

TMD factorization Bridging Large and Small x

Swagato Mukherjee

September 2024,
Palermo, Italy



MSTT(-erious) factorization of gluon TMDPDF operator

PHYSICAL REVIEW D **109**, 034035 (2024)

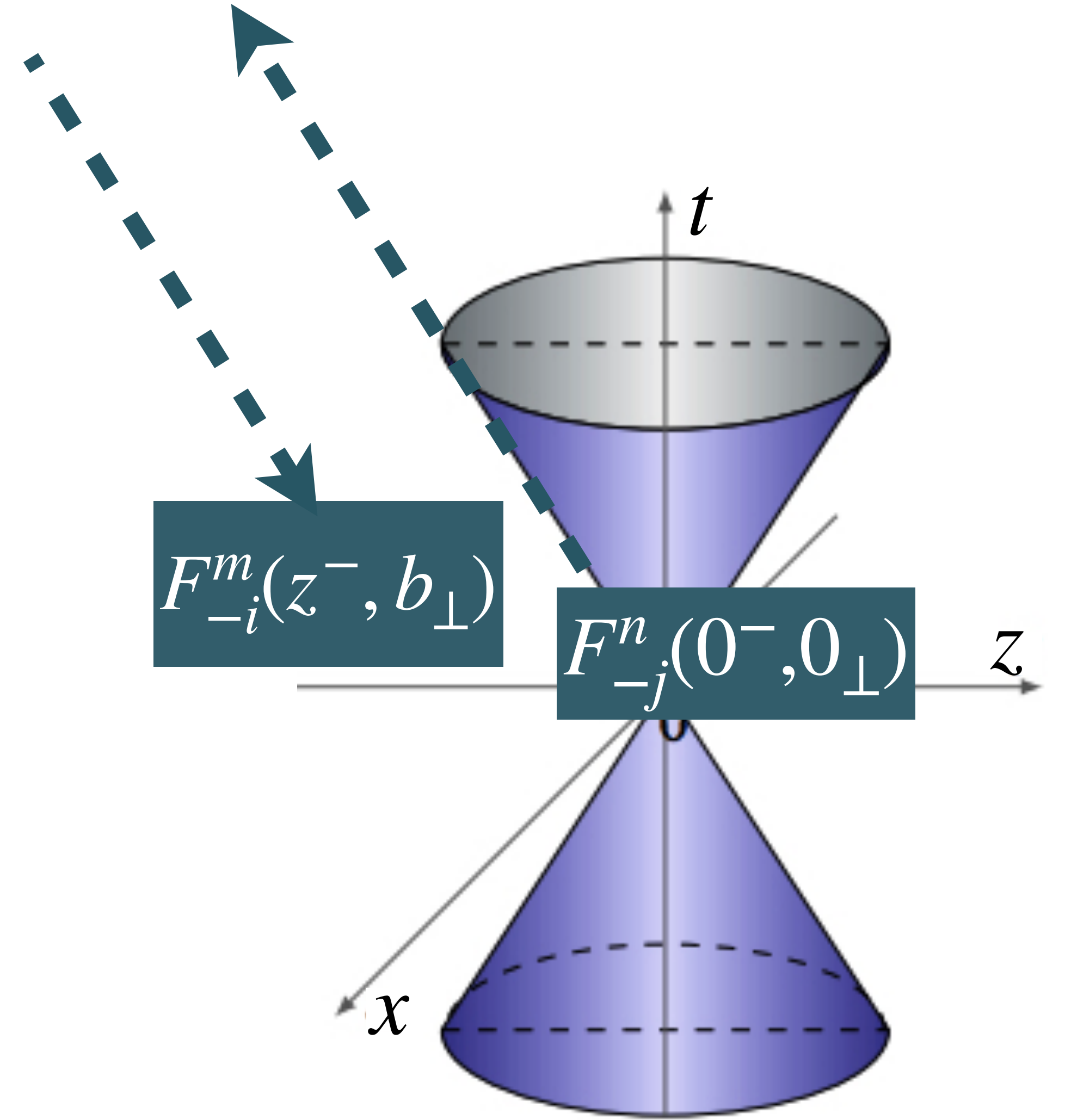
Unified description of DGLAP, CSS, and BFKL evolution: TMD factorization bridging large and small x

Swagato Mukherjee^{1,*}, Vladimir V. Skokov^{2,†}, Andrey Tarasov^{2,3,‡} and Shaswat Tiwari^{2,§}

new transverse-momentum dependent (TMD) factorization
unifying IR structures at large and small x

(1) gluon TMD beam function:

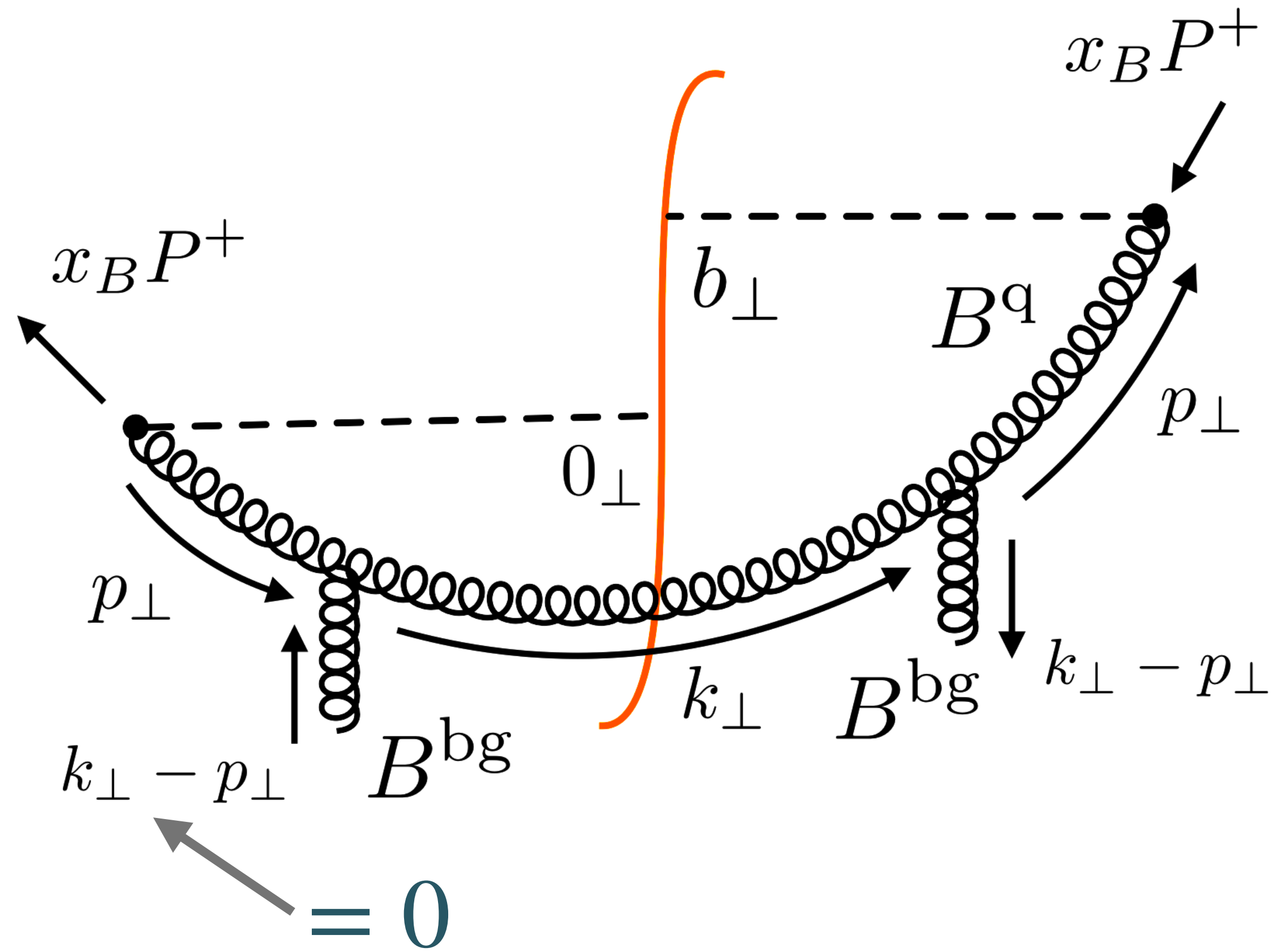
$$f_{ij} =$$



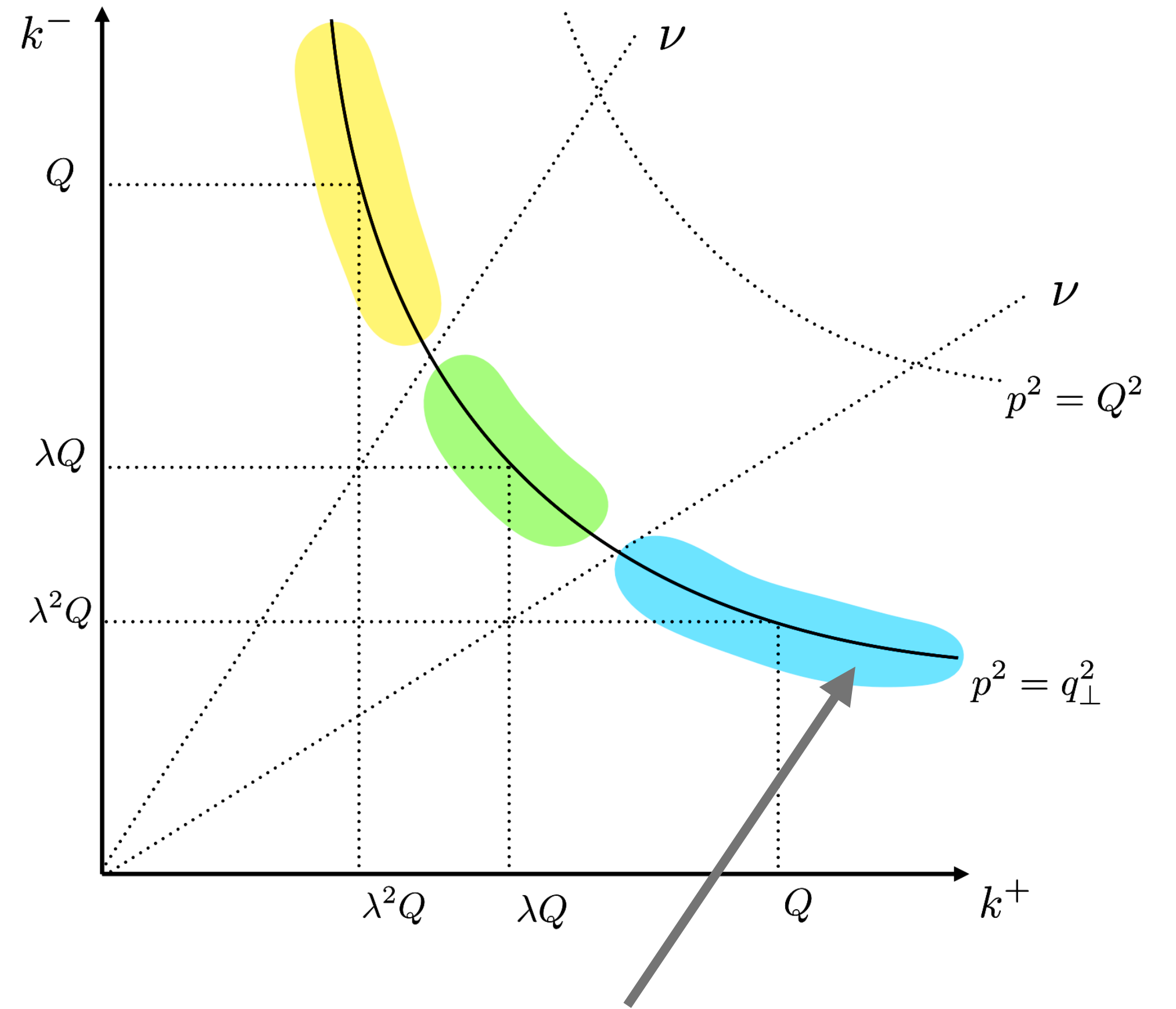
(1) gluon TMD beam function: f_{ij}

(2) $\mathcal{O}(\alpha_s)$ corrections in dilute background of 2 gluons

CSS/SCET factorization ...



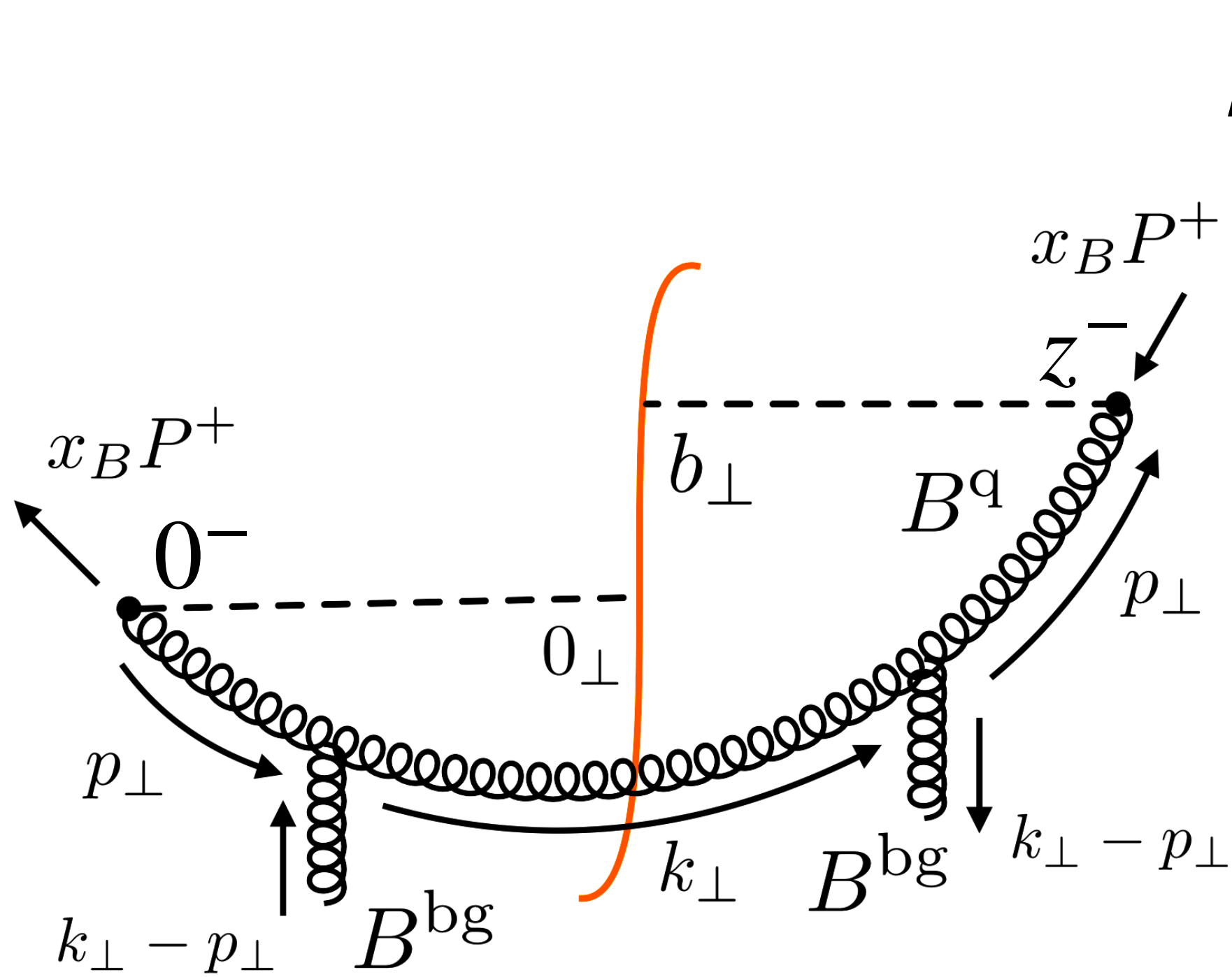
no trans. mom. supplied
by the background field



collinear modes are
approx. on mass-shell
no virtual corrections

... vs. MSTT

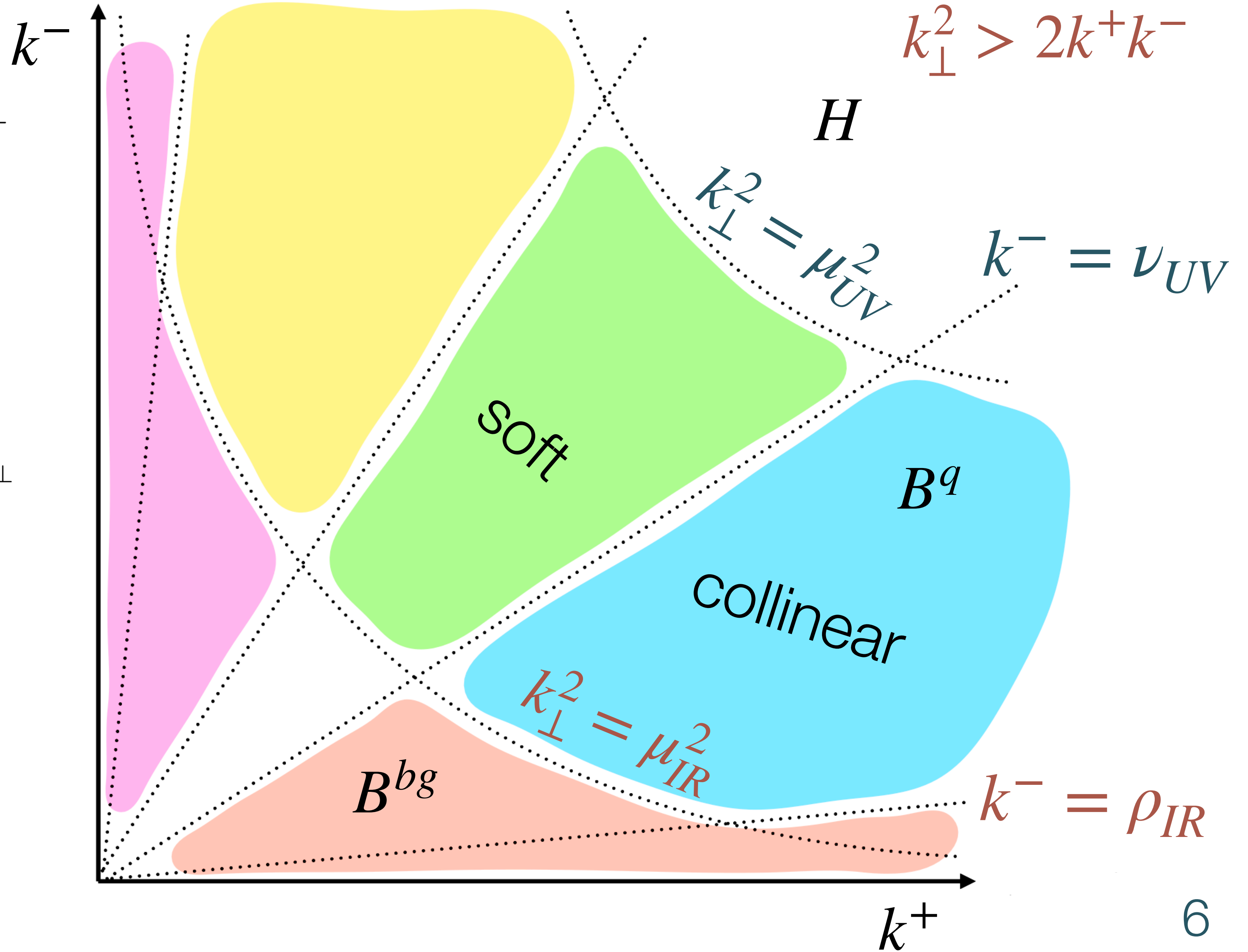
include virtual corrections



$$k_{\perp} - p_{\perp} > 0$$

trans. mom. supplied by the background field

$$k_{\perp}^2 > 2k^+k^-$$



(1) gluon TMD beam function: f_{ij}

(2) $\mathcal{O}(\alpha_s)$ corrections in background of 2 gluons

(3) f_{ij} @ $\mathcal{O}(\alpha_s)$:

(a) real + virtual corrections

(b) UV + IR divs. in k_{\perp} , UV + IR divs. in k^{-}

- (1) gluon TMD beam function: f_{ij}
- (2) dilute background of 2 gluons
- (3) f_{ij} @ $\mathcal{O}(\alpha_s)$: real+virtual, UV+IR divs. in k_{\perp} & k^{-}
- (4) regulate divergences:
 - (a) dim. reg. for k_{\perp}
 - (b) η -scheme for k^{-}

(1) gluon TMD beam function: f_{ij}

(2) dilute background of 2 gluons

(3) f_{ij} @ $\mathcal{O}(\alpha_s)$: real+virtual, UV+IR divs. in k_{\perp} & k^{-}

(4) regulate divs: dim. reg. for k_{\perp} , η -scheme for k^{-}

(5) renormalize $f_{ij} \rightarrow f_{ij}^R$:

(a) gluon wave function renormalization for UV div in k_{\perp}

(b) TMD soft function for UV div in k^{-}

- (1) gluon TMD beam function: f_{ij}
- (2) dilute background of 2 gluons
- (3) f_{ij} @ $\mathcal{O}(\alpha_s)$: real+virtual, UV+IR divs. in k_{\perp} & k^{-}
- (4) regulate divs: dim. reg. for k_{\perp} , η -scheme for k^{-}
- (5) renormalized f_{ij}^R : gluon wave function renormalization, TMD soft function

$$f_{ij}^R(x_B, b_{\perp}, \mu_{UV}, \zeta) = C^{NLO}(z, z_{\perp}, \mu_{UV}, \zeta, \mu_{IR}, \rho_{IR}) \otimes f_{ij}^{BG}(zx_B, z_{\perp} b_{\perp}, \mu_{IR}, \rho_{IR})$$

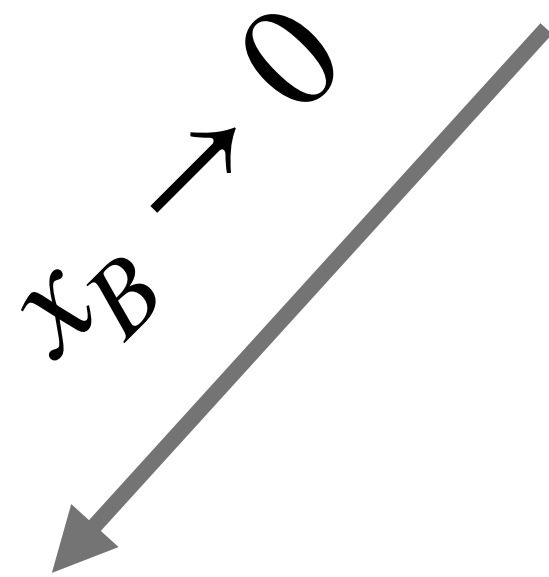
rapidity $\sim \zeta = x_B P^+$

summary: MSTT factorization

2-gluon background field corrections
to gluon TMDPDF operator

$$C^{NLO} \left(\ln[b_{\perp}\mu_{UV}], \ln[b_{\perp}\mu_{IR}], \ln[\mu_{UV}/\zeta], \ln[\rho_{IR}/\zeta] \right)$$

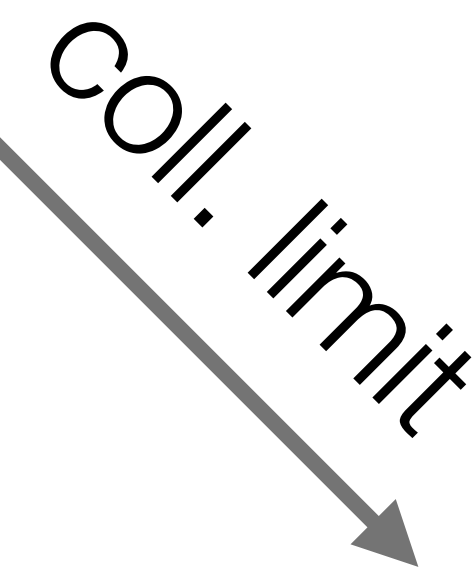
general structure involving IR & UV in rapidity,
new general evolution



$$\text{BFKL} \left(\ln[b_{\perp}\mu_{IR}], \ln[\rho_{IR}/\zeta] \right)$$

$$+ \text{CSS} \left(\ln[b_{\perp}\mu_{UV}], \ln[\mu_{UV}/\zeta] \right)$$

$$\otimes \text{Weizsaecker-Williams} \left(x_B = 0, b_{\perp} \right)$$



$$\text{DGLAP} \left(\ln[b_{\perp}\mu_{IR}] \right)$$

$$+ \text{CSS} \left(\ln[b_{\perp}\mu_{UV}], \ln[\mu_{UV}/\zeta] \right)$$

$$\otimes \text{PDF} \left(x_B, b_{\perp} = 0 \right)$$