Exclusive diffractive dijets at HERA and EIC using GTMDs

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Contents

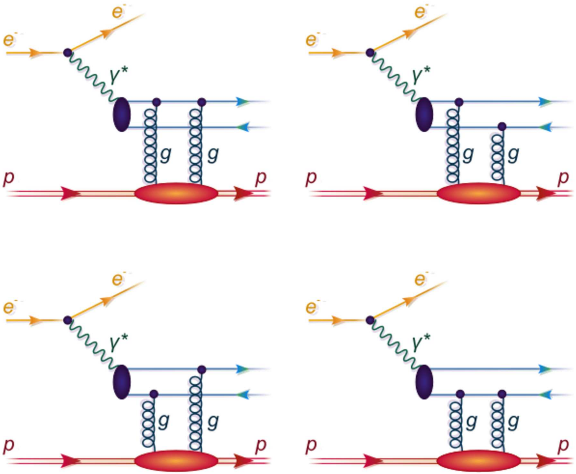
Introduction

- Diffractive production of dijets or pairs of heavy quarks and antiquarks in e+p collisions provide information on off-diagonal unintegrated gluon distributions in the target.
- The GTMDs may be (is) obtained in different ways.
  - One way is Fourier transform of so-called dipole amplitude.
  - Second way is an educated parametrization used in other calculations.
- GTMDs depend in general on:
  (a) $\vec{p}_\perp = \frac{1}{2}(\vec{p}_{\perp,q} - \vec{p}_{\perp,\bar{q}})$
  (b) $\vec{\Delta}_\perp = \vec{p}_{\perp,q} + \vec{p}_{\perp,\bar{q}}$
  (c) $\cos\phi = \frac{\vec{P}_\perp \cdot \vec{\Delta}_\perp}{P_\perp \Delta_\perp}$

Introduction

- The distributions dependent on $cos\phi$ (usually rather small) can be expanded and so-called elliptic gluon distributions can be obtained.
  Extraction of this term is often aim of some recent analyses for dijets or $c\bar{c}$ production in ep and pA collisions.
  No yet realistic distribution was obtained.
- Both H1 and ZEUS measured several observables associated with diffractive photoproduction of dijets in electron-proton collisions.
- There were some analyses trying to describe selected H1 data. Those communicated agreement with the data.
- In a recent analysis (arXiv:2403.15110, in print in Phys.Rev.D) we studied whether GTMD approach can explain the existing both H1 and ZEUS data.
- Somewhat similar analysis: Linek, Luszczak, Pasechnik, Schafer and Szczurek, for $pA \to pc\bar{c}A$, JHEP 10 (2023) 179.

Diffractive photoproduction of $q\bar{q}$ in ep collisions



Rysunek: Feynman diagrams for diffractive dijet production in $ep \to e' jjp$. Leading-order diagrams.

Kinematic variables

In this paper we will use the standard kinematic variables for incoming and outgoing lepton four-momenta k, k', we have the photon four-momentum $q = k - k'$, with $q^2 = -Q^2 < 0$. Let P be the four-momentum of the incoming proton, then the ep cm-energy squared is $s = (k+P)^2 \sim 2k \cdot P$ and the so-called inelasticity y is defined as

$$y = \frac{q \cdot P}{k \cdot P}, \tag{1}$$

so that the square of cms energy of the $\gamma^* p$ system $W^2_{\gamma p} = (q+P)^2$ is calculated from

$$W^2_{\gamma p} = ys - Q^2, \tag{2}$$

where we neglected the proton mass. The standard Bjorken variable x_{Bj}

$$x_{Bj} = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + W^2_{\gamma p} - m_p^2}. \tag{3}$$

Kinematic variables

The diffractive jets carry momentum fractions $z, 1-z$ of the photon and momenta $\vec{p}_{\perp 1}, \vec{p}_{\perp 2}$ transverse to the photon-proton collision axis. The square of the dijet invariant mass is

$$M_{jj}^2 = \frac{p_{\perp 1}^2 + m_q^2}{z} + \frac{p_{\perp 2}^2 + m_q^2}{1-z} - \Delta_\perp^2, \tag{4}$$

where $\vec{\Delta}_\perp = \vec{p}_{\perp 1} + \vec{p}_{\perp 2}$ is the transverse momentum of the dijet system, which for the exclusive limit of interest here, equals (negative of) the momentum transfer to the proton. We also use the standard diffractive variables

$$\beta = \frac{Q^2}{Q^2 + M_{jj}^2}, \tag{5}$$

which is the fraction of the Pomeron momentum carried by the struck quark. The fraction of the proton momentum carried by the pomeron is

$$x_{\mathbb{P}} = \frac{x_{Bj}}{\beta}. \tag{6}$$

Differential cross section

$$\frac{d\sigma^{ep}}{dydQ^2d\xi} = \frac{\alpha_{em}}{\pi y Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\frac{d\sigma_T^{\gamma^* p}}{d\xi}+(1-y)\frac{d\sigma_L^{\gamma^* p}}{d\xi}\right].$$

Here $d\xi = dzd^2\vec{P}_\perp d^2\vec{\Delta}_\perp$, and T, L stand for the transverse and longitudinal photons. We have neglected interferences between different photon polarizations, which vanish if one averages over the angle between electron scattering plane and hadronic plane.

Differential cross section

The $\gamma^* p \rightarrow q\bar{q}p$ cross sections for massive quarks/antiquarks read

$$\frac{d\sigma_T^{\gamma^* p}}{dzd^2\vec{P}_\perp d^2\vec{\Delta}_\perp} = 2N_c\alpha_{em}\sum_f e_f^2 \int d^2\vec{k}_\perp \int d^2\vec{k}_\perp{}' T\left(Y,\vec{k}_\perp,\vec{\Delta}_\perp\right) T\left(Y,\vec{k}_\perp{}',\vec{\Delta}_\perp\right)$$
$$\times \left\{\left(z^2+(1-z)^2\right)\left[\frac{(\vec{P}_\perp-\vec{k}_\perp)}{(\vec{P}_\perp-\vec{k}_\perp)^2+\epsilon^2}-\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2}\right]\cdot\left[\frac{(\vec{P}_\perp-\vec{k}_\perp{}')}{(\vec{P}_\perp-\vec{k}_\perp{}')^2+\epsilon^2}-\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2}\right]\right.$$
$$\left. + \; m_f^2\left[\frac{1}{(\vec{P}_\perp-\vec{k}_\perp)^2+\epsilon^2}-\frac{1}{P_\perp^2+\epsilon^2}\right]\cdot\left[\frac{1}{(\vec{P}_\perp-\vec{k}_\perp{}')^2+\epsilon^2}-\frac{1}{P_\perp^2+\epsilon^2}\right]\right\}, \tag{9}$$

$$\frac{d\sigma_L^{\gamma^* p}}{dzd^2\vec{P}_\perp d^2\vec{\Delta}_\perp} = 2N_c\alpha_{em}4Q^2z^2(1-z)^2 \times \sum_f e_f^2 \int d^2\vec{k}_\perp \int d^2\vec{k}_\perp{}' T\left(Y,\vec{k}_\perp,\vec{\Delta}_\perp\right) T\left(Y,\vec{k}_\perp{}',\vec{\Delta}_\perp\right)$$
$$\times \left[\frac{1}{(\vec{P}_\perp-\vec{k}_\perp)^2+\epsilon^2}-\frac{1}{P_\perp^2+\epsilon^2}\right]\cdot\left[\frac{1}{(\vec{P}_\perp-\vec{k}_\perp{}')^2+\epsilon^2}-\frac{1}{P_\perp^2+\epsilon^2}\right], \tag{10}$$

$$\epsilon^2 = z(1-z)Q^2 + m_f^2$$

GTMDs

The mentioned GTMD is a representation of the diffraction amplitude in momentum space often used in the literature (see e.g. Hagiwara et al. 2016, Hagiwara et al. 2017, Reinke Pelicer et al. 2018, Pasechnik et al. 2023 which is the Fourier transform of the dipole amplitude

$$T(Y,\vec{k}_\perp,\vec{\Delta}_\perp) = \int \frac{d^2\vec{b}_\perp}{(2\pi)^2}\frac{d^2\vec{r}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, e^{-i\vec{k}_\perp\cdot\vec{r}_\perp}\, N(Y,\vec{r}_\perp,\vec{b}_\perp)\,. \quad (11)$$

We decided to choose normalization that is consistent with Reinke Pelicer et al. 2018. However, such a Fourier transform does not converge to zero and requires regularization, which is often done by a Gaussian cutoff function

$$T(Y,\vec{k}_\perp,\vec{\Delta}_\perp) = \int \frac{d^2\vec{b}_\perp}{(2\pi)^2}\frac{d^2\vec{r}_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, e^{-i\vec{k}_\perp\cdot\vec{r}_\perp}\, N(Y,\vec{r}_\perp,\vec{b}_\perp)\, e^{-\varepsilon r_\perp^2}\,.$$

Results depend on the regularization parameter ε.

Amplitude

The value of the ε parameter has a significant impact on the distribution of the obtained cross-sections, which was shown in Linek et al:2023. For the purposes of this analysis, we decided to assume $\varepsilon = (0.5\,\mathrm{fm})^{-2}$, similarly to Reinke Pelicer et al. 2018, Boer et al. 2021, Linek et al. 2023. The Fourier expansion of the dipole amplitude allows to distinguish an isotropic contribution and an elliptical term depending on the orientation of the dipole,

$$N(Y, \vec{r}_\perp, \vec{b}_\perp) = N_0(Y, r_\perp, b_\perp) + 2\cos(2\phi_{br})\, N_\epsilon(Y, r_\perp, b_\perp) + \ldots\,,$$

however we only introduce the isotropic part.

We consider five different GTMD models. Two of them are parameterizations of off-forward gluon density matrices for diagonal unintegrated gluon distribution compatible with the Golec-Biernat-Wüsthoff (GBW model) and Moriggi-Paccini-Machado (MPM model) for which we use the diffractive slope $B = 4\ \mathrm{GeV}^{-2}$,

Amplitude

$$f\left(Y, \frac{\vec{\Delta}_\perp}{2} + \vec{k}_\perp, \frac{\vec{\Delta}_\perp}{2} - \vec{k}_\perp\right) = \frac{\alpha_s}{4\pi N_c} \frac{\mathcal{F}(x_{\mathbb{P}}, \vec{k}_\perp, -\vec{k}_\perp)}{k_\perp^4} \exp\left[-B\vec{\Delta}^2\right].$$

We also included models that are the Fourier transforms of the dipole amplitude described by equation (12). We use also the bSat model of Kowalski and Teaney (KT model) with parameters from Table I fit 3 of their paper and two models based on the McLerran-Venugopalan approach - proposed by Iancu-Rezaeian (MV-IR model) and the Boer-Setyadi (MV-BS model) fitted to the H1 experimental data. Both of these models are independent of the $x_{\mathbb{P}}$, therefore we modified the MV-IR model using $\lambda = 0.277$ as

$$T^{\rm mod}_{\rm MV-IR}(Y, \vec{k}_\perp, \vec{\Delta}_\perp) = T_{\rm MV-IR}(\vec{k}_\perp, \vec{\Delta}_\perp)\, e^{\lambda Y}, \qquad Y = \ln\left[\frac{0.01}{x_{\mathbb{P}}}\right]. \quad (12)$$

Amplitude

It should be noted that more realistic extensions of the MV-IR model have been proposed in the literature (see Mantysaari et al.). Adapting the MV-BS model according to Boer et al. 2021 we introduce $\chi = 1.25$ (see below),

$$N_0(r_\perp, b_\perp) = -\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2 \lambda^2} + e\right], \tag{13}$$

with

$$Q_s^2(b_\perp) = Q_0^2 \frac{4\pi\alpha_s C_F}{N_c} \exp\left[\frac{-b_\perp^2}{2R_p^2}\right]. \tag{14}$$

In principle, χ can be treated as a free parameter to be adjusted to experimental data.

Numerical results

Here we will present our results for both H1 and ZEUS cuts. The details of the cuts are described in original papers and are summarized in Table below.

H1 cuts	ZEUS cuts
$4 < Q^2 < 110\,\mathrm{GeV}^2$	$Q^2 > 25\,\mathrm{GeV}^2$
$x_{\mathbb{P}} < 0.1$	$x_{\mathbb{P}} < 0.01$
$0.05 < y < 0.7$	$0.1 < y < 0.64$
$-1 < \eta_{1,2} < 2.5$	$\eta_{1,2} < 2$
$p_{\perp 1} > 5\,\mathrm{GeV}$	$p_{\perp 1,2} > 2\,\mathrm{GeV}$
$p_{\perp 2} > 4\,\mathrm{GeV}$	$M_{jj} > 5\,\mathrm{GeV}$
$\lvert t \rvert < 1\,\mathrm{GeV}^2$	$90 < W_{\gamma p} < 250\,\mathrm{GeV}$

Tablica: Cuts used by the H1 and ZEUS collaborations.

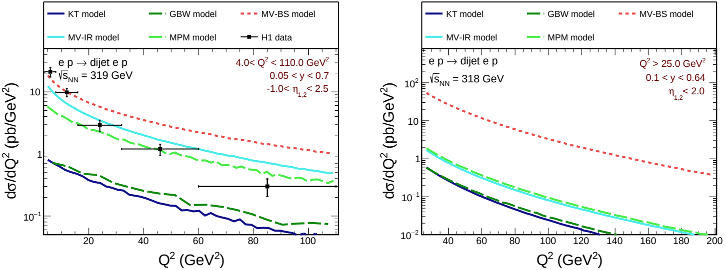
Total cross section

GTMD approach	H1, light $q\bar{q}$ σ (pb)	H1, $ep \longrightarrow c\bar{c}$ σ (pb)	H1, no $p_{\perp 1,2}$ cuts σ (pb)	ZEUS, light $q\bar{q}$ σ (pb)	ZEUS, $ep \longrightarrow c\bar{c}$ σ (pb)	ZEUS, no $p_{\perp 1,2}$ cuts σ (pb)
GBW	26.35	19.91	10900.86	13.57	6.67	337.11
MPM	147.94	108.26	10151.00	43.61	20.47	313.17
MV-BS	404.06	269.75	10999.73	1346.11	624.55	3117.95
KT	21.29	15.20	5957.65	12.57	5.67	52.60
MV-IR	243.20	155.21	11784.75	37.83	17.62	91.18
DATA	254		-	72		-

Tablica: Total cross section for H1 and ZEUS conditions and different approaches.

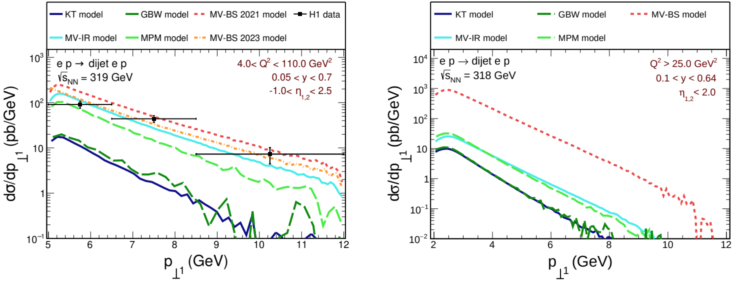
In the Table we present our phase space integrated cross sections for five different models of GTMDs. Quite different results are obtained for the different models. Results for light $q\bar{q}$ and $c\bar{c}$ dijets production are presented separately. For each model, the $c\bar{c}$ contribution is between 50 and 70% of the cross section for the light quark dijets. We placed in this table also experimental cross sections as measured by the H1 and ZEUS collaborations. All GTMDs except the MV-BS model underpredict experimental data, while the latter model is in the ballpark of H1 data but dramatically overpredicts the ZEUS results. Not taking into account cuts on jet momenta $p_{\perp 1,2}$ results in a several-fold increase in the total cross-sections. Here the $x_{\mathbb{P}}$-independent GTMD Boer et al. 2021 was used.

Q^2-dependence



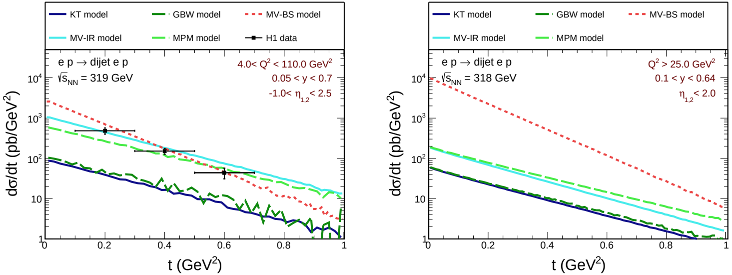
Rysunek: Q^2 dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

Transverse momentum dependence



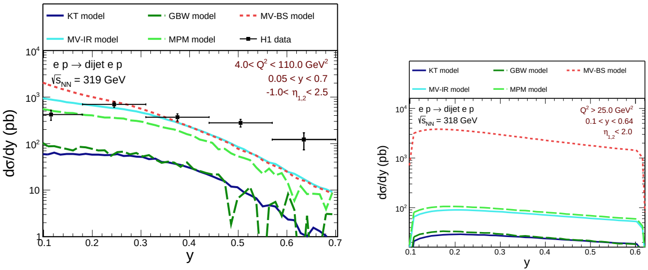
Rysunek: Jet transverse momentum dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

t-distributions



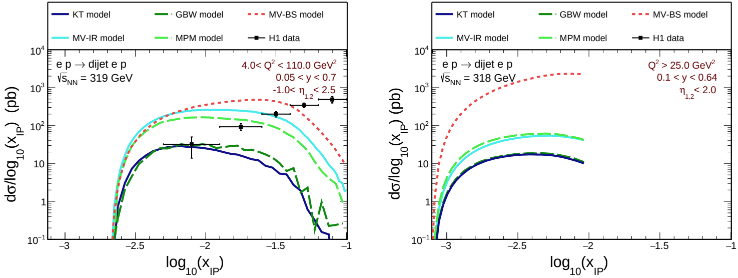
Rysunek: Mandelstam t dependence of of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

Inelasticity distributions



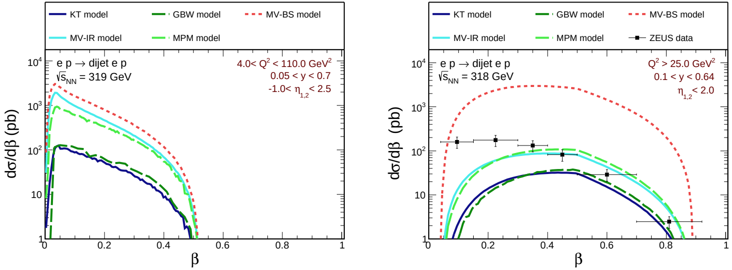
Rysunek: Inelasticity dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

$x_{\mathbb{P}}$ distributions



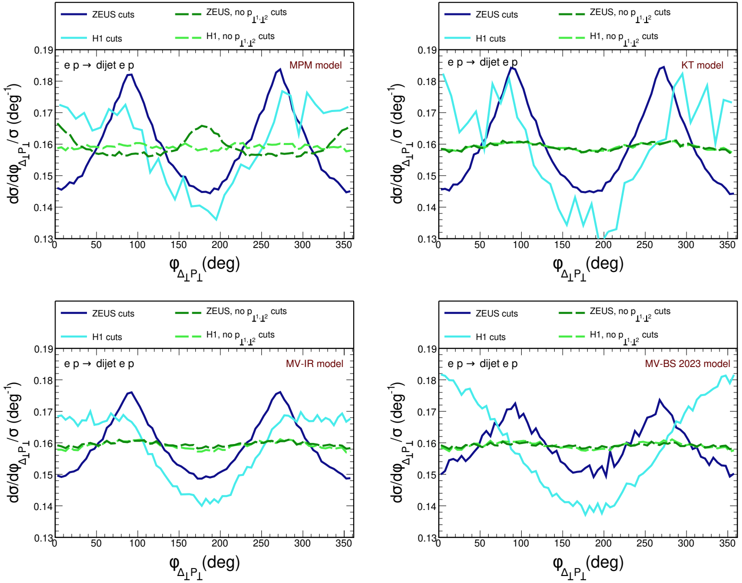
Rysunek: $x_{\mathbb{P}}$ distribution dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

β distributions



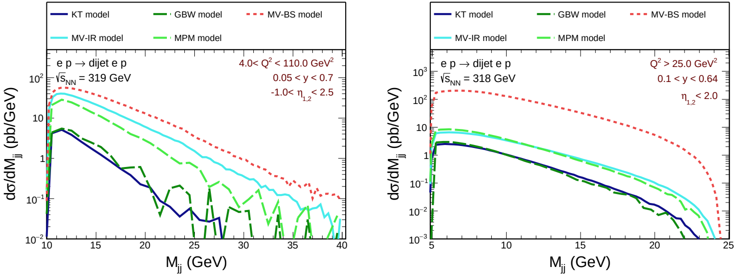
Rysunek: β dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

ϕ-angle distributions



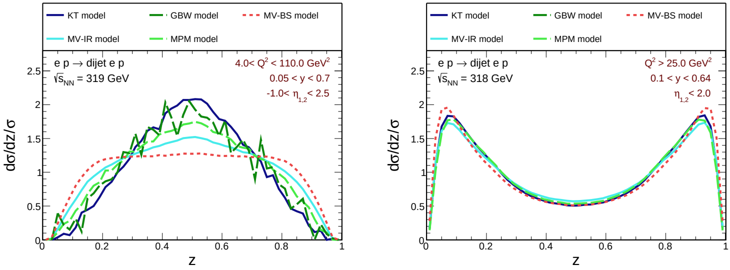
Rysunek: Distributions in the azimuthal angle ϕ between $\vec{P}_\perp$ and

Invariant mass distributions



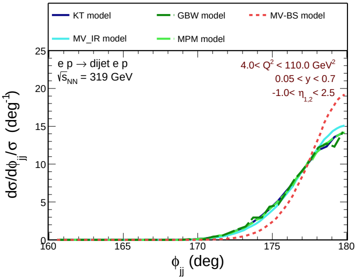
Rysunek: Invariant mass dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

z-distribution



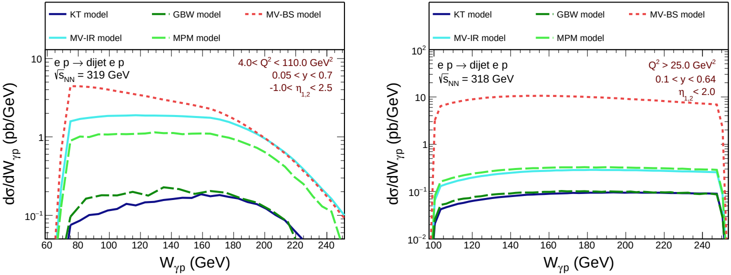
Rysunek: z dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs. Please note the normalization.

θ-distribution



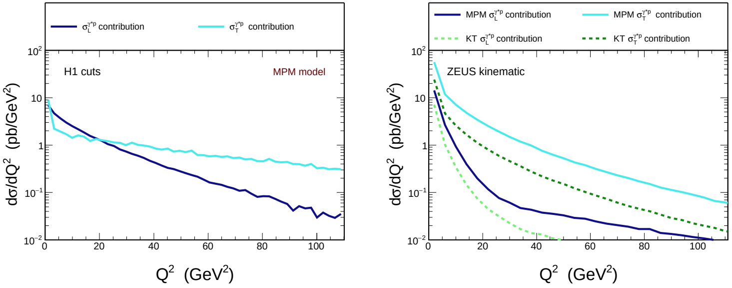
Rysunek: Azimuthal angle between produced jets dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs. Please note the normalization.

$W_{\gamma p}$ dependence



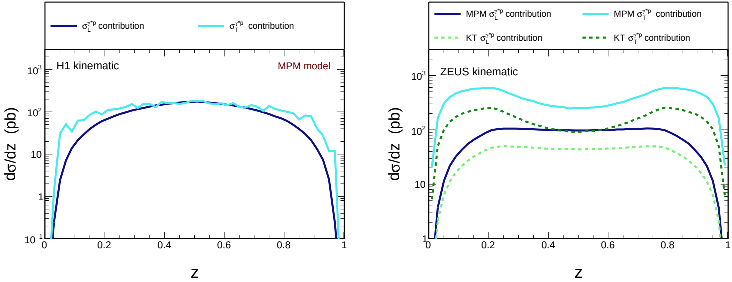
Rysunek: $W_{\gamma p}$ dependence of the cross section for H1 (left) and ZEUS (right) kinematics for different GTMDs.

MPM distribution



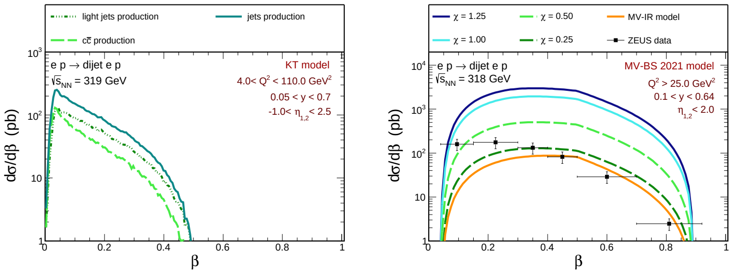
Rysunek: Q^2 distributions of transverse and longitudinal components of the cross section for the MPM and the KT GTMDs for H1 (left) and ZEUS (right) kinematics without Q^2 cuts.

MPM distribution



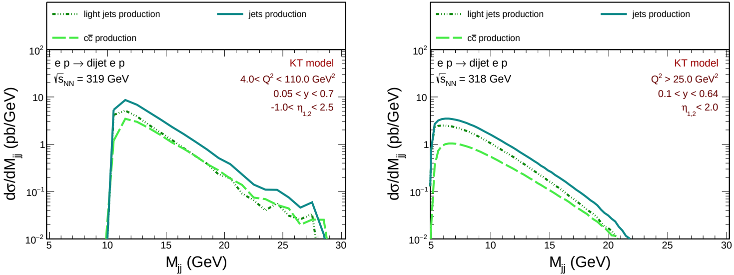
Rysunek: z distributions of transverse and longitudinal components of the cross section for the MPM and the KT GTMDs for H1 (left) and ZEUS (right) kinematics.

Contribution of $c\bar{c}$ component



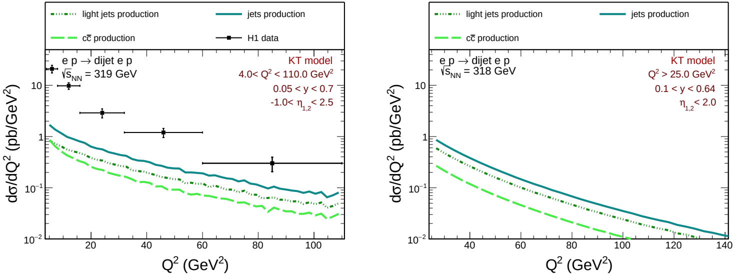
Rysunek: Contributions of light and $c\bar{c}$ dijets as a function of β for the KT GTMD for H1 (left) and ZEUS (right) kinematics.

Contribution of $c\bar{c}$ component



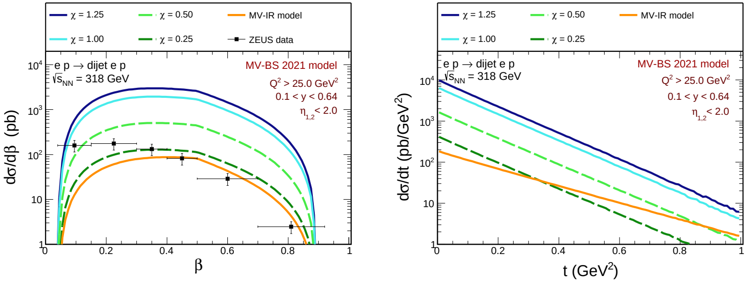
Rysunek: Contributions of light and $c\bar{c}$ dijets as a function of M_{jj} for the KT GTMD for H1 (left) and ZEUS (right) kinematics.

Contribution of $c\bar{c}$ component



Rysunek: Contributions of light and $c\bar{c}$ dijets as a function of Q^2 for the KT GTMD for H1 (left) and ZEUS (right) kinematics.

Contribution of $c\bar{c}$ component



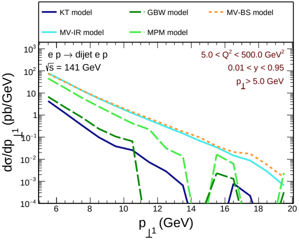
Rysunek: β and t distributions for ZEUS kinematic for the MV-BS model with different χ values.

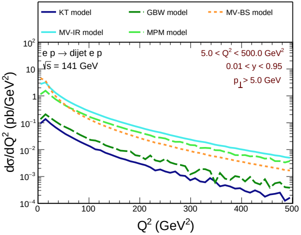
EIC predictions

The following conditions have been imposed:

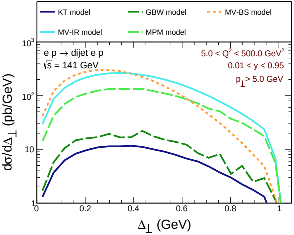
- the collision energy: $\sqrt{s} = 141$ GeV,
  electron energy: 18.8 GeV,
  proton energy: 275 GeV.
- the jet transverse momentum: $p_{\perp,1,2} > 5$ GeV.
- inelasticity range: $0.01 < y < 0.95$.
- pseudorapidity range: -3.5 $< \eta <$ 3.5.
- photon virtuality range: 5.0 GeV^2 $< Q^2 <$ 500 GeV^2.

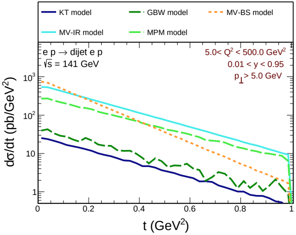
EIC predictions



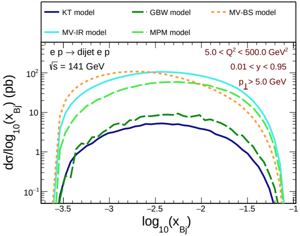


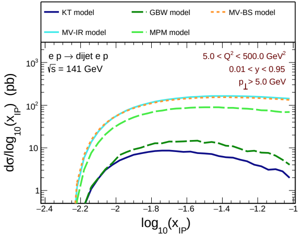
EIC predictions



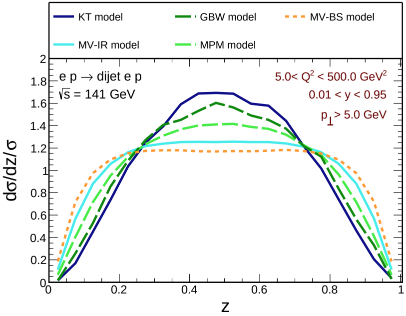


EIC predictions





EIC predictions



Summary and Conclusions

- Several differential distribution for diffractive photo-production of dijets in $ep \longrightarrow ejjp$ reaction at HERA have been presented.
- Different GTMDs models were used.
- Amplitudes for massive quarks have been obtained.
- We have used both educated well known parametrizations applied previously to describe (onset of) saturation effects as well as GTMDs obtained as Fourier transform of so-called dipole amplitudes.
- Some distributions have been compared to H1 and ZEUS data.

Summary and Conclusions

- Special attention has been paid to azimuthal correlations between the sum and the difference of jet transverse momenta. Even neglecting the so-called elliptic gluon distribution we generate fluctuations in the azimuthal angle distributions due to finite cuts on jet transverse momenta.
- We have analyzed the role of heavy $c\bar{c}$ component. It turned out nonnegligible.
- The MV-BS, MPM and MV-IR GTMDs models reasonably well describe observables for the H1 kinematics but fail to describe distributions in $x_{\mathbb{P}}$ and β for the ZEUS kinematics.
- Some predictions for the EIC kinematics have been also presented. They look similarly as for HERA.

Outlook

- Our findings suggests (in my opinion) that the simple leading-order GTMD approach may be not sufficient to explain the HERA as well as future EIC data. Most probably there are effects beyond that approach.
- In some configurations the pomeron exchange (gluonic formulation) may be not sufficient. There can be in addition subleading reggeon exchange (quark-induced effects).
- Also higher-order effects, as inclusion of $q\bar{q}g$ effects, may be necessary.
- Clearly further work is required.