3D nucleon structure from Lattice QCD

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

Supported by the National Science Center of Poland
SONATA BIS grant No. 2016/22/E/ST2/00013 (2017-2022)
OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

Outline:

Introduction

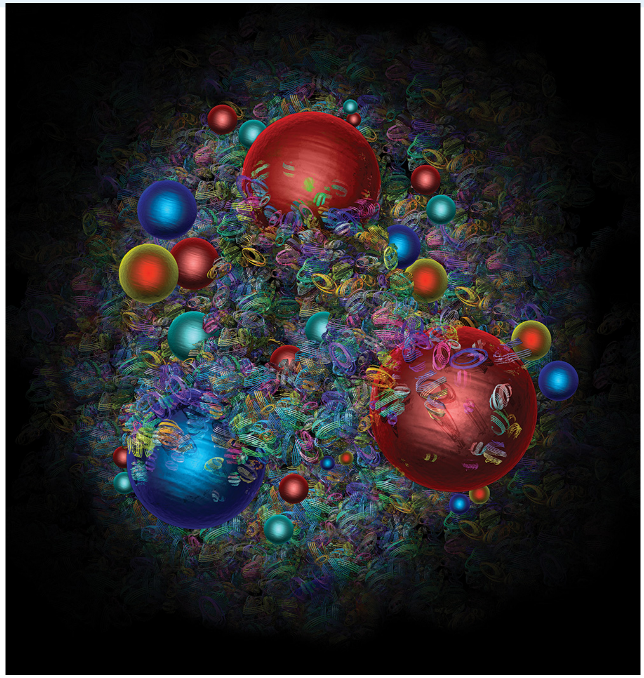
GPDs from lattice:
– how to access
– reference frames
– quasi and pseudo
– results

Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao
K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee
N. Nurminen, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

3D nucleon structure

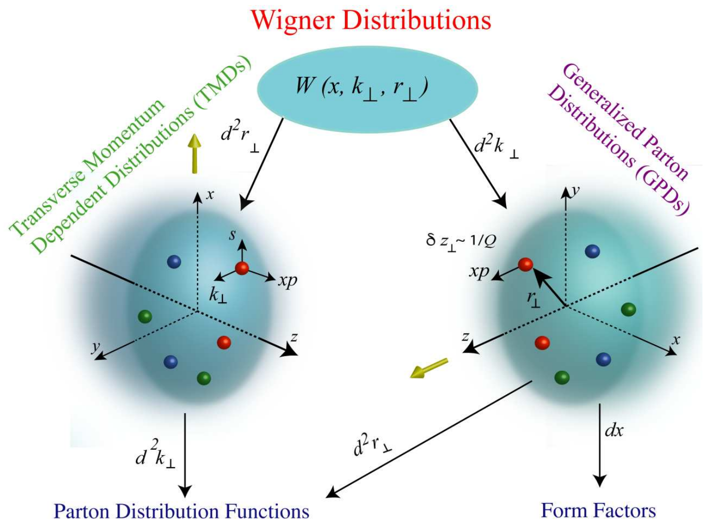


One of the central aims of hadron physics:
to understand better nucleon's 3D structure.

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise? NAS report 2018
- What are the emergent properties of dense systems of gluons?
- Answering these questions is one of the crucial expectations for the upcoming years!
- For this, we need to probe the 3D structure.
- Transverse position of partons: GPDs.
- Transverse momenta of partons: TMDs.
- Both theoretical and experimental input needed.

Generalized parton distributions (GPDs):

- much more difficult to extract than PDFs,
- but they provide a wealth of information:
  - ⋆ spatial distribution of partons in the transverse plane,
  - ⋆ mechanical properties of hadrons,
  - ⋆ hadron's spin decomposition,
- reduce to PDFs in the forward limit, e.g. $H(x, 0, 0) = q(x)$,
- their moments are form factors, e.g. $\int dx\, H(x, \xi, t) = F_1(t)$.

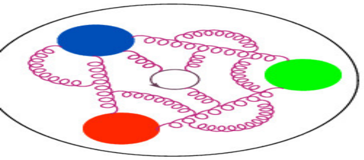


Partonic structure from Lattice QCD

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**

  (experiment) cross-section = perturbative-part $*$ partonic-distribution

  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution



Partonic structure from Lattice QCD

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**

  (experiment) cross-section = perturbative-part $*$ partonic-distribution

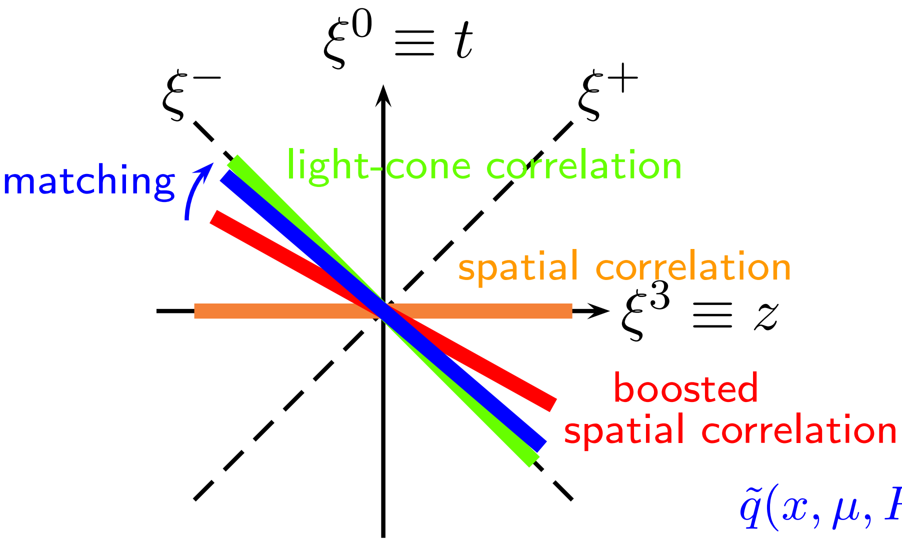
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution

  - ⋆ **hadronic tensor** – Liu, Dong, 1993
  - ⋆ **auxiliary scalar quark** – Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – Detmold, Lin, 2005
  - ⋆ **auxiliary light quark** – Braun, Müller, 2007
  - ⋆ **quasi-distributions** – Ji, 2013
  - ⋆ **"good lattice cross sections"** – Ma, Qiu, 2014
  - ⋆ **pseudo-distributions** – Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**

(experiment) cross-section = perturbative-part $*$ partonic-distribution
(lattice) lattice-observable = perturbative-part $*$ partonic-distribution

- ⋆ **hadronic tensor** – Liu, Dong, 1993
- ⋆ **auxiliary scalar quark** – Aglietti et al., 1998
- ⋆ **auxiliary heavy quark** – Detmold, Lin, 2005
- ⋆ **auxiliary light quark** – Braun, Müller, 2007
- ⋆ **quasi-distributions** – Ji, 2013
- ⋆ **"good lattice cross sections"** – Ma, Qiu, 2014
- ⋆ **pseudo-distributions** – Radyushkin, 2017
- ⋆ **"OPE without OPE"** – QCDSF, 2017



Euclidean matrix element:
$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

Its Fourier transform (quasi-distribution)
can be matched onto the light-cone distribution:
(Large Momentum Effective Theory (LaMET))

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF — pert.kernel — PDF — higher-twist effects

Partonic structure from Lattice QCD

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**

(experiment) cross-section = perturbative-part $*$ partonic-distribution
(lattice) lattice-observable = perturbative-part $*$ partonic-distribution

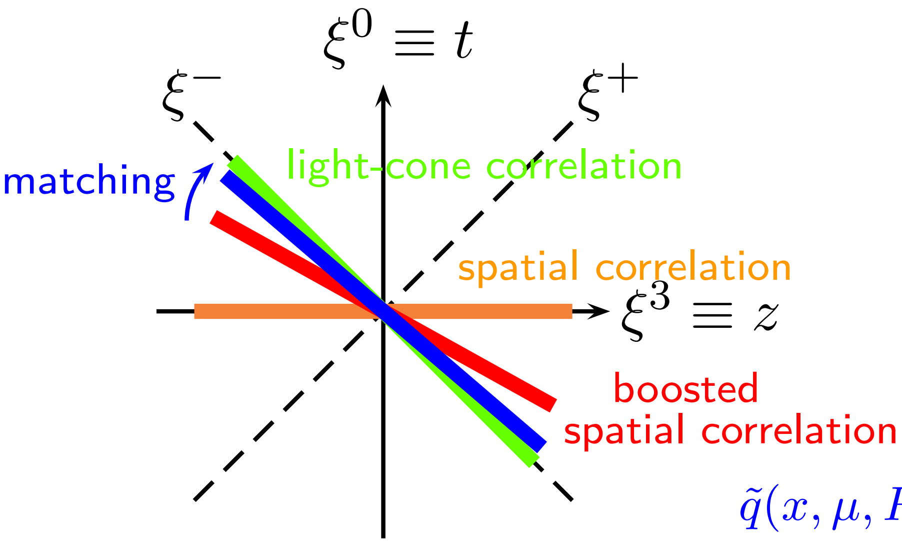
- ⋆ **hadronic tensor** – Liu, Dong, 1993
- ⋆ **auxiliary scalar quark** – Aglietti et al., 1998
- ⋆ **auxiliary heavy quark** – Detmold, Lin, 2005
- ⋆ **auxiliary light quark** – Braun, Müller, 2007
- ⋆ **quasi-distributions** – Ji, 2013
- ⋆ **"good lattice cross sections"** – Ma, Qiu, 2014
- ⋆ **pseudo-distributions** – Radyushkin, 2017
- ⋆ **"OPE without OPE"** – QCDSF, 2017

Dirac structures Γ for different GPDs:

VECTOR: γ_0, γ_3: H, E (unpolarized twist-2)
γ_1, γ_2: G_1, G_2, G_3, G_4 (vector twist-3)

AXIAL VECTOR: $\gamma_5\gamma_0, \gamma_5\gamma_3$: $\tilde{H}, \tilde{E}$ (helicity twist-2)
$\gamma_5\gamma_1, \gamma_5\gamma_2$: $\tilde{G}_1, \tilde{G}_2, \tilde{G}_3, \tilde{G}_4$ (axial vector twist-3)

TENSOR: $\gamma_1\gamma_3, \gamma_2\gamma_3$: $H_T, E_T, \tilde{H}_T, \tilde{E}_T$ (transversity twist-2)
$\gamma_1\gamma_2$: H_2', E_2' (tensor twist-3)



Parity projectors to disentangle 2/4 GPDs:

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$, POL-$k$: $\mathcal{P} = \frac{1+\gamma_0}{4} i\gamma_5\gamma_k$

Euclidean matrix element:

$\langle P_f | \overline{\psi}(z) \Gamma \mathcal{A}(z,0) \psi(0) | P_i \rangle$

Its Fourier transform (quasi-distribution) can be matched onto the light-cone distribution:
(Large Momentum Effective Theory (LaMET))

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda_{\rm QCD}^2/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF — pert.kernel — PDF — higher-twist effects

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

- nucleon boosts up to $P_3 = 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV^2, most data: $-t = 0.65,\ 0.69$ GeV^2,
- skewness: $\xi = 0,\ 1/3$.

up to $\mathcal{O}$(250K) measurements (≈ 500 confs, 32 src positions, 16 permut. of $\vec{\Delta}$).

Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001

Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501

Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512

Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501

Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 109(2024)034508

Twist-2 unpolarized GPDs (pseudo-GPDs) S. Bhattacharya et al. (ETMC/Temple) PRD110(2024)054502

Twist-2 transversity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) in preparation

GPDs in different frames of reference

Standard symmetric (Breit) frame:

source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,
sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

Lattice perspective:
construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator
preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.
Hence, **separate calculation for each momentum transfer $\vec{\Delta}$!**

Asymmetric frame:

source momentum: $P_i = (E_i, \vec{P} - \vec{\Delta})$,
sink momentum: $P_f = (E_f, \vec{P})$.

Lattice perspective:

Several momentum transfer vectors $\vec{\Delta}$ can be obtained within a single calculation!

Quasi- and pseudo-GPDs lattice procedures

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{\Delta}$, $\vec{\Delta}$ – momentum transfer
lattice computation of bare ME

different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

extraction of amplitudes and/or GPDs
frame-dependent formulas

amplitudes frame-invariant
possible different definitions of GPDs

QUASI

renormalization
e.g. RI scheme

reconstruction of x-dependence (F.T. in z)
e.g. Backus-Gilbert

matching to light cone
in x-space

PSEUDO

renormalization
e.g. double ratio

matching to light cone
in ν-space

reconstruction of x-dependence (F.T. in ν)
e.g. fitting ansatz

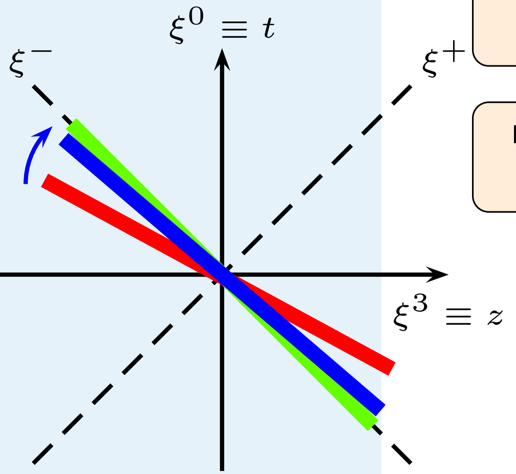
logarithmic and power divergences
in bare MEs/GPDs

reconstruction:
non-trivial ("inverse problem")

matching:
needs large boosts
valid up to HTEs

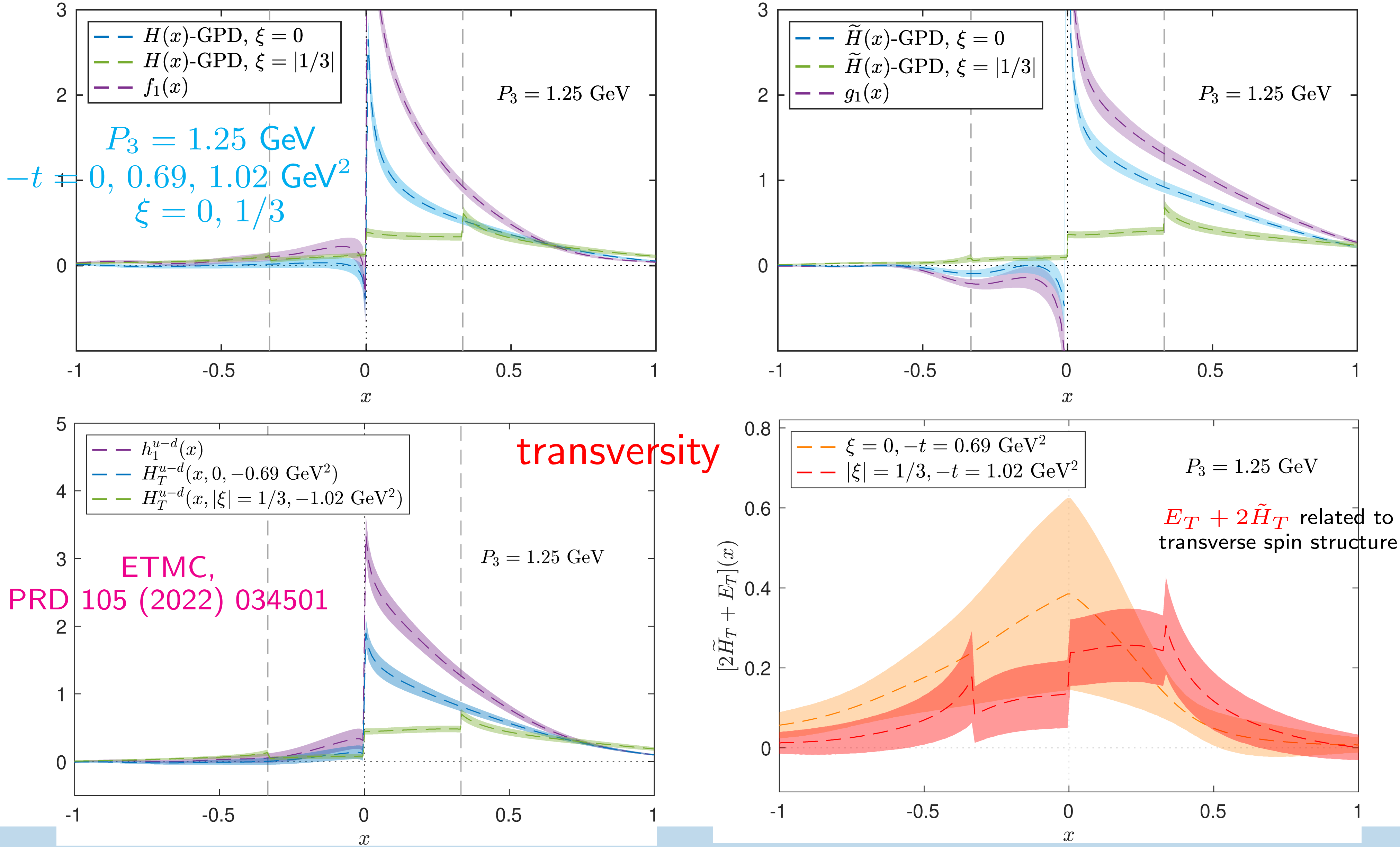
light-cone PDF

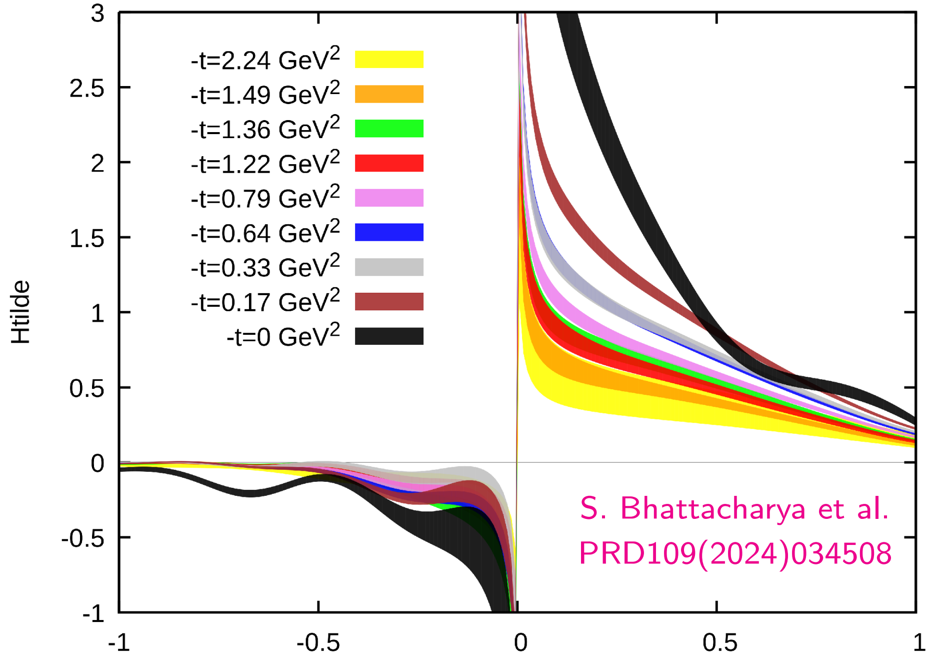
the final desired object!



First x-dependent nucleon GPDs (quasi, symm. frame)

unpolarized — ETMC, Phys. Rev. Lett. 125 (2020) 262001 — helicity

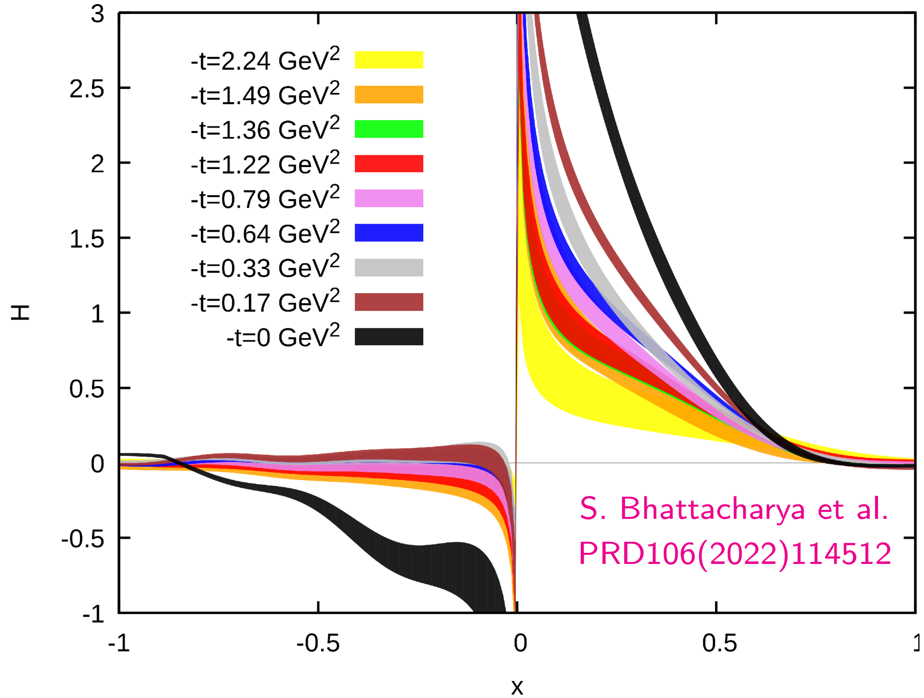


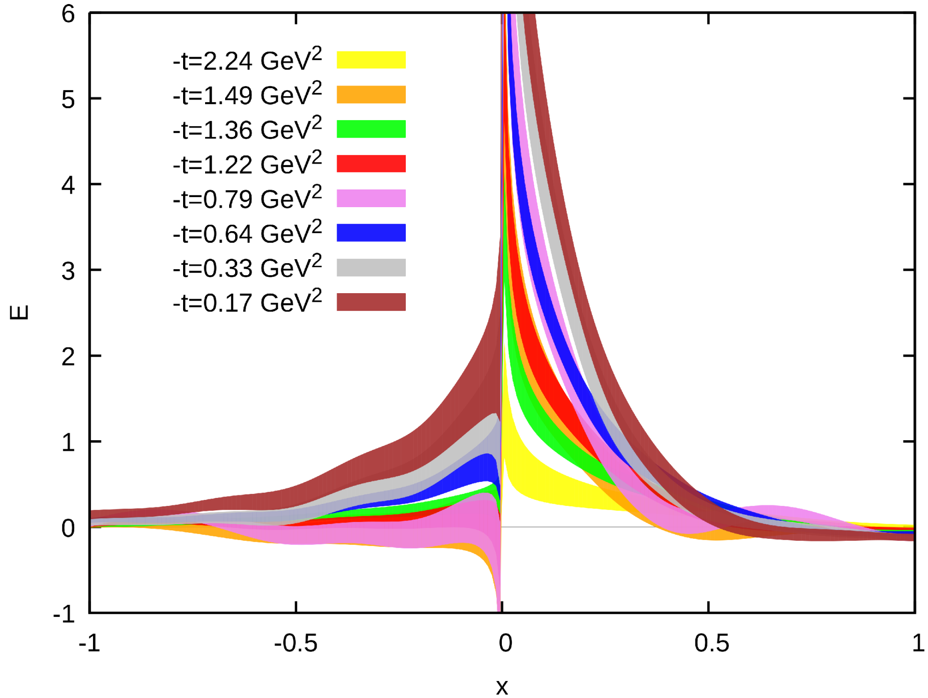


$\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2
$\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2
$\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2
$\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2
$\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2
$\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2
$\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2
$\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2

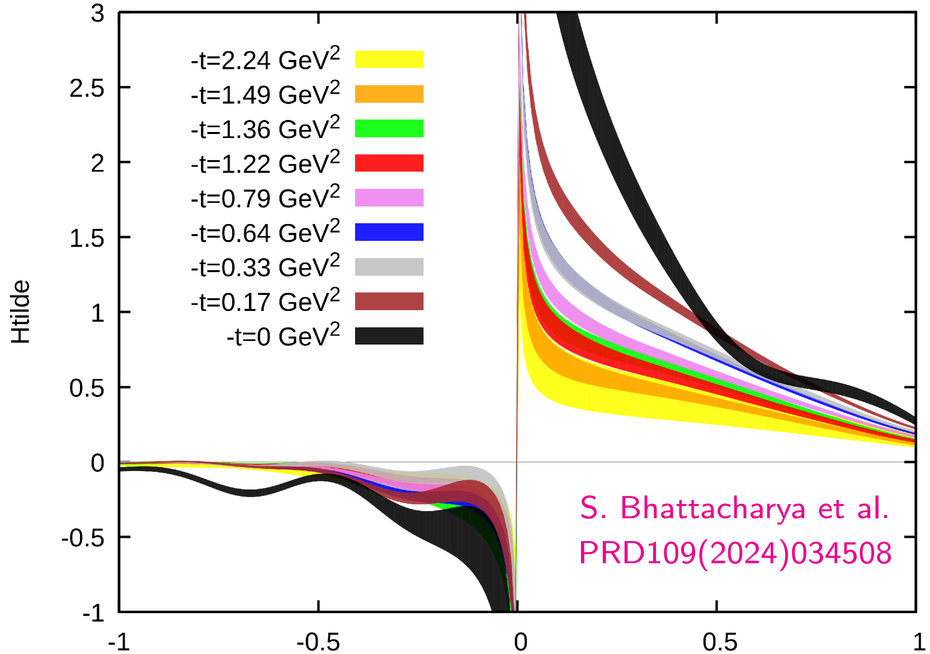
Impact parameter distribution:

$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp\cdot\Delta_\perp} GPD(x,t)$$





t-dependence of $\tilde{H}/H/E$ GPDs (quasi, asymm. frame)

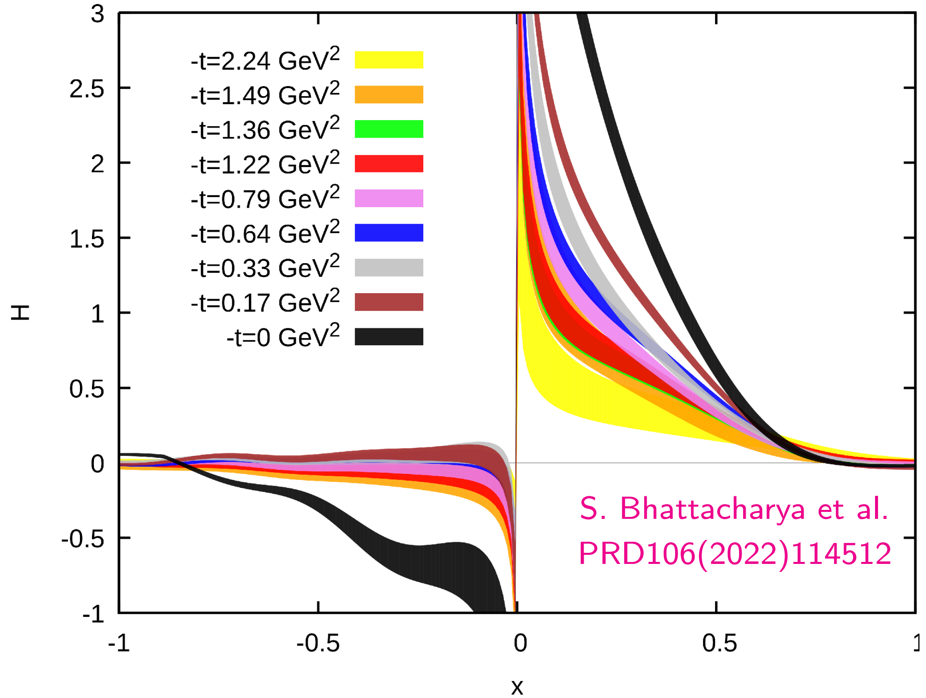


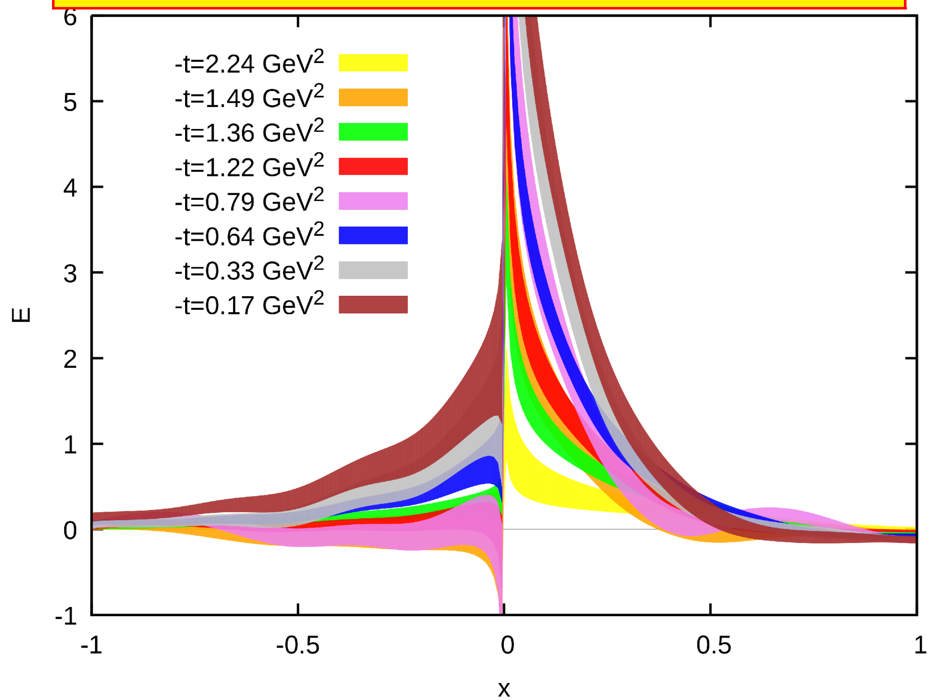
$\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2
$\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2
$\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2
$\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2
$\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2
$\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2
$\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2
$\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2

Impact parameter distribution:

$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} GPD(x,t)$$

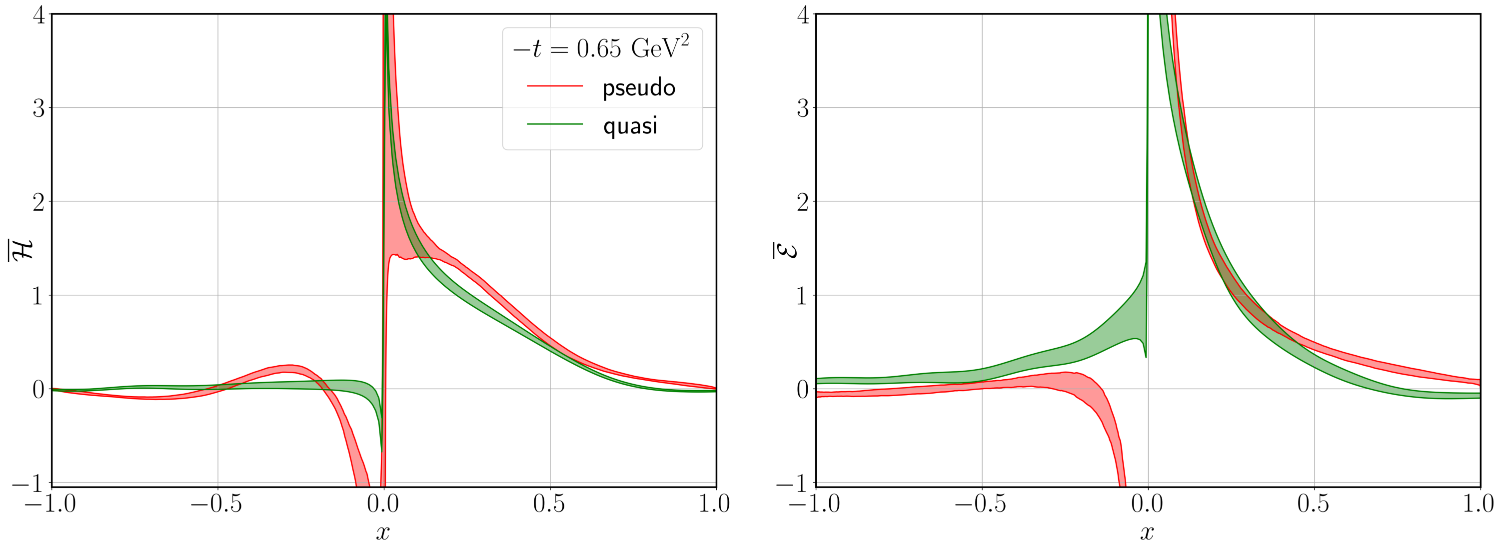
Soon results also for transversity GPDs!





H GPD from quasi vs. pseudo, $-t = 0.65$ GeV^2

S. Bhattacharya et al., PRD110(2024)054502



Qualitative agreement between pseudo and quasi.

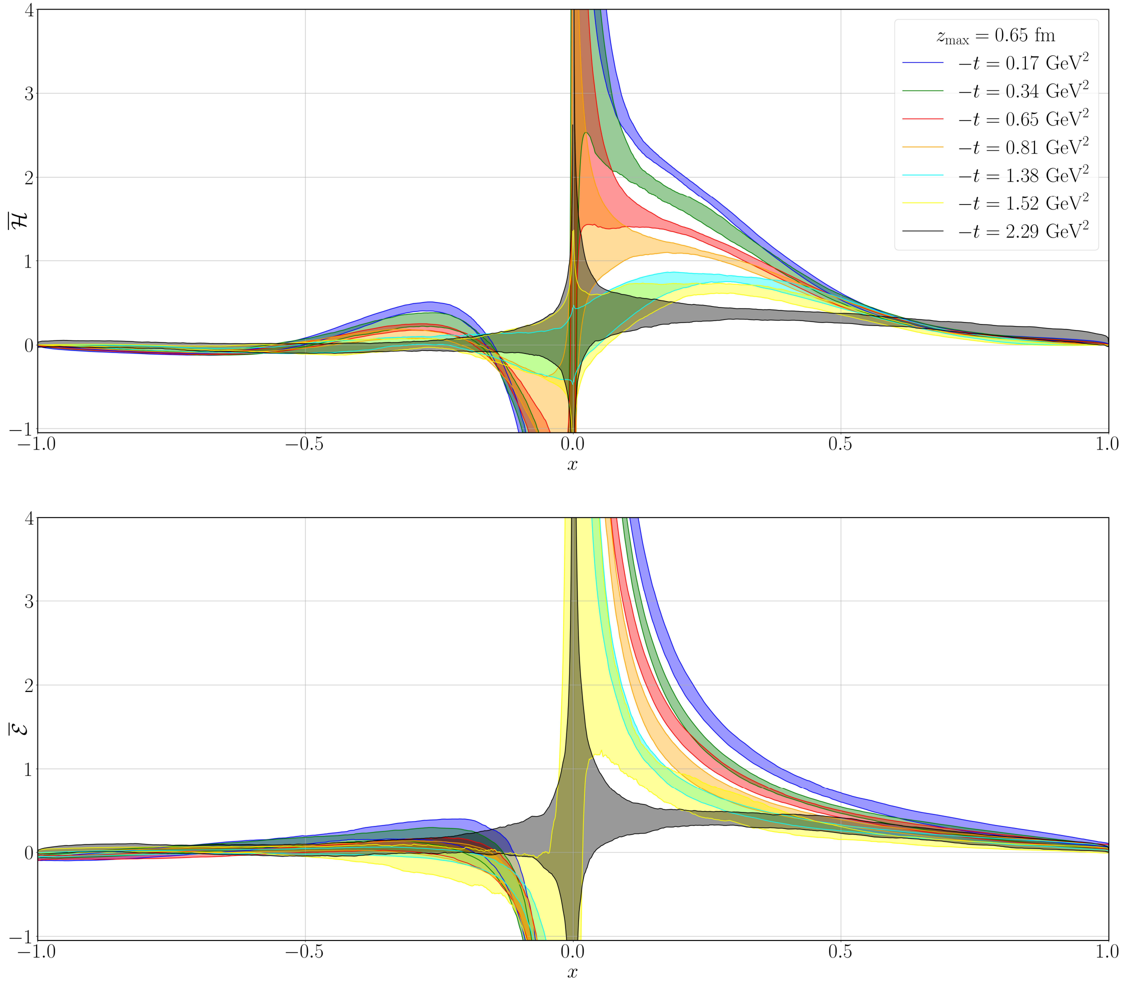
Evinced difference as measure of unquantified systematic effects.

Reminder:

- Main difference:
  quasi = factorization in x-space (LaMET),
  pseudo = short-distance factorization (SDF) in ν-space.
- Practical difference: reconstruction of x-dependence
  quasi = Backus-Gilbert,
  pseudo = fitting ansatz.

t-dependence of H/E GPDs (pseudo, asymm. frame)

S. Bhattacharya et al., PRD110(2024)054502



Qualitatively similar picture to the one from quasi-GPDs.
Quantitative conclusions after careful estimation of systematics!

Short-distance factorization (SDF) can also be used to extract moments of GPDs.

For ratio-renormalized H/E: $\mathcal{F}^{\overline{\mathrm{MS}}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\mathrm{MS}}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda_{\mathrm{QCD}}^2z^2)$,

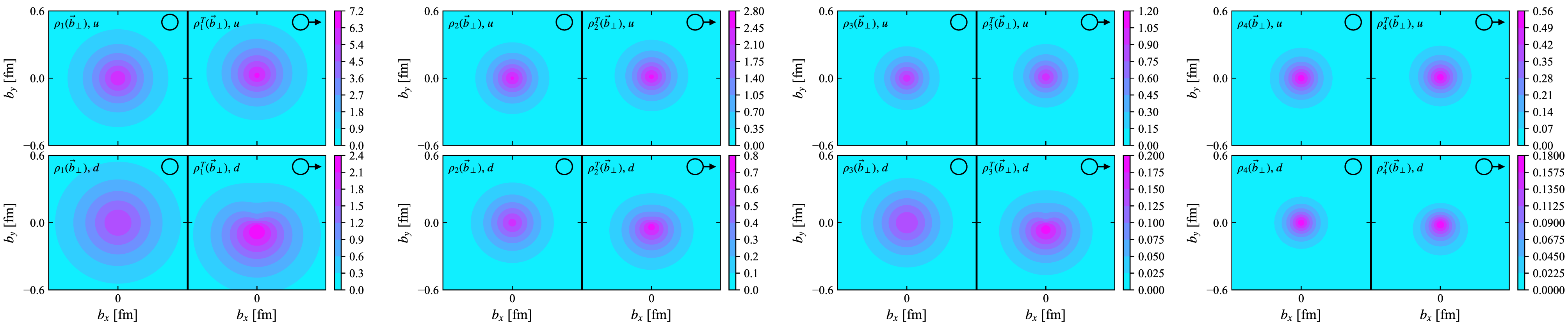
$C_n^{\overline{\mathrm{MS}}}(\mu^2z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}A_{n+1,0}(-\vec{\Delta}_\perp^2)e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho_{n+1}^T(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}[A_{n+1,0}(-\vec{\Delta}_\perp^2)+i\frac{\Delta_y}{2M}B_{n+1,0}(-\vec{\Delta}_\perp^2)]e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment | 2nd moment | 3rd moment | 4th moment



S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

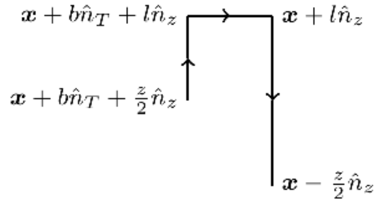
Unpolarized TMDs from lattice QCD

quasi-TMD · reduced soft function · matching kernel · Collins-Soper kernel · light-cone TMD · power corrections

$$\tilde{f}(x, b_T, \mu, \zeta_z)\sqrt{S_I(b_T, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_T,\mu)} f(x, b_T, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{\zeta_z}, \frac{M^2}{P_3^2}, \frac{1}{b_T^2\zeta_z}\right)$$

scales: μ – renorm. scale, ζ – rapidity scale, $\zeta_z = 2xP_3$

quasi-TMD: $\tilde{f}(x, b_T, \mu, \zeta_z) = P_3 \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixzP_3} \lim_{l\to\infty} \frac{\langle H(P_3)|\mathcal{O}_\Gamma(0,0,b_T,l,z)|H(P_3)\rangle}{\sqrt{Z_E(2|l|,|b_T|)}}$



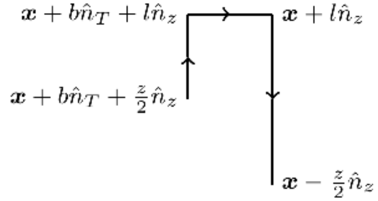
Unpolarized TMDs from lattice QCD

quasi-TMD · reduced soft function · matching kernel · Collins-Soper kernel · light-cone TMD · power corrections

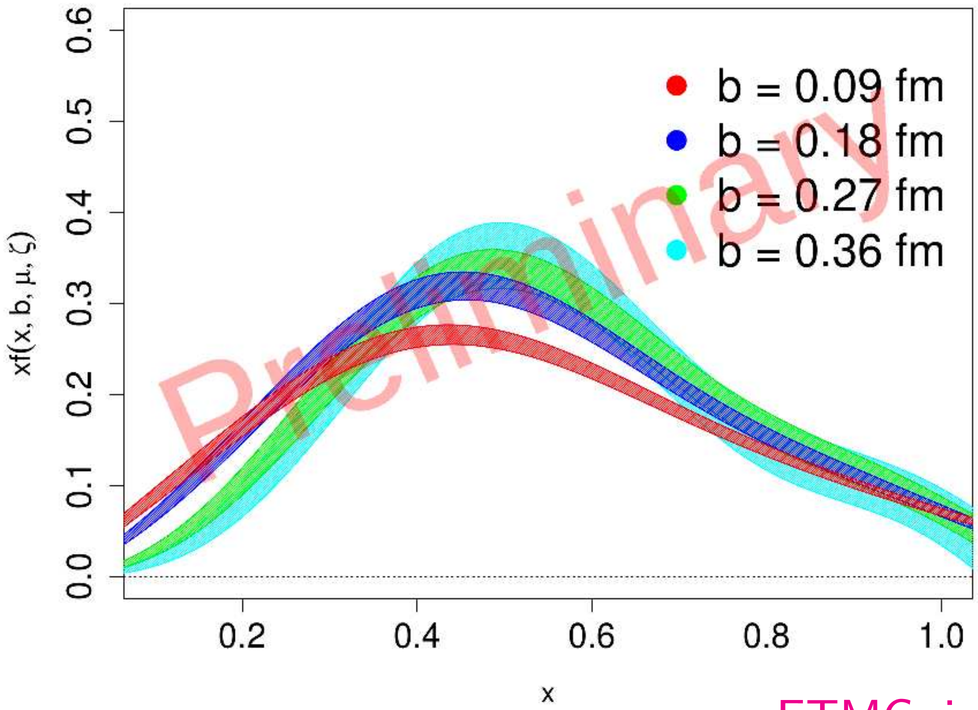
$$\tilde{f}(x,b_T,\mu,\zeta_z)\sqrt{S_I(b_T,\mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_T,\mu)} f(x,b_T,\mu,\zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{\zeta_z},\frac{M^2}{P_3^2},\frac{1}{b_T^2\zeta_z}\right)$$

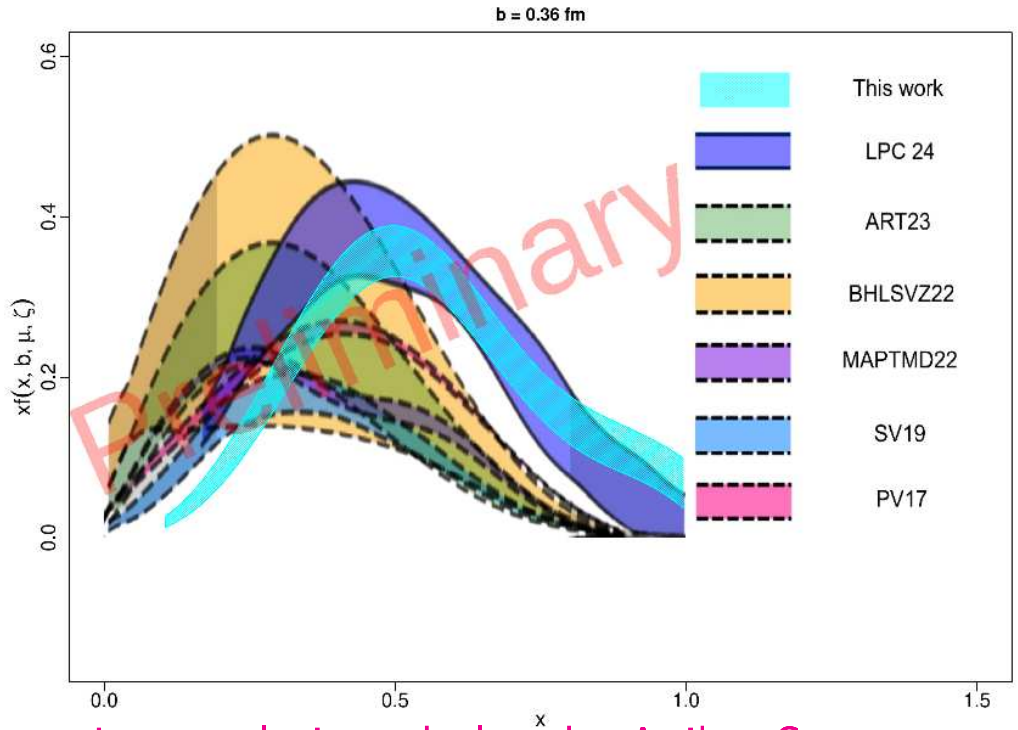
scales: μ – renorm. scale, ζ – rapidity scale, $\zeta_z = 2xP_3$

quasi-TMD: $\tilde{f}(x,b_T,\mu,\zeta_z) = P_3\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{ixzP_3}\lim_{l\to\infty}\frac{\langle H(P_3)|\mathcal{O}_\Gamma(0,0,b_T,l,z)|H(P_3)\rangle}{\sqrt{Z_E(2|l|,|b_T|)}}$



Lattice setup: size $24^3\times 48$, $a\approx 0.093$ fm, pion mass 350 MeV, 600 confs





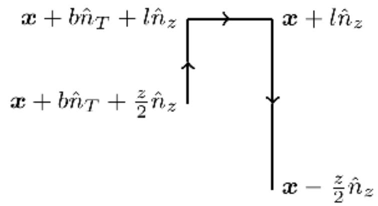
ETMC, in preparation, analysis and plots by Aniket Sen

Unpolarized TMDs from lattice QCD

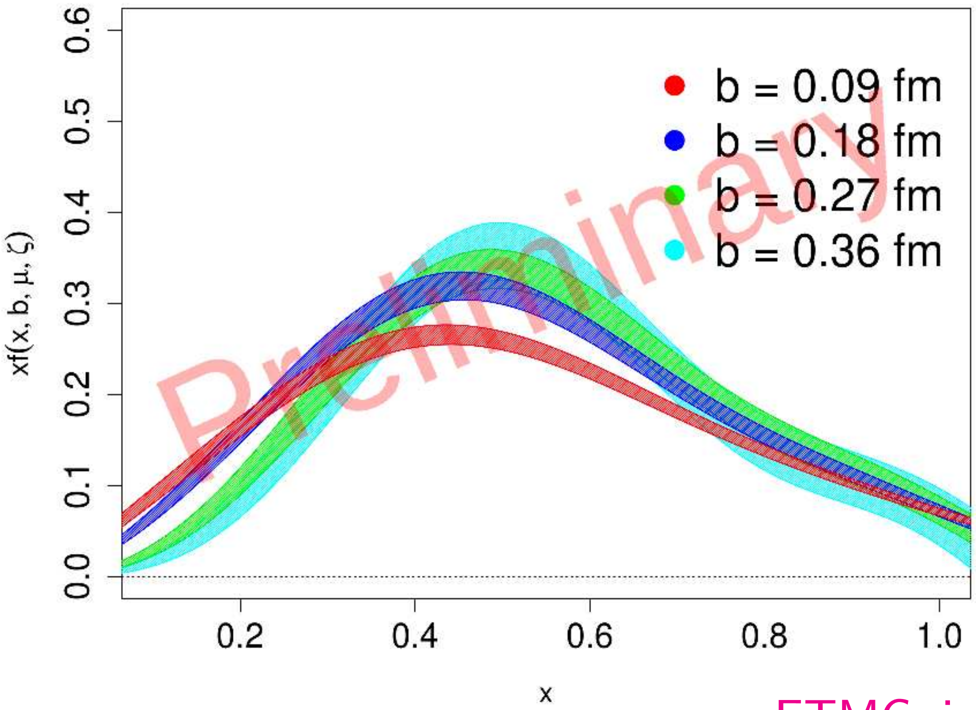
$$\underbrace{\tilde{f}(x,b_T,\mu,\zeta_z)}_{\text{quasi-TMD}}\underbrace{\sqrt{S_I(b_T,\mu)}}_{\text{reduced soft function}} = \underbrace{H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right)}_{\text{matching kernel}} e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)\underbrace{K(b_T,\mu)}_{\text{Collins-Soper kernel}}} \underbrace{f(x,b_T,\mu,\zeta)}_{\text{light-cone TMD}} + \underbrace{\mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{\zeta_z},\frac{M^2}{P_3^2},\frac{1}{b_T^2\zeta_z}\right)}_{\text{power corrections}}$$

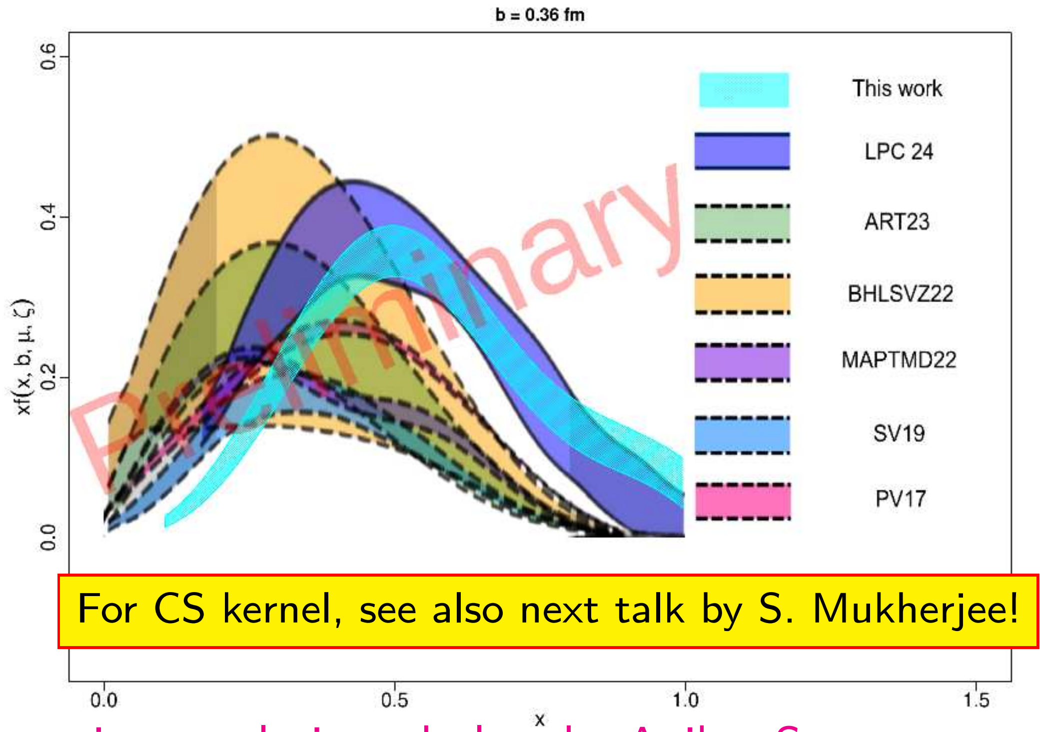
scales: μ – renorm. scale, ζ – rapidity scale, $\zeta_z = 2xP_3$

quasi-TMD: $\tilde{f}(x,b_T,\mu,\zeta_z) = P_3\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{ixzP_3}\lim_{l\to\infty}\frac{\langle H(P_3)|\mathcal{O}_\Gamma(0,0,b_T,l,z)|H(P_3)\rangle}{\sqrt{Z_E(2|l|,|b_T|)}}$



Lattice setup: size $24^3 \times 48$, $a \approx 0.093$ fm, pion mass 350 MeV, 600 confs





For CS kernel, see also next talk by S. Mukherjee!

ETMC, in preparation, analysis and plots by Aniket Sen

Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- A lot of follow-up work in progress: transversity GPDs, pion and kaon GPDs, other twist-3 GPDs, extension of kinematics.
- Encouraging prospects also for TMDs!
- Obviously, GPDs/TMDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Quantification of systematics very laborious, but crucial.
- Consistent progress will ensure complementary role to pheno!

Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- A lot of follow-up work in progress: transversity GPDs, pion and kaon GPDs, other twist-3 GPDs, extension of kinematics.
- Encouraging prospects also for TMDs!
- Obviously, GPDs/TMDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Quantification of systematics very laborious, but crucial.
- Consistent progress will ensure complementary role to pheno!

Thank you for your attention!

Backup slides

Main theoretical tool: unpolarized: S. Bhattacharya et al., PRD106(2022)114512

Lorentz-covariant parametrization of matrix elements:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\Big[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\Big]u(p,\lambda),$$

$$F^{[\gamma^{\mu}\gamma_5]}=\bar{u}(p',\lambda')\Big[\frac{i\epsilon^{\mu Pz\Delta}}{m}\widetilde{A_1}+\gamma^{\mu}\gamma_5\widetilde{A_2}+\gamma_5\Big(\frac{P^{\mu}}{m}\widetilde{A_3}+mz^{\mu}\widetilde{A_4}+\frac{\Delta^{\mu}}{m}\widetilde{A_5}\Big)+m\not{z}\gamma_5\Big(\frac{P^{\mu}}{m}\widetilde{A_6}+mz^{\mu}\widetilde{A_7}+\frac{\Delta^{\mu}}{m}\widetilde{A_8}\Big)\Big]u(p,\lambda)$$

helicity: S. Bhattacharya et al., PRD109(2024)034508

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$ or $\widetilde{A_i}(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

$$\Pi_0^a(\Gamma_0)=C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3z}{4m}A_4$$
$$+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).$$

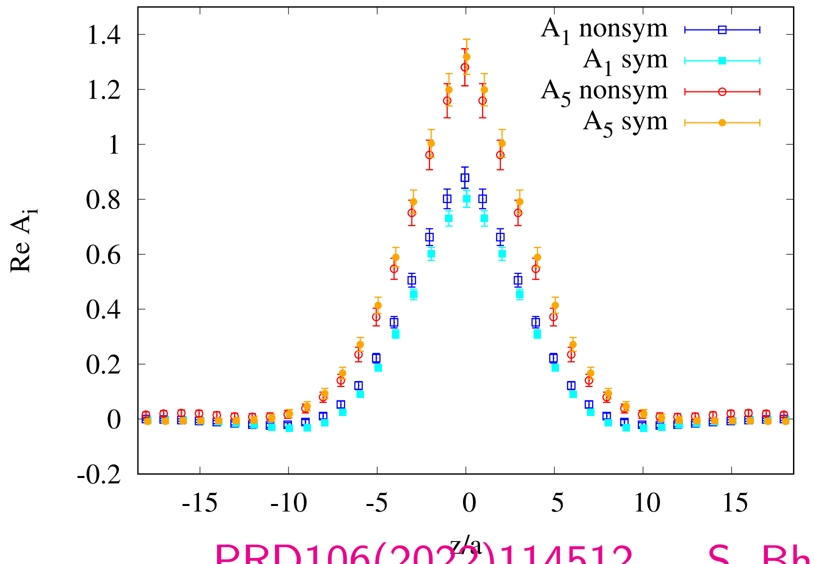
- matrix elements $\Pi_{\mu}(\Gamma_{\nu})$ or $\Pi_{\mu 5}(\Gamma_{\nu})$ are **frame-dependent**,
- but the amplitudes A_i or $\widetilde{A_i}$ are **frame-invariant**.

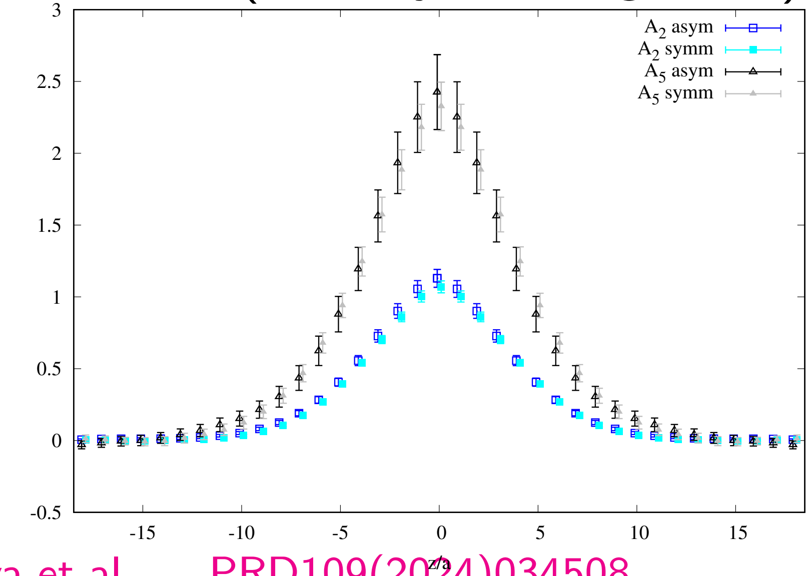
Proof of concept (comparison between frames)

A_1, A_5 (unpolarized leading ones)

$\widetilde{A_2}$, $\widetilde{A_5}$ (helicity leading ones)

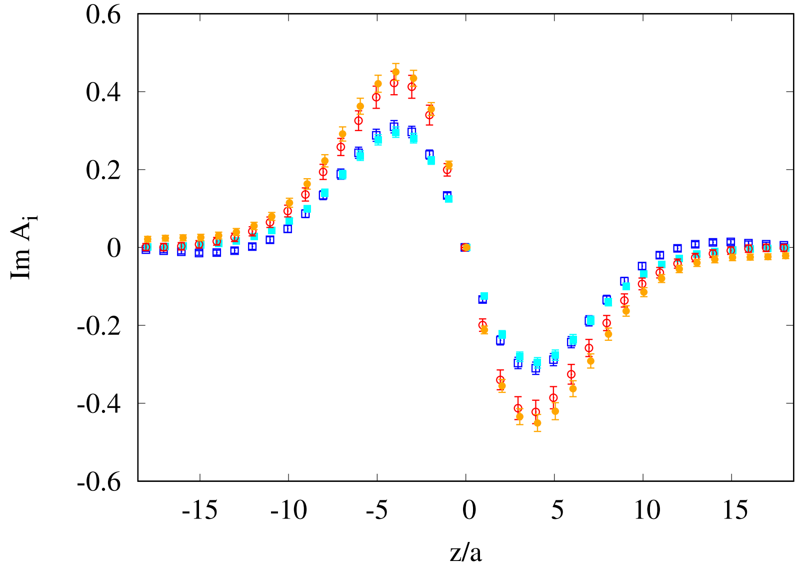
Re

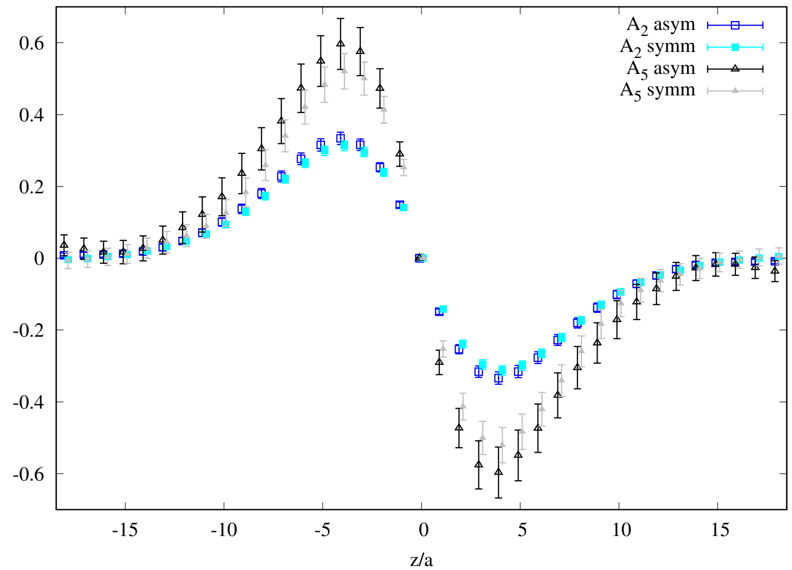




PRD106(2022)114512 S. Bhattacharya et al. PRD109(2024)034508

Im





Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3} A_6\,, \qquad F_{E(0)} = -A_1 + 2A_5 + \frac{z\left(4E^2-\Delta_1^2-\Delta_2^2\right)}{2P_3} A_6\,.$$

ASYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2+\Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3+\Delta_0\Delta_\perp^2)}{2P_0P_3} A_8\,,$$

$$F_{E(0)} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0+2P_0)}{2P_0P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2+2P_0\Delta_0+4P_0^2+\Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2+2\Delta_0P_0+4P_0^2+\Delta_\perp^2\right)}{2P_0P_3} A_8\,.$$

One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1\,, \qquad F_E = -A_1 + 2A_5 + 2zP_3A_6\,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

Two definitions of $\widetilde{H}$:

standard ($\gamma_5\gamma_3$ operator): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6} - m^2z^2\widetilde{A_7}\,,$

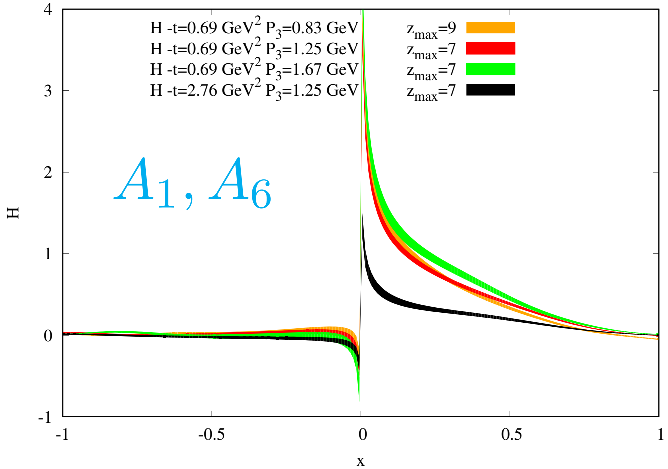
another ($\gamma_5\gamma_i$ operators, $i=0,1,2$): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6}\,.$

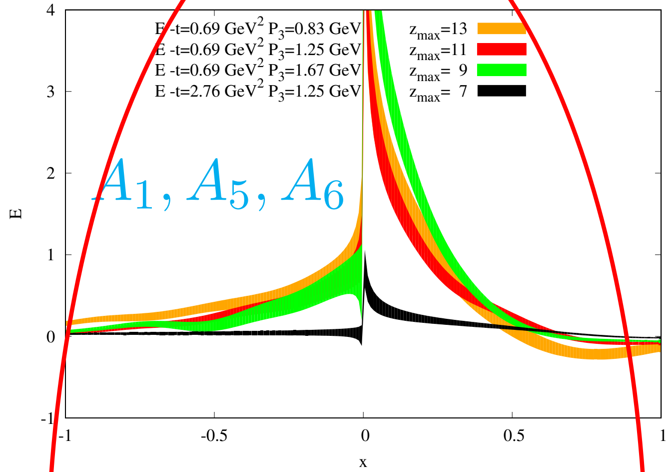
$\widetilde{E}$ impossible to extract at zero skewness: $F_{\tilde{E}} = 2\,\frac{P\cdot z}{\Delta\cdot z}\widetilde{A_3} + 2\widetilde{A_5}\,.$

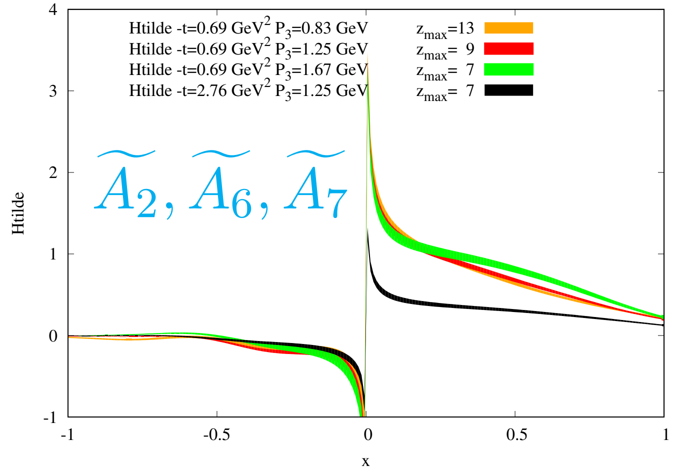
Convergence of alternative definitions of $\tilde{H}/H/E$

UNPOLARIZED | HELICITY

STANDARD





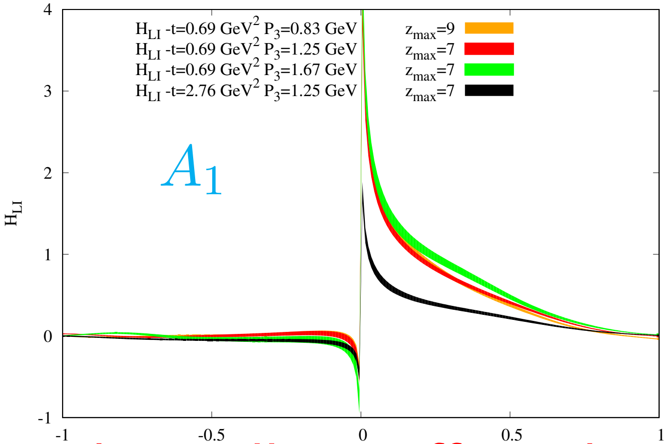


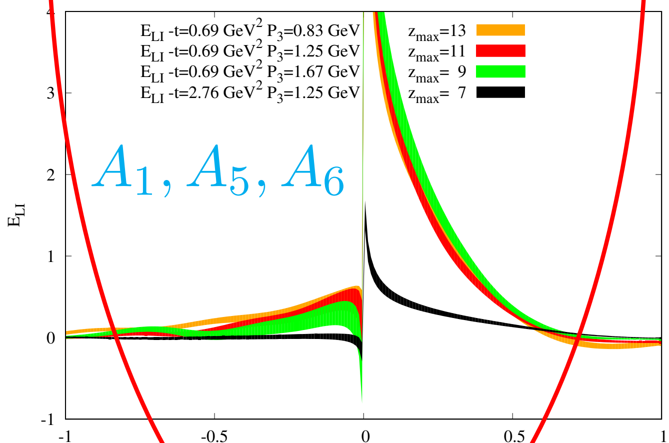
γ_0 operator (non-LI) | $\gamma_5\gamma_3$ operator (LI)

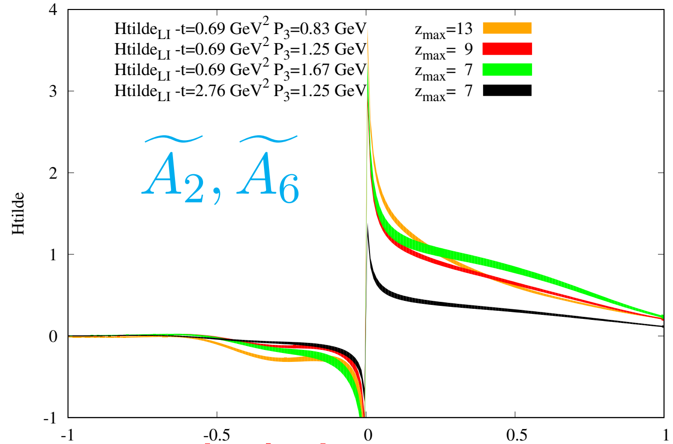
H-GPD | E-GPD | $\widetilde{H}$-GPD

γ_0, γ_T operators (LI) | $\gamma_5\gamma_0$, $\gamma_5\gamma_T$ operators (LI)

ALTERNATIVE





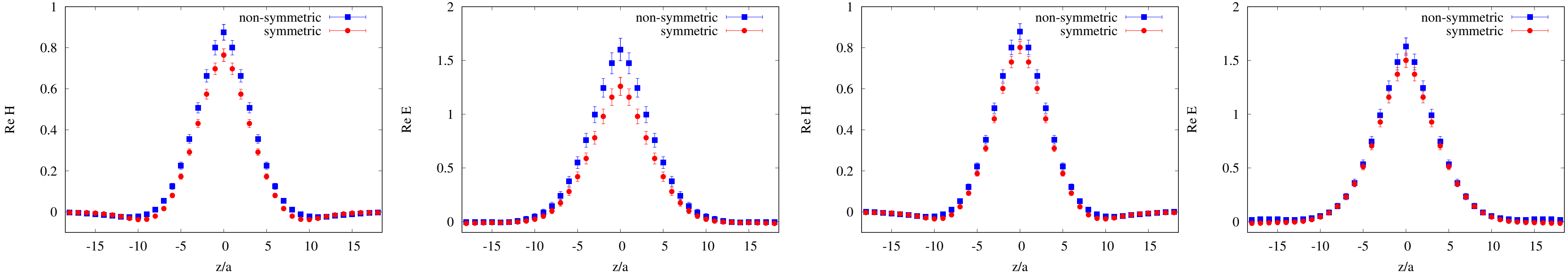


basically unaffected | highly-improved! | slightly worse

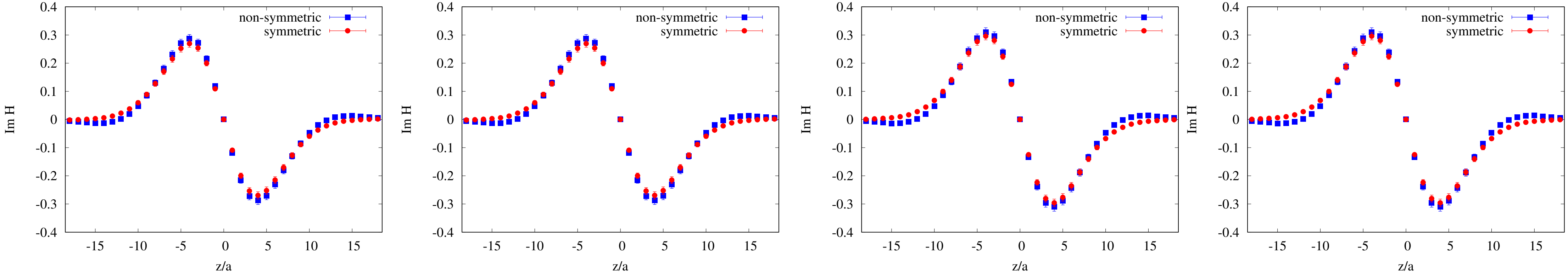
H and E GPDs – comparison of definitions

STANDARD DEFINITION | LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD



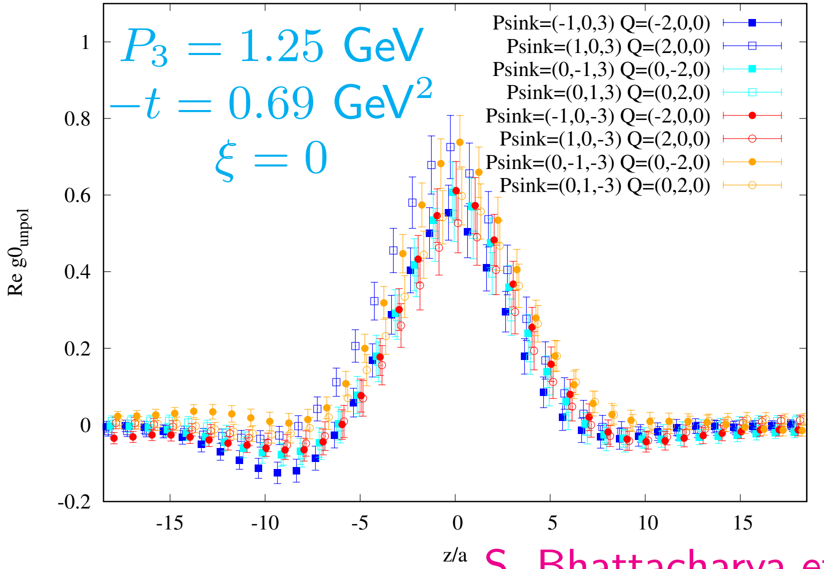
S. Bhattacharya et al., PRD106(2022)114512

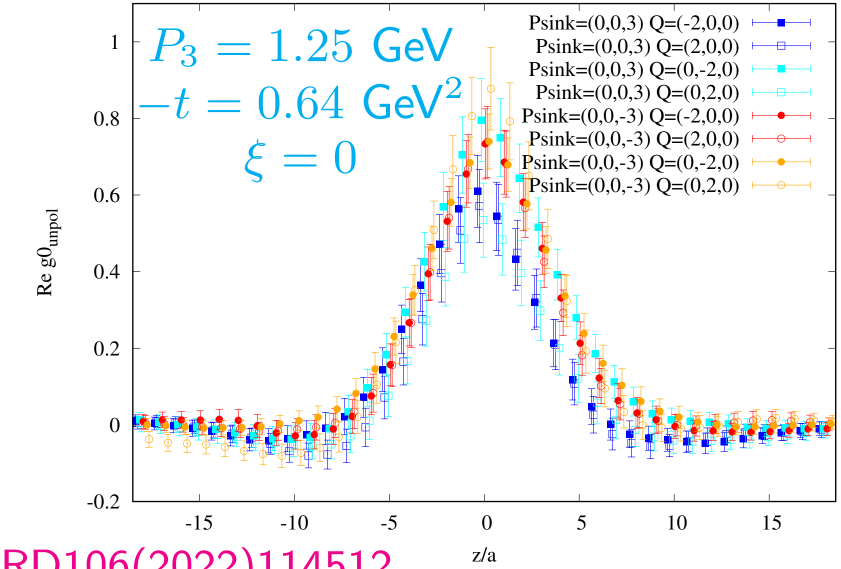


Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame | non-symmetric frame

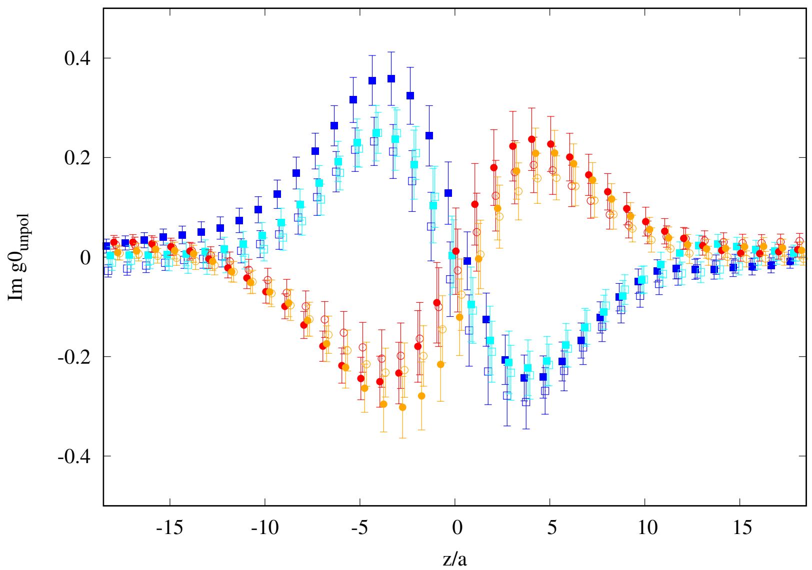
Re

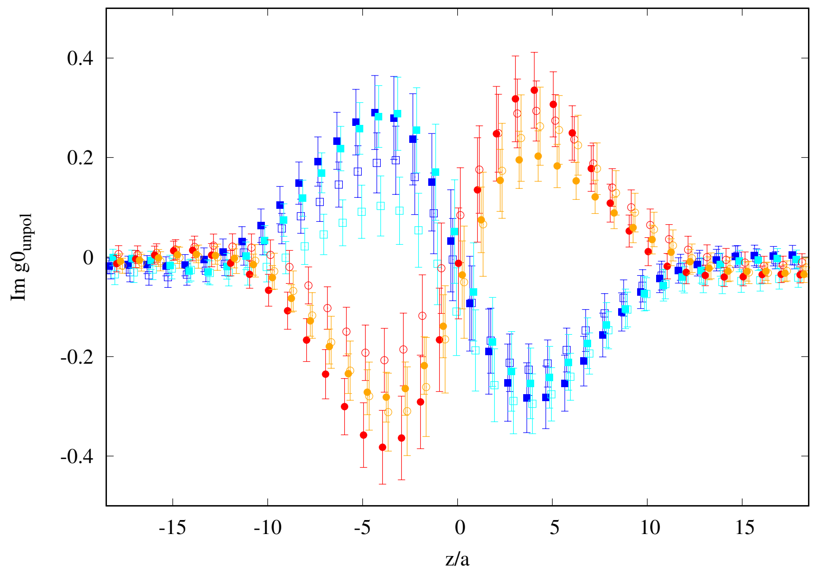




S. Bhattacharya et al., PRD106(2022)114512

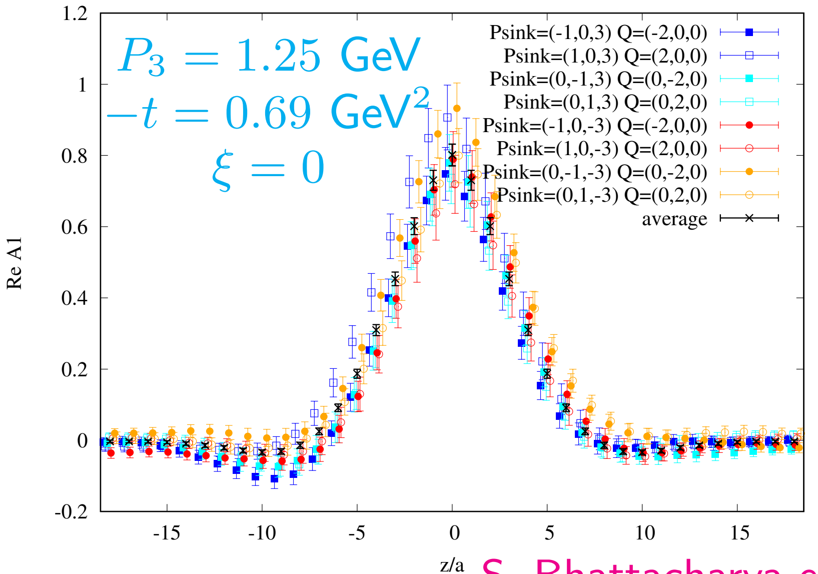
Im



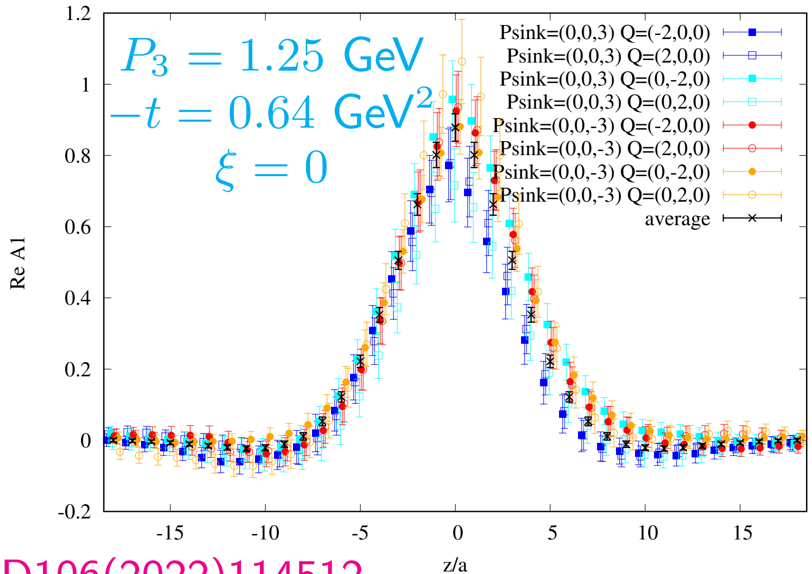


Example amplitude A_1

symmetric frame

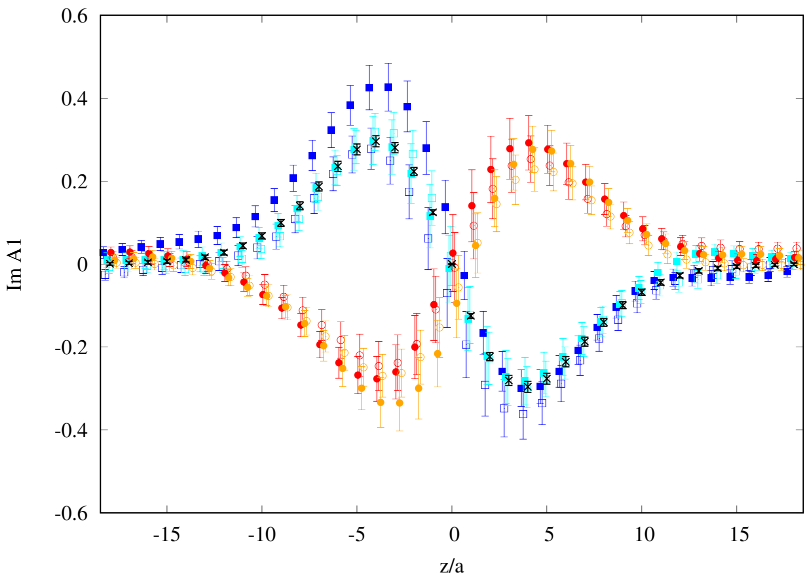


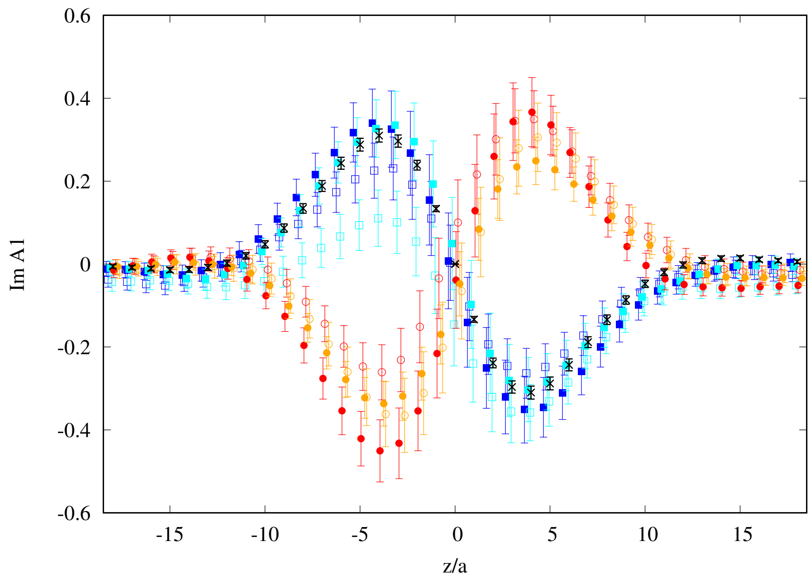
non-symmetric frame



Re

S. Bhattacharya et al., PRD106(2022)114512

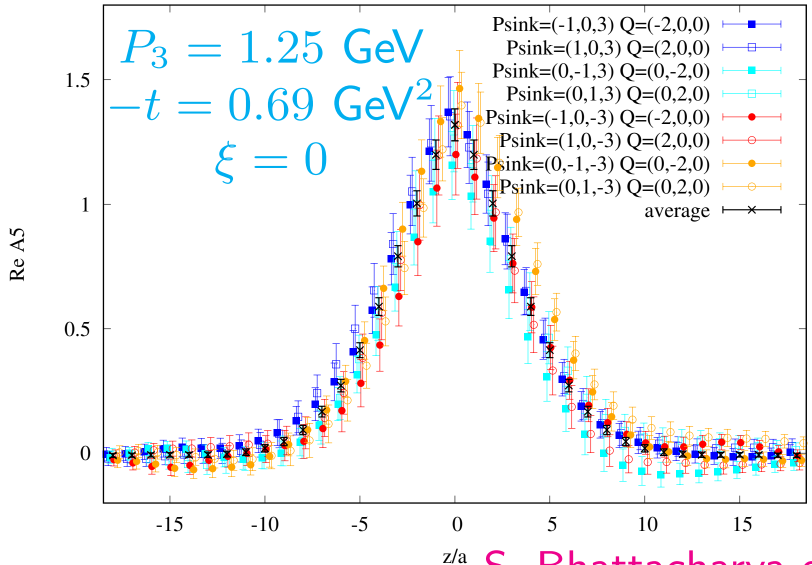


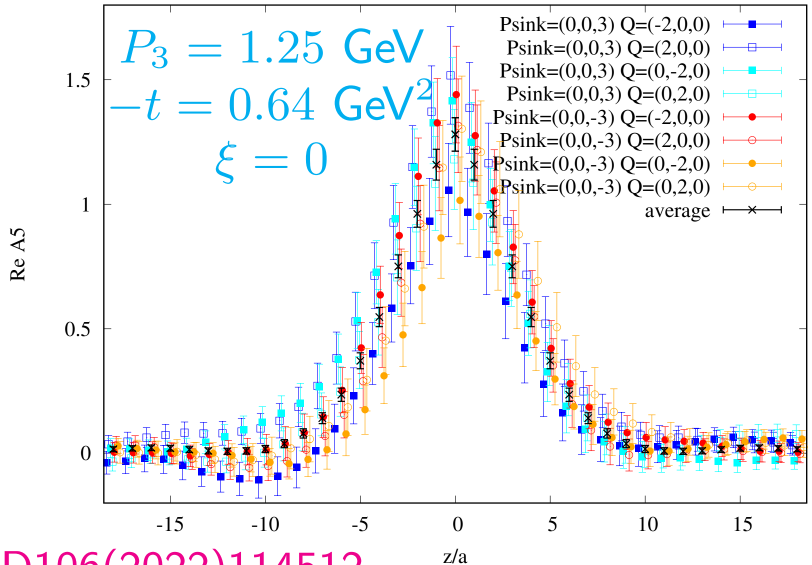


Im

Example amplitude A_5

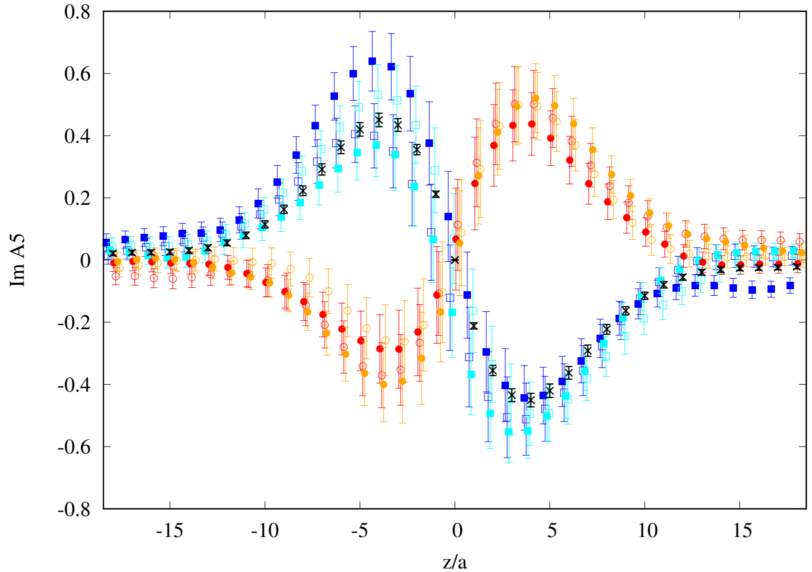
symmetric frame | non-symmetric frame

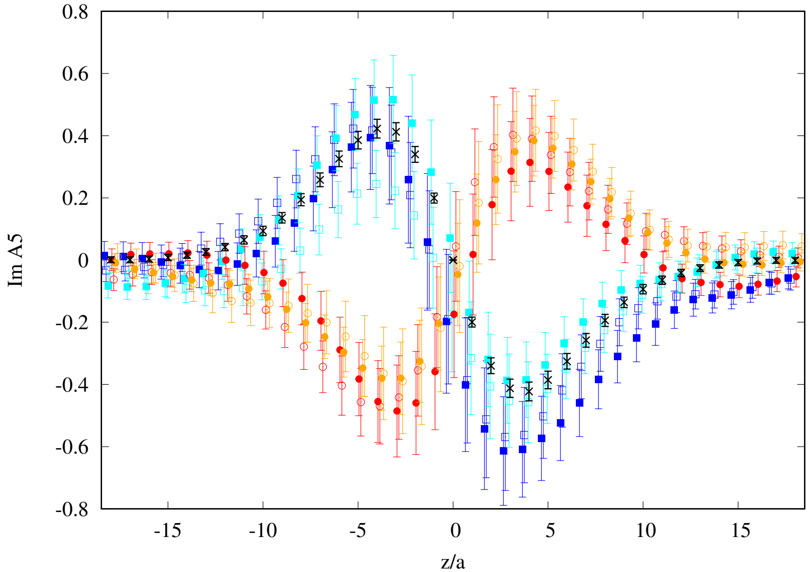




Re

S. Bhattacharya et al., PRD106(2022)114512

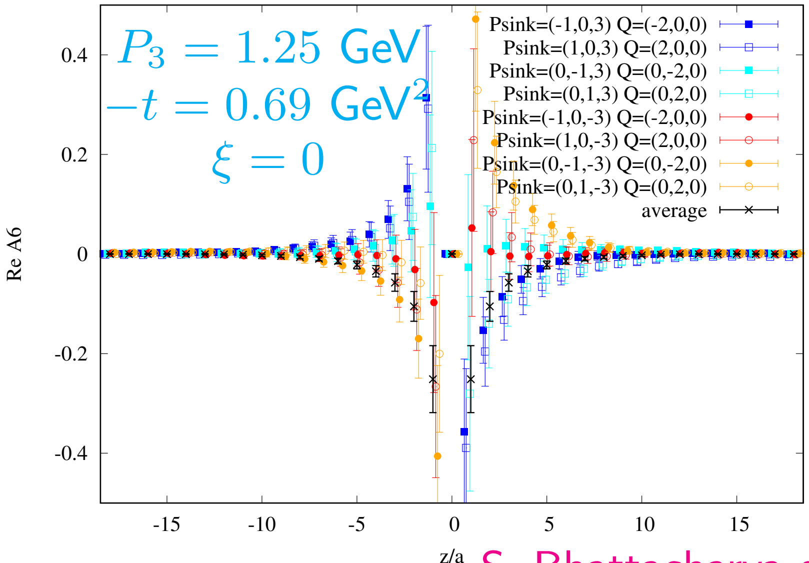


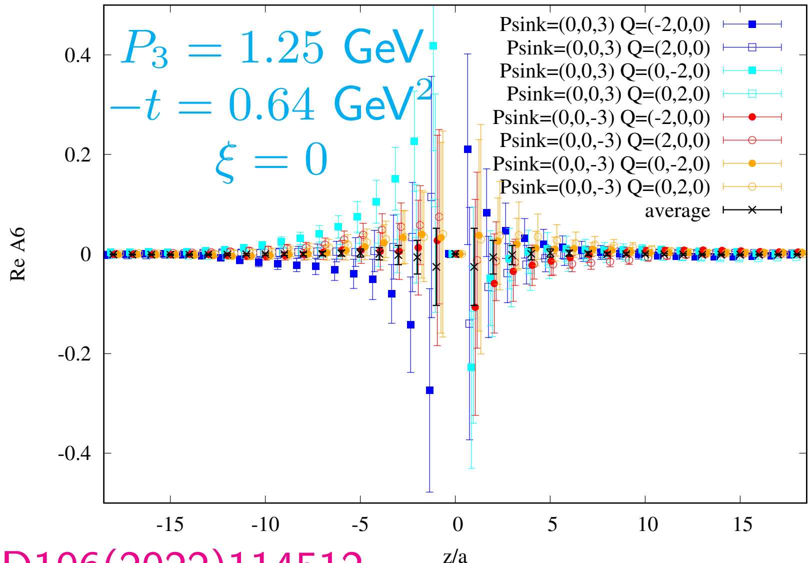


Im

Example amplitude A_6

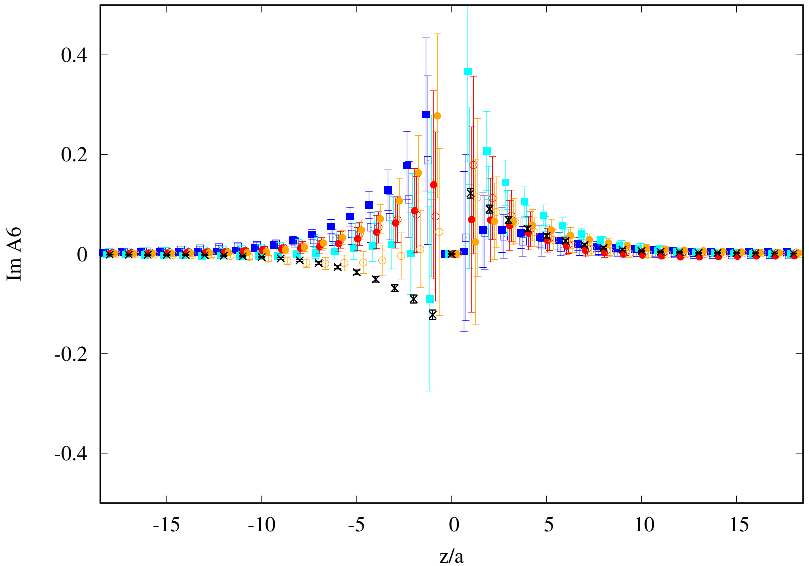
symmetric frame | non-symmetric frame

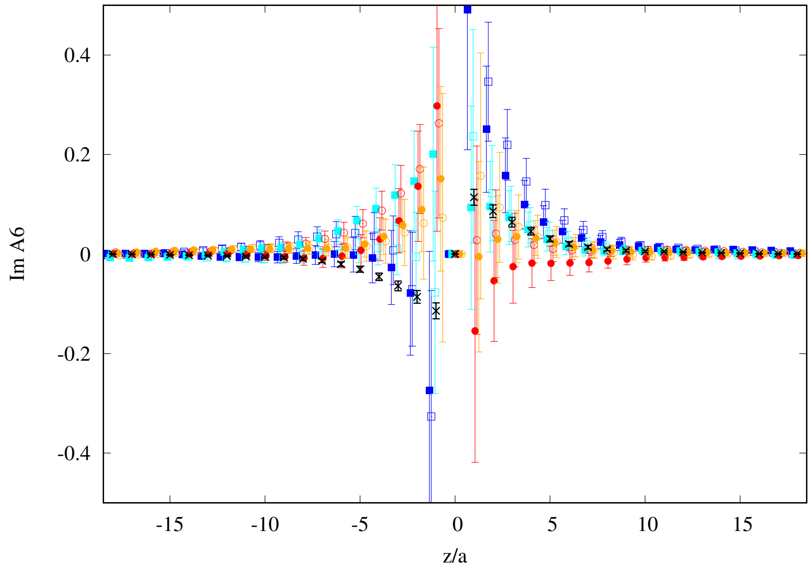




Re

S. Bhattacharya et al., PRD106(2022)114512



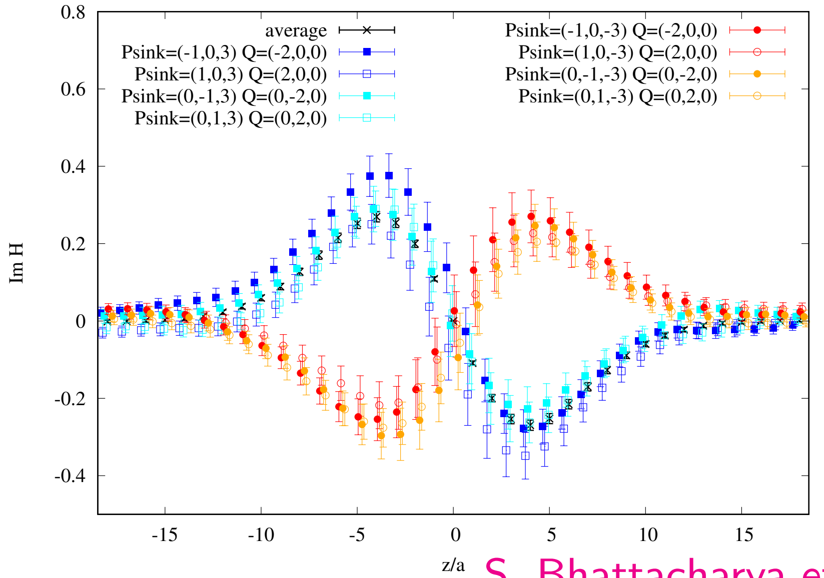


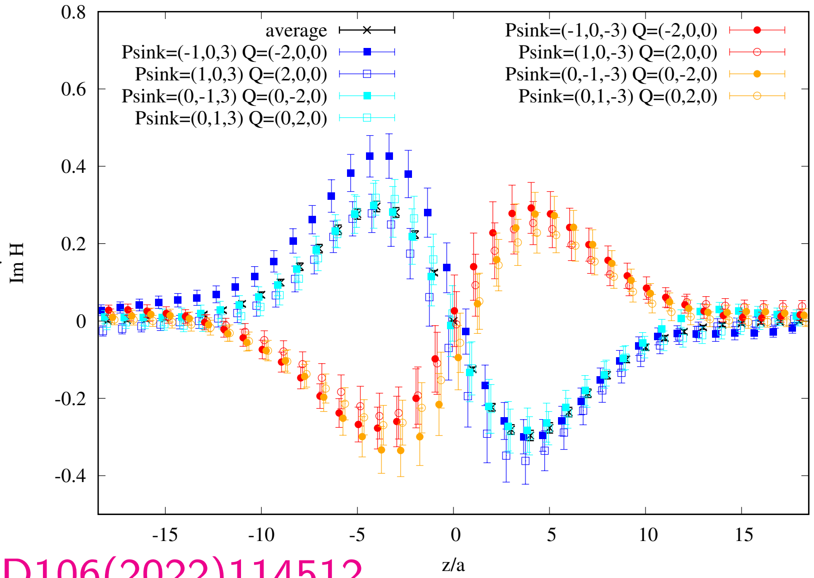
Im

H and E GPDs – signal improvement

standard | Lorentz-invariant

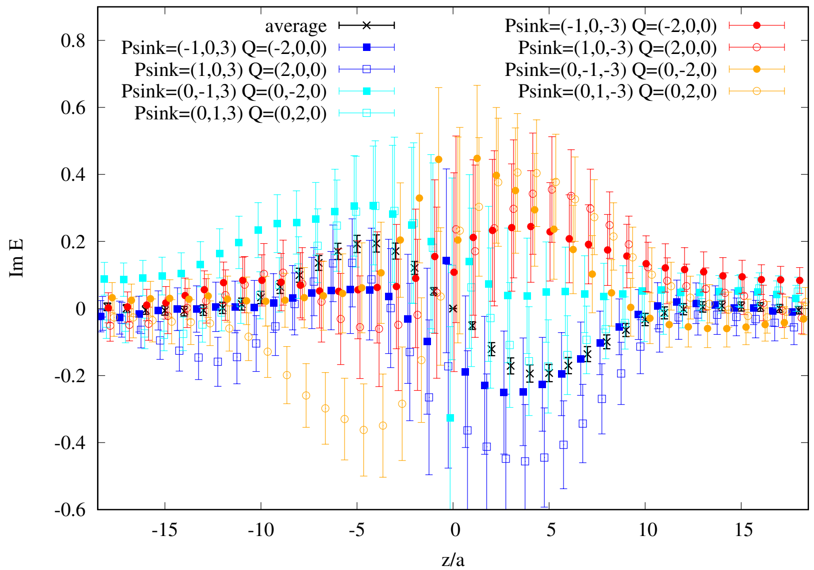
Im H

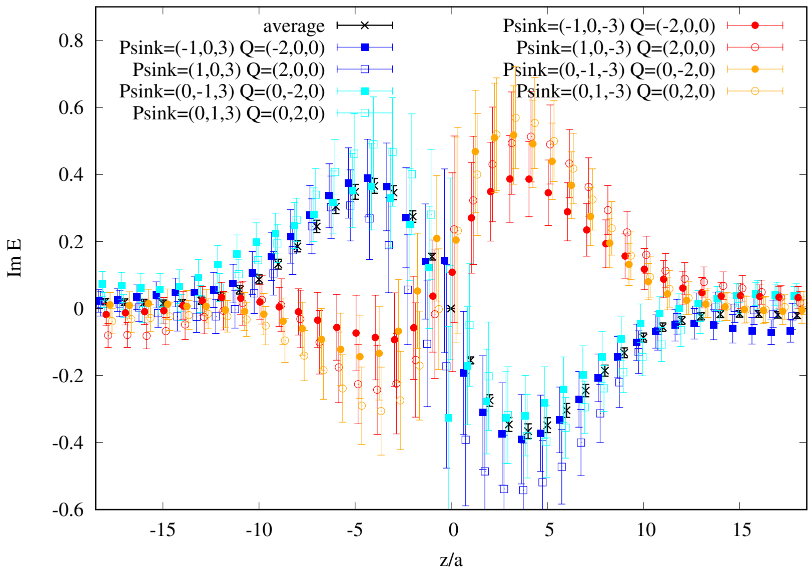




S. Bhattacharya et al., PRD106(2022)114512

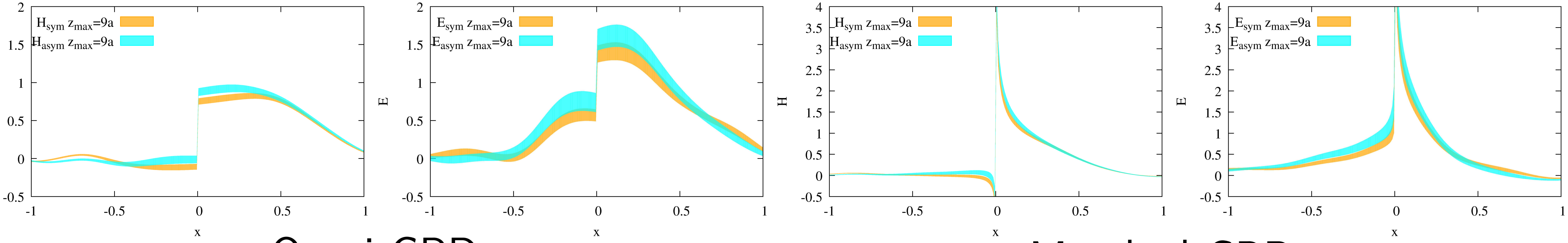
Im E





Quasi- and matched H and E GPDs

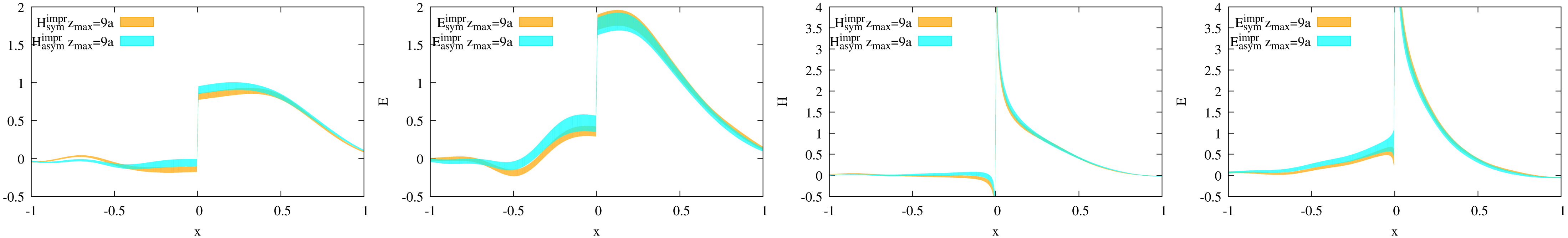
STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



Pseudo-ITDs (double-ratio renormalization):

A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3, z) = \frac{F(P_3, z)}{f(0, z)} \frac{f(0,0)}{f(P_3, 0)}$$

$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs ($-t > 0$)

f – MEs of PDFs ($t = 0$)

Pseudo-GPDs

Pseudo-ITDs (double-ratio renormalization):

A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3,z) = \frac{F(P_3,z)}{f(0,z)}\,\frac{f(0,0)}{f(P_3,0)}$$

$\mathcal{F} = \{\mathcal{H},\mathcal{E}\}$ – MEs of GPDs ($-t>0$)
f – MEs of PDFs ($t=0$)

can be matched to light-cone ITDs at short distances:

$$\overline{\mathcal{F}}(P_3,z) = \mathcal{F}(P_3,z) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du\; C(u)\left(\mathcal{F}(uP_3,z) - \mathcal{F}(P_3,z)\right),$$

$$C(u) = \frac{1+u^2}{u-1}\ln\frac{z^2\mu^2 e^{2\gamma_E+1}}{4} + 4\frac{\ln(1-u)}{u-1} - 2(u-1).$$

Pseudo-GPDs

Nucleon structure
Partonic structure in LQCD
Setup
Reference frames
Quasi vs. pseudo
GPDs(symm. frame)
GPDs (asymm. frame)
GPDs moments
TMDs
Summary

Backup slides
GPDs definitions
Pseudo-GPDs
GPDs moments
GPDs moments

Pseudo-ITDs (double-ratio renormalization): A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3, z) = \frac{F(P_3, z)}{f(0, z)} \frac{f(0,0)}{f(P_3, 0)}$$

$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs ($-t > 0$)
f – MEs of PDFs ($t = 0$)

can be matched to light-cone ITDs at short distances:

$$\overline{\mathcal{F}}(P_3, z) = \mathcal{F}(P_3, z) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, C(u) \left(\mathcal{F}(uP_3, z) - \mathcal{F}(P_3, z)\right),$$

$$C(u) = \frac{1+u^2}{u-1} \ln \frac{z^2\mu^2 e^{2\gamma_E+1}}{4} + 4\frac{\ln(1-u)}{u-1} - 2(u-1).$$

Given matched ITDs one can reconstruct x-dependent GPDs:

$$\mathrm{Re}\,\overline{\mathcal{F}}(\nu, \mu) = \int_0^1 dx\, \cos(\nu x) \overline{\mathcal{F}}_v(x, \mu),$$

$$\mathrm{Im}\,\overline{\mathcal{F}}(\nu, \mu) = \int_0^1 dx\, \sin(\nu x) \overline{\mathcal{F}}_{v2s}(x, \mu)$$

Pseudo-GPDs

Pseudo-ITDs (double-ratio renormalization):

A. Radyushkin, Phys. Rev. D100 (2019) 116011

$$\mathcal{F}(P_3, z) = \frac{F(P_3, z)}{f(0, z)} \frac{f(0,0)}{f(P_3, 0)}$$

$\mathcal{F} = \{\mathcal{H}, \mathcal{E}\}$ – MEs of GPDs ($-t > 0$)
f – MEs of PDFs ($t = 0$)

can be matched to light-cone ITDs at short distances:

$$\overline{\mathcal{F}}(P_3, z) = \mathcal{F}(P_3, z) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, C(u) \left(\mathcal{F}(uP_3, z) - \mathcal{F}(P_3, z)\right),$$

$$C(u) = \frac{1+u^2}{u-1} \ln \frac{z^2 \mu^2 e^{2\gamma_E + 1}}{4} + 4\frac{\ln(1-u)}{u-1} - 2(u-1).$$

Given matched ITDs one can reconstruct x-dependent GPDs:

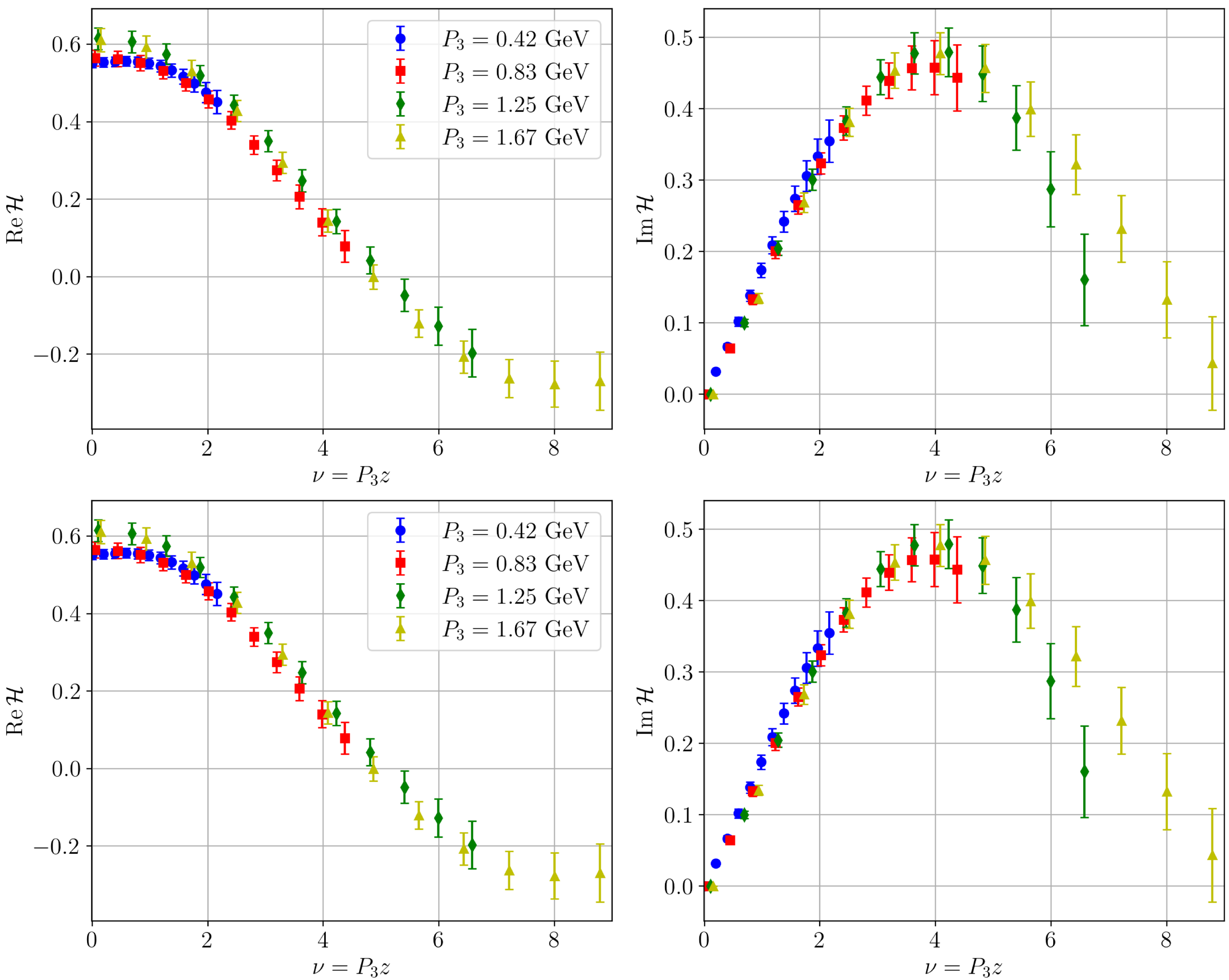
$$\mathrm{Re}\, \overline{\mathcal{F}}(\nu, \mu) = \int_0^1 dx\, \cos(\nu x) \overline{\mathcal{F}}_v(x, \mu),$$

$$\mathrm{Im}\, \overline{\mathcal{F}}(\nu, \mu) = \int_0^1 dx\, \sin(\nu x) \overline{\mathcal{F}}_{v2s}(x, \mu)$$

using a fitting ansatz:

$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b \left(1 + c x^{d_1} (1-x)^{d_2}\right).$$

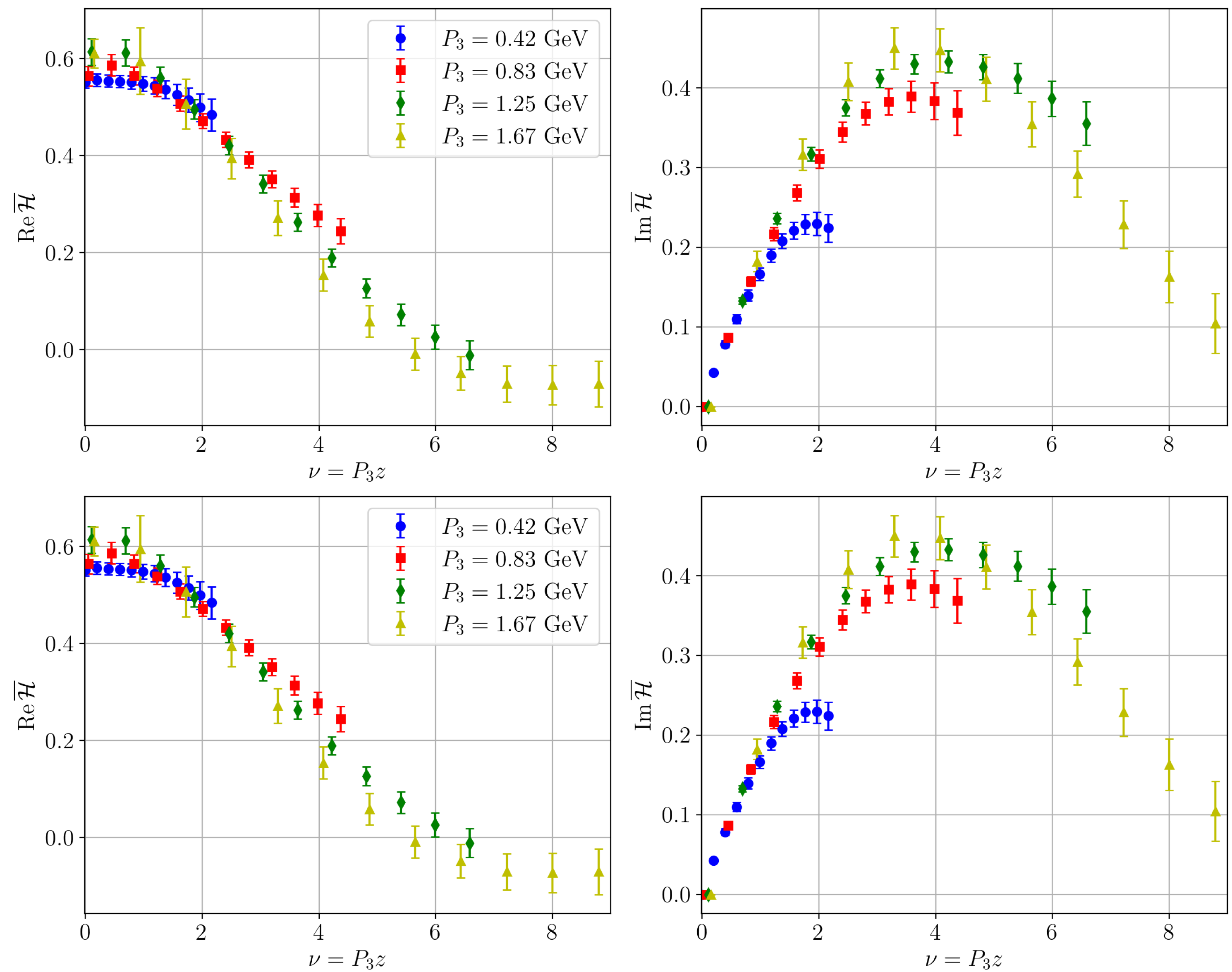
Reduced ITDs



$-t$ [GeV^2]	P_3 [GeV]	(Δ_1, Δ_2) [$2\pi/L$]	N_Δ	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm meas}$
0	±0.42	(0,0)	2	100	8	1600
0	±0.83	(0,0)	2	100	8	1600
0	±1.25	(0,0)	2	269	16	8608
0	±1.67	(0,0)	2	506	32	32384
0.17	±0.42	(±1,0), (0,±1)	8	100	8	6400
0.17	±0.83	(±1,0), (0,±1)	8	100	8	6400
0.17	±1.25	(±1,0), (0,±1)	8	269	8	17216
0.17	±1.67	(±1,0), (0,±1)	8	506	32	129536
0.34	±0.42	(±1, ±1)	8	100	8	6400
0.34	±0.83	(±1, ±1)	8	100	8	6400
0.34	±1.25	(±1, ±1)	8	195	8	12480
0.34	±1.67	(±1, ±1)	8	506	32	129536
0.65	±0.42	(±2,0), (0,±2)	8	100	8	6400
0.65	±0.83	(±2,0), (0,±2)	8	100	8	6400
0.65	±1.25	(±2,0), (0,±2)	8	269	8	17216
0.65	±1.67	(±2,0), (0,±2)	8	506	32	129536
0.81	±0.42	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±0.83	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±1.25	(±1,±2), (±2,±1)	16	195	8	24960
0.81	±1.67	(±1,±2), (±2,±1)	16	506	32	259072
1.24	±0.42	(±2, ±2)	8	100	8	6400
1.24	±0.83	(±2, ±2)	8	100	8	6400
1.24	±1.25	(±2, ±2)	8	195	8	12480
1.24	±1.67	(±2, ±2)	8	506	32	129536
1.38	±0.42	(±3,0), (0,±3)	8	100	8	6400
1.38	±0.83	(±3,0), (0,±3)	8	100	8	6400
1.38	±1.25	(±3,0), (0,±3)	8	269	8	17216
1.38	±1.67	(±3,0), (0,±3)	8	506	32	129536
1.52	±0.42	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±0.83	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±1.25	(±1,±3), (±3,±1)	16	195	8	24960
1.52	±1.67	(±1,±3), (±3,±1)	16	506	32	259072
2.29	±0.42	(±4,0), (0,±4)	8	100	8	6400
2.29	±0.83	(±4,0), (0,±4)	8	100	8	6400
2.29	±1.25	(±4,0), (0,±4)	8	269	8	17216
2.29	±1.67	(±4,0), (0,±4)	8	506	32	129536

S. Bhattacharya et al., PRD110(2024)054502

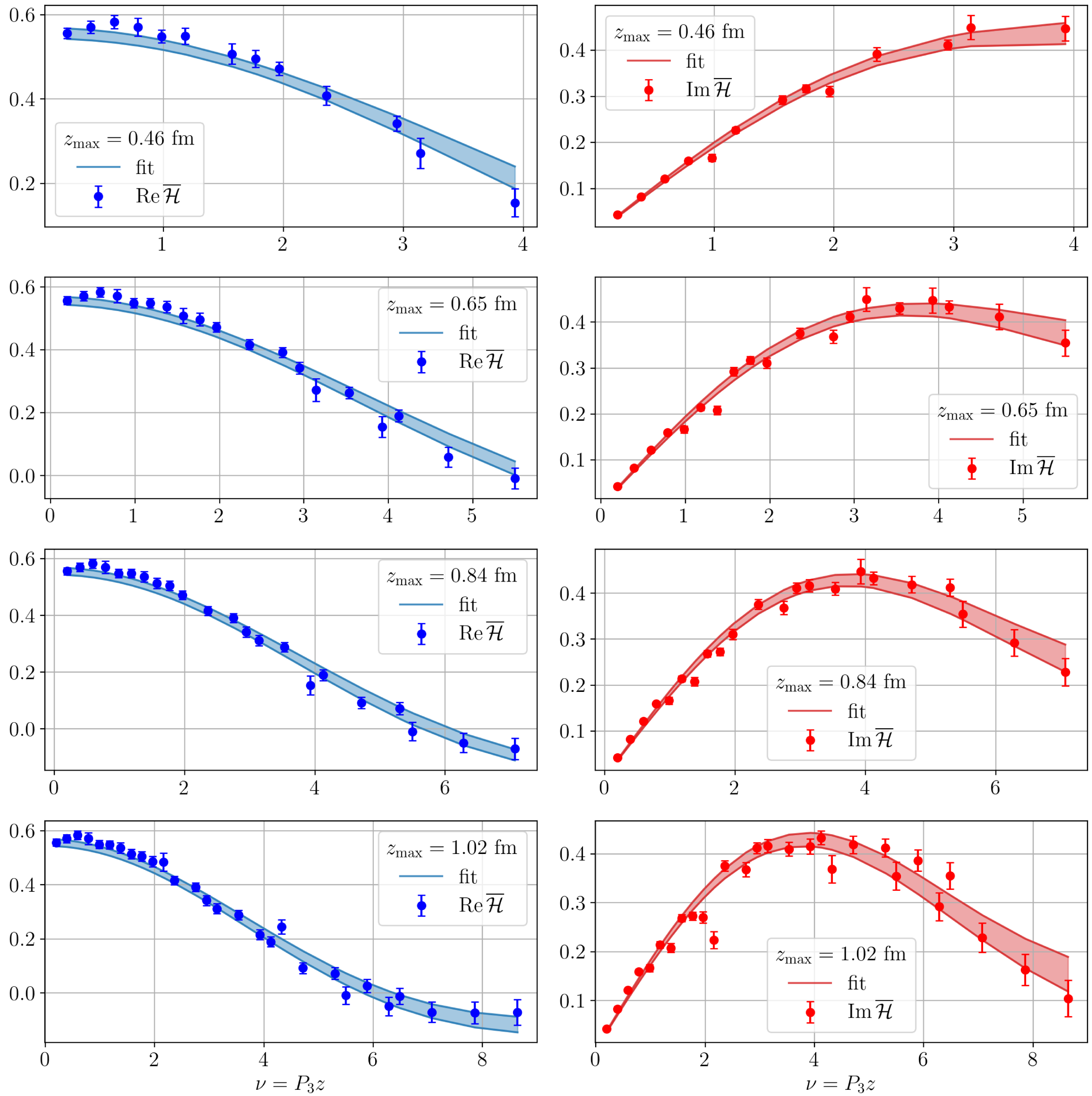
Light-cone (matched) ITDs



$-t$ [GeV^2]	P_3 [GeV]	(Δ_1, Δ_2) [$2\pi/L$]	N_Δ	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm meas}$
0	±0.42	(0,0)	2	100	8	1600
0	±0.83	(0,0)	2	100	8	1600
0	±1.25	(0,0)	2	269	16	8608
0	±1.67	(0,0)	2	506	32	32384
0.17	±0.42	(±1,0), (0,±1)	8	100	8	6400
0.17	±0.83	(±1,0), (0,±1)	8	100	8	6400
0.17	±1.25	(±1,0), (0,±1)	8	269	8	17216
0.17	±1.67	(±1,0), (0,±1)	8	506	32	129536
0.34	±0.42	(±1, ±1)	8	100	8	6400
0.34	±0.83	(±1, ±1)	8	100	8	6400
0.34	±1.25	(±1, ±1)	8	195	8	12480
0.34	±1.67	(±1, ±1)	8	506	32	129536
0.65	±0.42	(±2,0), (0,±2)	8	100	8	6400
0.65	±0.83	(±2,0), (0,±2)	8	100	8	6400
0.65	±1.25	(±2,0), (0,±2)	8	269	8	17216
0.65	±1.67	(±2,0), (0,±2)	8	506	32	129536
0.81	±0.42	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±0.83	(±1,±2), (±2,±1)	16	100	8	12800
0.81	±1.25	(±1,±2), (±2,±1)	16	195	8	24960
0.81	±1.67	(±1,±2), (±2,±1)	16	506	32	259072
1.24	±0.42	(±2, ±2)	8	100	8	6400
1.24	±0.83	(±2, ±2)	8	100	8	6400
1.24	±1.25	(±2, ±2)	8	195	8	12480
1.24	±1.67	(±2, ±2)	8	506	32	129536
1.38	±0.42	(±3,0), (0,±3)	8	100	8	6400
1.38	±0.83	(±3,0), (0,±3)	8	100	8	6400
1.38	±1.25	(±3,0), (0,±3)	8	269	8	17216
1.38	±1.67	(±3,0), (0,±3)	8	506	32	129536
1.52	±0.42	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±0.83	(±1,±3), (±3,±1)	16	100	8	12800
1.52	±1.25	(±1,±3), (±3,±1)	16	195	8	24960
1.52	±1.67	(±1,±3), (±3,±1)	16	506	32	259072
2.29	±0.42	(±4,0), (0,±4)	8	100	8	6400
2.29	±0.83	(±4,0), (0,±4)	8	100	8	6400
2.29	±1.25	(±4,0), (0,±4)	8	269	8	17216
2.29	±1.67	(±4,0), (0,±4)	8	506	32	129536

S. Bhattacharya et al., PRD110(2024)054502

Fitted ITDs

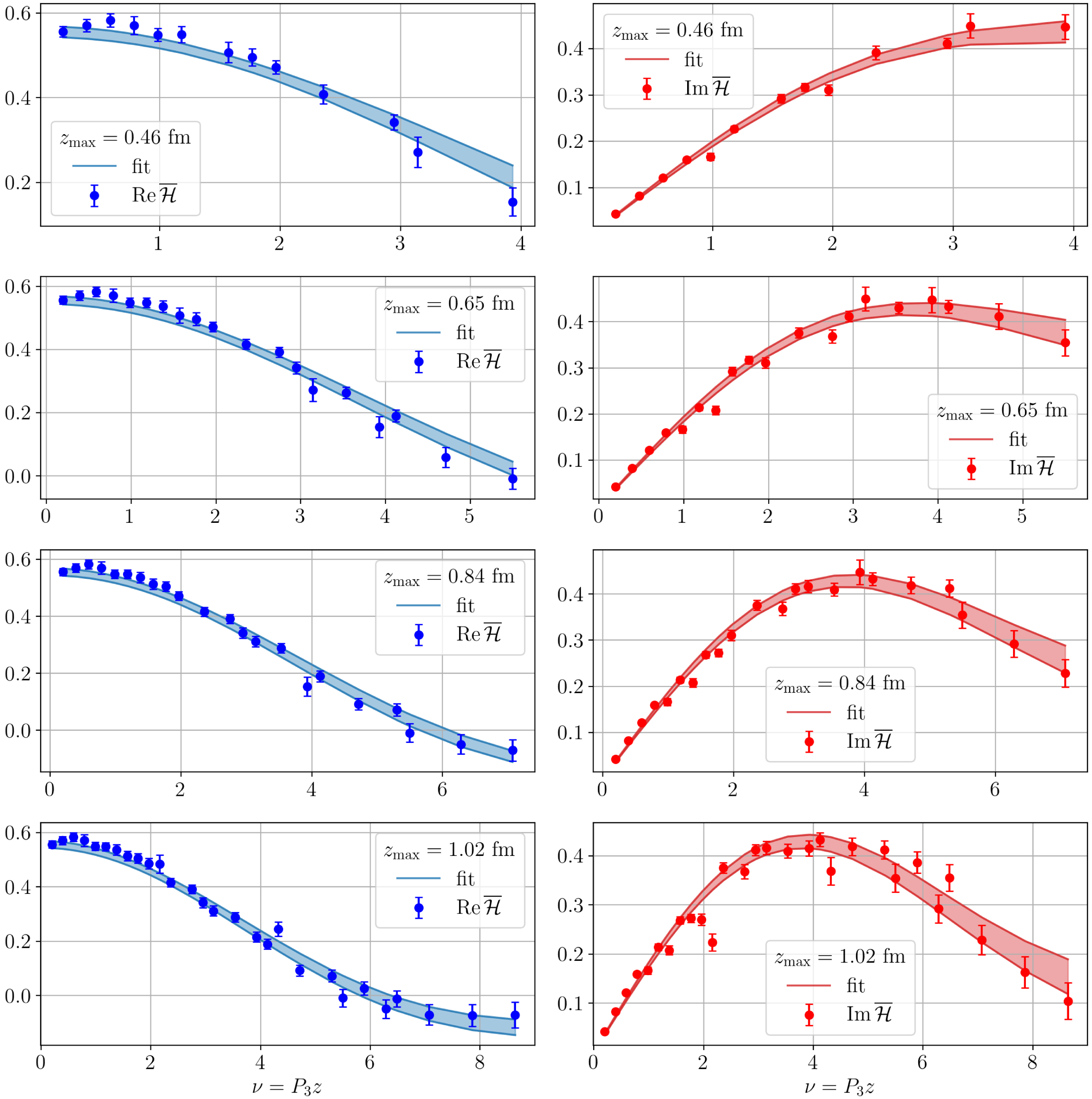


$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b$$

marginal or absent sensitivity to additional fit parameters

S. Bhattacharya et al., PRD110(2024)054502

Fitted ITDs



$$\overline{\mathcal{F}}(x) = N x^a (1-x)^b$$

marginal or absent sensitivity to additional fit parameters

$z_{\max} = 7a \approx 0.65$ fm – rather conservative choice

$z_{\max} = 9a \approx 0.84$ fm – also plausible (particularly for valence)

S. Bhattacharya et al., PRD110(2024)054502

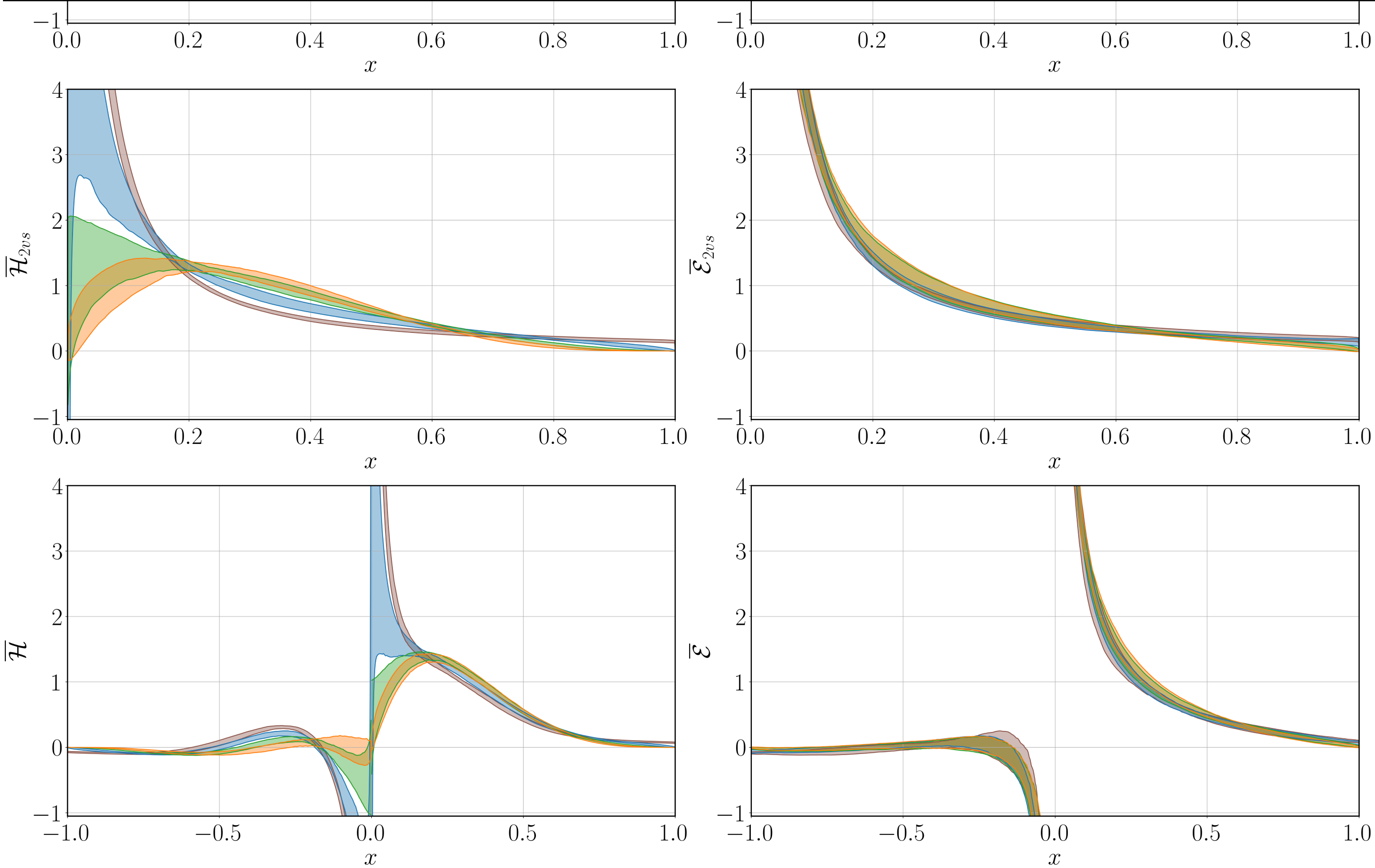
Fitting-reconstructed GPDs in x-space

S. Bhattacharya et al., PRD110(2024)05450

$\overline{\mathcal{H}}$ $\overline{\mathcal{E}}$

[redacted]

VALENCE

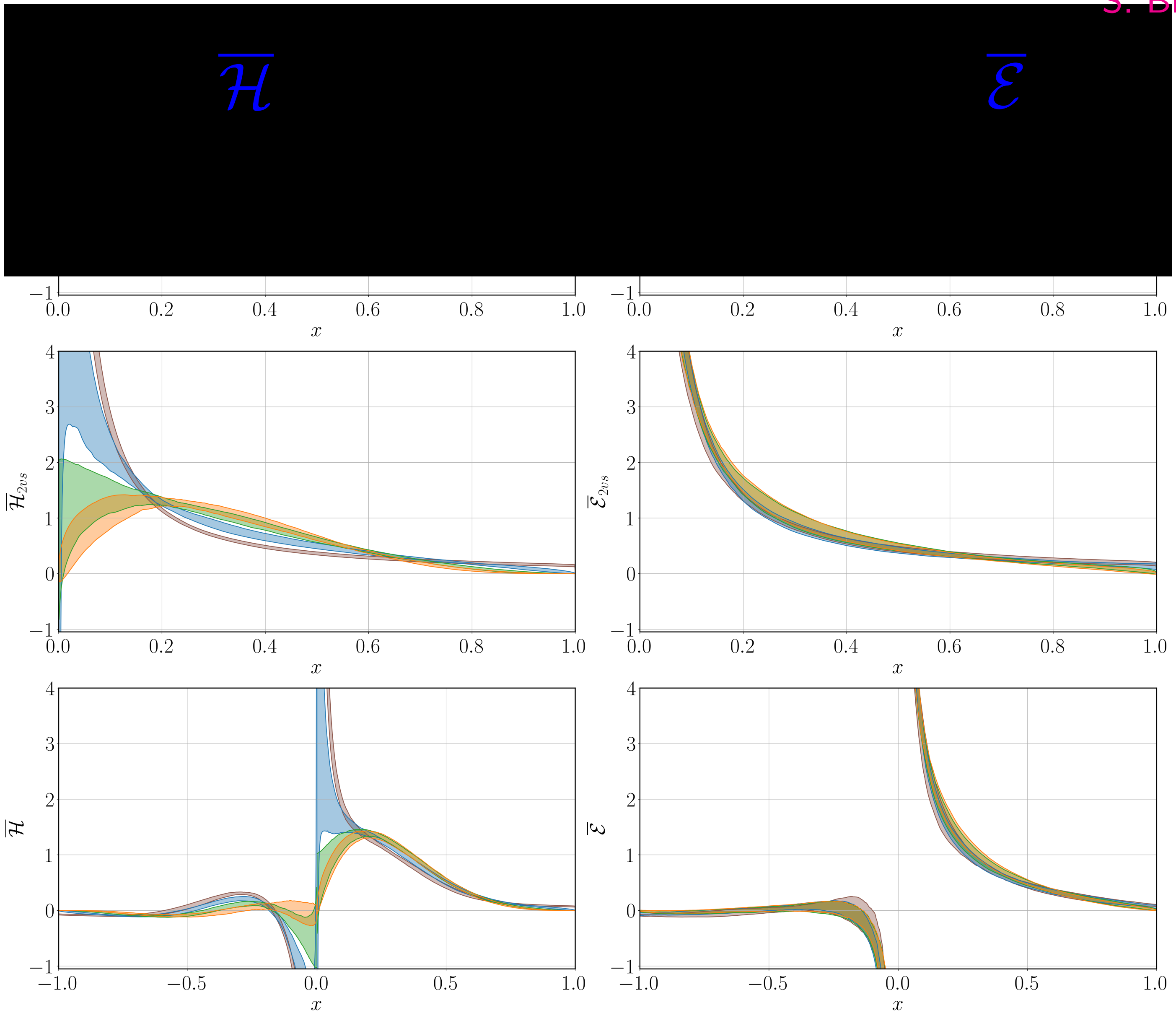


VALENCE + 2*SEA

VALENCE + SEA

Fitting-reconstructed GPDs in x-space

S. Bhattacharya et al., PRD110(2024)05450



VALENCE

VALENCE + 2*SEA

VALENCE + SEA

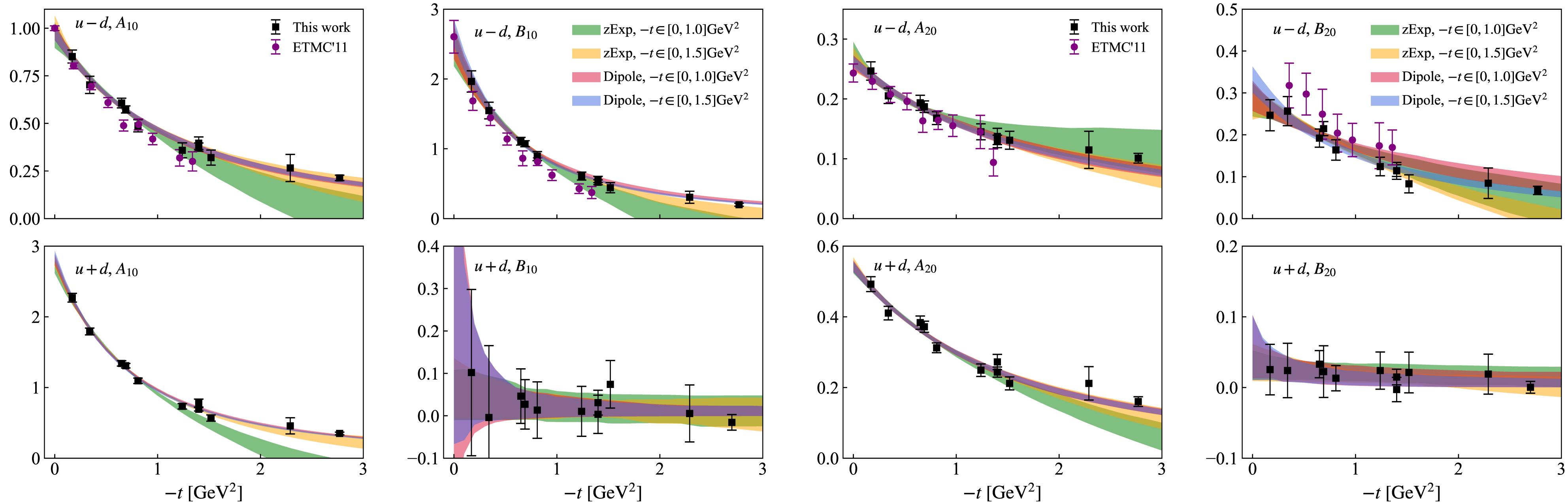
In all cases $z_{\text{max}} = 7a$ and $9a$ nicely compatible

Interesting:
$\overline{\mathcal{E}}$ obviously better-behaved!

Short-distance factorization of ratio-renormalized H/E:

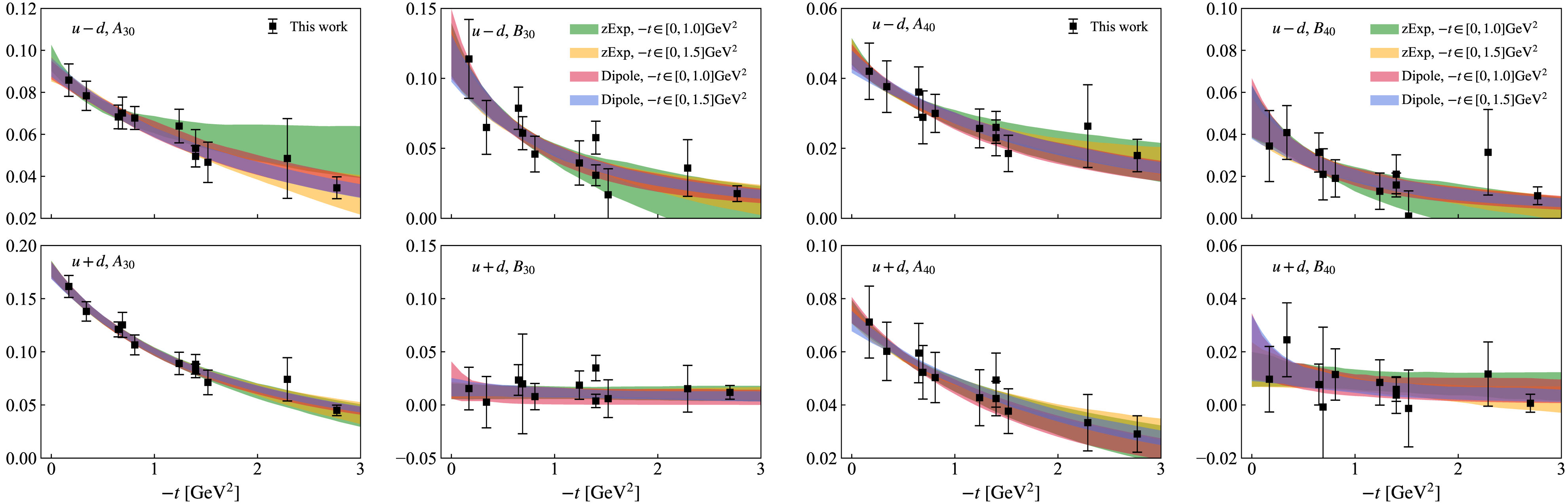
$$\mathcal{F}^{\overline{\rm MS}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\rm MS}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda^2_{\rm QCD}z^2),$$

$C_n^{\overline{\rm MS}}(\mu^2z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

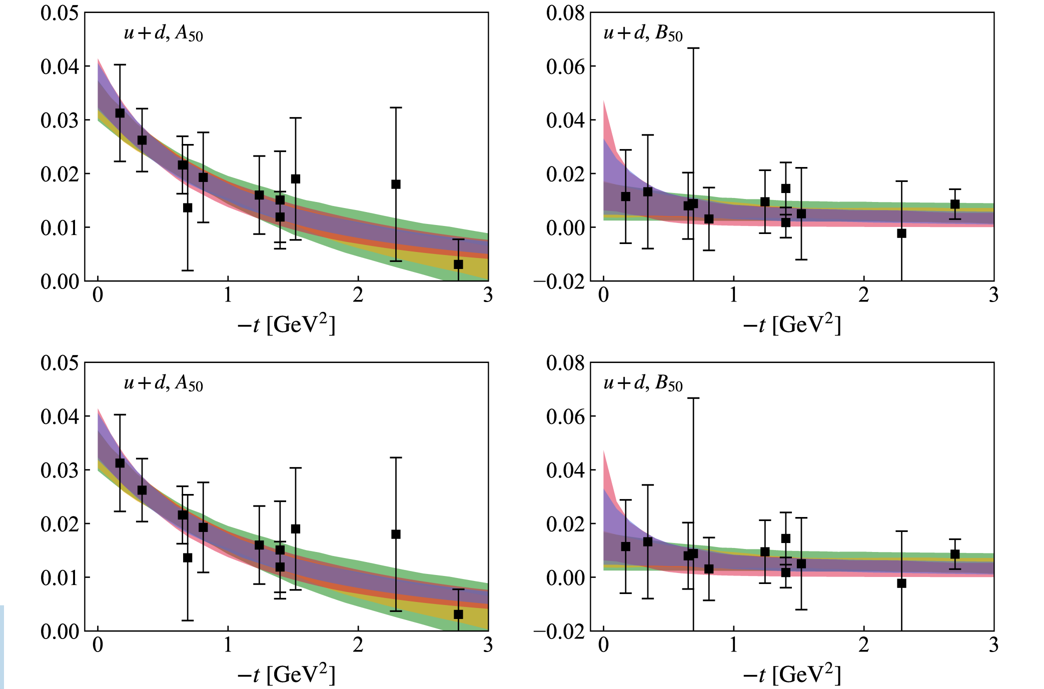


S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

GPDs moments from OPE of non-local operators



Also
higher moments!



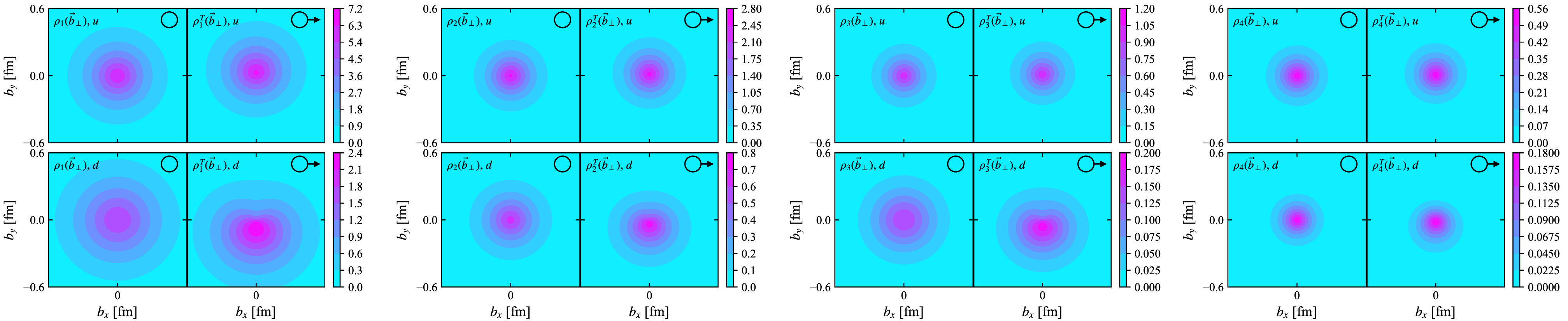
S. Bhattacharya et al.
(ETMC/BNL/ANL)
PRD 108(2023)014507

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}_\perp^2) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}_\perp^2)] e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment 2nd moment 3rd moment 4th moment



S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507