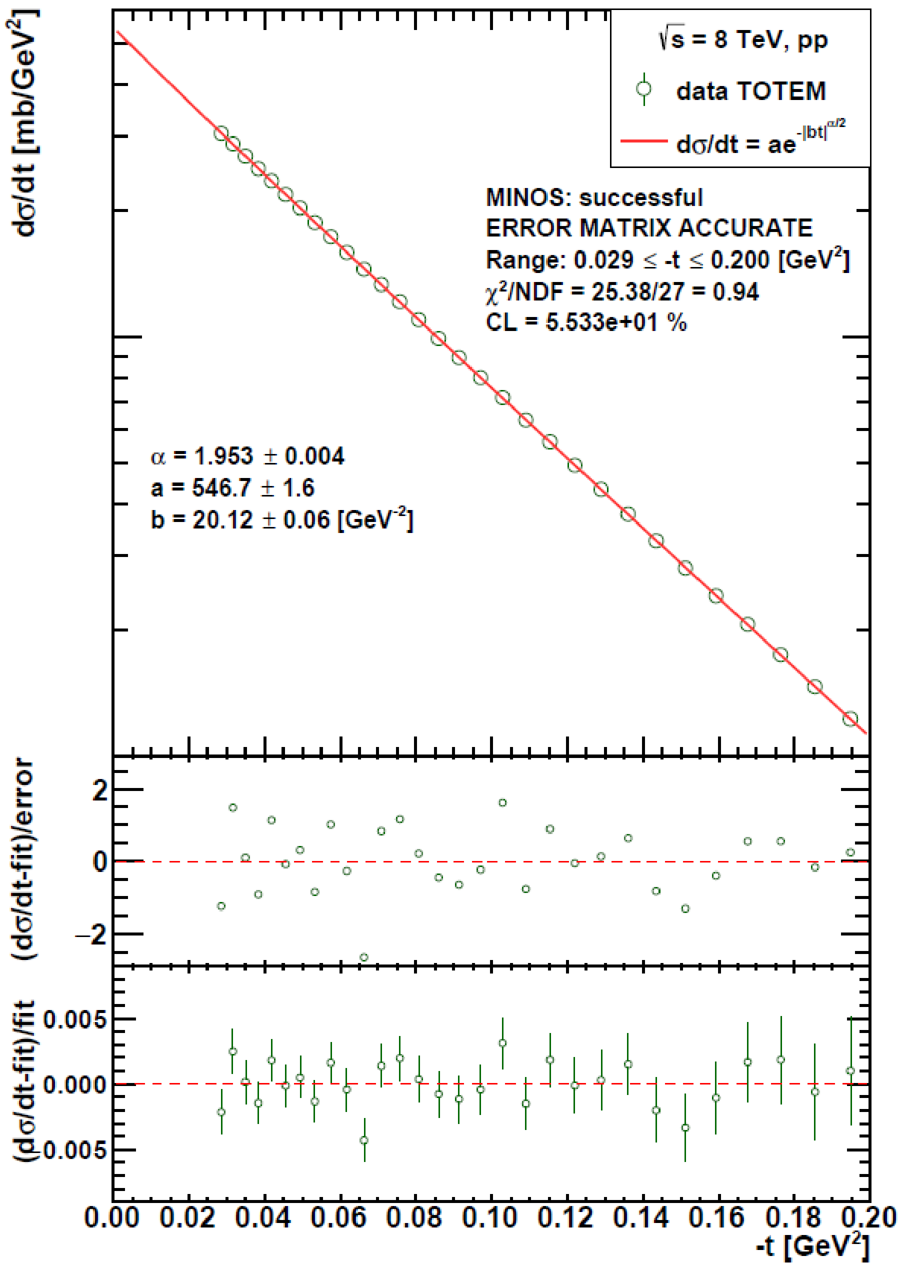

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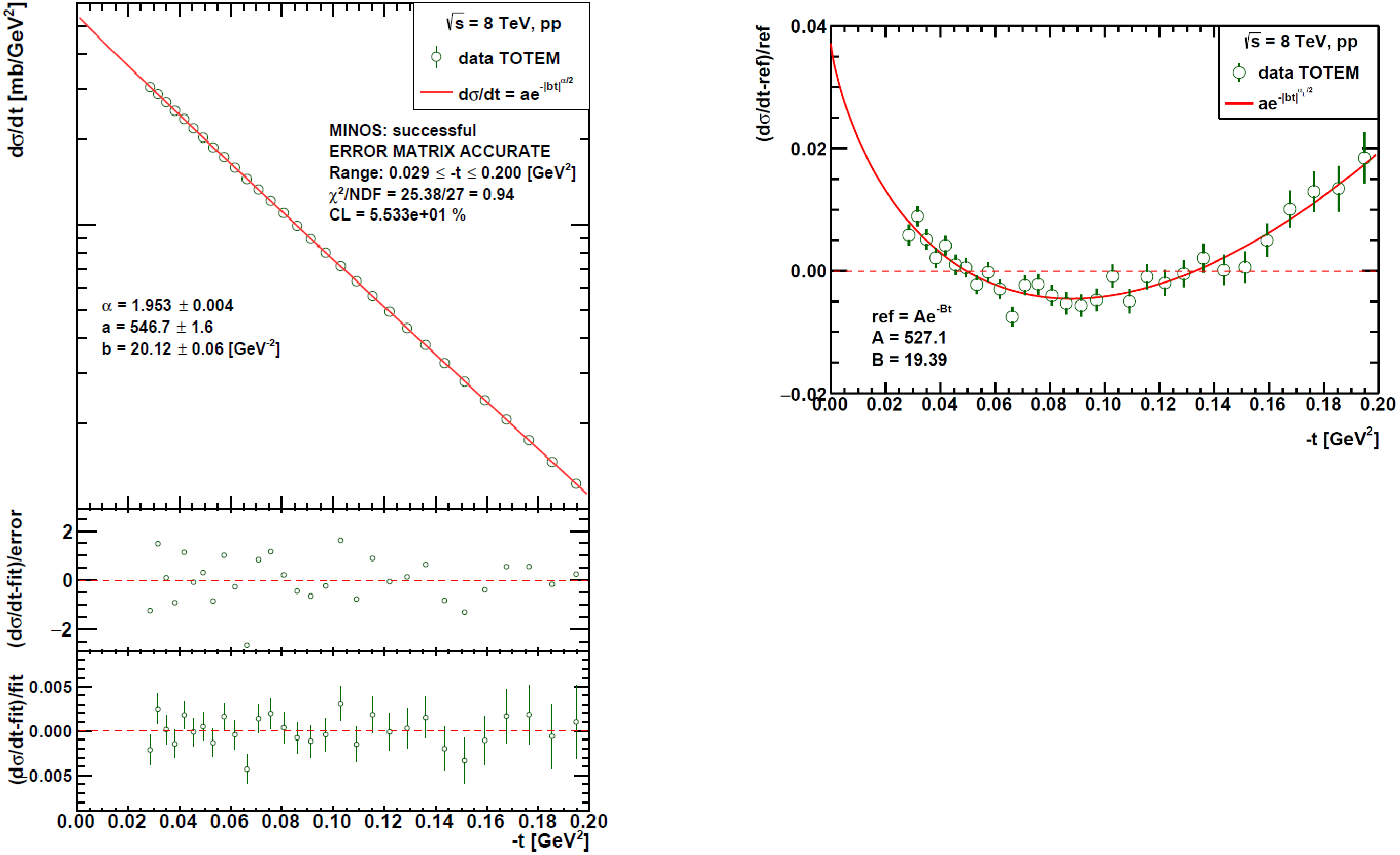

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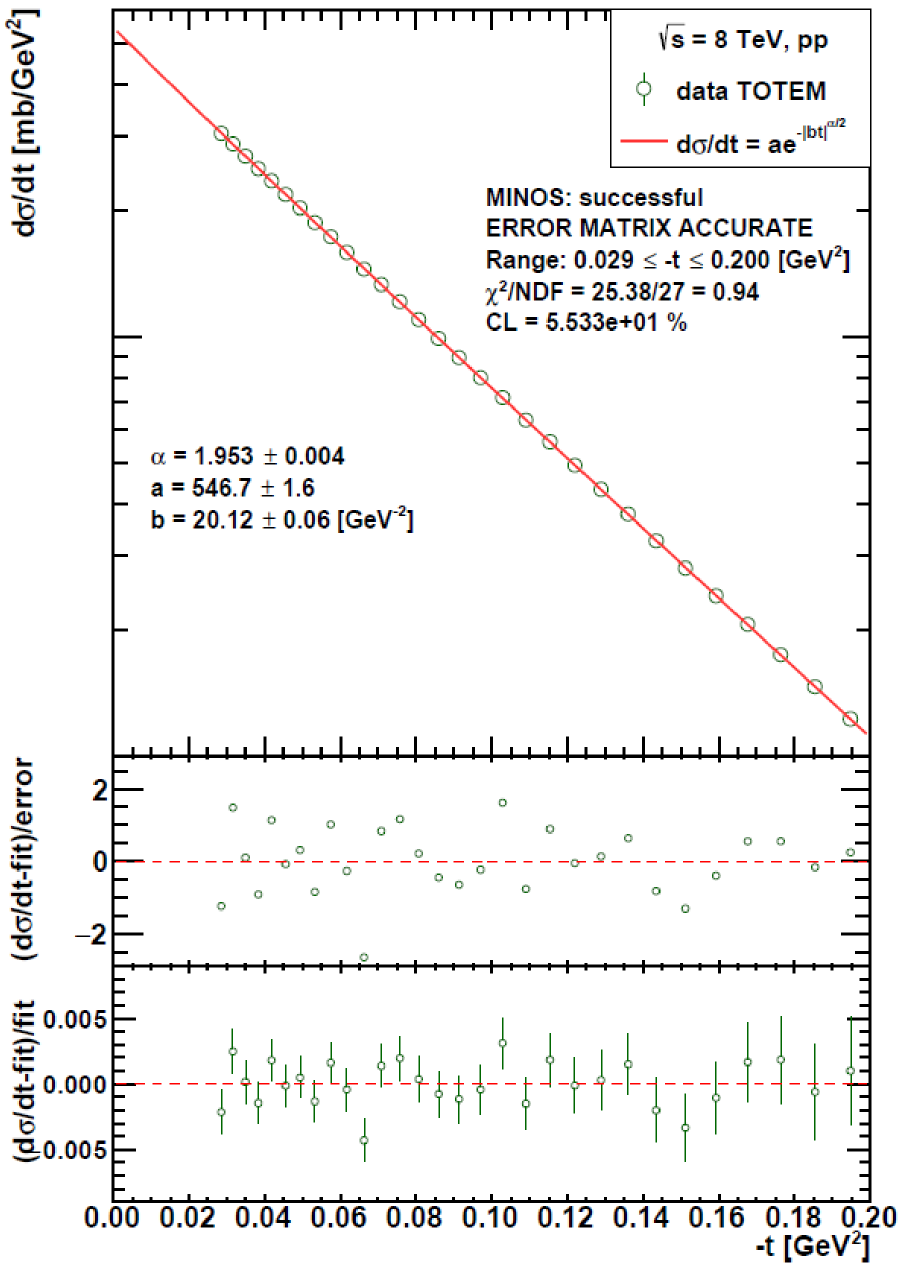

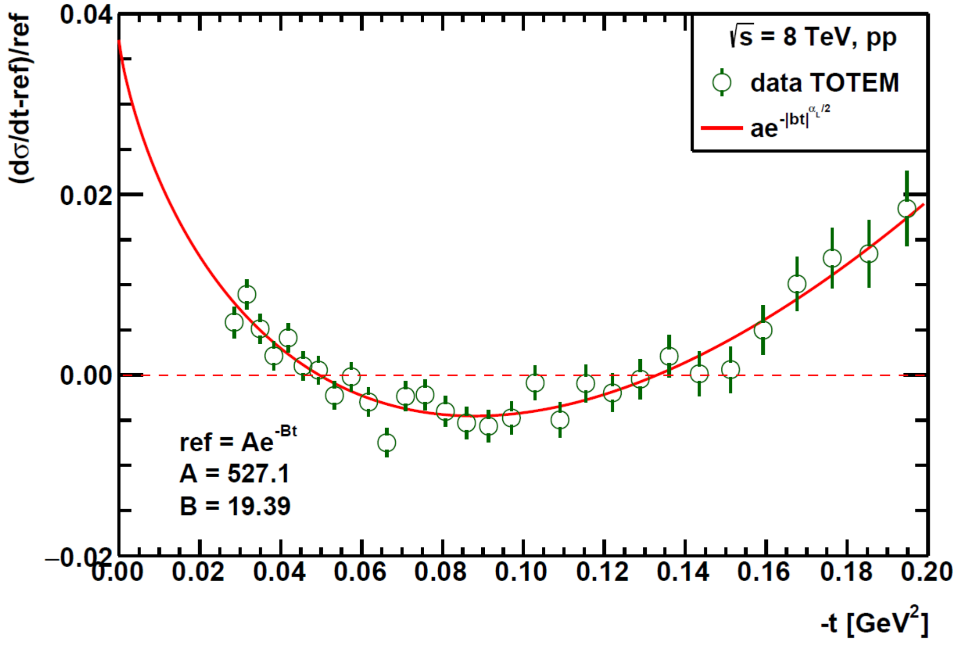

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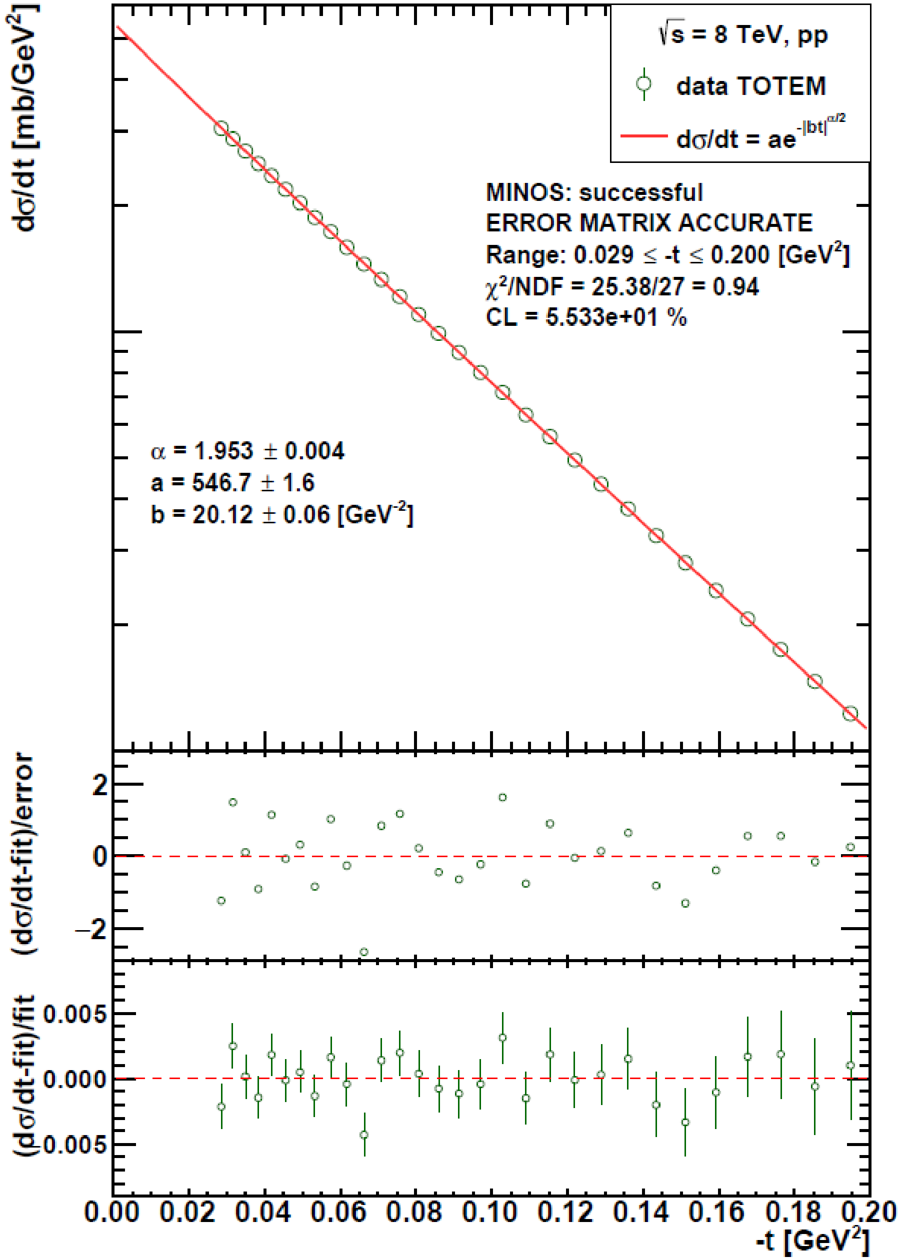

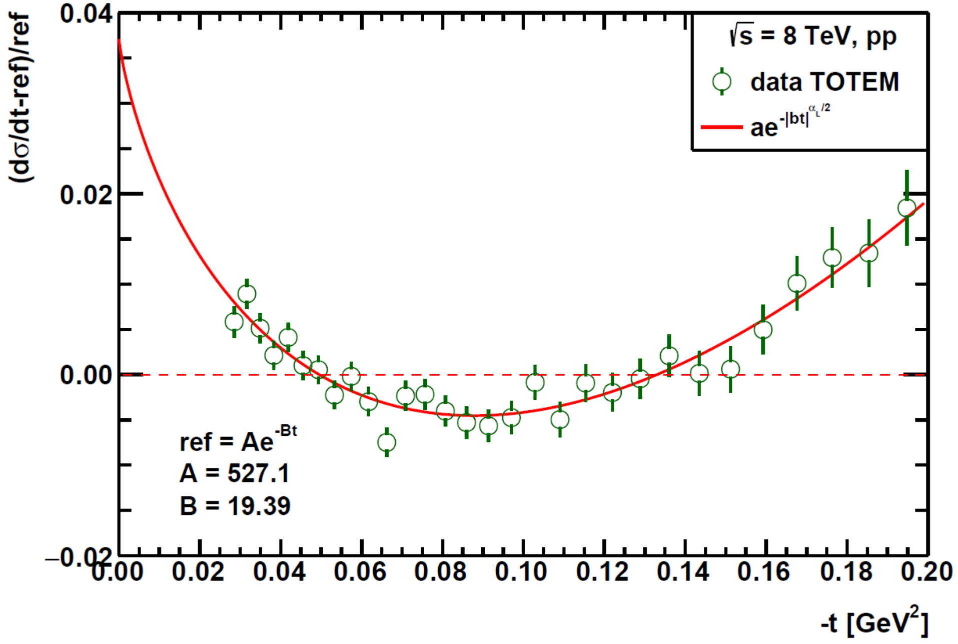

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- similarly good descriptions are obtained to all the LHC data on low-|t| pp (and p$\bar{\text{p}}$) $\boldsymbol{d\sigma/dt}$

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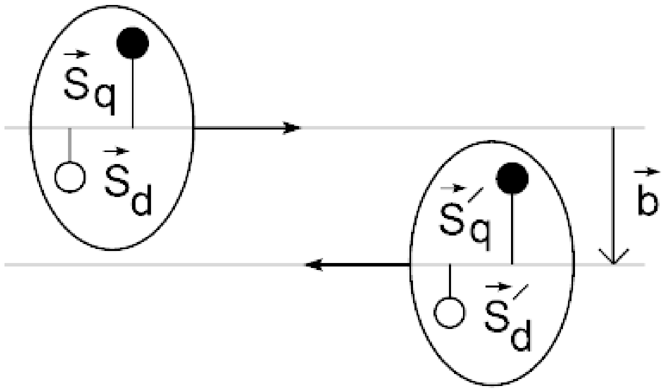

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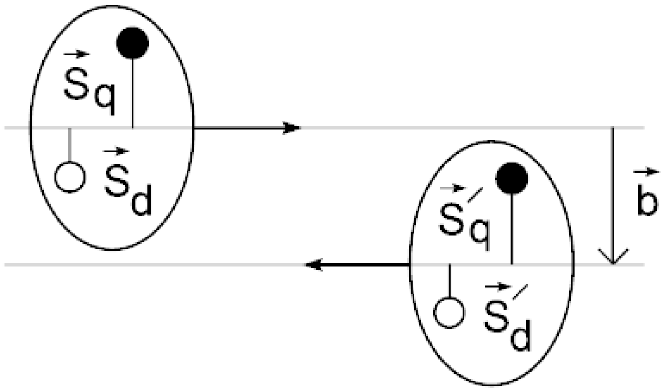

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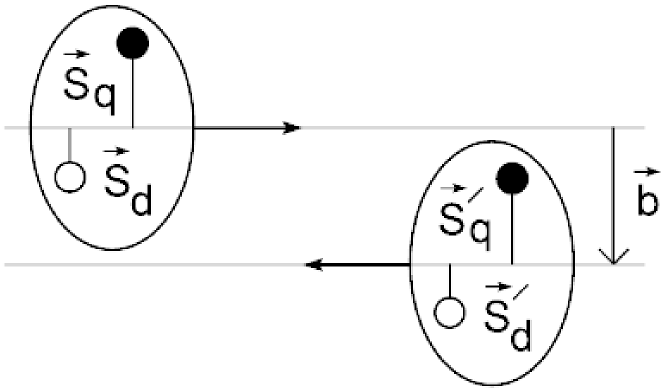

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$$D(\vec{s}_q, \vec{s}_d) = L(\vec{s}_q - \vec{s}_d|\alpha_L, R_{qd}/2) \times \\ \times (1+\lambda)^2 \delta^2(\vec{s}_q + \lambda\vec{s}_d)$$

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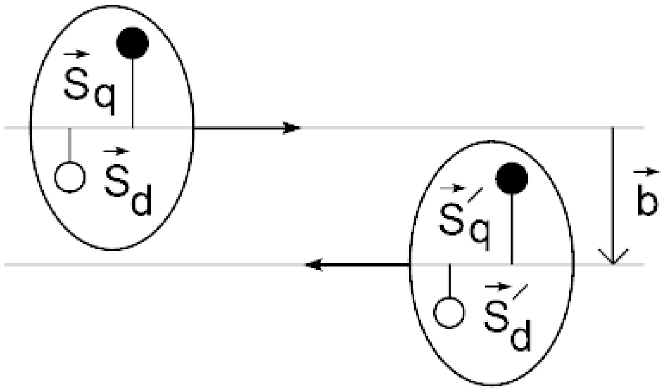

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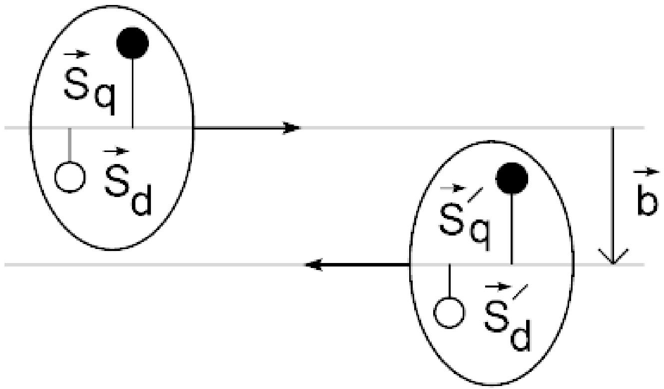

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a new free parameter: α_L, the Lévy index of stability;
if $\alpha_L = 2$, the ReBB model with Gaussian distributions is recovered

Thank you for your attention!

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