Top quark production in hadron colliders at NNLL accuracy

Diffraction and Low x 2024
Trabia, Palermo, Sicily, 8-14 September 2024

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Outline

- In the talk I will review physics beyond and results on soft gluon resummation in top quark hadroproduction at hadron colliders
- The focus will be on production associated top-antitop quark production and a heavy electroweak boson: the Higgs, Z or W, where we contributed a lot
- The core: results of long-term physics program initiated about 10 years ago by Anna Kulesza and LM about 2014, carried out in parallel by SCET collaboration
- The main goal has been to extend the soft gluon resummation to the three-particle final state and to provide precise prediction for LHC measurements
- The project is active: we expect new results (NNLO + NNLL) this year
- The group: **R. Balsach, A. Kulesza, D. Schwartlander, T. Stebel, V. Theeuwes, LM**

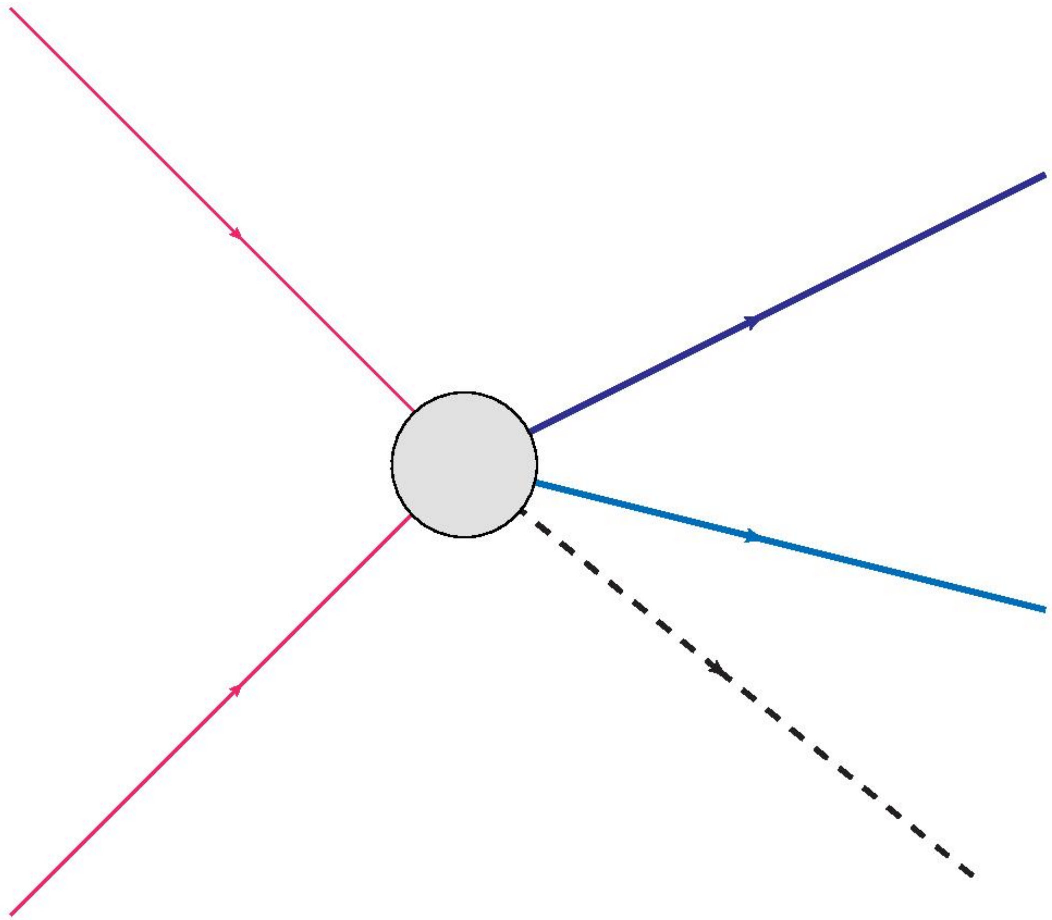
Motivation: the precision path explored at the LHC

- Calculation based on perturbative quantum field theory. Need for 2 or 3 loop calculations in QCD and EW. QCD corrections are usually larger than EW
- The most robust calculational technique: fixed order calculations. Difficulty grows quickly with increasing number of external legs and with the number of important mass scales.
- With more than 2 final state particles dimension of phase space grows from 2 to 5 (or more) at the LO. Also, more complicated topologies appear, and higher order calculations become much more difficult

- The current theoretical frontier for calculations of cross sections for hadroproduction of massive particles:
  - NNNLO calculations for 2 → 1 e.g.: $pp \rightarrow H + X$ (since 2015): precision O(3%)
  - NNLO calculations for 2 → 2 e.g.: $\mathrm{pp} \rightarrow t\bar{t} + X$: precision O(10%)
  - NLO calculation for 2 → 3 e.g.: $\mathrm{pp} \rightarrow t\bar{t} + H + X$: precision O(20%)
  - Approximate NNLO calculation for $\mathrm{pp} \rightarrow t\bar{t} + H + X$ (2023)
- Possible improvement of precision: all order resummation of enhanced corrections in perturbative series → soft gluon resummation

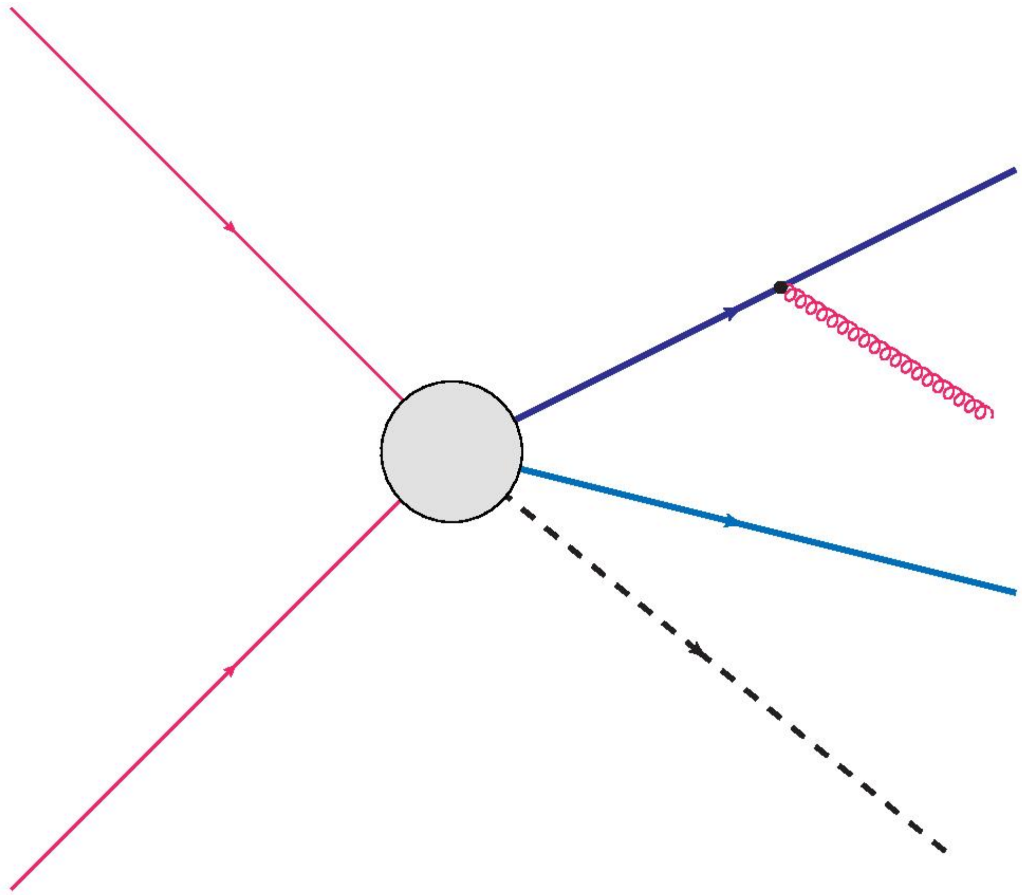
Soft gluons - physics picture: the process

- Consider a process with 2 light-like colored lines going to 2 n-time-like colored lines + some neutral lines e.g. $gg \rightarrow t\bar{t} + H$



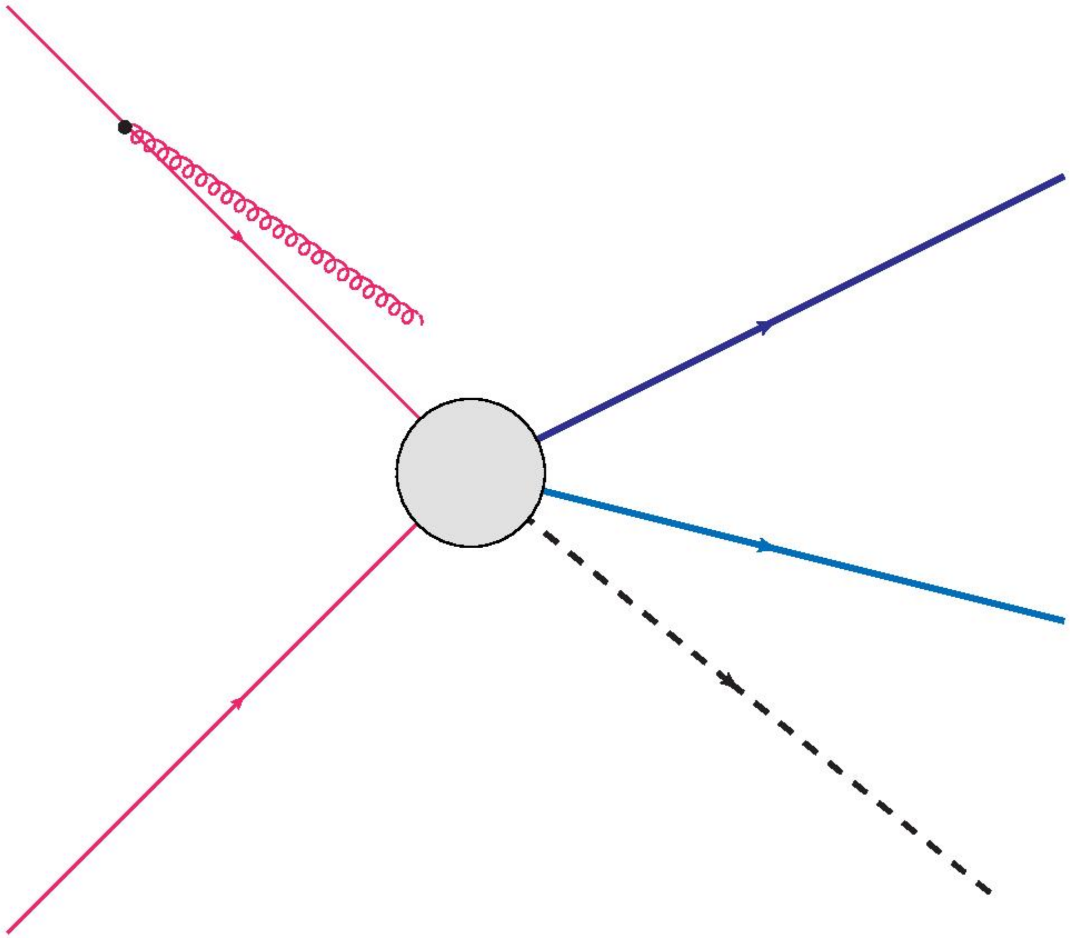
Physics picture: soft gluon radiation

- Compute higher order QCD corrections:
  real wide angle gluon emissions → single IR divergence (soft)



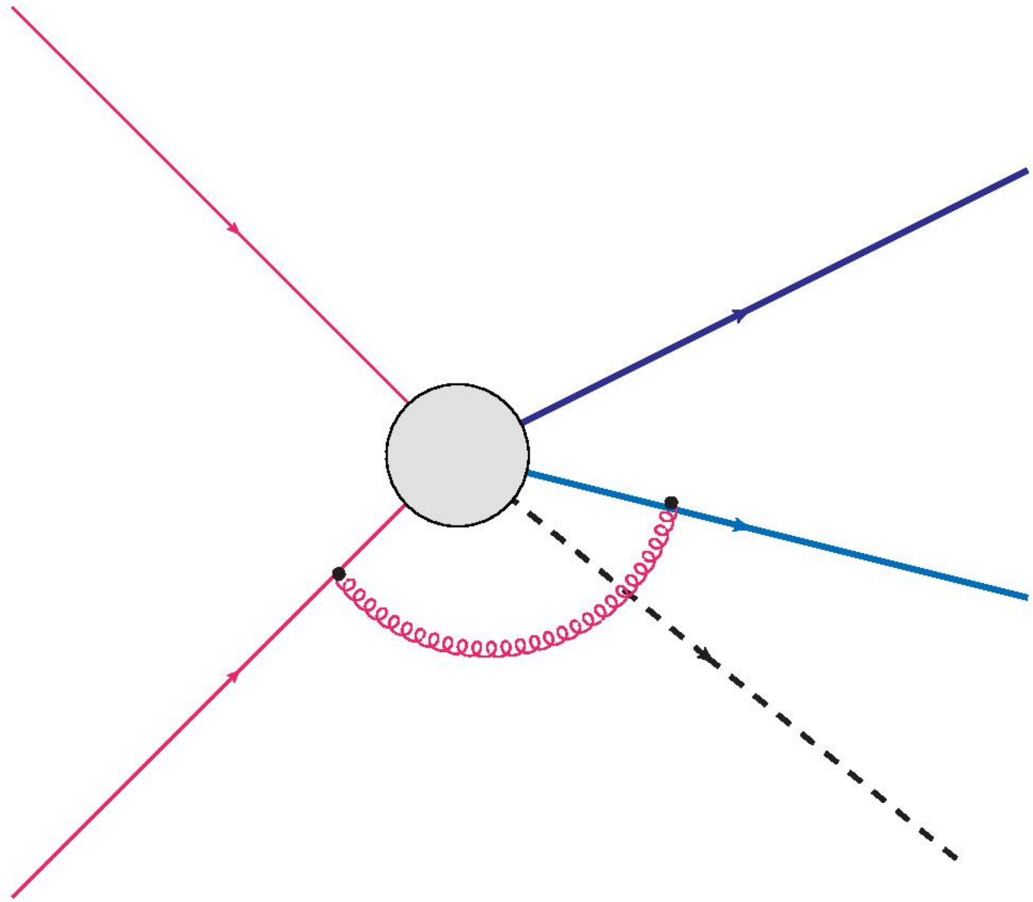
Physics picture: collinear gluon radiation

- Compute higher order QCD corrections:
  real collinear gluon emissions → double and single IR divergences (coll. & soft)



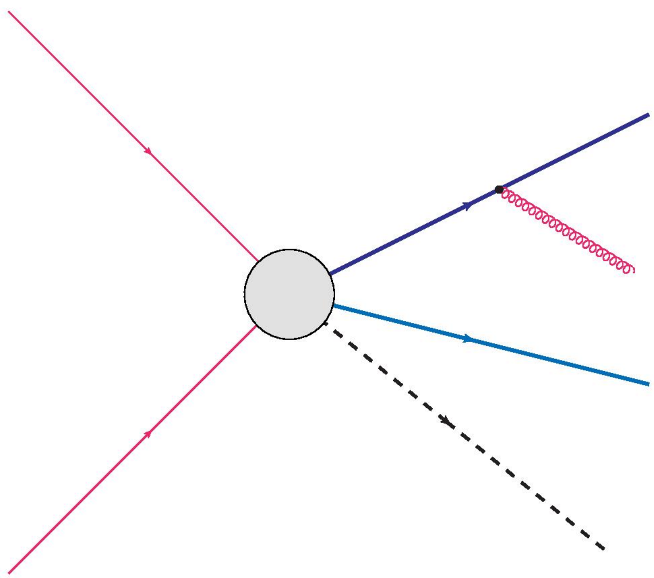
Physics picture: virtual corrections

- Compute higher order QCD corrections:
  virtual gluon emissions → double and single IR divergences (coll. & soft)

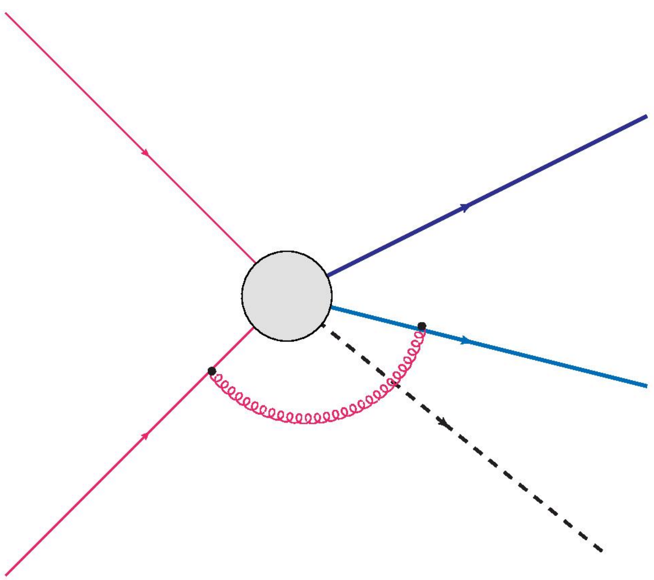


Physics picture: cross section at NLO

- Kinoshita-Lee-Naunberg theorem: in cross section IR divergencies cancel after phase space integration but leave double and single logarithms of scale ratio



squared



times LO

Soft gluon corrections at NLO

- Close to absolute threshold s_0 for heavy particle production $t\bar{t}B$ or tB: $s_{part} \rightarrow s_0$ soft gluon corrections are enhanced: cancellation of collinear and soft IR singular terms from virtual corrections and **real emissions only in phase space region squeezed by kinematics →**

  emergence of double and single logarithms of $\beta = (1 - s_0 / s_{part})^{1/2}$

- For example, for $t\bar{t}H$ production the threshold $s_0 = (2m_t + M_H)^2$ and:

$$\delta\hat{\sigma}_{NLO}|_{\log} = \hat{\sigma}_{\rm Born}\frac{2\alpha_s}{\pi}\left\{C_{ab}\left[2\log^2\beta - 3\log\beta - 2\log\beta\log\left(\frac{\mu_F}{2m_t + M_H}\right) - C^{ab}_{\rm FSR}\ln\beta\right]\right\}$$

- Soft gluon effects at the absolute threshold are less relevant in the total NLO cross-section as the approach to the threshold $\sigma_{Born} \sim \beta^{\alpha}$ due to volume measure of massive particle phase space

Picking enhanced terms: soft gluon resummation

- Framework based on proofs of hard factorization by Collins, Soper and Sterman and by Catani and Trentadue (1980-s)

- Logarithmically enhanced soft gluon corrections may factorized and resummed to all order of perturbation theory keeping:

  $(\alpha_s \log^2 \beta)^n$ at LL, $(\alpha_s^n \log^{2n-1} \beta)$ at NLL, $(\alpha_s^n \log^{2n-2} \beta)$ at NNLL accuracy

- We work up to NNLL accuracy: hard scattering at NLO, soft-collinear logs at NNLO + collinear logs (NNLL) + soft logs (NNLL)

  Collinear logs → incoherent, they redefine massless partons

  Soft wide-angle logs → coherent, depend on total color current

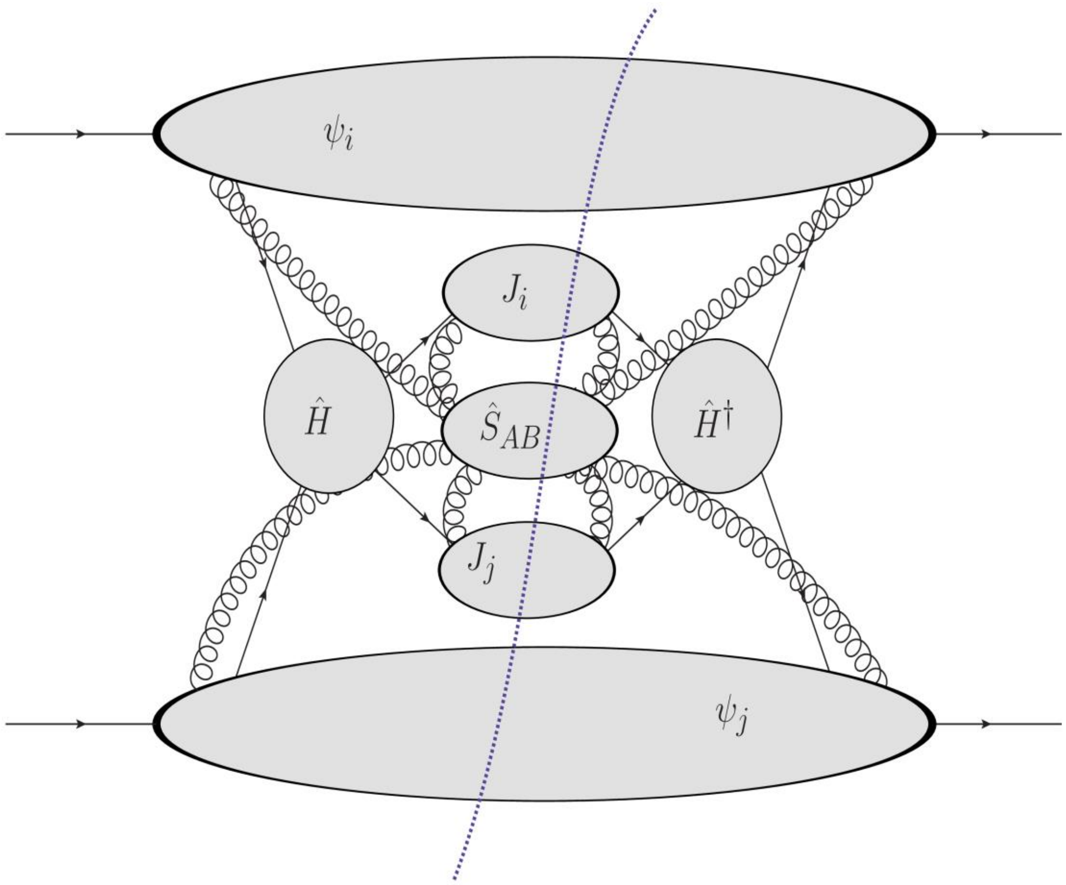
- Resummation may be performed using Renormalization Group technique

- Those corrections may be factorized in the Mellin moment space:

  particularly simple picture of resummation → multiplicative factors at given value of the Mellin moment N

All order approach: general factorization procedure

$$\hat{\sigma} = \hat{H}^\dagger \otimes \hat{S} \otimes \hat{H} \otimes \psi_i \otimes \psi_j \otimes J_i \otimes J_j$$



S_{AB} — soft gluon matrix

H — hard amplitude matrix

ψ_i — initial state "jet factors"
collinear radiation of incoming partons

J_i — collinear "jet factors"
for final state massless partons

Compact summary of the resummation formalism

Ingredients of the NLL calculation:

- LO hard matrix in color tensor basis (computed)
- LO soft matrix at initial scale (trivial)
- One loop soft anomalous dimension matrix (computed)
- NLL collinear factors (known)

NNLL calculation:

- Hard matrix in color tensor basis at one loop (extracted from NLO QCD calculation)
- One-loop soft matrix at one initial scale (computed)
- Two-loop soft anomalous dimension matrix
- NNLL collinear factors (known)

Scale evolution of the soft gluon matrix: renormalization group equations

- The anomalous dimension matrix in color tensor space governs the Soft Matrix evolution:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\hat{S}(\mu,g)=-\hat{\Gamma}^{\dagger}_{S}\hat{S}(\mu,g)-\hat{S}(\mu,g)\hat{\Gamma}_{S}$$

$$\hat{\Gamma}_{S}(g)=-\frac{g}{2}\frac{\partial}{\partial g}\,\mathrm{Res}_{\epsilon\to 0}\hat{Z}_{S}(g,\epsilon)$$

$$\beta(g)=-g^{3}\frac{\beta_0}{(4\pi)^2}-g^{5}\frac{\beta_1}{(4\pi)^4}-\dots$$

$$\hat{\Gamma}_{S}=\frac{\hat{\gamma}^{(0)}}{(4\pi)^2}+\frac{\hat{\gamma}^{(1)}}{(4\pi)^4+\dots}$$

- The evolution between the process scale and the lower cutoff scale on the soft gluon energy: generation of logs of the scale ratio
- Solution in the Mellin space

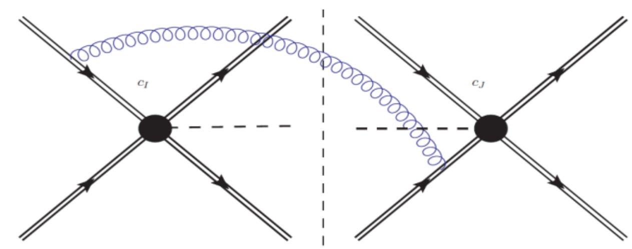
Soft anomalous dimension – computation

- Colour structures: standard $2 \to 2$ s-channel basis of color tensors is sufficient:

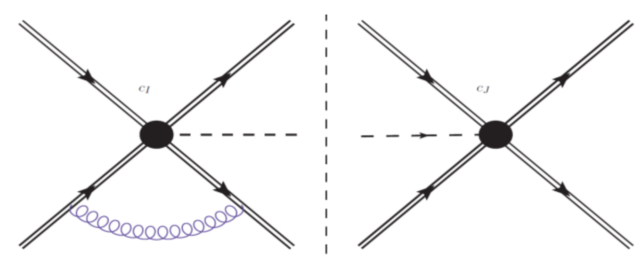
$$q\bar{q}: \quad 1_{\alpha_1\alpha_2} \otimes 1_{\alpha_3\alpha_4} \quad t^a_{\alpha_1\alpha_2} \otimes t^a_{\alpha_3\alpha_4}$$

$$gg: \quad 1^{a_1a_2} \otimes 1_{\alpha_3\alpha_4}, \quad i f^{a_1a_2b} \otimes t^b_{\alpha_3\alpha_4}, \quad d^{a_1a_2b} \otimes t^b_{\alpha_3\alpha_4}$$

- Eikonal integrals (up to power corrections in soft gluon energy)



- Soft anomalous dimension matrices determined from IR singularities (in dimensional regularization) of the virtual diagrams



- Mixing between the color tensors: anomalous dimension matrices

Matching to NLO

- In order to make full use of available information:
  matching of soft gluon resummation to the existing NLO calculations

- The NLO cross-section implemented in MC codes:
  PowHEG BOX, aMC@NLO and SHERPA

- From the cross-section at the NLL / NNLL accuracy the part beyond fixed order NLO expansion is taken and combined with the exact NLO result

  → **NLL / NNLL cross-section matched to NLO**

- Matching to NNLO fixed order results also possible

Beyond NLO, towards NNLL: NLO hard matrix element

- The available NLO calculations give access to the value of NLO correction in the threshold limit

$$\mathbf{H}_{ij\to klB} = \mathbf{H}^{(0)}_{ij\to klB} + \frac{\alpha_s}{\pi}\mathbf{H}^{(1)}_{ij\to klB} + \dots$$

- Part of this correction coincides with the LL and NLL soft gluon logarithms → taken care by the soft gluon resummation

- The reminder of the NLO correction constant at the threshold limit (as function of the Mellin moment *N*) → hard matrix at NLO

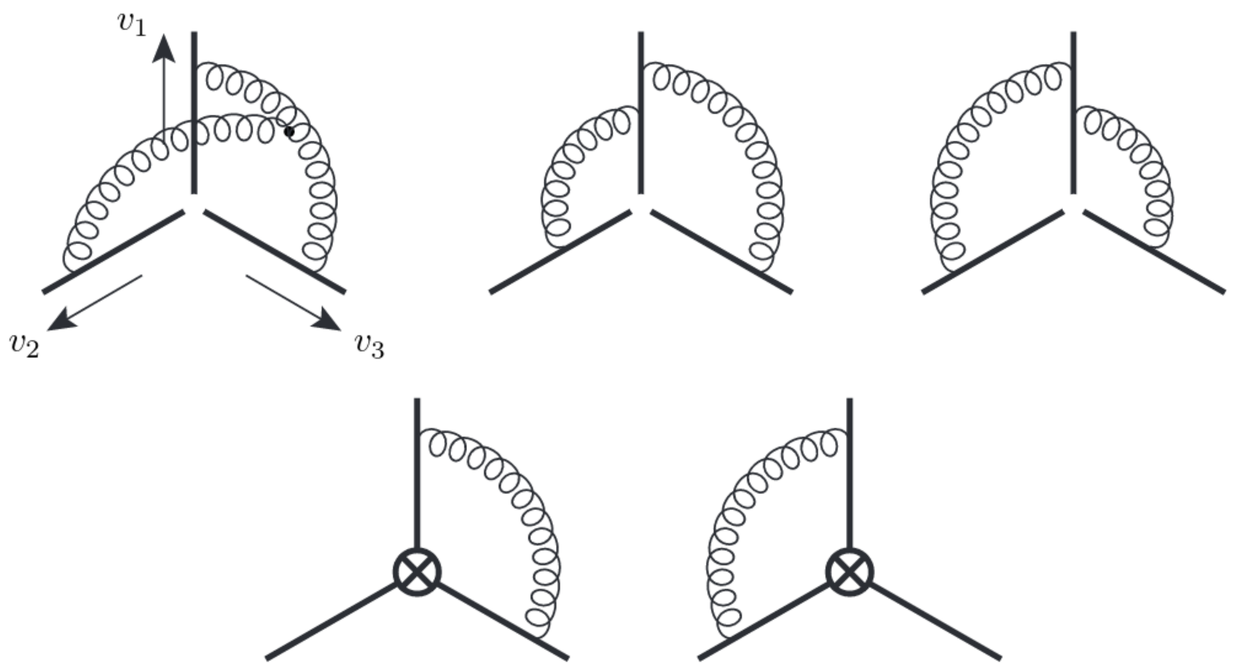
- Inclusion of NLO correction in the hard matrix element → necessary part of NNLL resummation, but it may be also used as an improvement of the NLL resummation → customary in soft gluon resummation in Higgs boson physics

Soft anomalous dimension at NNLL

- Soft anomalous dimension – new topologies with three eikonal lines

- Color structures containing 3 SU(3) generators: $T^a T^b T^c$
  [A. Ferroglia, M. [Neubert, B. Pecjak, L. L. Yang, Phys.Rev.Lett. 103 (2009) 201601,

JHEP 0911 (2009) 062]



Organizing the perturbative series:

- In Mellin representation ***log*β -s** translate into Mellin moments logs: ***log N***
- **Hence the LL corresponds to $(\alpha_s \log^2 N)^n$,**
  NLL to $\alpha_s \log N\,(\alpha_s \log^2 N)^{n-1}$, NNLL to $\alpha_s\,(\alpha_s \log^2 N)^{n-1}$
- The LL terms, come from the LO soft–collinear corrections
- The soft (wide angle) gluon matrix enters at NLL (exponentiating result), and its first correction at NNLL
- The Mellin space resumming factors organized into functions at given level of accuracy: $g_1(\alpha_s, N)$, $g_2(\alpha_s, N)$ *and* $g_3(\alpha_s, N)$

Resummation for differential distributions: Q-variable

- Dynamical threshold: resummation of differential distributions picks soft (collinear and soft wide angle) logarithms depending on the section of the phase space

- The simplest variable: total invariant mass Q of final state particles, for $t\bar{t}H$

$$Q^2 = (p_t^2 + p_{\bar{t}}^2 + p_H^2)$$

- Emergence of logs: cut-off on the available emitted gluon energy is given by difference of the invariant masses of the initial partonic and the final state

- In differential distributions terms $\sim \log^n(z) / (1-z)_+$ appear, that lead to absolute threshold $\log^{n+1}\beta$ after integration

- *Nomenclature: absolute threshold: M-scheme; invariant mass: Q-scheme*

What has been done to date?

- $t\bar{t}$ **at NLL:** S. Catani, M. Mangano, P. Nason, L. Trentadue, 1996
- $t\bar{t}$ **at NNLL** M**.** Cacciari, M. Czakon, M. Mangano, A. Mitov and P.Nason, 2011

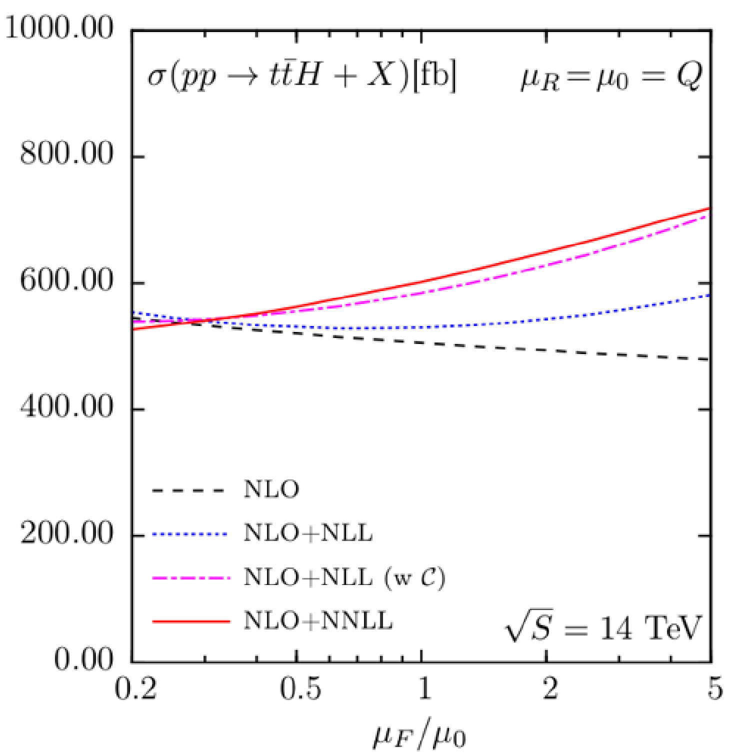
- $t\bar{t}H$ **at NNL' , direct QCD:** A. Kulesza, D. Schwartlander, T. Stebel, V. Theeuwes, R. Balsach, LM, 2015, 2016
- $t\bar{t}H$ **at NNLL , Soft Collinear Effective Theory (SCET):** A. Broggio, A. Ferroglia, B.D. Pecjak, A. Signer and L.L.Yang, 2015, 2016
- $t\bar{t}H$ **at NNLL:** direct QCD 2017
- $t\bar{t}Z, t\bar{t}W$ **at NNLL** direct QCD, SCET 2018-2020

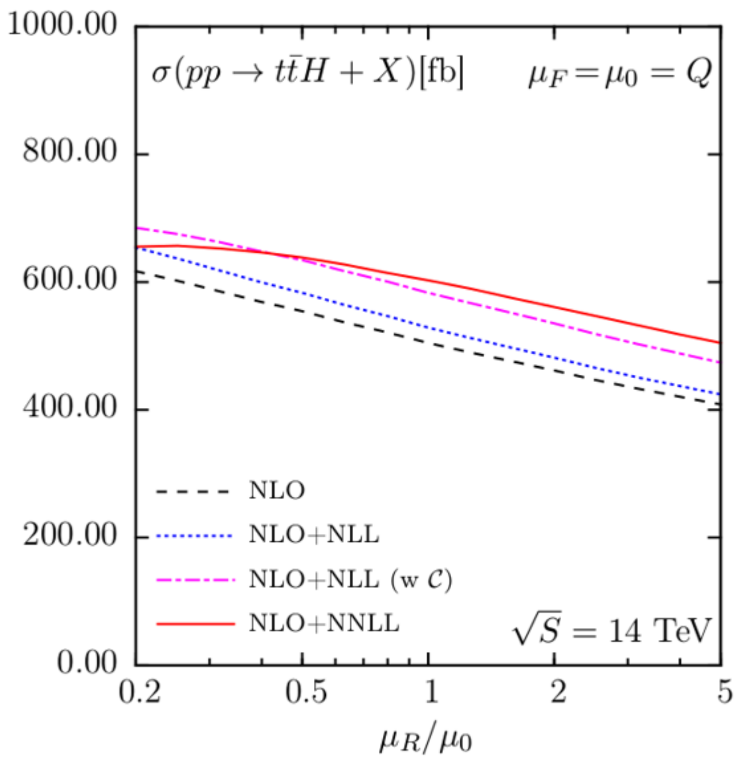
- tH **at NLL'** A. Kulesza, L.M. Valero and V. Theeuwes 2021
- $t\bar{t}t\bar{t}$ **at NLL'** M. van Beekveld, A. Kulesza and L. M.Valero 2022

- $t\bar{t}H$: **approx. NNLO calculation:** S. Catani, S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli, 2023

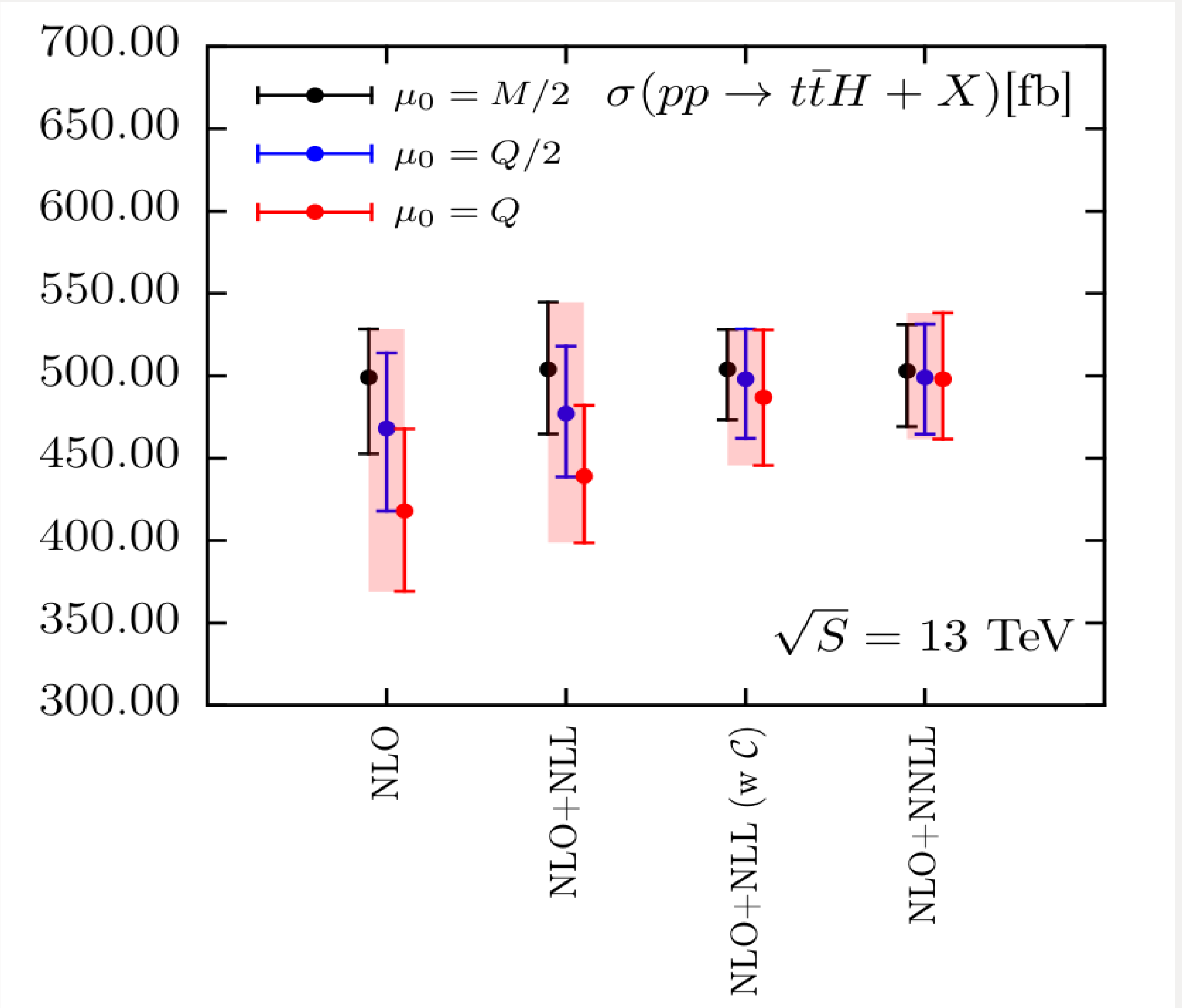
Theoretical uncertainty estimation

- The scheme is based on scale variation. This includes variation of the central scale and variation of the factorization and renormalization scales around it
- The renormalization and factorization scales are varied independently by factor of 2 up and down → the 7-point method



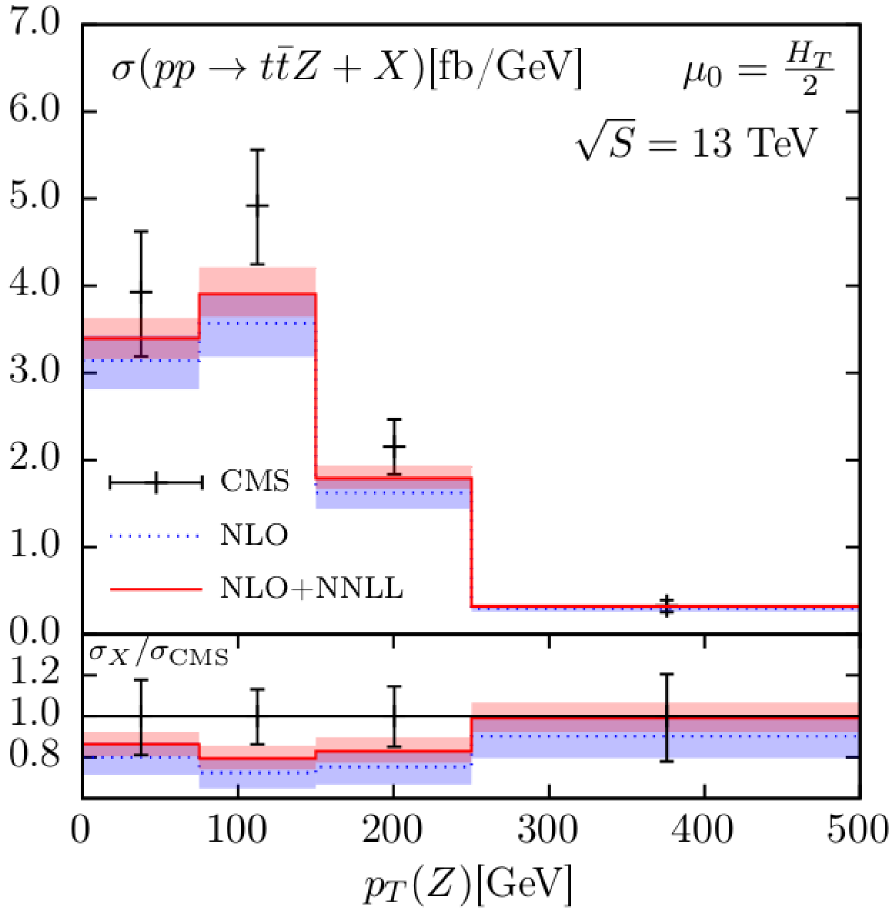


Predictions: NLO + NNLL invariant mass dependent resummation



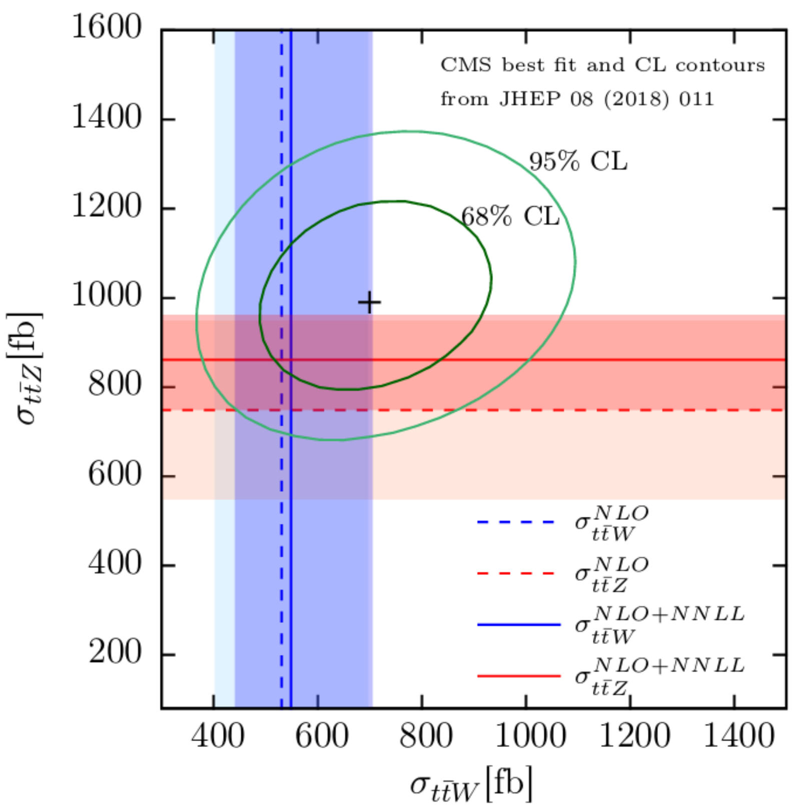
NLO + NNLL resummation for Z boson pT distribution in $t\bar{t}Z$ hadroproduction vs CMS data

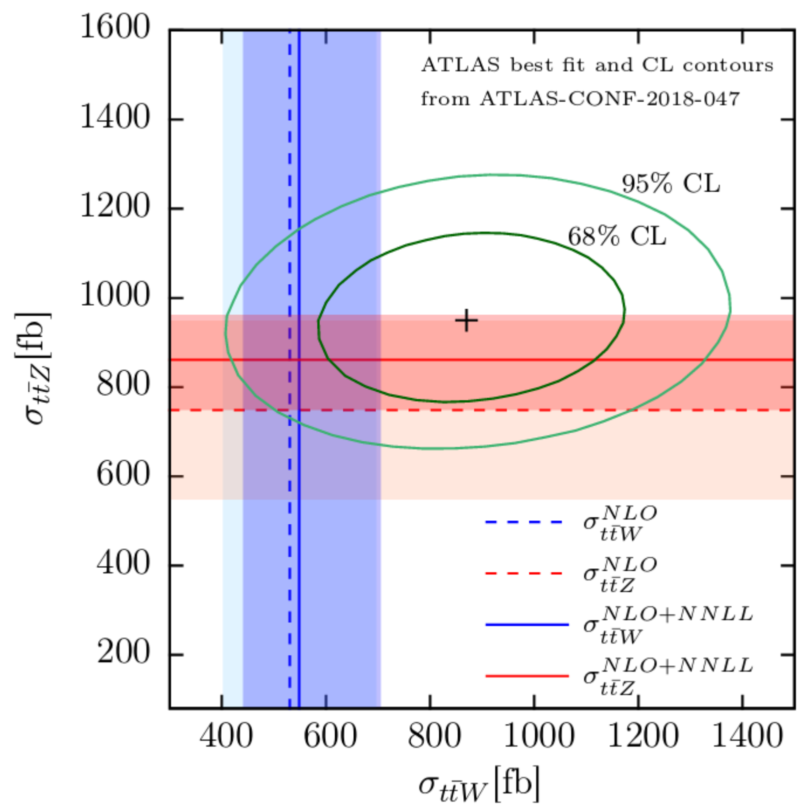
- Accuracy: NLO QCD + NLO EW / NLO+NNLL QCD + NLO EW



- NLO + NNLL results: closer to data, smaller theoretical uncertainty

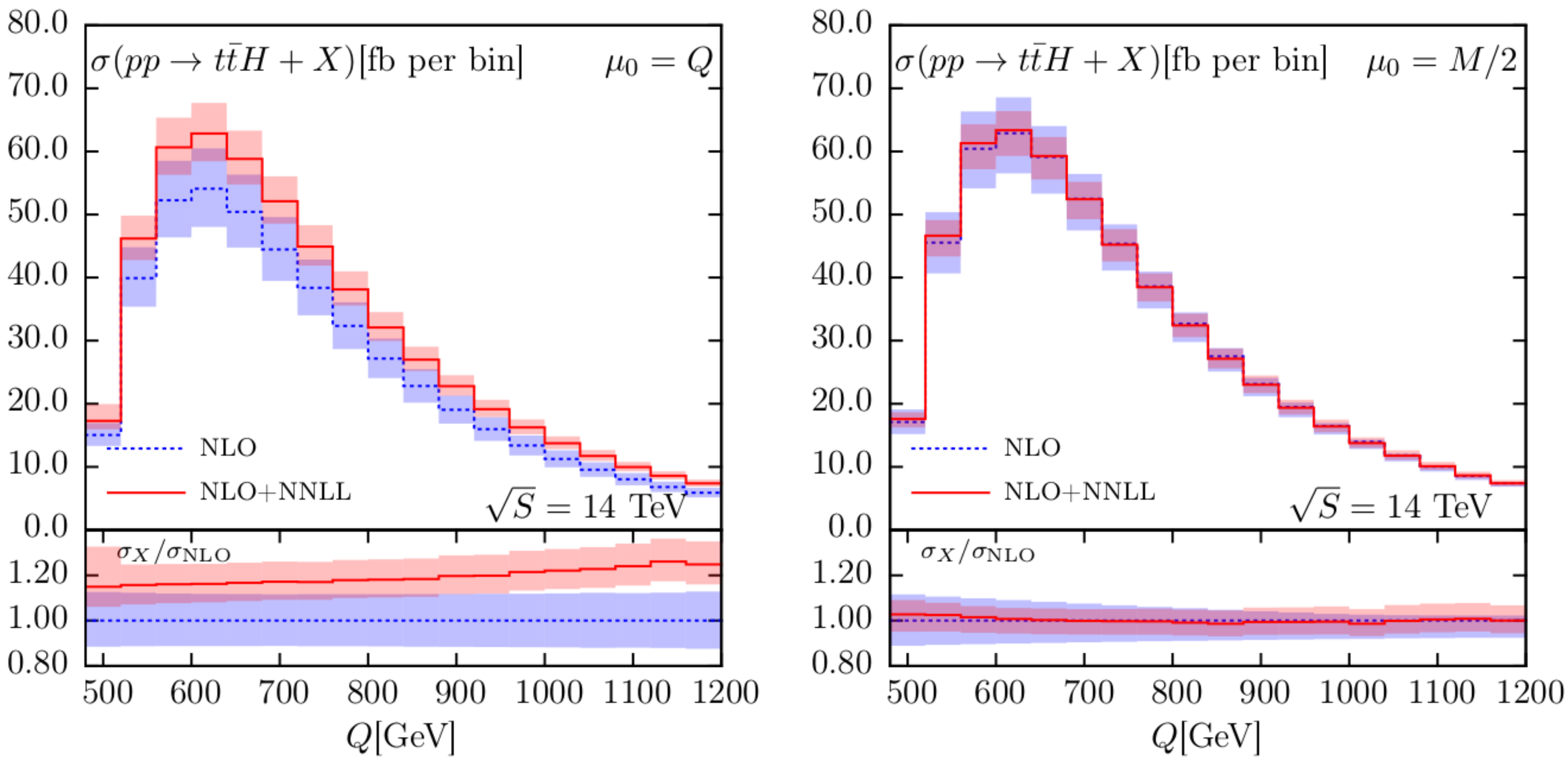
NLO + NNLL invariant mass dependent resummation for $t\bar{t}Z$ and $t\bar{t}W$ vs CMS and ATLAS data





- Theory errors reduced, and for Z made lower than experimental ones
- Measurements consistent with the Standard Model predictions

Results: effects of the NNLL resummation on the invariant mass (Q) distribution



- Sizable improvement of theoretical precision of NLO+NNLL w.r.t. the NLO
- Improvement of the stability of results w.r.t. the choice of the central scale

Most recent news for $t\bar{t}H$: approximate NNLO calculation

- The exact two-loop calculation for $t\bar{t}H$ is not available yet, but an approximate NNLO calculation improves the theoretical accuracy

[S. Catani et al. *Phys.Rev.Lett.* 130 (2023) 11, 111902] (Oct. 2022)

- The key ingredient: approximation the Higgs boson as a soft particle

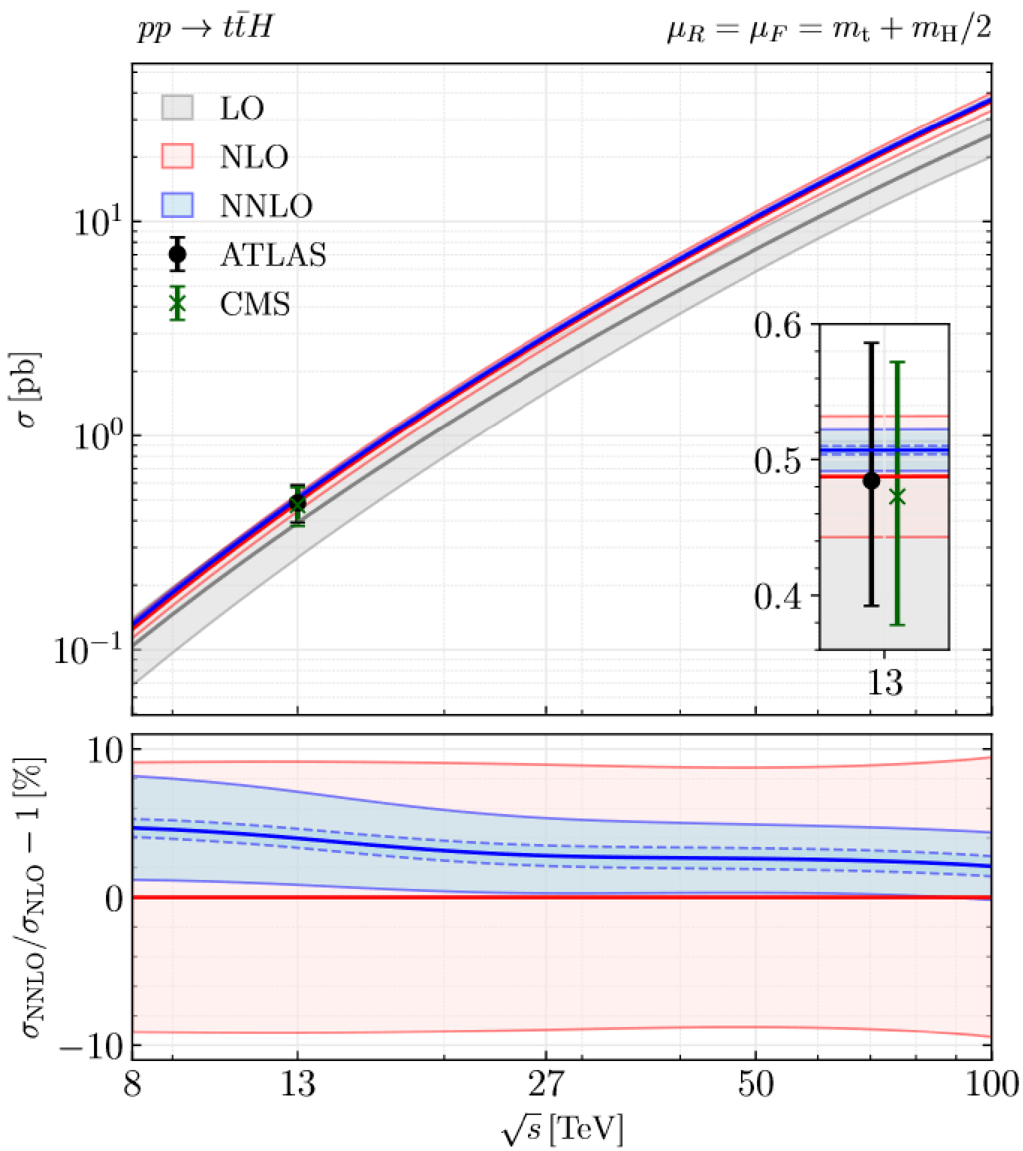
$$\mathcal{M}(\{p_i\},k) \simeq F(\alpha_S(\mu_R);\frac{m_t}{\mu_R})\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\,\mathcal{M}(\{p_i\})$$

- Accuracy of this approximation was tested against LO and NLO calculations, the overall effect estimated to results with ~ 0.6% additional uncertainty
- Results:

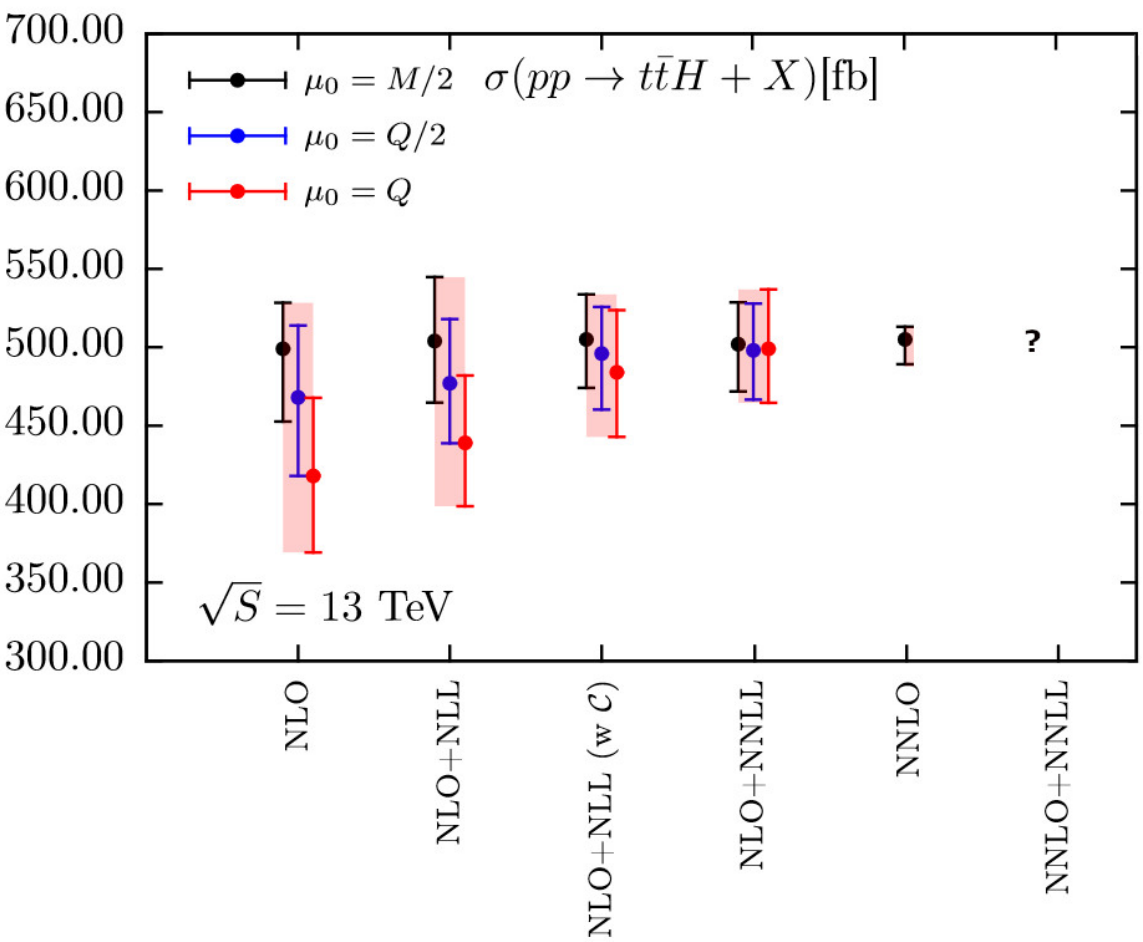
σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)\,^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

Approximate NNLO predictions

[S. Catani et al. *Phys.Rev.Lett.* 130 (2023) 11, 111902]



Ongoing: Approximate NNLO + NNLL
NNLO + NNLL (direct QCD) / NNLL (SCET) + NLO (EW)



- Approximate NNLO greatly improves the theoretical precision
- Consistency of approximate NNLO results with NLO+NNLL
- Room for further improvement:

  NNLO + NNLL

Summary

- Soft gluon resummation in $t\bar{t}H$, $t\bar{t}Z$ and $t\bar{t}W$ production in proton – proton collisions have been performed for absolute threshold and mass dependent schemes at NNLL accuracy matched to NLO QCD
- Extension has been performed of the soft gluon resummation to three- (and four-) body final state with non-trivial colour structure
- Significant improvement of the NLO+NNLL theoretical precision w.r.t. the NLO results
- Recently approximate NNLO calculation become available, consistent with NLO+NNLL predictions, high precision, theory uncertainty at +-3.5%
- Further improvement of the theoretical precision by resummation possible and ongoing!

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THANK YOU!