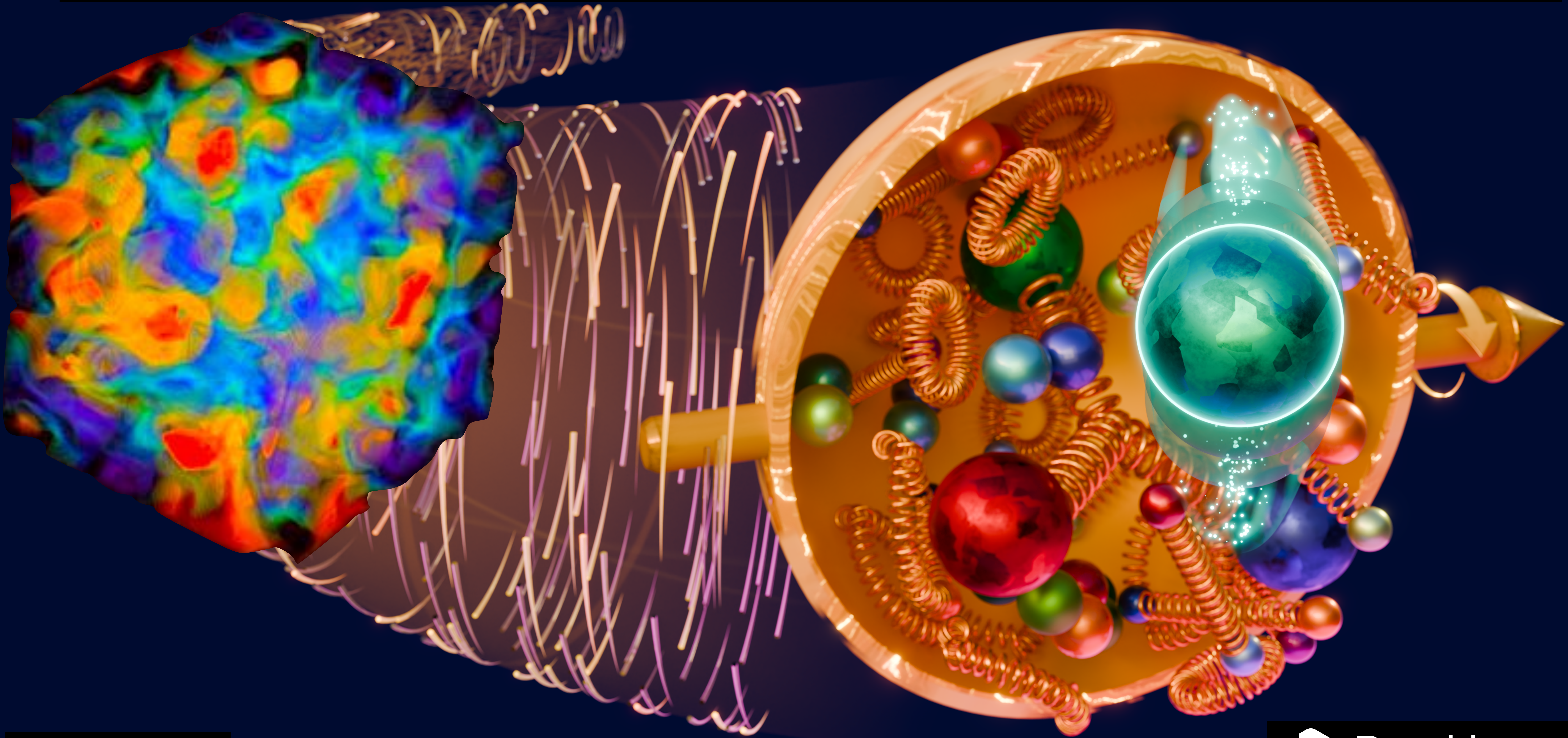
Nonperturbative Collins-Soper Kernel from Lattice QCD

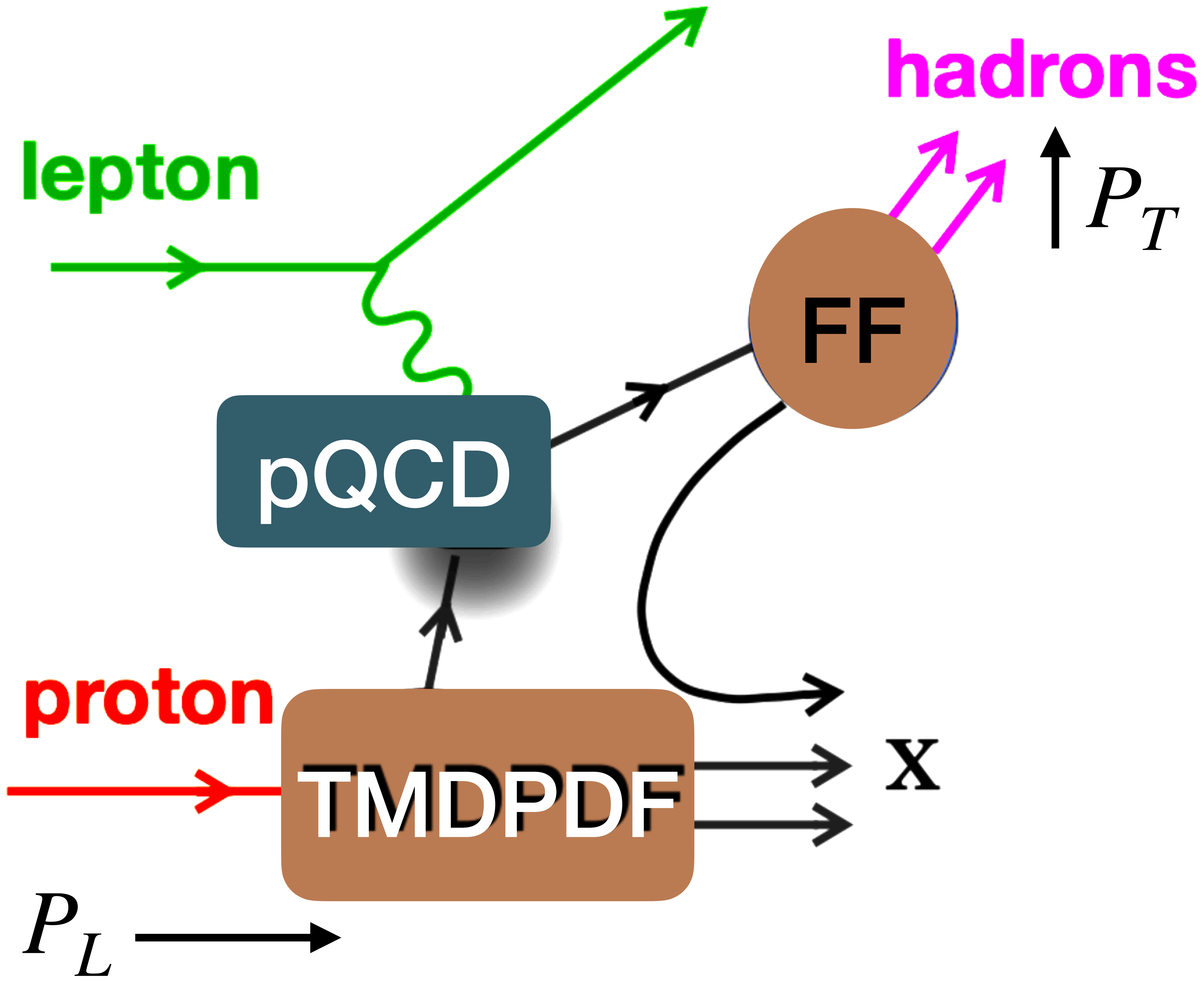


September 2024,
Palermo, Italy

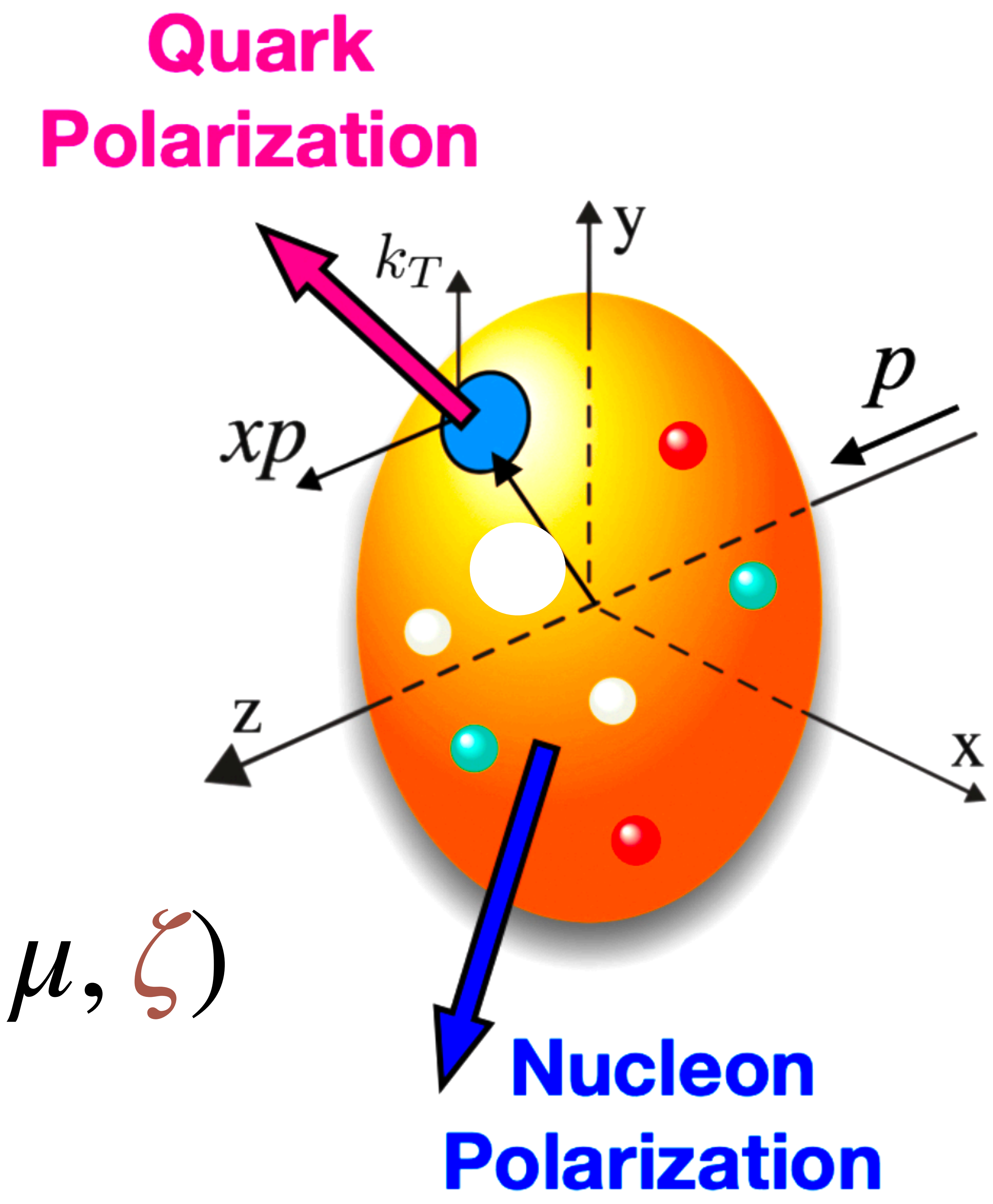
Swagato Mukherjee

Brookhaven National Laboratory

semi-inclusive deep inelastic scattering: JLAB, EIC, ...



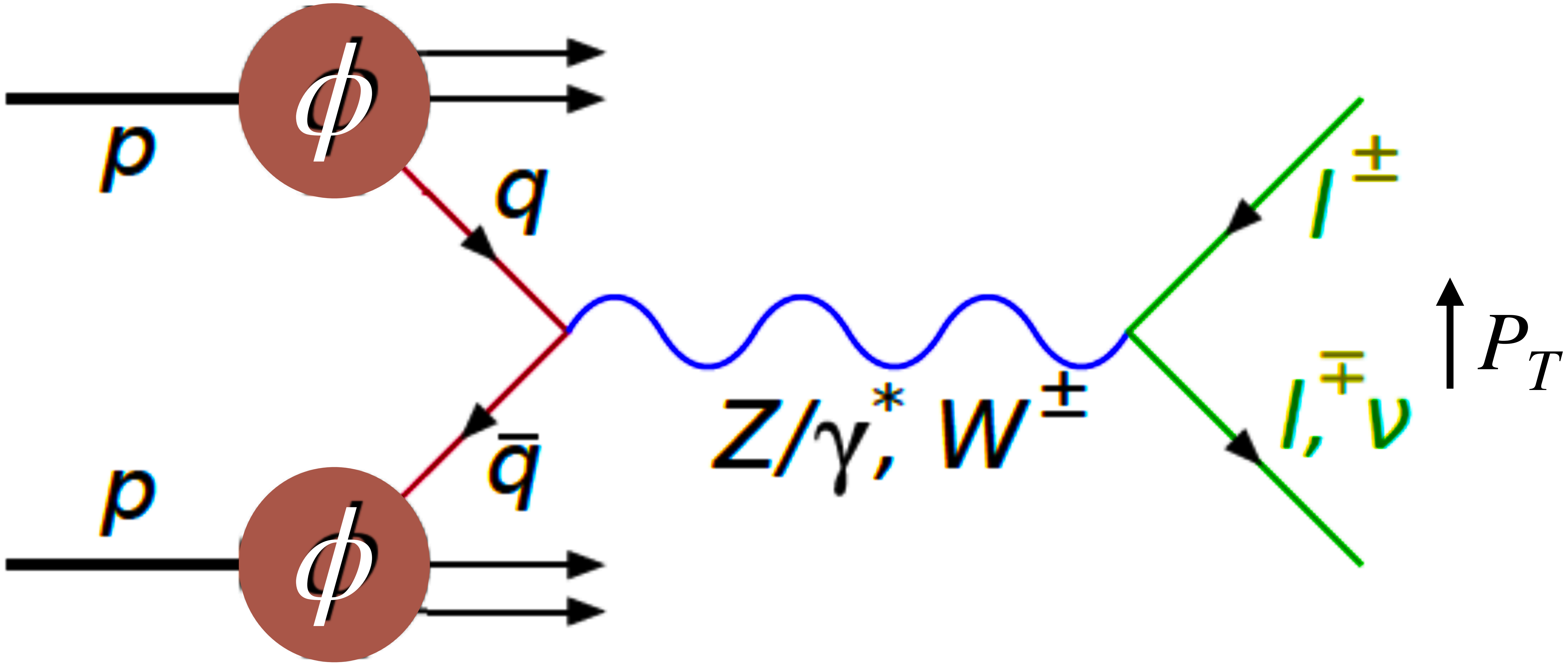
$$\phi(x, k_T, \mu, \zeta)$$



transverse momentum-dependent PDF (TMDPDF)

b_T is Fourier conjugate of k_T

Drell-Yan processes: RHIC, LHC, …



$$\sigma \sim \phi(x, k_T, \mu, \eta) \otimes \phi(x, k_T, \mu, \zeta)$$

evolution of TMD functions across collision energies

Collins-Soper kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_T, \mu) = \frac{\partial \phi(x, b_T, \zeta, \mu)}{\partial \ln \sqrt{\zeta}}$$

property of QCD vacuum — independent of hadronic state

nonperturbative Collins-Soper kernel

b_T

$b_T \sim \Lambda_{\mathrm{QCD}}^{-1}$

nonperturbative

$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$

$b_T \ll \Lambda_{\mathrm{QCD}}^{-1}$

perturbative

JLAB

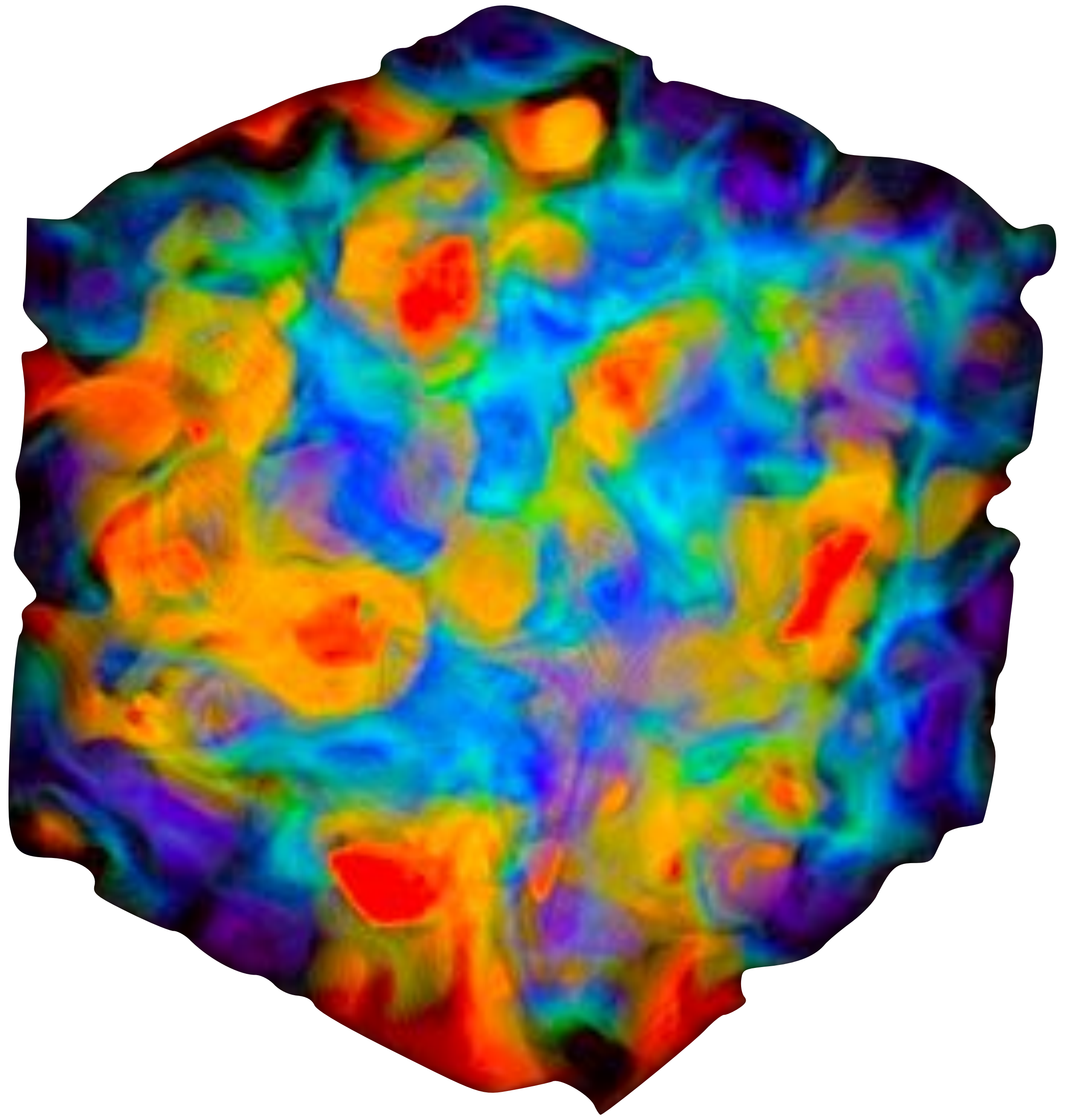
EIC

RHIC

LHC

$\sqrt{\zeta}$

our objective

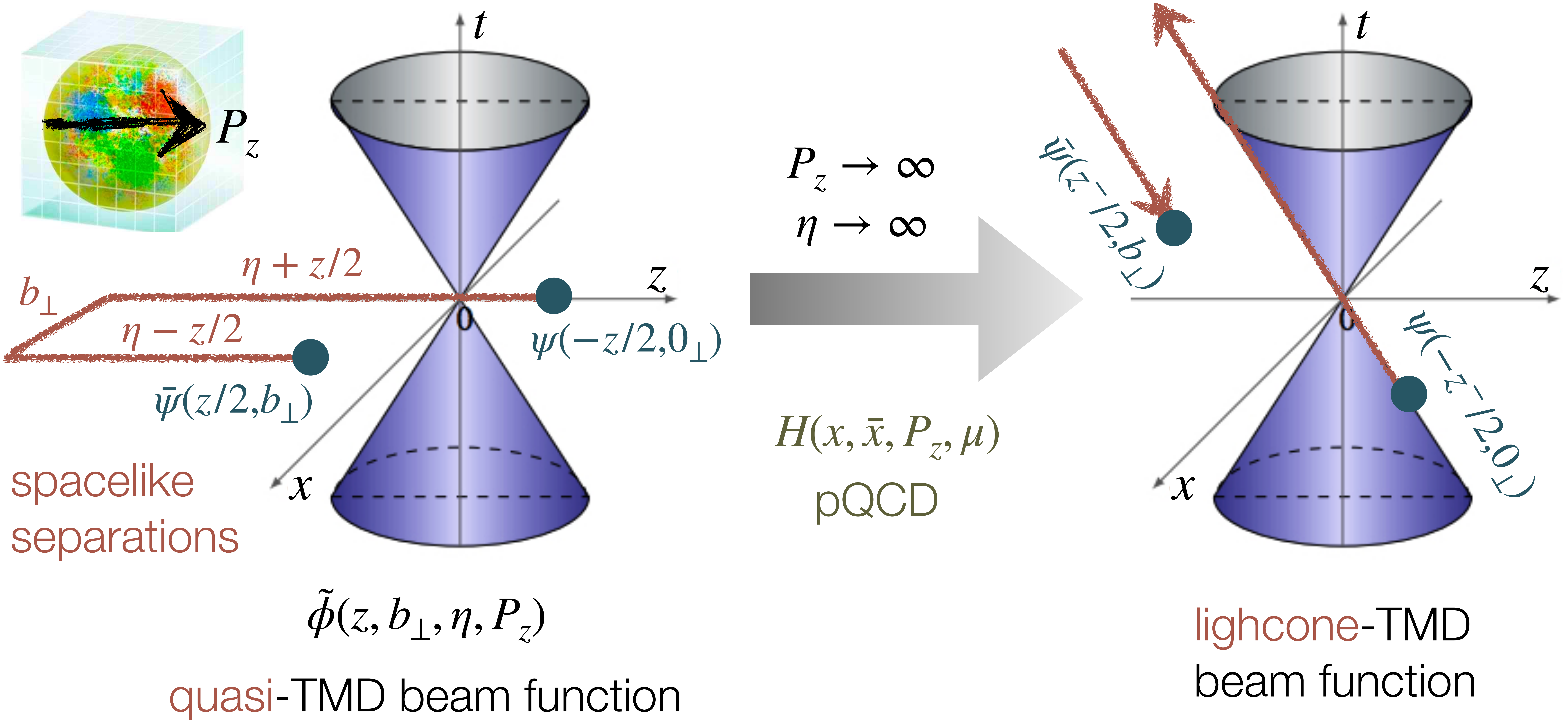


lattice QCD

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$$

nonperturbative CS kernel

TMD distributions from lattice QCD



- large η; check for η-independence: $\tilde{\phi}(b_z, b_\perp, \eta, P_z) \longrightarrow \tilde{\phi}(b_z, b_\perp, P_z)$

- renormalize: $\tilde{\phi}(b_z, b_\perp, P_z) \longrightarrow \tilde{\phi}(b_z, b_\perp, P_z, \mu)$

- Fourier transform to momentum (x) space: $\tilde{\phi}(b_z, b_\perp, P_z, \mu) \longrightarrow \tilde{\phi}(x, b_\perp, P_z, \mu)$

- Perturbative matching:

Collins Soper kernel

$$\frac{\tilde{\phi}_\Gamma(x, b_\perp, P_z, \mu)}{\sqrt{S_r(b_\perp, \mu)}} = H(x, \bar{x}, P_z, \mu)\phi(x, b_\perp, \zeta, \mu) \exp\left[\frac{1}{4}\left(\ln\frac{(2xP_z)^2}{\zeta} + \ln\frac{(2\bar{x}P_z)^2}{\zeta}\right)\gamma^{\overline{\text{MS}}}(b_\perp, \mu)\right]$$

$$+\,\mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}, \frac{\Lambda^2_{\text{QCD}}}{(\bar{x}P_z)^2}, \frac{1}{(b_\perp(\bar{x}P_z))^2}\right)$$

soft function

perturbative kernel

$\bar{x} = 1 - x$

lightcone TMD distribution

power corrections

CS kernel from lattice QCD

- ratios of quasi-TMD beam functions for 2 different boost momenta, P_1 & P_2

Collins Soper kernel

perturbative kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\mathrm{MS}}}(x,\mu,P_1,P_2)$$

independent of x, P_1, P_2

+ power corrections

soft function cancels

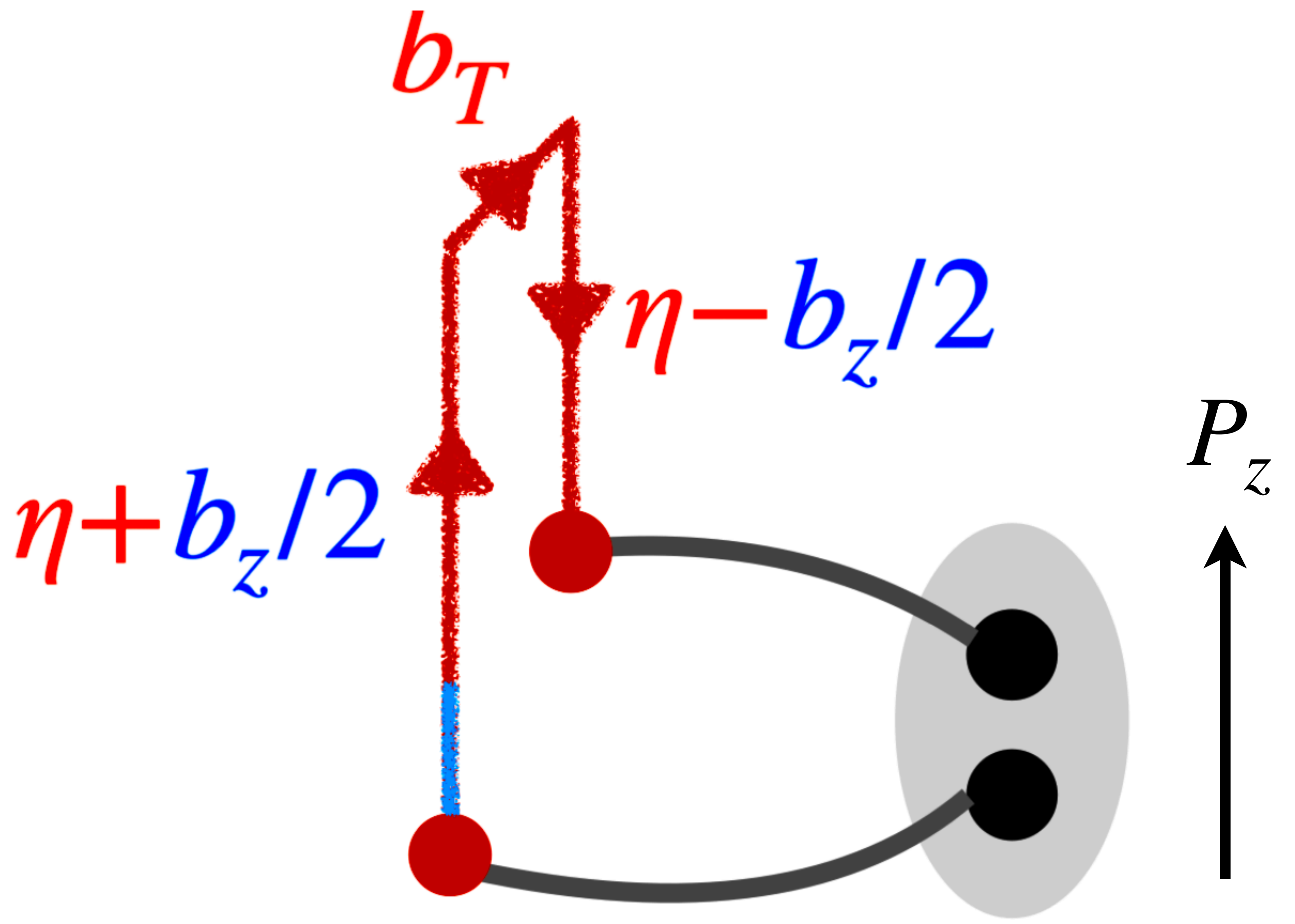
P_1 & P_2 both must be large to suppress power corrections, such that CS kernel is indep. of those

lattice QCD calculations of CS kernel

- simplest choice for the quasi-TMD beam function $\tilde{\phi}(b_z, b_\perp, \eta, P_z)$

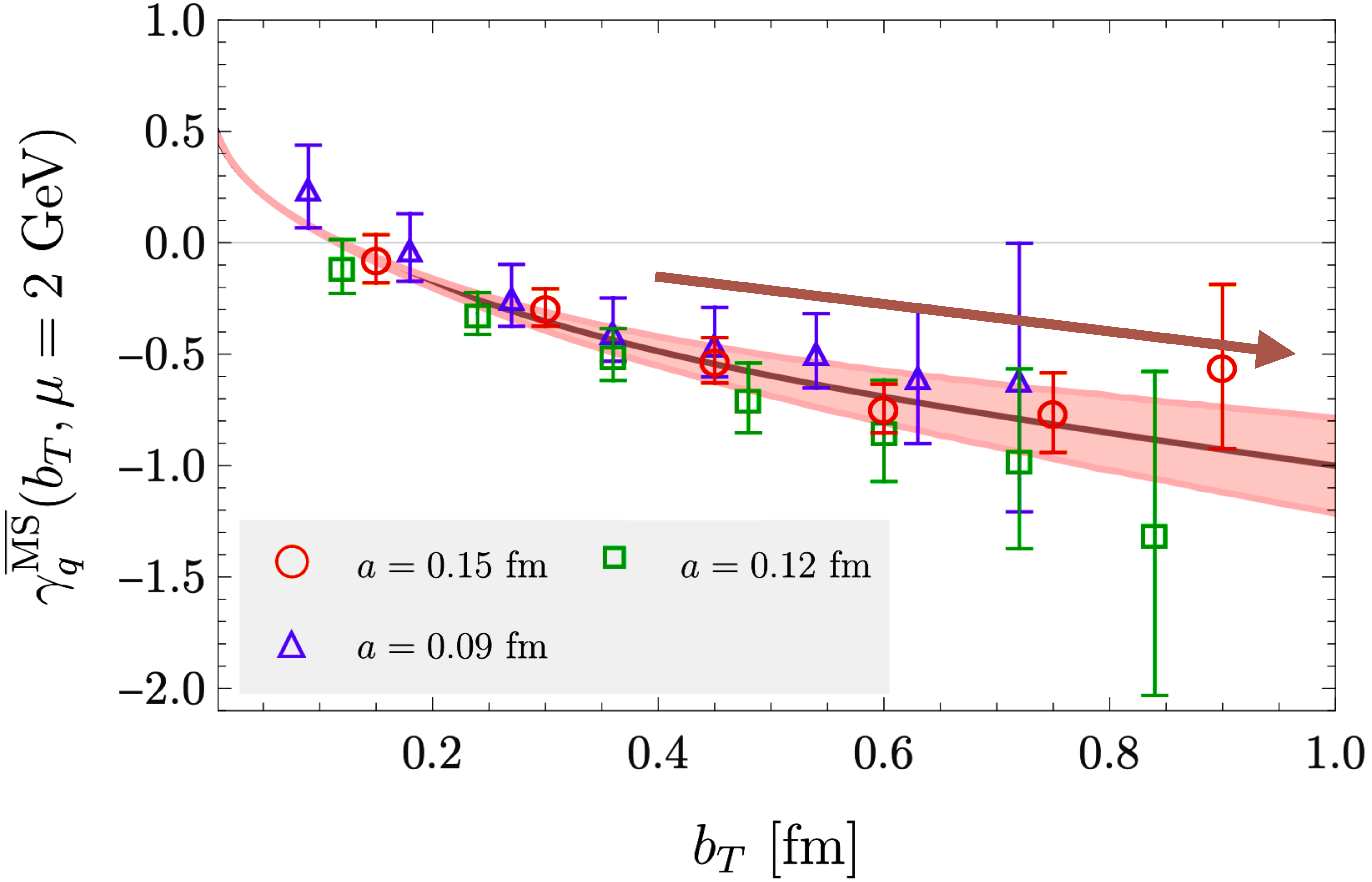
pion TMD wave function (TMDWF)

$$\langle \Omega | \bar{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_\sqsupset(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta) \psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$



the challenge

rapidly growing errors with increasing $b_\perp$

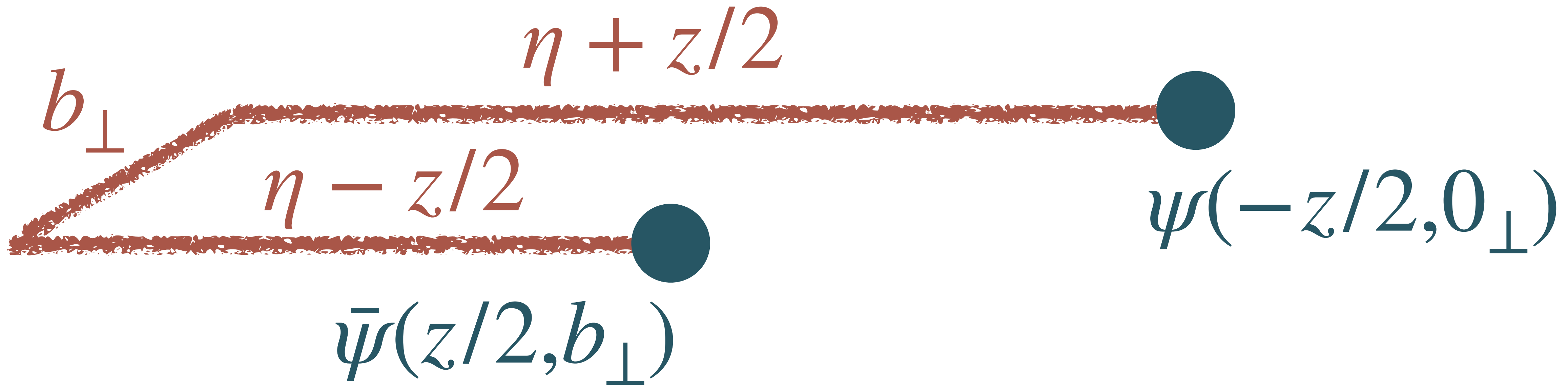


Avkhadiev et al., Phys. Rev. Lett. 23, 231901 (2024)

understanding the challenge

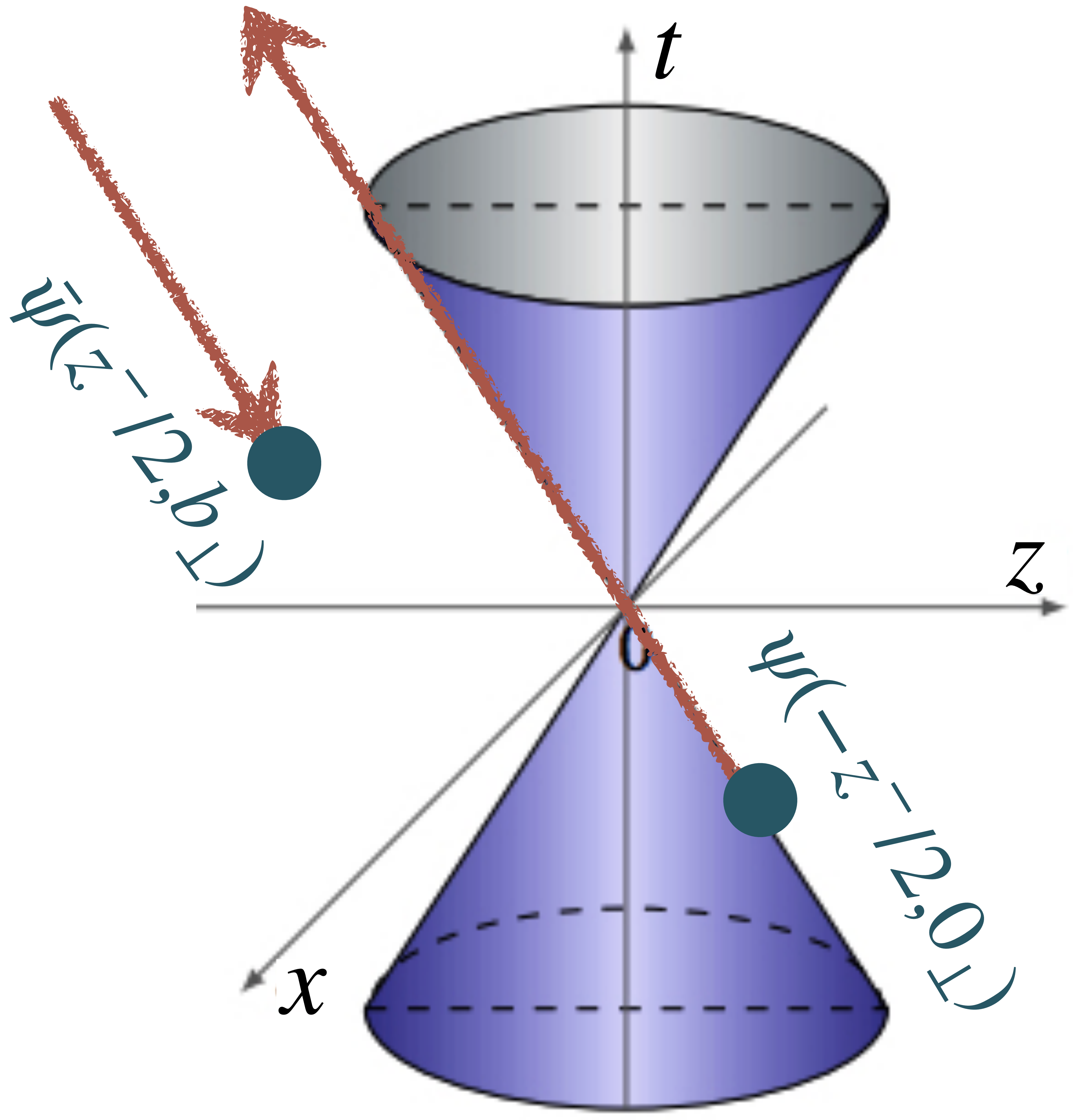
multiplicative renormalization factor of the Wilson line:

$$\sim e^{-\delta m(\eta + b_\perp)}$$



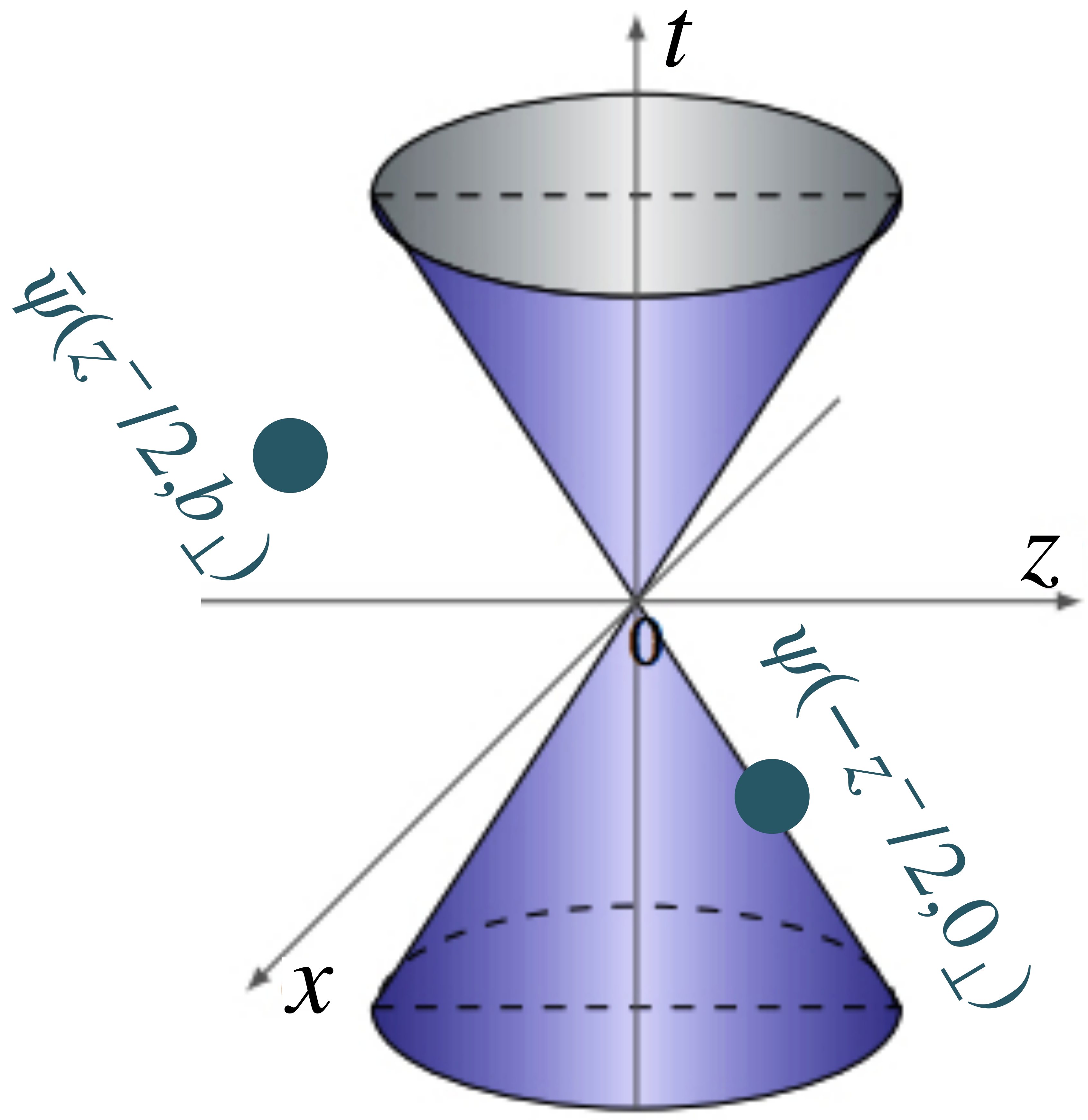
exponential decrease of signal for large η and increasing $b_\perp$

overcoming the challenge



$$\equiv$$

physical lightcone gauge $A^+ = 0$



- how can we access $A^+ = 0$ in lattice QCD calculations ?

find a gauge that becomes equivalent to $A^+ = 0$ in the limit $P_z \to \infty$

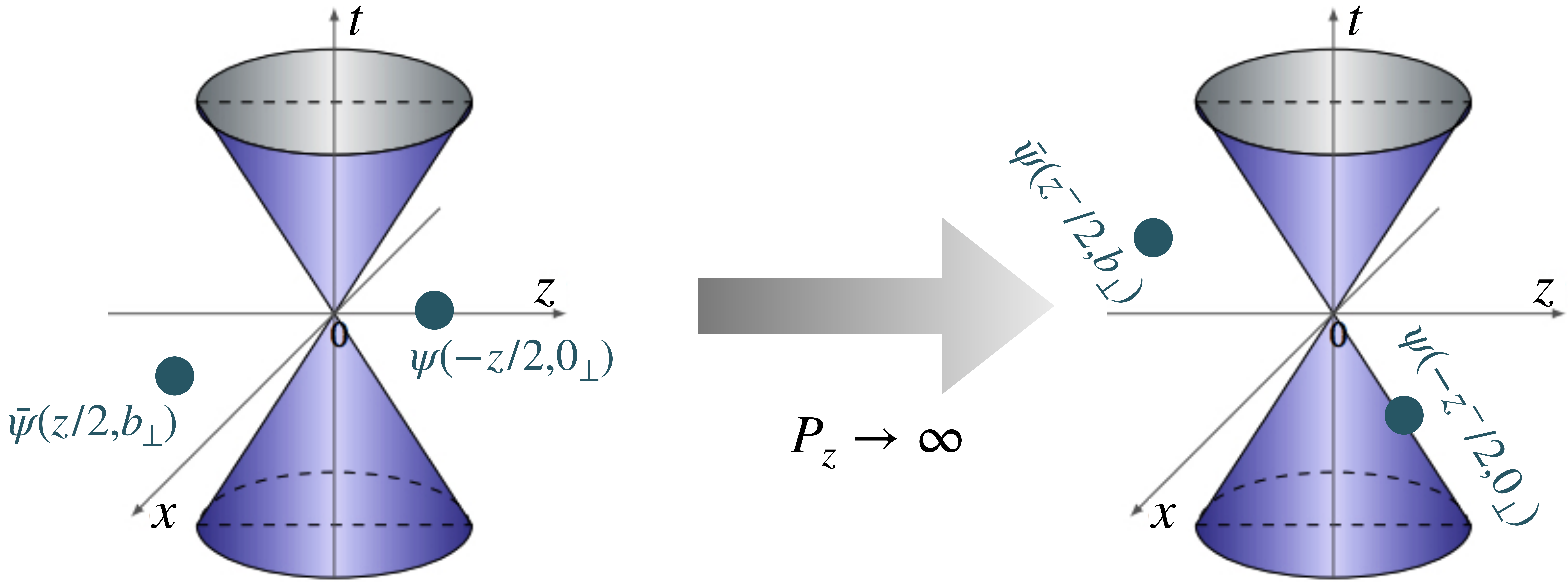
Coulomb gauge	$P_z \to \infty$	lightcone gauge
$\vec{\nabla} \cdot \vec{A} = 0$	→	$A^+ = 0$

Ji et al., Phys. Rev. Lett. 111, 112002 (2013)

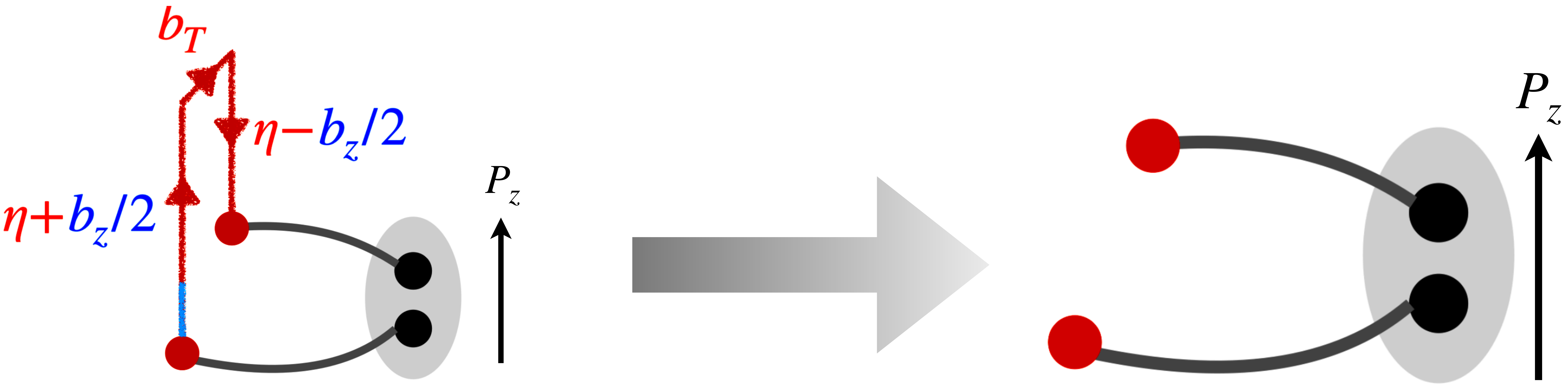
Hatta et al., Phys. Rev. D 89, no.8, 085030 (2014)

quasi-TMD beam function in Coulomb gauge (CG)

$$\overline{\psi}(\frac{\mathbf{b}}{2})\Gamma\psi(-\frac{\mathbf{b}}{2})|_{\nabla\cdot\mathbf{A}=0}$$



CG quasi-TMD beam function



$$\langle \Omega | \bar{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_{\sqsupset}(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta)\psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$

$$\langle \Omega | \bar{\psi}(\frac{b_z}{2}, b_\perp)\Gamma \psi(-\frac{b_z}{2}, 0) |_{\vec{\nabla}\cdot\vec{A}=0} | \pi^+, P_z \rangle$$

+ re-computation of pQCD matching function $\delta\gamma^{\overline{\mathrm{MS}}}(x, \mu, P_1, P_2)$

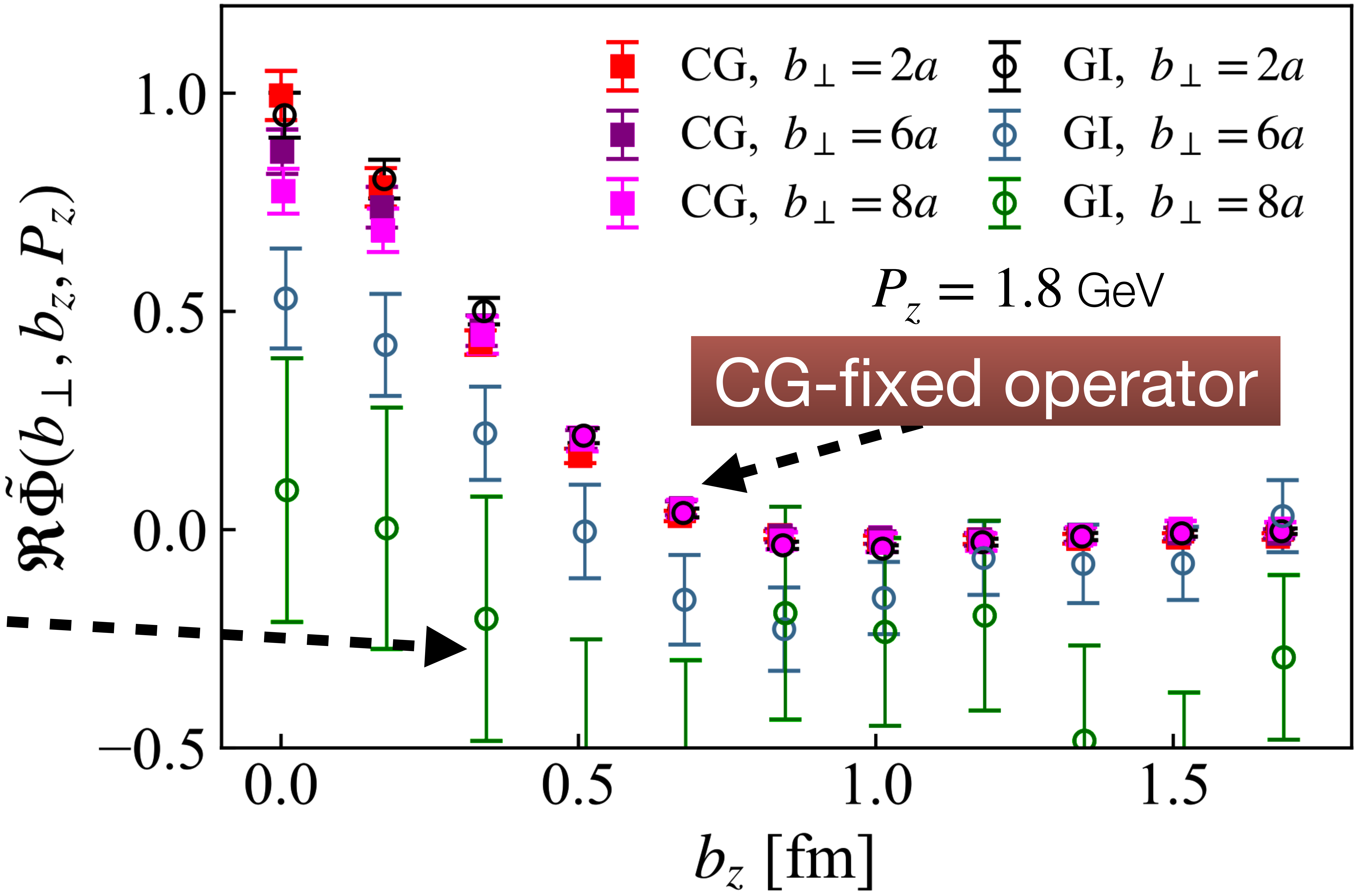
next-to-leading-log (NLL) accuracy

Zhao: 2311.01391

renormalized quasi-TMD beam functions

unitary chiral (Domain Wall) fermions, physical pion mass

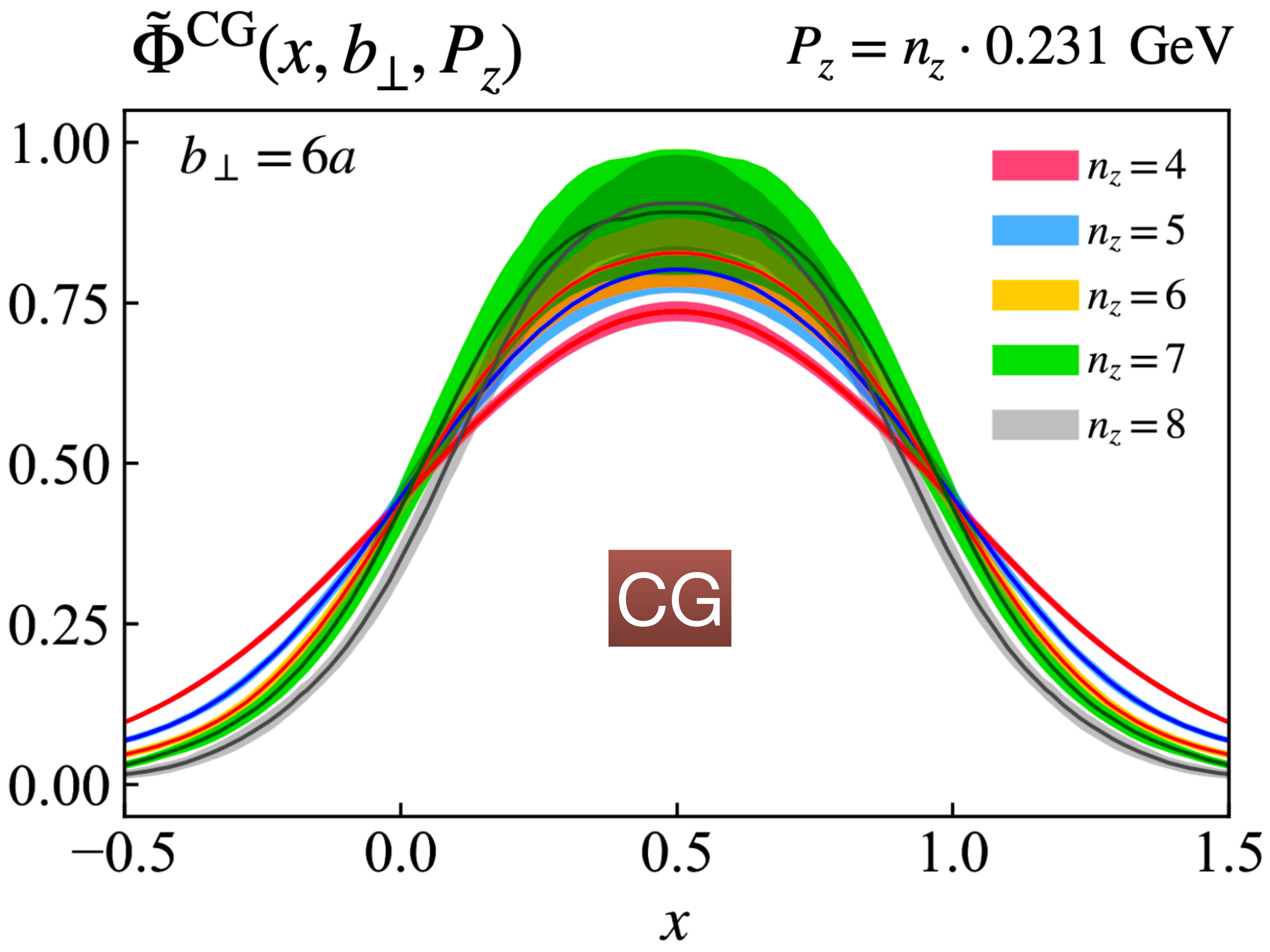
gauge invariant (GI) operator



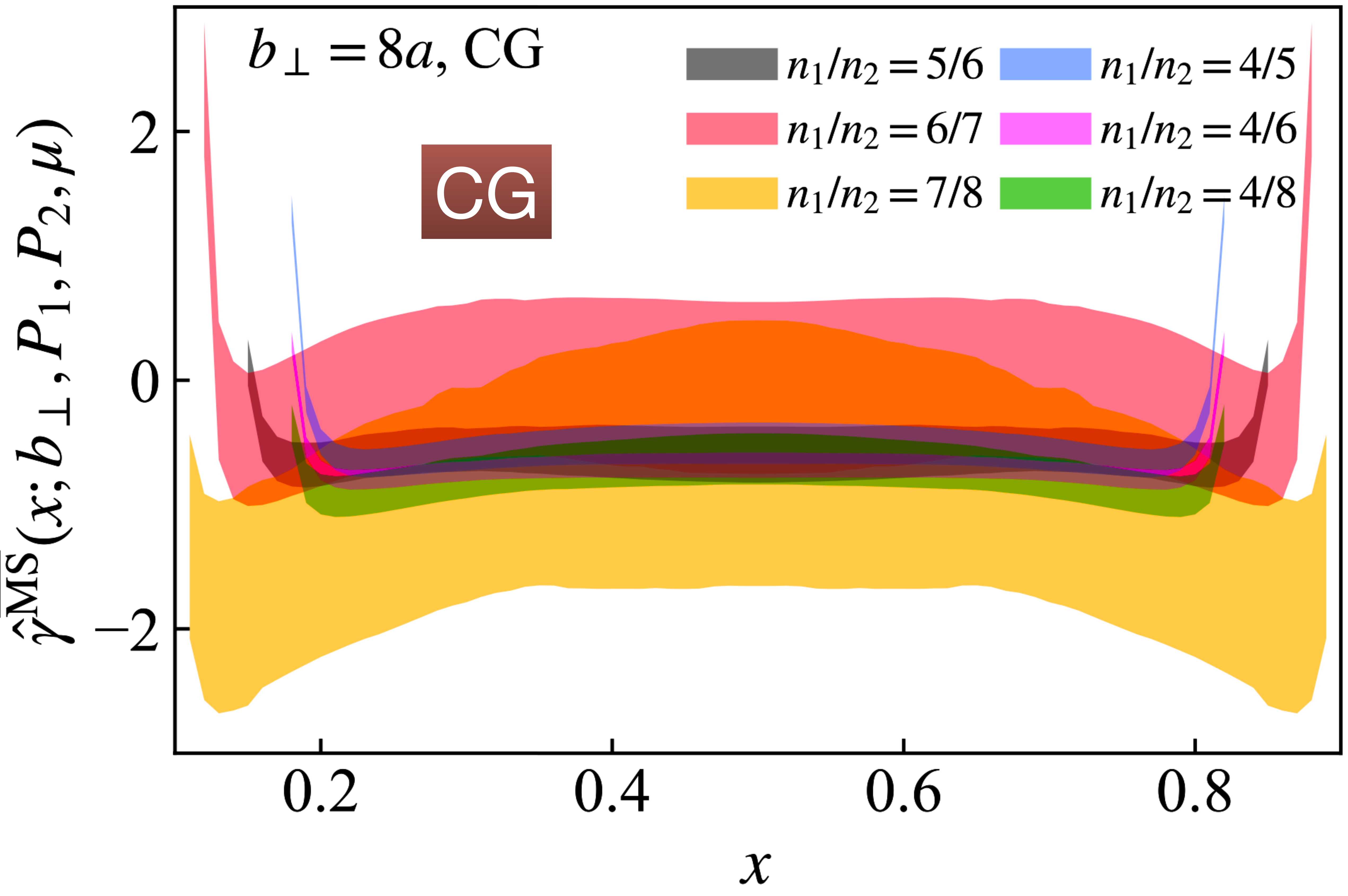
lattice spacing a=0.085 fm

Bollweg et al.: Phys. Lett. B 852, 138617 (2024)

quasi-TMD beam functions in momentum space



x and P independence of CS kernel



$P_i = 0.231 n_i$ GeV, μ = 2 GeV

Summary: nonperturbative CS kernel from lattice QCD