NLO calculation for inclusive back-to-back dihadron in DIS in the saturation regime

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Why is TMD factorisation at small x interesting?

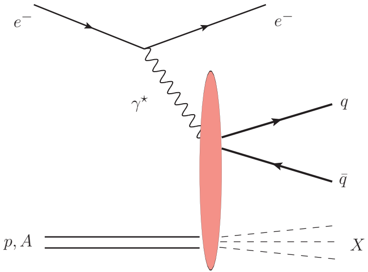
- Study the matching between small x and moderate x.
  $\Rightarrow$ relevant for EIC phenomenology since x_g not necessarily $\ll \mathcal{O}(0.01)$.
- Simplify the numerical evaluation of cross-sections at small x.
  $\Rightarrow$ TMDs are in general easier to evaluate than CGC correlators of Wilson lines.
- Provide analytic expressions for the various TMDs in the saturation regime.
  $\Rightarrow$ give pQCD motivated initial conditions to CSS evolution.

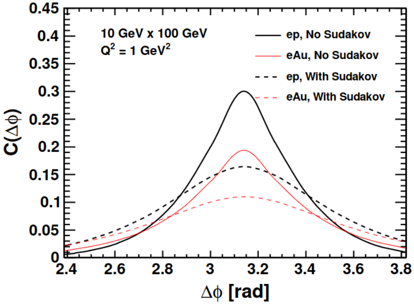
Outline

- TMD factorisation well established at LO for many processes at small x.
  $pA \to H + X$, $pA \to \gamma j + X$, $eA \to j + X$, $eA \to jj + X$, etc
- Improve the state of the art to NLO / NLO+Sudakov resummation. In this talk:
  - dihadron production in DIS
  - single-inclusive jet production in DIS. *(if enough time)*

Back-to-back di-jets in DIS

⇒ access to the Weizsäcker-Williams gluon TMD $G(x, \boldsymbol{q}_\perp)$ in the back-to-back limit.



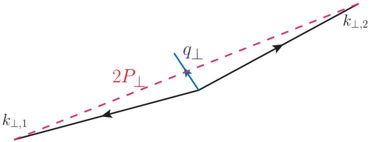


[Zheng, Aschenauer, Lee, Xiao, PRD 89 (2014) 7]

⇒ Similar pheno signal of saturation as in pA, cf STAR results and Zilong Chang's talk.

Kinematics and LO cross-section

- Def: $|\boldsymbol{P}_\perp| = |z_2\boldsymbol{k}_{\perp,1} - z_1\boldsymbol{k}_{\perp,2}| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_{\perp,1} + \boldsymbol{k}_{\perp,2}|$



- In this limit, LO CGC x-section yields TMD factorization [Dominguez, Marquet, Xiao, Yuan, PRD 83 (2011)]

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}^{ij}(\boldsymbol{P}_\perp)G^{ij}(x,\boldsymbol{q}_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

See also [del Castillo, Echevarria, Makris, Scimemi, JHEP 01 (2021) 088]

- $G(x,\boldsymbol{q}_\perp)$: WW gluon TMD

$$G^{ij}(x,\boldsymbol{q}_\perp) = \frac{-2}{\alpha_s}\int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}\left\langle \mathrm{Tr}\left[\partial^i V^\dagger(\boldsymbol{b}_\perp)V(\boldsymbol{b}'_\perp)\partial^j V^\dagger(\boldsymbol{b}'_\perp)V(\boldsymbol{b}_\perp)\right]\right\rangle_x$$

$\Rightarrow$ can be obtained from the usual operator definition by taking the limit $x \to 0$.

Dominant NLO contributions

- High energy logs $\ln(1/x_g)$, with $x_g = (M_{q\bar{q}}^2 + Q^2)/(W^2 + Q^2)$.

  $\Rightarrow$ resummed via JIMWLK evolution of $G(x, \boldsymbol{q}_\perp)$.

- Sudakov logarithms $\ln^2(P_\perp^2/q_\perp^2)$, $\ln(P_\perp^2/q_\perp^2)$

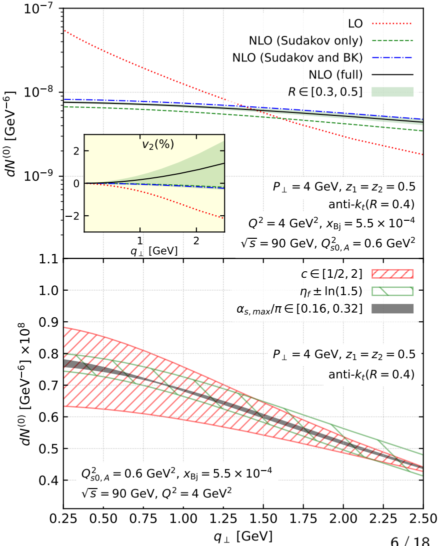
  $\Rightarrow$ first computed at DLA in [Mueller, Xiao, Yuan, PRD 88 (2013)], calculation revisited in more details in [Taels, Altinoluk, Beuf, Marquet, JHEP 10 (2022) 184 and PC, Salazar, Schenke, Venugopalan, JHEP 11 (2022) 169]

  $\Rightarrow$ resummed in a Sudakov soft factor/CSS evolution of the WW gluon TMD.

TMD factorisation at NLO for back-to-back dijet production at small x

[PC, Salazar, Schenke, Stebel, Venugopalan, PRL 132 (8), 081902]

$$\langle \mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}} \rangle_{x_f} = \mathcal{H}^{\lambda,ii}_{\mathrm{LO}} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}^0(x_f, \boldsymbol{r}_{bb'})$$
$$\times \left\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \left[-\frac{N_c}{4} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) - s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) \right.\right.$$
$$\left.\left. + \beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \mathcal{C}^{\lambda}(Q/M_{q\bar{q}}, z_1, R, x_f/x_g) \right] \right\}$$



- x_f dependence of the gluon TMD given by collinearly improved BK-JIMWLK evolution (or collinear improved BFKL in the limit $q_\perp \gg Q_s$).
  Orthogonal approach to small-x improved DGLAP, cf Federico Silvetti talk.
- First line should be exponentiated à la CSS to resum large double and single Sudakov logs.
- $s_L = -C_F \ln(z_1 z_2) + N_c \ln(1 + Q^2/M^2_{q\bar{q}}) - C_F \ln(R^2)$
  $\Rightarrow$ agreement with [Hatta, Xiao, Yuan, Zhou, PRD 104 (2021) 5]

What about dihadron production?

- Instead of measuring two jets, measure two back-to-back hadrons.
- Phenomenologically more interesting at the EIC, as $P_\perp$ can be smaller, $\mathcal{O}(1)$ GeV.
- For dihadron, final state collinear divergence is canceled by the DGLAP evolution of the collinear fragmentation function.
- Schematically, after rapidity factorisation thanks to collinearly improved BK-JIMWLK,

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^\star+A\to h_1 h_2+X}}{\mathrm{d}^2\boldsymbol{k_{h1\perp}}\,\mathrm{d}^2\boldsymbol{k_{h2\perp}}\,\mathrm{d}\eta_{h1}\mathrm{d}\eta_{h2}}\right|_{\mathrm{NLO}} = \int_0^1 \frac{\mathrm{d}\zeta_1}{\zeta_1^2}\int_0^1 \frac{\mathrm{d}\zeta_2}{\zeta_2^2} D_{h_1/q}(\zeta_1,\mu_F^2) D_{h_2/q}(\zeta_2,\mu_F^2)$$
$$\times\left[\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^\star+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\,\mathrm{d}^2\boldsymbol{k_{2\perp}}\,\mathrm{d}\eta_1\mathrm{d}\eta_2}\right|_{\mathrm{LO}} + \underbrace{\alpha_s\mathcal{I}(\boldsymbol{k_{1\perp}},\boldsymbol{k_{2\perp}},z_1,z_2;x_f,\mu_F)}_{\textbf{finite}\ \text{NLO impact factor}}\right]_{k_i^\mu=k_{h,i}^\mu/\zeta_i}$$

[See Bergabo, Jalilian-Marian, JHEP 01 (2023) 095], Fucilla, Grabovsky, Li, Szymanowski, Wallon, JHEP 03 (2023) 159 (diffractive case)]

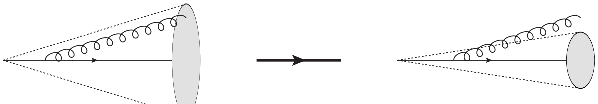
Sudakov double logs in dihadron production

[PC, Salazar, arXiv:2405.19404]

- The limit $P_\perp \gg q_\perp$ of the NLO impact factor $\mathcal{I}$ contains the leading double Sudakov logarithm:

$$\alpha_s \mathcal{I} = -\frac{\alpha_s}{\pi}\left[\frac{C_F}{4} + \frac{C_F}{4} + \frac{N_c}{4}\right]\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \ldots$$

- Sudakov double log **differs** from the dijet case via the two red $C_F/4$ coefficients.
- Physical interpretation: in-cone gluon radiations now contribute to the imbalance. For $R^2 \sim q_\perp/P_\perp$, dijet $\sim$ dihadron.



$q_\perp^2/P_\perp^2 \ll R \ll 1$ $\quad\longrightarrow\quad$ $R \sim q_\perp^2/P_\perp^2 \ll 1$

NLO result in the TMD limit

- The back-to-back limit of the NLO impact factor for dihadron production gives

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^\star+A\to h_1 h_2+X,(0)}}{\mathrm{d}^2\boldsymbol{k_{h1\perp}}\mathrm{d}^2\boldsymbol{k_{h2\perp}}\mathrm{d}\eta_{h1}\mathrm{d}\eta_{h2}}\right|_{\mathrm{NLO}} = \int_0^1 \frac{\mathrm{d}\zeta_1}{\zeta_1^2}\int_0^1 \frac{\mathrm{d}\zeta_2}{\zeta_2^2} D_{h_2/\bar{q}}(\zeta_2,\mu_F^2) D_{h_1/q}(\zeta_1,\mu_F^2)$$
$$\times\, \mathcal{H}_{\mathrm{LO}}^{\lambda,0}\int\frac{\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}}\hat{G}^0(x_f,\boldsymbol{r}_{bb'})\frac{\alpha_s}{\pi}\left\{-\left[\frac{C_F}{2}+\frac{N_c}{4}\right]\ln^2\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right)\right.$$
$$\left.+\left[C_F\left(\ln(z_1 z_2)+\frac{3}{2}\right)-N_c\ln(1+\chi^2)\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{bb'}^2}{c_0^2}\right)+\mathcal{C}_{\overline{\mathrm{MS}}}^{\lambda}\left(\frac{x_g}{x_f},\frac{\mu_F r_{bb'}}{c_0};Q/M_{q\bar{q}},z_1,z_2\right)\right\},$$

- Single log computed as well, different from the jet case.
- No large logs in $\mathcal{C}_{\overline{\mathrm{MS}}}^{\lambda}$.

Resummation of new double log requires TMD fragmentation function!

- Preserve the universality of the Sudakov factor for the WW gluon TMD.
- Promote the collinear FF into TMD one.

$$D_{h/q}(z,\mu_F) \to \overline{D}_{h/q}(z,\boldsymbol{q}_\perp;\mu_F,\xi)$$

- CSS evolution with ξ of the TMD fragmentation function takes care of the new double logs! In coordinate space,

$$\overline{D}_{h/q}(\zeta,\boldsymbol{r}_{bb'};\mu_F,P_\perp) = D_{h/q}(\zeta,\mu_F)\exp\left(-\frac{\alpha_s C_F}{\pi}\int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\boldsymbol{P}_\perp^2}\frac{\mathrm{d}\mu^2}{\mu^2}\left[\frac{1}{2}\ln\left(\frac{Q^2}{\mu^2}\right)-\frac{3}{4}\right]\right)$$

with $\mu_F \sim c_0/r_{bb'} \sim q_\perp$.

- DGLAP evolution of FF from $\mathcal{O}(\Lambda_{\mathrm{QCD}}) \to \mu_F \sim c_0/r_{bb'}$ and CSS evolution from $c_0/r_{bb'} \to P_\perp$.

Final NLO+resummation result for back-to-back dihadron in DIS

- Resummed expression $\mathrm{d}\sigma = \mathcal{H} \otimes G_{\mathrm{WW}} \otimes \overline{D}_{h_1/q} \otimes \overline{D}_{h_2/\bar{q}}$ involves 3 resummations:
  - small x + CSS evolution of $\hat{G}^0(x_f, \boldsymbol{r}_{bb'}; P_\perp)$
  - DGLAP + CSS evolution of $\overline{D}_{h/q}(\zeta, \boldsymbol{r}_{bb'}; \mu_F, P_\perp)$

$$\mathrm{d}\sigma^{\gamma^\star + A \to h_1 h_2 + X}\Big|_{\mathrm{NLO}} = \int_0^1 \frac{\mathrm{d}\zeta_1}{\zeta_1^2} \int_0^1 \frac{\mathrm{d}\zeta_2}{\zeta_2^2} \mathcal{H}^\lambda_{\mathrm{LO}}(\boldsymbol{P}_\perp, Q, z_1, z_2)$$
$$\times \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}^0(x_f, \boldsymbol{r}_{bb'}; P_\perp) \overline{D}_{h_2/\bar{q}}(\zeta_2, \boldsymbol{r}_{bb'}; \mu_F, P_\perp) \overline{D}_{h_1/q}(\zeta_1, \boldsymbol{r}_{bb'}; \mu_F, P_\perp)$$
$$\times \left\{ 1 + \frac{\alpha_s}{\pi} \mathcal{C}^\lambda_{\overline{\mathrm{MS}}}\left(\frac{x_g}{x_f}, \frac{\mu_F r_{bb'}}{c_0}; Q/M_{q\bar{q}}, z_1, z_2 \right) + \mathcal{O}(\alpha_s^2) \right\}$$

- NLO coefficient function in $\overline{\mathrm{MS}}$ scheme $\mathcal{C}^\lambda_{\overline{\mathrm{MS}}}$ analytically known.
  $\Rightarrow$ straightforward (future) numerical implementation.

Single inclusive jet production in DIS

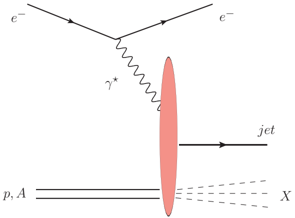
[PC, Iancu, Mueller, Yuan, 2408.03129]

⇒ Measure **jets** in DIS events and bin in terms of $P_\perp$ measured in Breit of dipole frame:

$$\frac{\mathrm{d}\sigma^{eA\to \mathrm{e'+jet}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}P_\perp}$$

⇒ In the case of a hadron measurement, see Cyrille Marquet's talk on Monday.

⇒ Also accesses the sea quark TMD at small x in the limit $Q^2 \gg \boldsymbol{P}_\perp^2$.

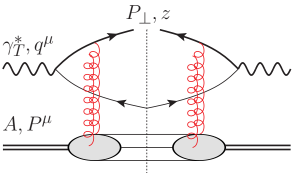


TMD factorisation in SIDIS at LO

For $Q^2 \gg P_\perp^2$, the small x expression [Mueller (1990), Nikolaev and Zakharov (1991)]

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^{\star}A\to\mathrm{jet}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}z}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^4}\,2[z^2+(1-z)^2]\int_{\boldsymbol{b}_\perp,\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y}} e^{-i\boldsymbol{P}_\perp\cdot(\boldsymbol{r}_{xy}-\boldsymbol{r}_{x'y})}\left\langle 1-D_{yx'}-D_{xy}+D_{xx'}\right\rangle_x$$

$$\times\ \bar{Q}^2K_1(\bar{Q}r_{x'y})K_1(\bar{Q}r_{xy})\frac{\boldsymbol{r}_{x'y}\cdot\boldsymbol{r}_{xy}}{r_{x'y}r_{xy}}\ ,\quad \text{with }\ \bar{Q}^2=z(1-z)Q^2$$



$\Rightarrow$ factorises in terms of the (sea) quark TMD depending, which itself depends on the gluon dipole $\mathcal{D}(x,\boldsymbol{q}_\perp)$ [Marquet, Xiao, Yuan, PLB 682, 207 (2009)]

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^{\star}+A\to\mathrm{jet}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\times\underbrace{\frac{N_c}{\pi^2}\int_{\boldsymbol{b}_\perp}\int\frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2}\mathcal{D}(x,\boldsymbol{q}_\perp)\left[1-\frac{\boldsymbol{P}_\perp\cdot(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)}{(P_\perp^2-(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2)}\ln\frac{P_\perp^2}{(\boldsymbol{P}_\perp-\boldsymbol{q}_\perp)^2}\right]}_{\text{sea quark TMD}}$$

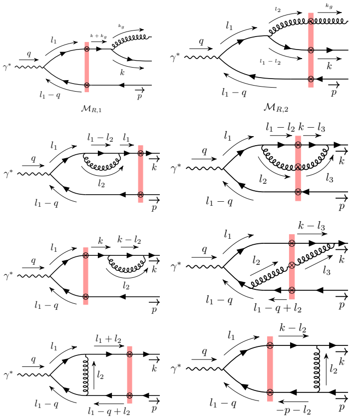
$\Rightarrow$ is dominated by aligned jet configurations, i.e. $1-z\sim\boldsymbol{P}_\perp^2/Q^2$ or $z\sim\boldsymbol{P}_\perp^2/Q^2$.

Outline of the NLO computation

- NLO calculation at small x for general **jet** kinematics performed in [PC, Ferrand, Salazar, JHEP 05 (2024) 110].
  For single hadron, see [Bergabo, Jalilian-Marian, JHEP 01 (2023) 095 (inclusive), Fucilla, Grabovsky, Li, Szymanowski, Wallon, JHEP 02 (2024) 165 (diffractive)]
- Compute the NLO impact factor in the limit $Q^2 \gg P_\perp^2 \gg Q_s^2$.

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \left[1+\alpha_s\mathcal{I}(\boldsymbol{P}_\perp, Q, R, x_f)\right]$$

- High energy factorisation with collinearly improved BK/BFKL. [Altinoluk, Jalilian-Marian, Marquet, 2406.08277]
- NLO impact factor depends on the jet definition.



NLO Feynman graphs

Jet definition in DIS

- Several jet definitions are possible in DIS.
- Not all of them ensure factorisation of the fully inclusive **jet** cross section.

  [Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)]
- Same phenomenon arises for TMD factorisation.

Jet def	distance measure	dipole frame NLO clustering condition ($R \ll 1$)
LI C/A	$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$	$\frac{M_{ij}^2 z^2}{\boldsymbol{P}_\perp^2 R^2} \leq 1$
SI C/A	$d_{ij} = \frac{1-\cos(\theta_{ij})}{1-\cos(R)}$ **in Breit frame**	$\frac{M_{ij}^2 z^2}{Q^2 R^2} \leq 1$
new LI jet def	$d_{ij} = M_{ij}^2/(z_i z_j Q^2 R^2)$	$\frac{M_{ij}^2}{z_i z_j Q^2 R^2} \leq 1$

- Goal: find a jet definition which ensures TMD factorisation of the single inclusive jet cross-section.

Jet definition and Sudakov logarithms

- NLO Sudakov logs $L = \ln(Q^2/P_\perp^2)$ depend on the jet definition!

For LI C/A (or anti-k_t):

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j(A)+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{3}{4}L^2 + \left(\frac{3}{4} - \ln(R)\right)L + \mathcal{O}(1)\right]$$

while for SI C/A ($\beta = 2$) and our new jet definition ($\beta = 0$)

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j(B)+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \left(\frac{3(1-\beta/2)}{4} + \ln(R)\right)L + \mathcal{O}(1)\right]$$

- From CSS evolution of the quark TMD alone, we expect the log structure

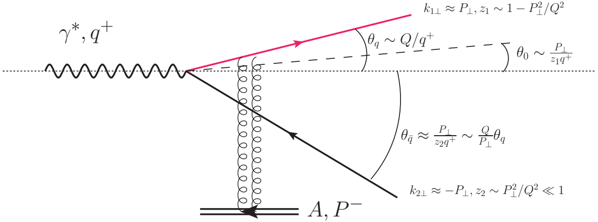
$$\frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \frac{3}{4}L\right]$$

$\Rightarrow$ TMD factorisation implies $\beta = 0$.
New LI jet definition in DIS suitable for TMD factorisation with jets.

- Note that Sudakov DL for a jet measurement is half the DL for hadron measurement.

Physical interpretation

- Our new clustering condition equivalent to $\theta_{ij} \leq R\theta_{\text{jet}}$ with $\theta_{\text{jet}} \sim Q/q^+$.
- Angle of the jet set by its virtuality rather by its transverse momentum.
  (Naively, $\theta_{\text{jet}} \sim \frac{P_\perp}{zq^+}$.)
- Soft gluons contributing to Sudakov must have $\theta_g \gg \theta_{\text{jet}}$.
  $\Rightarrow$ stronger constraint than $\theta_g \gg \frac{P_\perp}{zq^+}$!



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Aligned jet geometric configuration in dipole frame.

Conclusion

- NLO small x calculations are consistent with the TMD formalism at moderate x (CSS) provided
  - $\Rightarrow$ one uses collinearly improved small x evolution equations,
  - $\Rightarrow$ TMD fragmentation functions are used for final state hadronization.

- NLO small x calculations in the TMD limit can help us improve on TMD factorisation theorems at moderate x.

- Example with single inclusive jet production in DIS where TMD factorisation demands specific jet definition.

- Recent NLO calculation at small x open new interesting questions: interplay between small x/DGLAP/CSS evolution [Mukherjee, Skokov, Tarasov, Tiwari, PRD 109 (2024), PC, Iancu, 2408.03129], functional form of the NLO coefficient function in high energy factorisation, etc