~~Un~~Expected breakdown of collinear factorisation at leading twist in exclusive $\pi^0 - \gamma$ photoproduction due to Glauber pinch

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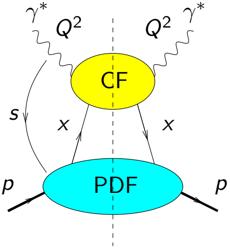
Based on 2311.09146 with Saad Nabeebaccus, Jakob Schönleber, Lech Szymanowski

- Deep Inelastic Scattering DIS: inclusive process

  $\Rightarrow$ 1-dimensional structure

  $\Rightarrow$ Collinear factorisation at the *cross section* level

Coefficient Function $\otimes$ Parton Distribution Function
(hard) (soft)



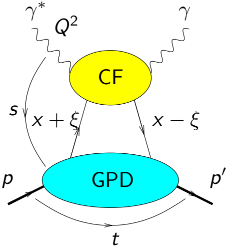
DVCS: exclusive process (non forward amplitude)

Fourier transf.: $t \leftrightarrow$ impact parameter

$\Rightarrow$ 3-dimensional structure

Collinear factorisation implies,
at the *amplitude* level:

Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)



x: *Average* mom. fraction of the nucleon carried by the parton
ξ: Mom. fraction of the nucleon *transferred* to hard part

[X. Ji: hep-ph/9609381]

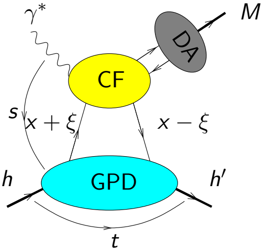
[A. Radyushkin: hep-ph/9604317, hep-ph/9704207]

[J. Collins, A. Freund: hep-ph/9801262]

[D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi: hep-ph/9812448]

DVMP: γ replaced by $\rho,\ \pi, \cdots$

GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)



[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

[A. Radyushkin: hep-ph/9704207]

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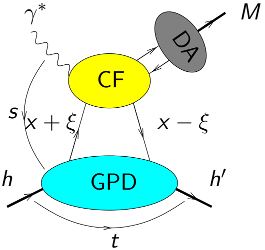


[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

[A. Radyushkin: hep-ph/9704207]

proofs valid only for some restricted cases

Original motivation: Extraction of chiral-odd GPDs at *leading* twist.

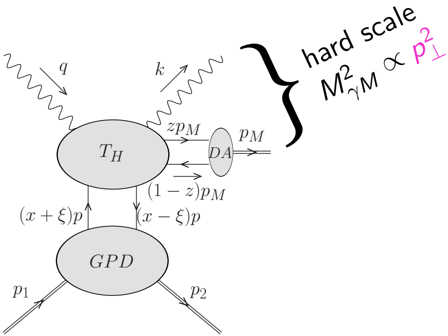
- $\gamma N \rightarrow \rho^0_T \pi^+ N'$:
  M. El Beiyad, B. Pire, M. Segond, L. Szymanowski, SW: [1001.4491]
- $\gamma N \rightarrow \gamma M N'$:
  - $M = \rho^0$: R. Boussarie, B. Pire, L. Szymanowski, SW: [1609.03830]
  - $M = \pi^\pm$: G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, SW: [1809.08104]
  - $M = \pi^\pm$, $\rho^{0,\pm}$, wider kinematical coverage, various observables:
    G. Duplančić, S. Nabeebaccus, K. Passek-Kumerički, B. Pire, L. Szymanowski, SW: [2212.00655, 2302.12026]

Richer kinematics of 3-body final state processes allows the sensitivity of GPDs wrt x to be probed (beyond moment-type dependence, e.g. in DVCS)
J. Qiu, Z. Yu: [2305.15397]

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M) + N'(p_2)$$

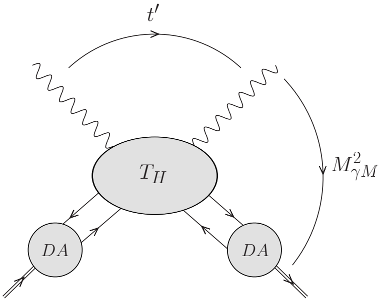
$$\mathcal{A} = \int_{-1}^{1} dx \int_0^1 dz \; T(x,\xi,z) \; H(x,\xi,t) \; \Phi_M(z)$$

- *Fully differential* cross-section differential covering $S_{\gamma N}$ from $\sim 4\,\text{GeV}^2$ to $20000\,\text{GeV}^2$.
- Good statistics at various experiments, particularly at *JLab*.
- Polarisation asymmetries also sizeable.
- *Small* ξ limit of quark GPDs can be studied at collider experiments.



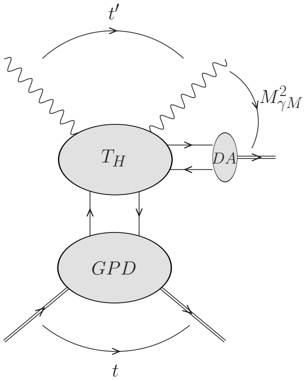
Introduction

Is collinear factorisation justified?



large angle factorisation

à la Brodsky Lepage



We thus argue *collinear factorisation* of the amplitude at large $M^2_{\gamma M}$, t', u', and small t.

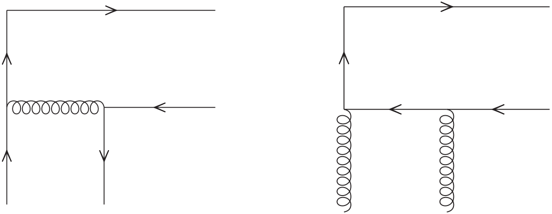
$$t = (p_2 - p_1)^2 \,, \qquad u' = (p_M - q)^2 \,,$$

$$t' = (k - q)^2 \,, \qquad S_{\gamma N} = (q + p_1)^2 \,.$$

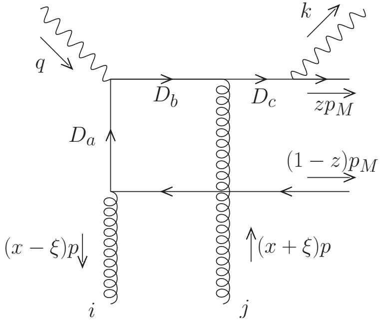
- Recently, factorisation has been proved for the process $\pi N \to \gamma\gamma N'$ by J. Qiu, Z. Yu [2205.07846].
- This was extended to a wide range of $2 \to 3$ exclusive processes by J. Qiu, Z. Yu [2210.07995]
- The proof relies on having large p_T, rather than large invariant mass (e.g. photon-meson pair).
- In fact, NLO computation has been performed for $\gamma N \to \gamma\gamma N'$ by O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, J. Wagner [2110.00048, 2204.00396]
- Also, NLO computation for $\gamma\gamma \to \pi^+\pi^-$ by crossing symmetry G. Duplancic, B. Nizic: [hep-ph/0607069].

Issues with exclusive $\pi^0\gamma$ photoproduction...

- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.
- A total of 24 diagrams contribute in this case (compared to 20 diagrams from quark GPD contributions), with 6 groups of 4 related by symmetries ($x \to -x$ and $z \to 1-z$ separately).
- Diagrams amount to connecting photons to the following two topologies:



Result assuming collinear factorisation
Specific diagram



$$CF \sim \frac{\mathrm{Tr}\left[\not{p}_M \gamma^5 \not{\epsilon}_k \left(\not{k} + z\not{p}_M\right)\gamma^j \left(\not{q} - (x-\xi)\not{p} - \bar{z}\not{p}_M\right)\not{\epsilon}_q \left(-(x-\xi)\not{p} - \bar{z}\not{p}_M\right)\gamma^i\right]}{[2z\, kp_M]\left[-2\,(x-\xi)\,qp - 2\bar{z}\,qp_M + 2\bar{z}\,(x-\xi)\,pp_M + i\epsilon\right]\left[2\bar{z}\,(x-\xi)\,pp_M + i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0} \propto \frac{x-\xi}{\left[(x-\xi) + A\bar{z} - i\epsilon\right]\left[\bar{z}\,(x-\xi) + i\epsilon\right]}, \qquad A \equiv \frac{q\cdot p_M}{q\cdot p} > 0.$$

(Assuming p_M is along minus direction)

Result assuming collinear factorisation
Specific diagram

Need to dress coefficient function CF with gluon GPD $\left(\frac{H_g(x)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)}\right)$, and DA $(z\bar{z})$. This gives

$$\mathcal{A} \sim \frac{\bar{z}\,(x-\xi)\,H_g(x)}{(x-\xi+i\epsilon)\,[(x-\xi)+A\bar{z}-i\epsilon]\,[\bar{z}\,(x-\xi)+i\epsilon]}$$

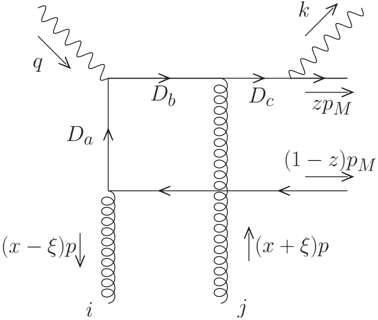
$$\longrightarrow \frac{H_g(x)}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

The integral over z and x diverges if the GPD $H_g(x)$ is non-vanishing at $x=\xi$:

$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$$\supset \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \quad \Longrightarrow \text{divergent imaginary part!}$$

Result assuming collinear factorisation
Specific diagram



$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$\Longrightarrow$ The "*pinching*" is caused by propagators D_a and D_b.

What about the sum of diagrams?

$$\sum \mathcal{A} \sim \frac{z\bar{z}\left(x^2-\xi^2\right)\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+8x^2\xi^2 z\bar{z}\right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{z\bar{z}\left[x-\xi+i\epsilon\right]^2\left[\bar{z}\left(x+\xi\right)-\alpha z\left(x-\xi\right)-i\epsilon\right]\left[z\left(x-\xi\right)+\alpha\bar{z}\left(x+\xi\right)-i\epsilon\right]}$$

$$\times\frac{1}{\left[x+\xi-i\epsilon\right]^2\left[\bar{z}\left(x-\xi\right)+\alpha z\left(x+\xi\right)-i\epsilon\right]\left[z\left(x+\xi\right)-\alpha\bar{z}\left(x-\xi\right)-i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0}\propto \frac{\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+8x^2\xi^2 z\bar{z}+\right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{\left[x-\xi+i\epsilon\right]\left[2\xi\bar{z}-\alpha\left(x-\xi\right)-i\epsilon\right]\left[\left(x-\xi\right)+2\xi\alpha\bar{z}-i\epsilon\right]}$$

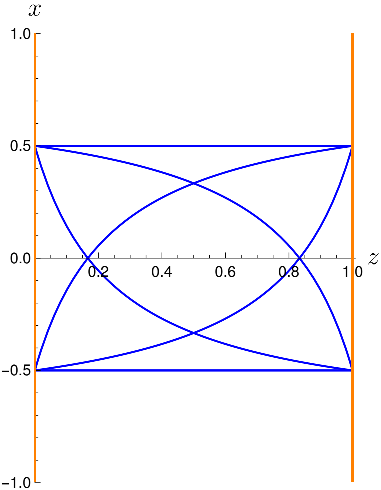
Full amplitude (anti)-symmetric in $x \to -x$ and $z \to \bar{z}$ for (anti)-symmetric GPD (only symmetric result shown above).

$\Longrightarrow$ *divergence survives*, and actually adds up.

Result assuming collinear factorisation

Singularity structure of the full amplitude

'Phase Space' for amplitude

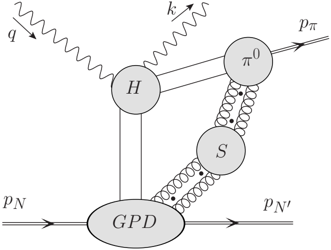


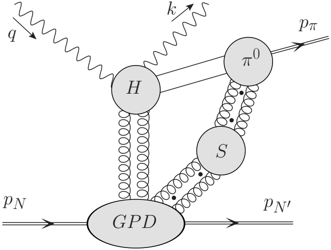
$\xi = 0.5$

- Unfortunately, no cancellations between the 4 corners.
- In $\gamma\gamma \to MM$, only ERBL region exists, no poles are crossed, and endpoint contributions are suppressed by DAs.
- Indication of problem with naive collinear factorisation? At twist-2??
- *Can this divergence be understood from a theoretical point of view?*
  YES! $\Longrightarrow$ [S. Nabeebaccus, J. Schönleber, L. Szymanowski, SW: 2311.09146]

- How to obtain the dominant contribution of an amplitude (in QCD) in a certain specific kinematics (e. g. collinear)?
  $\Longrightarrow$ Libby-Sterman power counting rule [Phys.Rev.D 18 (1978) 3252; Phys.Rev.D 18 (1978) 4737]
- Extensively used in factorisation proofs [Collins: Foundations of perturbative QCD]
- Basic idea is to identify regions of loop momenta of partons (also number of partons), which gives the dominant contribution to the full amplitude.
- Collect all contributions to the *smallest* α:

$$\mathcal{A} = Q^{\beta} \sum_{\alpha} f_{\alpha} \lambda^{\alpha} \,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_{\pi},\, m_N}{Q} \ll 1$$





In both of the above cases, the power counting is:

$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1\,, \qquad \alpha = 1$$

Collinear factorisation at *all orders* and *leading power* provided:

- the above collinear pinch diagrams (standard) are the *only ones contributing to the leading power of $\alpha = 1$*
- the *soft factor S 'cancels'*

Pinches
Landau conditions

Pinches correspond to regions of loop momentum which cannot be avoided through contour deformations.

They can be identified efficiently through Landau conditions:

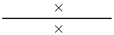
$$I(z) = \lim_{\epsilon \to 0^+} \int_{\mathbb{R}^{dL}} d^{dL}\omega \, \frac{N(\omega, z)}{\prod_{j=0}^{n} (D_j(\omega, z) + i\epsilon)} \, .$$

Given z, $\omega_S \in \mathbb{R}^{dL}$ such that the set

$$\mathcal{D} = \{ j \in \{1, ..., n\} \mid D_j(\omega_S, z) = 0 \}$$

is non-empty, we have a pinch at ω_S iff there exist real and non-negative numbers α_j for $j \in \mathcal{D}$ such that

- $\forall i \in \{1, ..., dL\} \; : \; \sum_{j \in \mathcal{D}} \alpha_j \frac{\partial D_j}{\partial \omega_i}(\omega_S; z) = 0.$

  pinch:  no pinch: 
- At least one of the α_j is non-zero

Note: Existence of pinch does *not* imply existence of a singularity: Need to also perform *power counting*.

Consider the bubble integral, with massless internal lines:

$$I_1(p^2) = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$

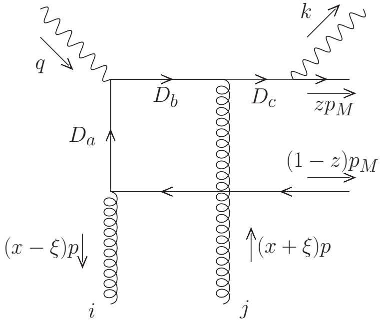
According to the Landau conditions, there is *always* a pinch related to soft momentum k, independent of p.

This is because when $k = 0$, both the propagator $k^2 + i\epsilon$ and its first derivative are zero.

$\implies$ *Landau conditions for a pinch at $k = 0$ are satisfied.*

However, note that the power counting does not give an IR divergence for $p^2 \neq 0$:

$$\implies \frac{[\lambda^4]}{[\lambda^2][1]} \sim \lambda^2$$



Divergence obtained when $(x - \xi)\, p$ and $(1 - z)\, p_M$ lines become soft:

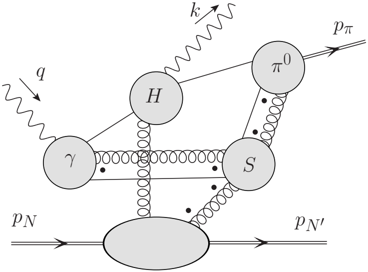
$\implies$ D_a becomes soft and D_b becomes collinear with respect to q.

Is there a *leading* pinch diagram that corresponds to this region?

Yes!

Reduced diagram analysis

Other leading pinch surfaces?



$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \alpha = 1$$

$\Longrightarrow$ power counting is the same as the collinear region!

Note: Corresponding reduced diagram for quark GPD case is power suppressed.

What exactly does the pinch surface correspond to?

- Use Sudakov basis $(+,-,\perp)$:

$$\text{Collinear} \quad k \sim Q\left(1, \lambda^2, \lambda\right) \quad (\text{or} \quad k \sim Q\left(\lambda^2, 1, \lambda\right))$$

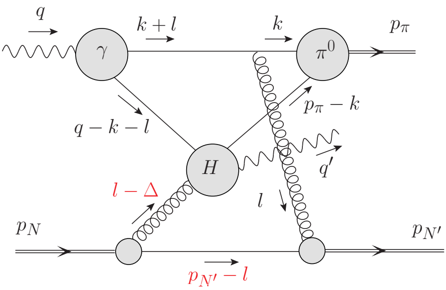
- Need to distinguish between *ultrasoft*, *soft* and *Glauber* gluons:

$$\begin{aligned} &\text{Ultrasoft} && k \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right) \\ &\text{Soft} && k \sim Q\left(\lambda, \lambda, \lambda\right) \\ &\text{Glauber} && k \sim Q\left(\lambda^2, \lambda^2, \lambda\right) \quad (\text{or similar with} \quad |k_\perp^2| \gg k^+ k^-) \end{aligned}$$

note: typical of a small-x gluon in k_T-factorisation

- Libby-Sterman power counting formula strictly applies for *ultrasoft gluons* only.
- However, these are typically eliminated by the use of *Ward identities*.
- Glauber gluons *cannot* be eliminated/suppressed by the use of Ward identities.
- Key Question: Is there a Glauber pinch that contributes at leading power?

Glauber pinch



(Notation: $(+, -, \perp)$)

$p_N,\, p_{N'},\, \Delta \sim Q\left(1, \lambda^2, \lambda\right), \quad \Delta^+ < 0.$

$p_\pi \sim Q\left(\lambda^2, 1, \lambda\right)$

$q,\, k \sim Q\,(1, 1, 1), \quad q^2,\, k^2 \sim \lambda^2 Q^2$

[Loop] $l \sim Q\,(\lambda, \lambda, \lambda)$

[Loop] $r \sim Q\,(\lambda, \lambda, \lambda)$

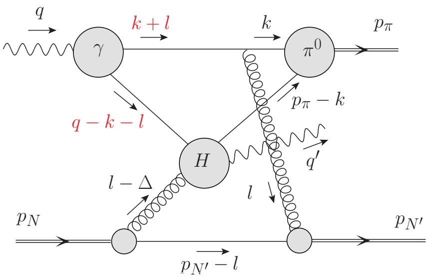
Recall: Soft loop momenta k and l *always* need to be considered.

- *l^- pinch*:

$$(l - \Delta)^2 + i0 = -2\Delta^+ l^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^- = \mathcal{O}(\lambda^2) - i0\,.$$

$$(p_{N'} - l)^2 + i0 = -2p_{N'}^+ l^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^- = \mathcal{O}(\lambda^2) + i0\,.$$

Glauber pinch

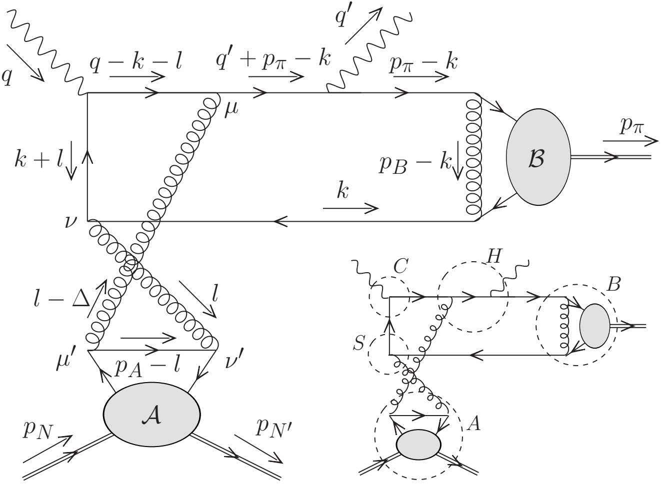


l^+ pinch:

$$(q-k-l)^2 + i0 = -2q^+k^- - 2q^-l^+ + \mathcal{O}(\lambda) + i0$$
$$\implies l^+ = \mathcal{O}(\lambda) + i0\,.$$

$$(k+l)^2 + i0 = 2l^+k^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies l^+ = \mathcal{O}(\lambda) - \mathrm{sgn}(k^-)i0\,.$$

Conclusion: l^+ is pinched to be $\mathcal{O}(\lambda)$, and l^- is pinched to be $\mathcal{O}(\lambda^2)$.
$\implies$ Glauber pinch, since $k^+k^- \ll |k_\perp|^2$.

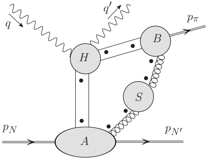


Explicit 2-loop analysis shows that the Glauber pinch demonstrated previously is leading, i.e. it scales as λ^α, with $\alpha = 1$.

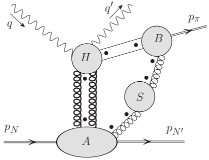
Leading and non leading reduced diagrams

Landau equations $\leftrightarrow$ Coleman-Norton thm $=$ classical scattering process

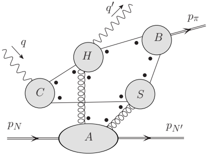
$\Rightarrow$ reduced superficially-leading and super-leading diagrams:



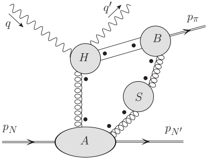
λ^1



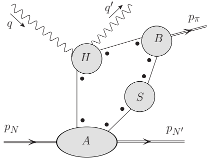
λ^1



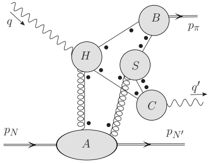
$\lambda^0 \to \lambda^1$ (Ward id. $+$ Glauber)



$\lambda^{-1} \to \lambda^2$ (Ward id.)



$\lambda^1 \to \lambda^2$ (soft-end sup.)

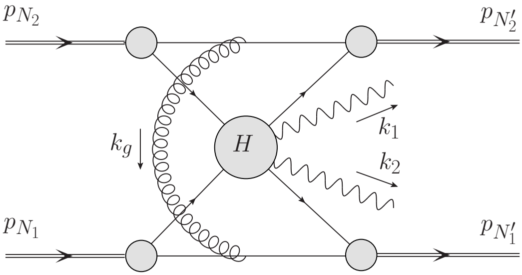


$\lambda^0 \to \lambda^2$ (Ward id.)

dots $=$ any number of scalar polarized gluons $\Rightarrow$ usually Wilson lines in GPD and DA

Very similar to the exclusive double diffractive process, where the Glauber gluon is pinched between the two pairs of incoming and outgoing collinear hadrons.

$$p(p_{N_1}) + p(p_{N_2}) \longrightarrow p(p_{N_1'}) + p(p_{N_2'}) + \gamma(k_1) + \gamma(k_2)$$



Here, the Glauber pinch corresponds to $l \sim (\lambda^2, \lambda^2, \lambda)$

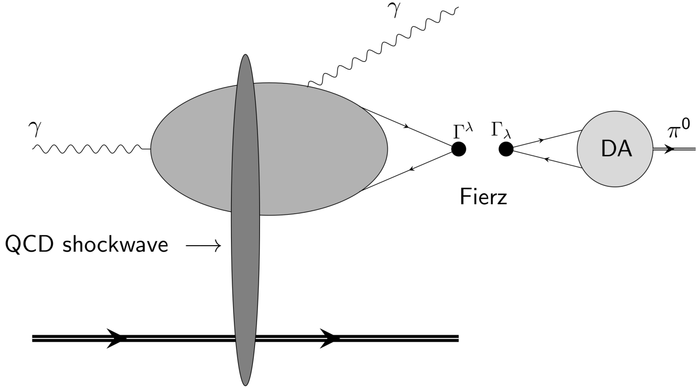
Instead, in our case, the Glauber gluon (which corresponds to one of the active partons) is pinched between *a pair of collinear hadrons*, and *a soft line joining the outgoing pion and the incoming photon*.

$\gamma p \to \gamma \pi^0 p$ at high energy

The same exclusive process can be described at high energy using the QCD shockwave approach.

No divergencies anymore

[M. Fucilla, S. Nabeebaccus, L. Szymanowski, SW, J. Yarwick in progress]



Conclusions

- Collinear factorisation for the exclusive $\pi^0\gamma$ photoproduction *fails* due to *Glauber pinch* in the *gluon exchange channel*.
- Direct calculation assuming collinear factorisation diverges already at leading order and leading twist.
- The same thing happens for the exclusive process $\pi^0 N \to N\gamma\gamma$.
- Channels where *2-gluon exchanges are forbidden* ($\pi^\pm$ and $\rho^{0,\pm}$) are *safe from the effects discussed here*.
- Factorisation breaking effects also expected to occur in specific channels that allow for 2-gluon exchanges in exclusive di-meson photoproduction: $\gamma N \to M_1 M_2 N'$. [ongoing]
- Compute $\gamma N \to \gamma\pi^0 N$ in high-energy (k_T) factorisation. [ongoing]

BACKUP SLIDES

Consider the *triangle* integral, with *massless* internal lines:

$$I_2 = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((k - p_1)^2 + i\epsilon)((k + p_2)^2 + i\epsilon)}.$$

Again, Landau conditions predict the existence of a pinch at $k = 0$.

If $p_1^2 = m_1^2$ and $p_2^2 = m_2^2$, then the power counting predicts a *logarithmic divergence*:

$$\implies \frac{[\lambda^4]}{[\lambda^2][\lambda][\lambda]} \sim \lambda^0$$

This is of course the well-known soft singularity of triangle integrals, where the massless particle connects to two on-shell legs.

Consider the bubble integral, with *massless* internal lines:

$$I_1(p^2) = \lim_{\epsilon \to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$

We apply the Landau conditions:

$$k^2 = 0, \qquad p^2 - 2p \cdot k = 0, \qquad \alpha_1 k + \alpha_2 (k - p) = 0$$
$$\alpha_1, \alpha_2 \geq 0, \quad \alpha_1 + \alpha_2 > 0$$

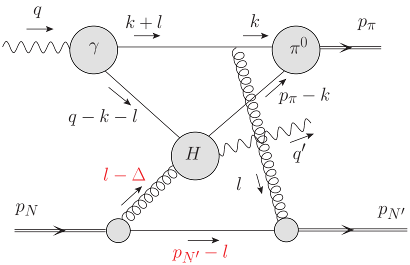
This implies

$$k^2 = 0, \qquad p^2 - 2p \cdot k = 0, \qquad k = \alpha p,$$

where $1 \geq \alpha \geq 0$. This only has a solution if $p^2 = 0$. This is of course nothing but the well-known collinear singularity.

The power counting indicates a *logarithmic divergence*:

$\implies \dfrac{[\lambda^4]}{[\lambda^2][\lambda^2]} \sim \lambda^0$, as expected



Start with $k \sim Q(\lambda_s, \lambda_s, \lambda_s)$, where $\lambda_s \ll 1$, but completely general wrt λ. *Study pole in k^+*:

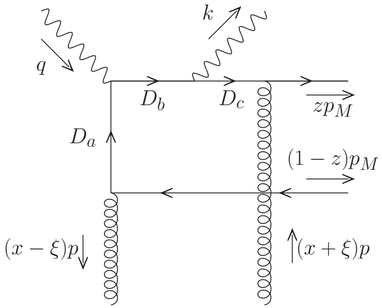
$$k^2 + i0 = 2k^+k^- - |k_\perp|^2 + i0\,,$$
$$\implies k^+ = \mathcal{O}(\lambda_s) - \mathrm{sgn}(k^-)\, i0\,.$$

$$(p_\pi - k)^2 + i0 = -2p_\pi^- k^+ + \mathcal{O}(\max(\lambda^2, \lambda_s^2)) + i0\,,$$
$$\implies k^+ = \mathcal{O}(\max(\lambda^2, \lambda_s^2)) + i0\,.$$

Non-analyticity at $k^- = 0$, and k^+ pinched to be $\mathcal{O}(\lambda_s)$ for $\lambda_s \geq \lambda^2$, or k^+ pinched to be $\mathcal{O}(\lambda^2)$ for $\lambda_s \leq \lambda^2$

Factorisation breaking effects in $\pi^0\gamma$ photoproduction

Gluon GPD contributions



$$D_a = ((x-\xi)p + \bar{z}p_M)^2 + i\epsilon$$
$$= s\bar{\alpha}\bar{z}\left[x - \xi + i\epsilon\right] \ ,$$

$$D_b = (k + zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\left[z\,(x-\xi-i\epsilon) + \alpha\bar{z}\,(x+\xi-i\epsilon)\right] \ ,$$

$$D_c = (zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\bar{\alpha}z\left[x+\xi-i\epsilon\right]$$

$\Longrightarrow$ *pinching of poles in the propagators (D_a and D_b) in the limit of $z \to 1$*