

Fragmentation of heavy quarks

*Boris Kopeliovich
Valparaiso, Chile*

Specific features of heavy quarks

Fragmentation functions:
heavy vs light quarks

Data show that a heavy quark loses only a small fraction $1-z$ of its initial energy

$$\Delta E = (1 - z)E \ll E$$

Why?

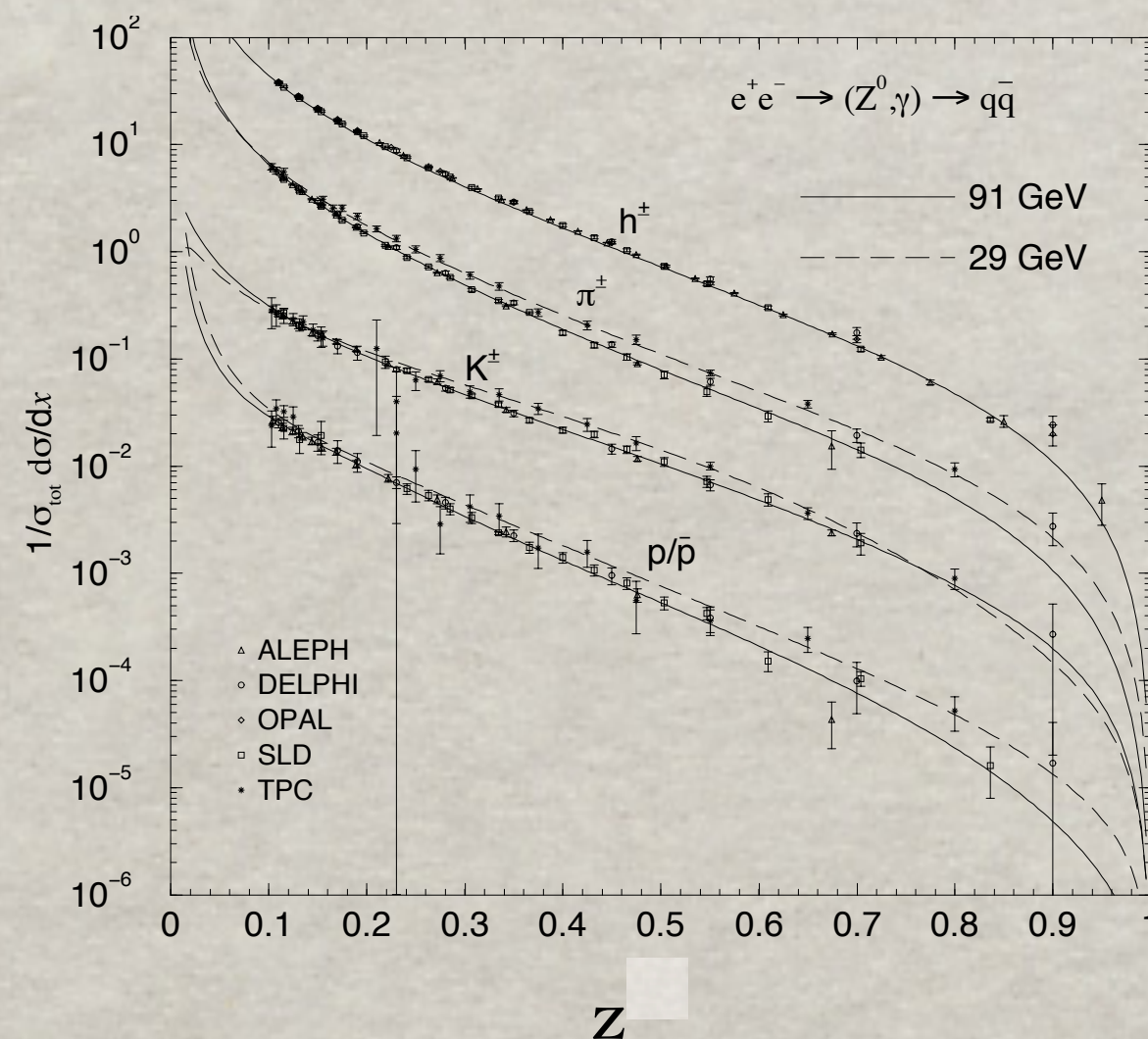
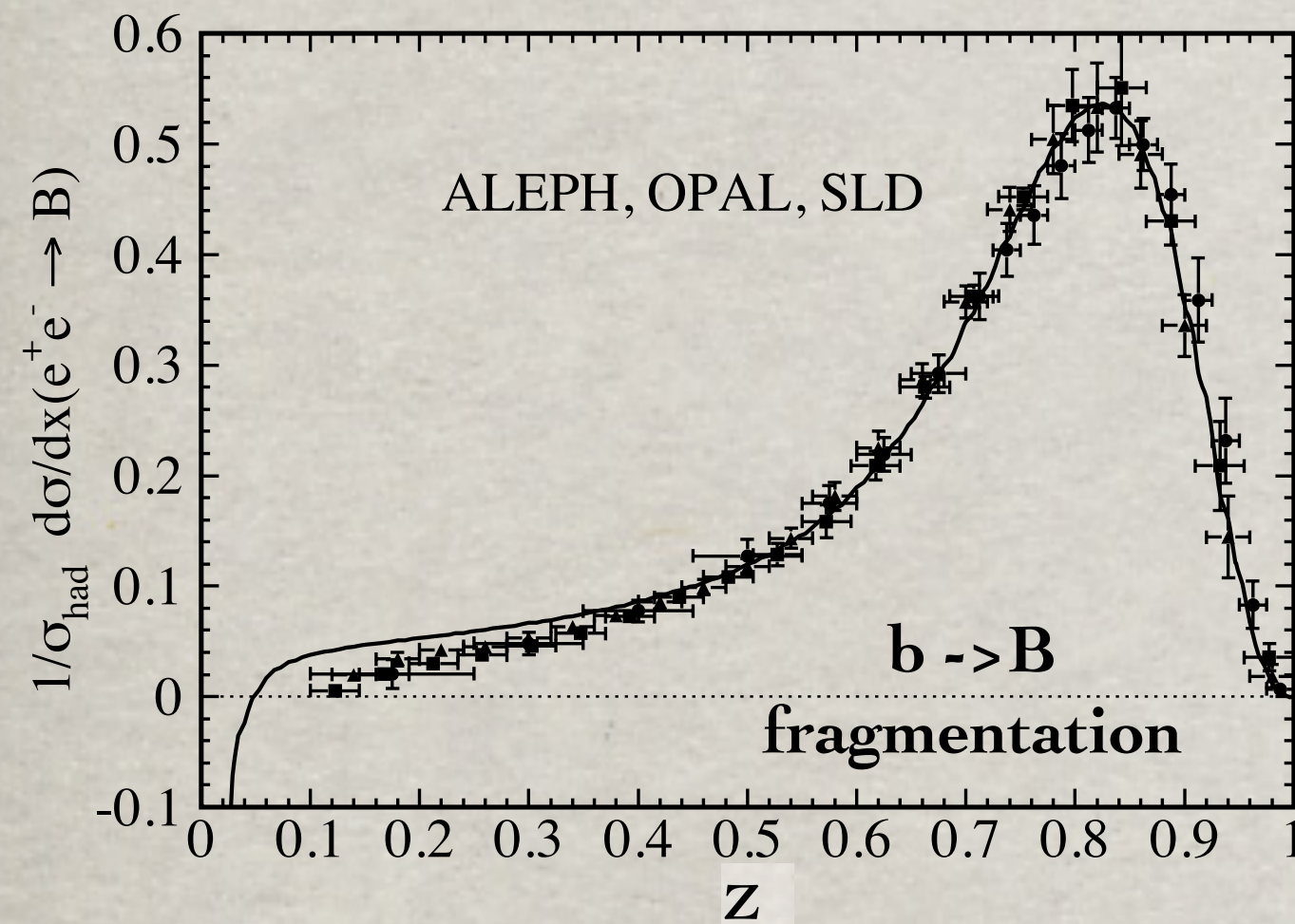
This implies a short hadronization time
(on the contrary to the usual assumption)

The quark regenerates its stripped-off color field by means of gluon radiation, which are emitted sequentially, rather than burst simultaneously.

The radiation (coherence) length of a gluon

$$L_c^g = \frac{2Ex(1-x)}{k_T^2 + x^2m_Q^2}$$

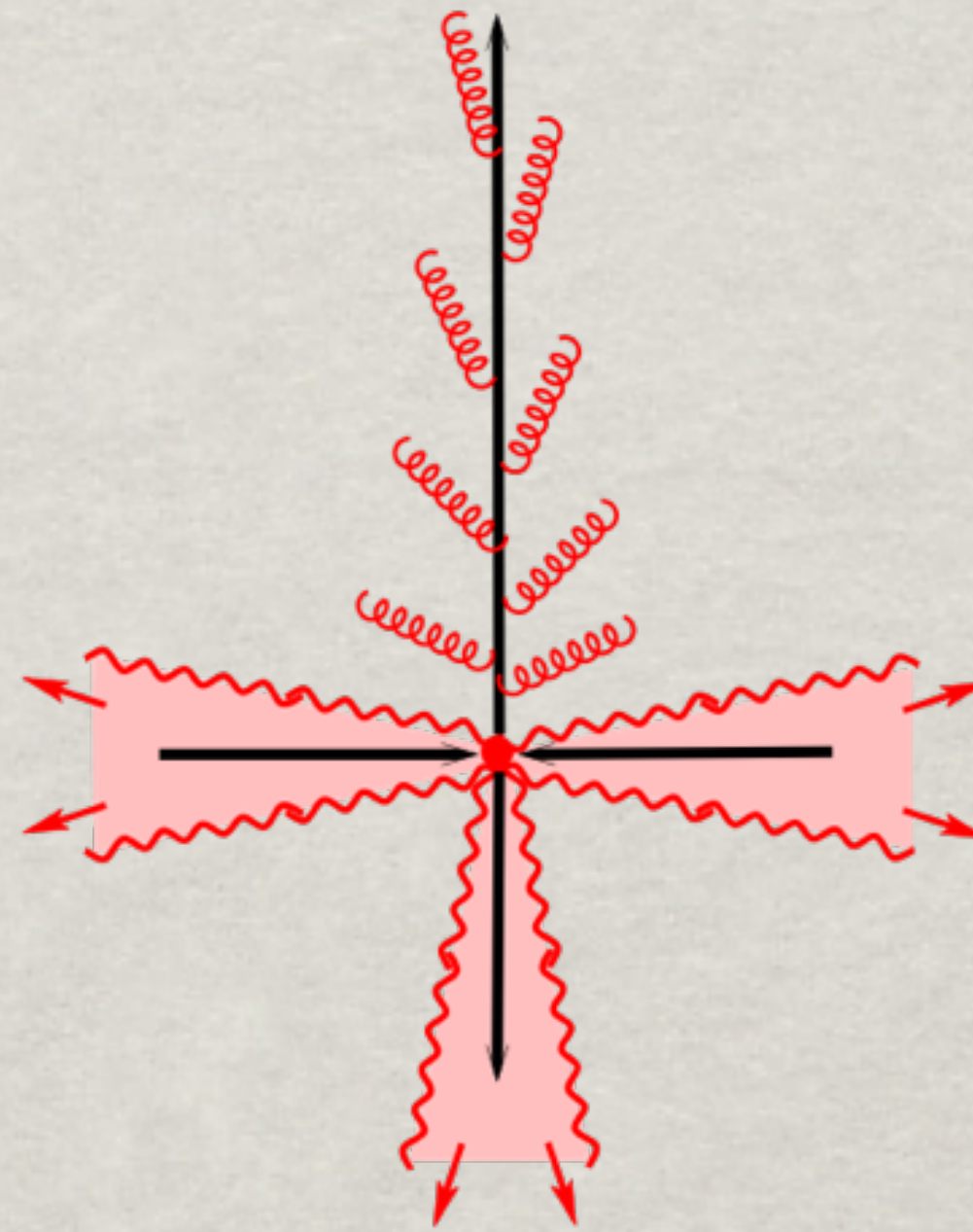
One can trace the radiational dissipation of energy according to the radiation length ordering. First are radiated soft gluons with small x and large k_T .



Radiated energy

B.K., I.Potashnikova, I.Schmidt,
PRC 82(2010)037901

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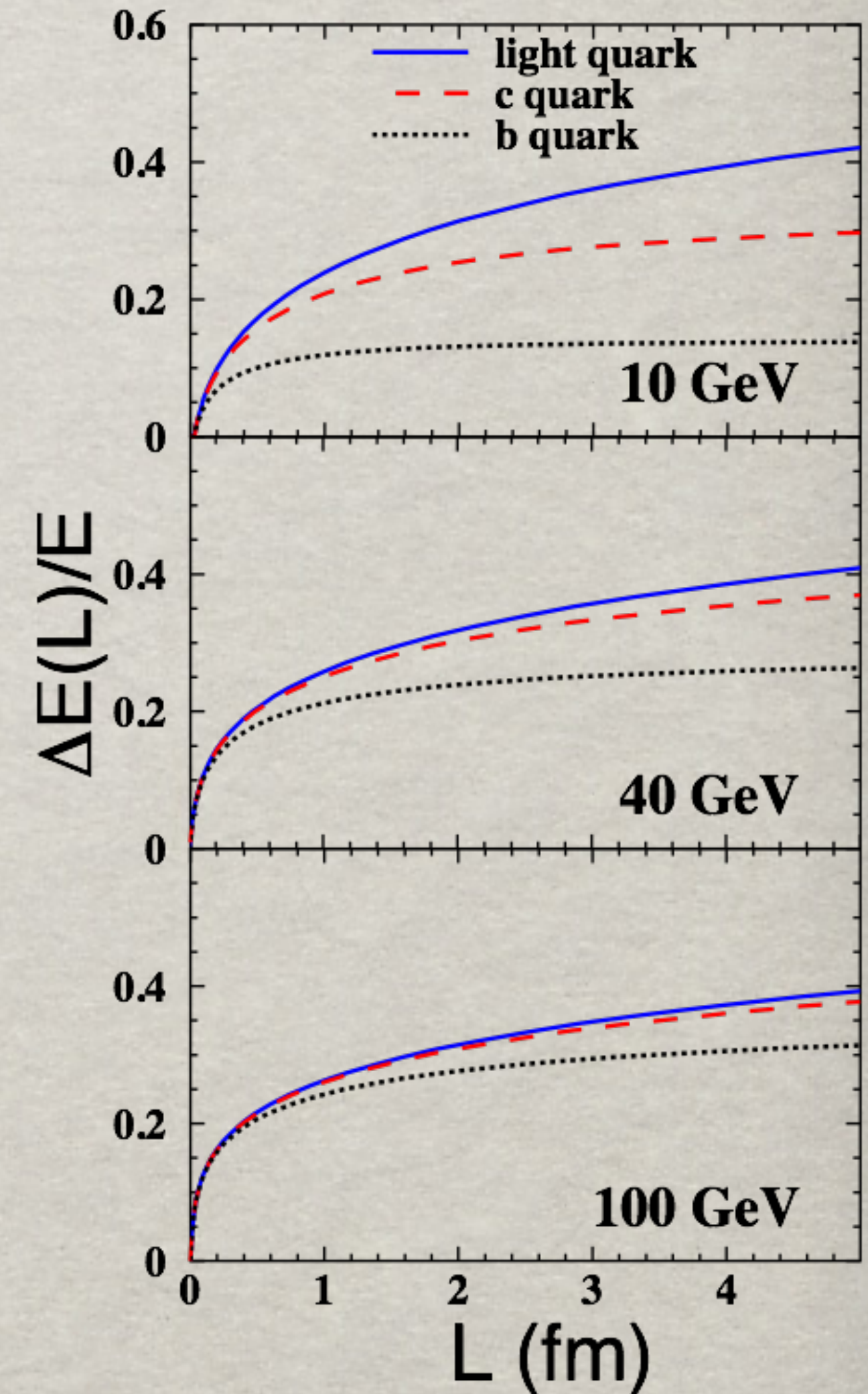


How much energy is radiated along path length L ?

$$\Delta E(L) = E \int_{\Lambda^2}^{Q^2} dk^2 \int_0^1 dx x \frac{dn_g}{dx dk^2} \Theta(L - l_c)$$

Dead-cone: gluons with $k^2 < x^2 m_q^2$ are suppressed.

Heavy quarks radiate less energy than the light ones.



Specific features of heavy quarks

Energy loss in vacuum

B.K., I.Potashnikova, I.Schmidt

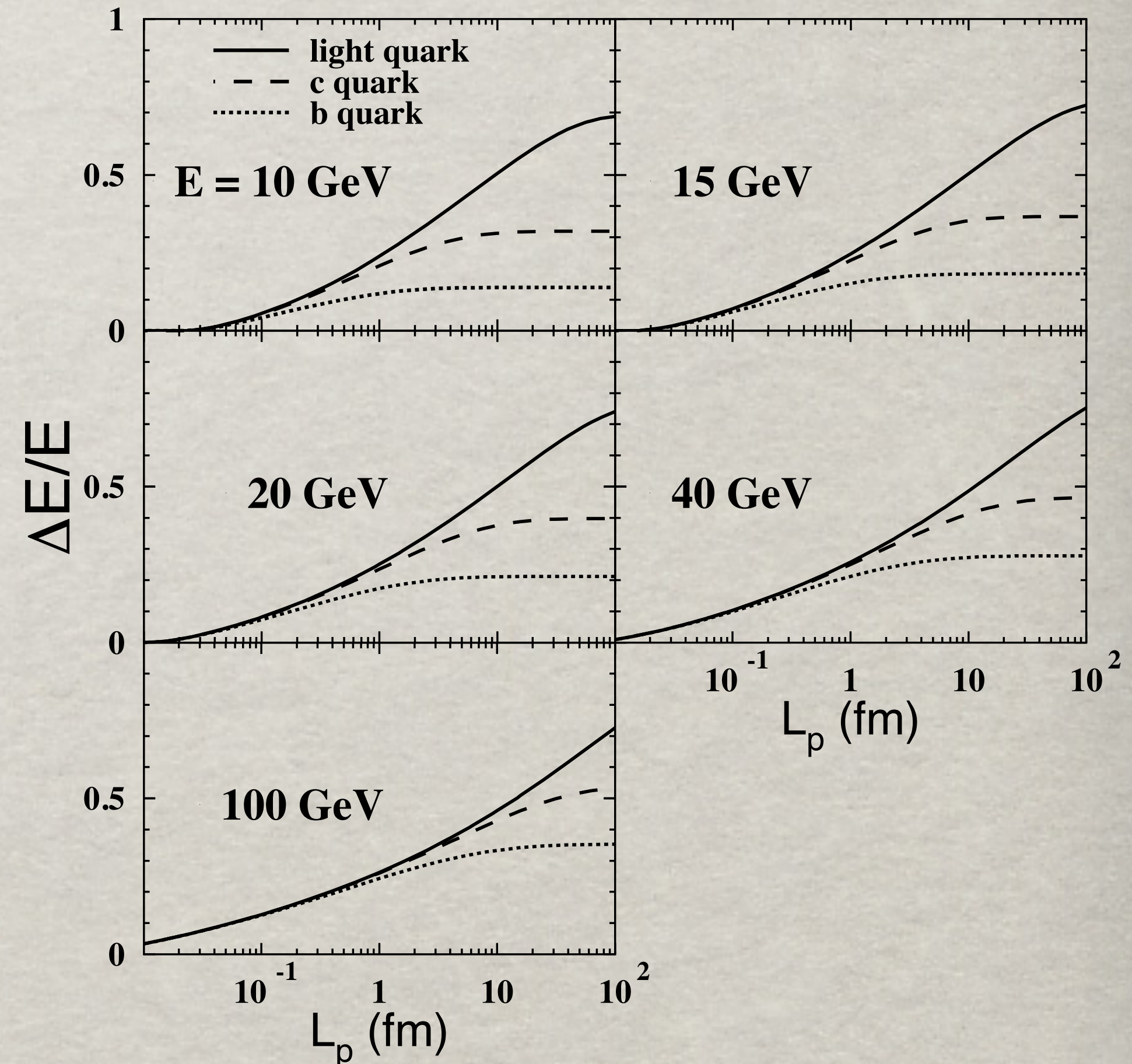
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How much energy is radiated over path length L ?

$$\Delta E(L) = E \int_{\Lambda^2}^{Q^2} dk^2 \int_0^1 dx x \frac{dn_g}{dx dk^2} \Theta(L - L_c^g)$$

$$\frac{dn_g}{dx dk^2} = \frac{2\alpha_s(k^2)}{3\pi x} \frac{k^2 [1 + (1-x)^2]}{[k^2 + x^2 m_q^2]^2}$$

Dead-cone effect: gluons with $k^2 < x^2 m_q^2$ are suppressed. Heavy quarks radiate less energy than the light ones. They restore their color field and promptly stop radiating.



Specific features of heavy quarks

This explains the observed specific shape of the fragmentation function $D_{b/B}(z)$

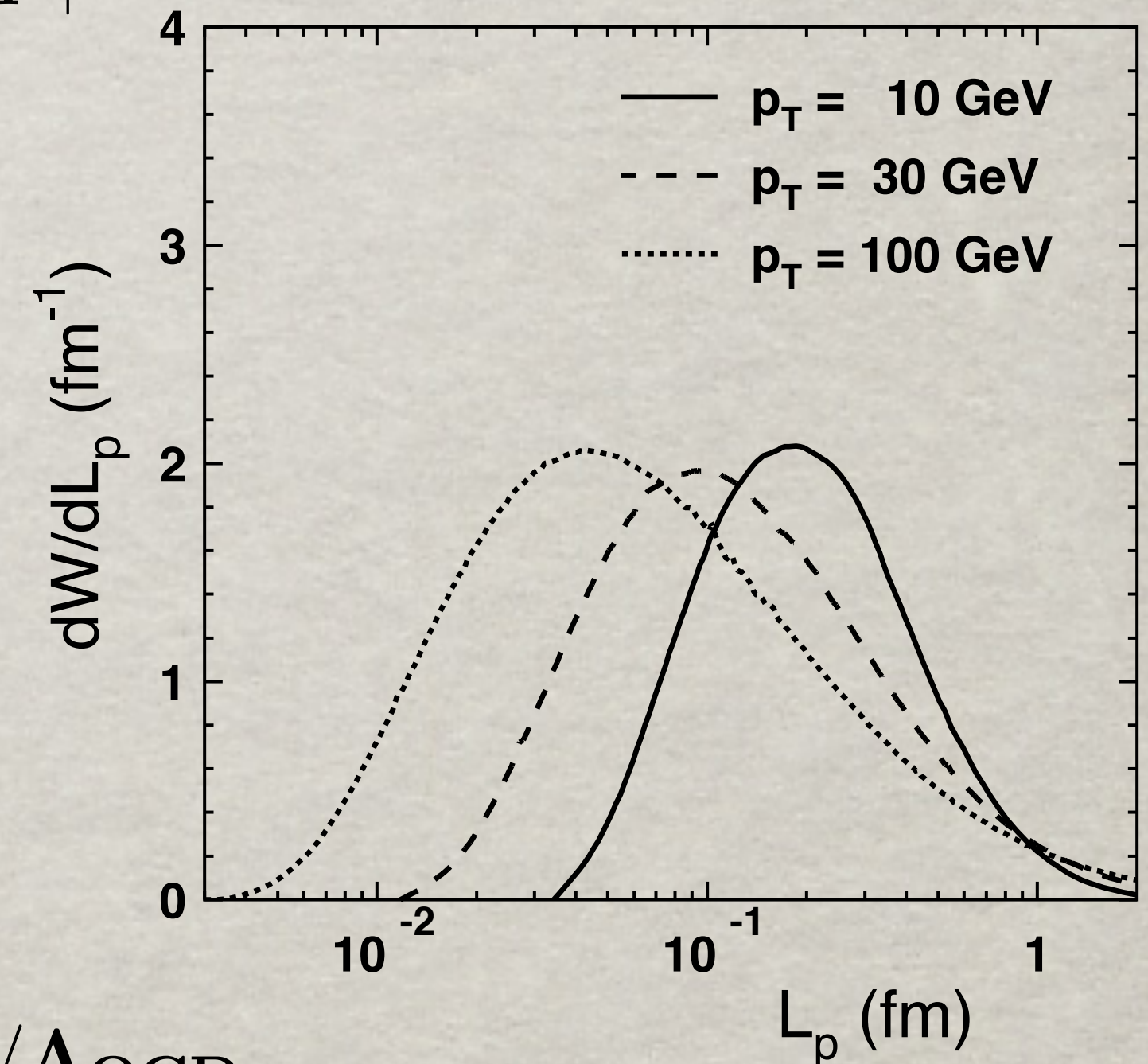
The fractional light-cone momentum $z \equiv \frac{p_+^B}{p_+^b} = 1 - \frac{\Delta p_+^b(L_p)}{p_+^b}$

As far as we can calculate $\Delta E(L)/E$, the production length distribution can be extracted directly from $D_{b/B}(z)$

$$\frac{dW}{dL_p} = \frac{1}{p_+^b} \left. \frac{\partial \Delta p_+^b}{\partial L} \right|_{L=L_p} D_{b/B}(z)$$

Remarkably, the mean value of L_p is extremely short and shrinks with rising p_T

It is much shorter than the confinement radius, $L_p \ll 1/\Lambda_{\text{QCD}}$
i.e the fragmentation mechanism is pure perturbative.
 Not a large size $Q\bar{q}$ meson is produced at $L=L_p$,
 but a small-size dipole, with no certain mass.



B.K., J.Nemchik, I.Potashnikova, I.Schmidt
 arXiv:1909.08831

Attenuation in a hot medium

Characteristic length scales

Medium modified production

A b-quark propagates through the hot medium, easily picking up and losing accompanying light quarks. Meanwhile the b-quark keeps losing energy with a rate, slightly enhanced by medium-induced effects. Eventually the detected B-meson is produced at the dilute surface of the medium at $L=L_p$ with probability and exponentially attenuates further at $L>L_p$. Thus, the medium-modified L_p distribution reads

$$\frac{dW^{AA}}{dL_p} = \frac{\langle r_B^2 \rangle}{2} \hat{q}(L_p) \exp \left[-\frac{\langle r_B^2 \rangle}{2} \int_{L_p}^{\infty} dL \hat{q}(L) \right] \propto \langle r_B^2 \rangle \hat{q}(L_p) / 2$$

Attenuation in a hot medium

Formation length of a $Q\bar{q}$ meson

The light quark in the B-meson carries a tiny fraction of the momentum,
 $x \sim m_q/m_b \approx 5\%$

The produced b-q dipole has a small transverse separation, but its size expands with a high speed, enhanced by $1/x$. It quickly reaches the large hadronic size.

$$L_f \sim \frac{1}{2}x(1-x)\langle r_T^2 \rangle p_T$$

This is the early, perturbative stage of the dipole expansion.

The further evolution filters out the states with large relative phase shifts. The longest time takes discrimination between the two lightest hadrons, the ground state B and the first radial excitation B', which concludes the formation process. Correspondingly the full formation path length is,

$$L_f = \frac{2p_T}{m_{B'}^2 - m_B^2}$$

E.g. for oscillatory potential $m_{B'} - m_B = 2\omega = 0.6 \text{ GeV}$,
so $L_f = 0.06 \text{ fm}[p_T / 1 \text{ GeV}]$.



Attenuation in a hot medium

Mean free path in the medium

The mean free path of such a meson in a hot medium with transport coefficient \hat{q} is

$$\lambda_B \sim \frac{1}{\hat{q} \langle r_T^2 \rangle}, \quad \text{where} \quad \langle r_T^2 \rangle = \frac{8}{3} \langle r_{ch}^2 \rangle$$

B meson is nearly as big as a pion, $\langle r_{ch}^2 \rangle_B = 0.378 \text{ fm}^2$ [Ch.-W. Hwang (2001)]

$$\text{E.g. at } \hat{q} = 1 \text{ GeV}^2/\text{fm} \quad \lambda_B = 0.04 \text{ fm}$$

A b-quark propagates through the hot medium, easily picking up and losing accompanying light quarks. Meanwhile the b-quark keeps losing energy with a rate, slightly enhanced by medium-induced effects. Eventually the detected B-meson is produced at the dilute surface of the medium.



Attenuation in a hot medium

Assuming factorization and fragmentation mechanism of $b \rightarrow B$ production

$$\frac{d^2\sigma_{pp \rightarrow BX}}{d^2p_T} = \frac{1}{2\pi p_T E_T} \int d^2q_T \frac{d^2\sigma_{pp \rightarrow bX}}{d^2q_T} \int_0^\infty dL_p \frac{dW}{dL_p} \frac{\Delta E(L_p)}{E} \delta\left(1 - z - \frac{\Delta E(L_p)}{E}\right).$$

Here the fragmentation function $D_{b/B}(z)$ is replaced by the production length distribution dW/dL_p , which peaks at extremely short distances $L_p < 1\text{fm}$.

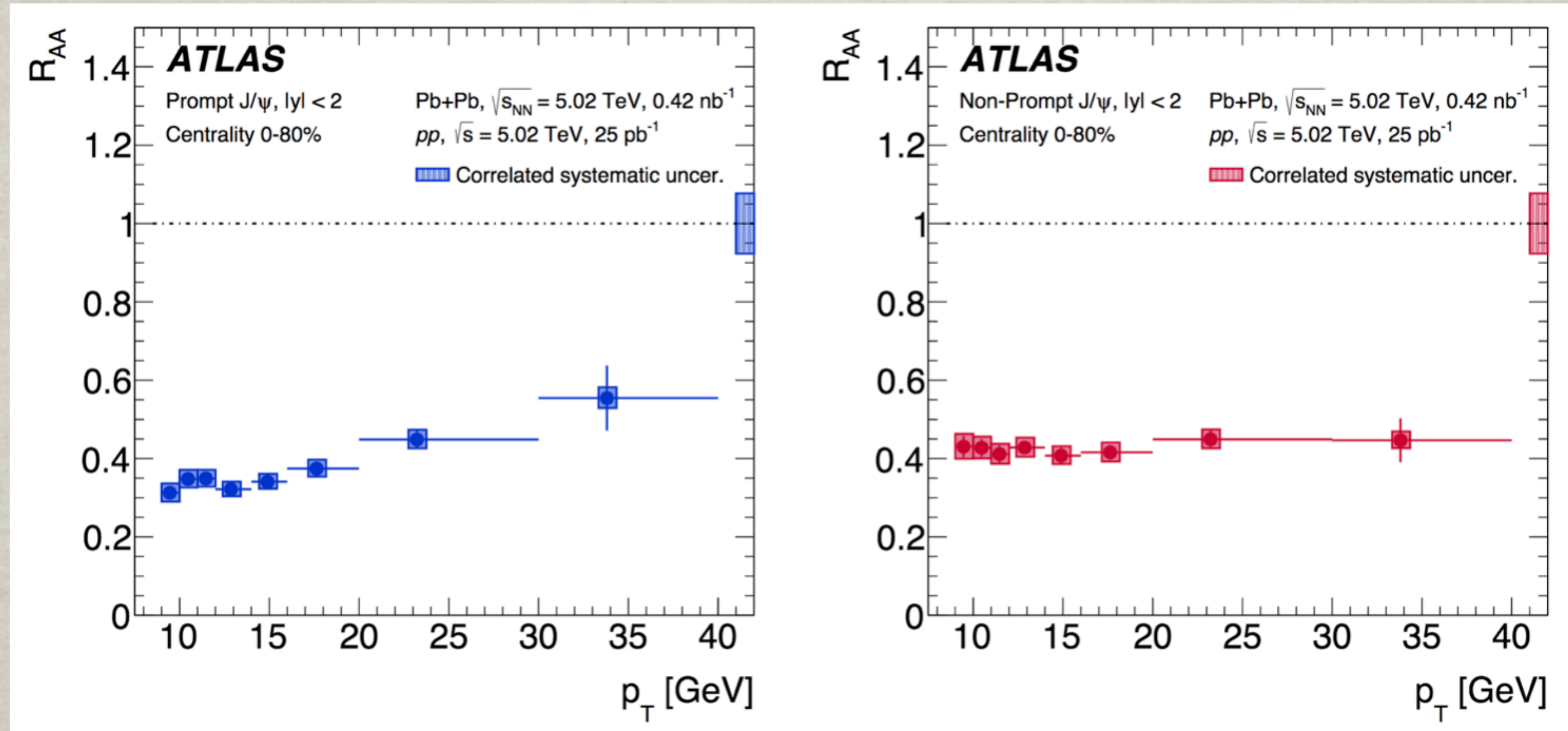
In the case of AA collisions one can employ the same formula, but with a modified production length distribution.

$$\frac{dW^{AA}}{dL_p} = \frac{\langle r_B^2 \rangle}{2} \hat{q}(L_p) \exp\left[-\frac{\langle r_B^2 \rangle}{2} \int_{L_p}^\infty dL \hat{q}(L)\right]$$

L_p turns out to be much longer, because the B-meson is produced mainly at the medium border,

$$\frac{d^2\sigma_{AA \rightarrow BX}}{d^2p_T d^2s} = \frac{1}{2\pi p_T E_T} \int d^2q_T \frac{d^2\sigma_{pp \rightarrow bX}}{d^2q_T} \int d^2\tau T_A(s) T_A(\tilde{s} - \tilde{\tau}) \int_0^\infty dL_p \frac{dW^{AA}}{dL_p} \frac{\Delta E(L_p)}{E} \delta\left(z - \frac{\Delta E(L_p)}{E}\right)$$

M.Arratia, W.Brooks et al. ATLAS, *Eur.Phys.J.C* 78 (2018) 9



Why the AA/NN ratio rises with pT for prompt J/ψ, but is hardly varies for non-prompt?

Slow expansion of $\bar{Q}Q$ separation

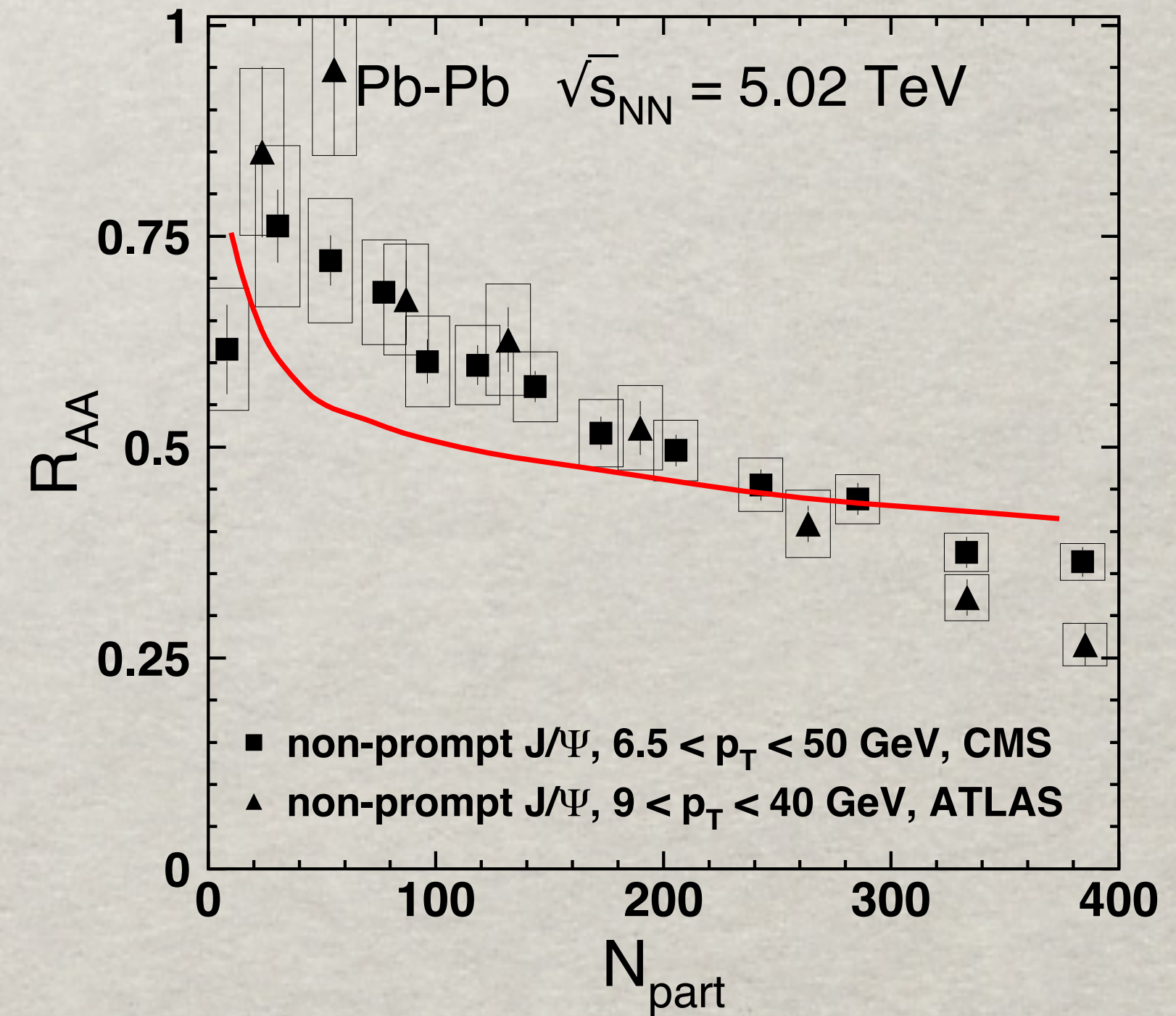
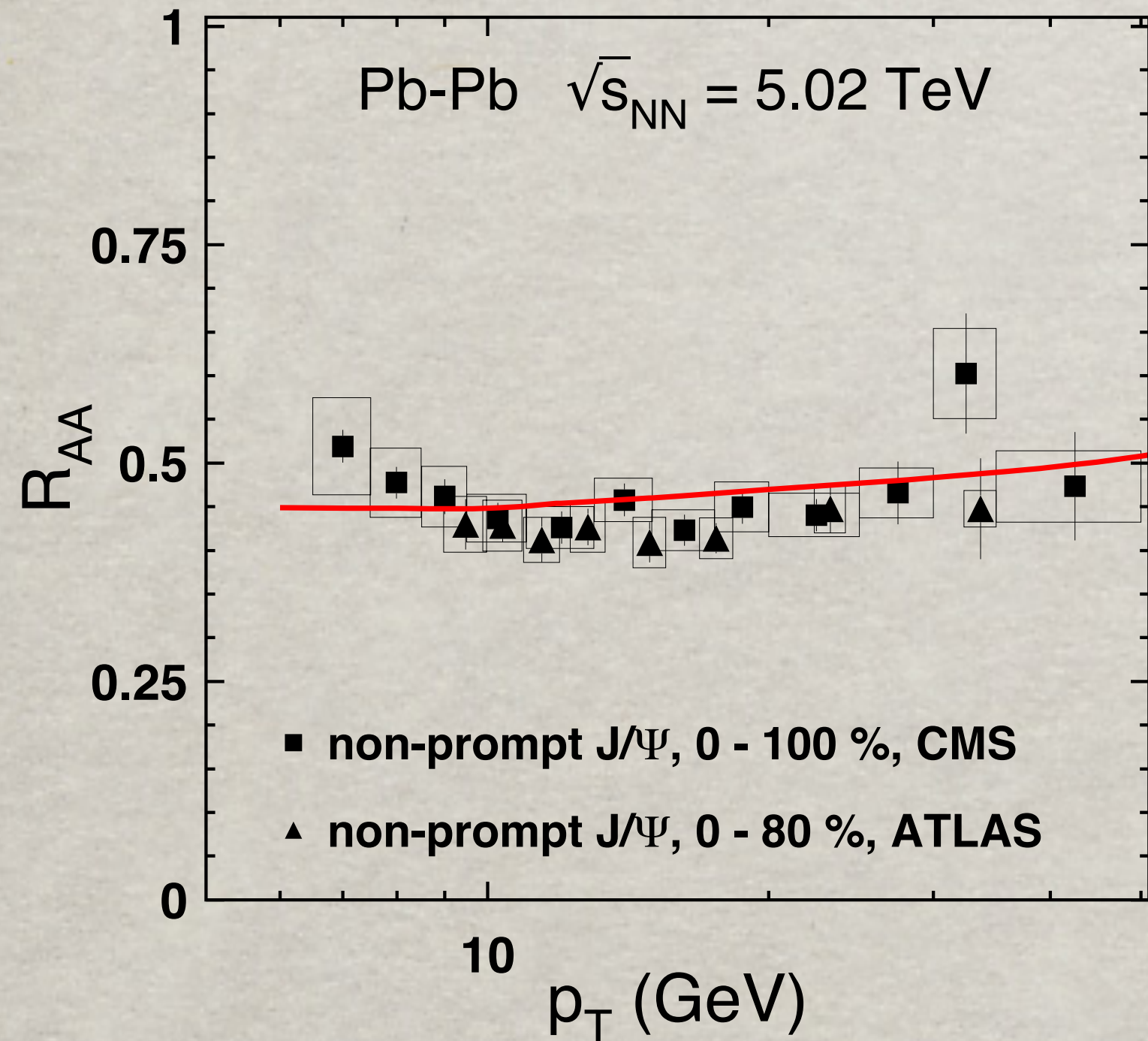
Color transparency

Fast expansion of $\bar{q}Q$ separation

No color transparency

Results: B mesons

Different sources of time-dependent energy loss should be added up. Medium-induced energy loss is much smaller than the vacuum one, and should not produce a dramatic effect. They are particularly small for heavy flavors (Yu.Dokshitzer & D.Kharzeev (2001))

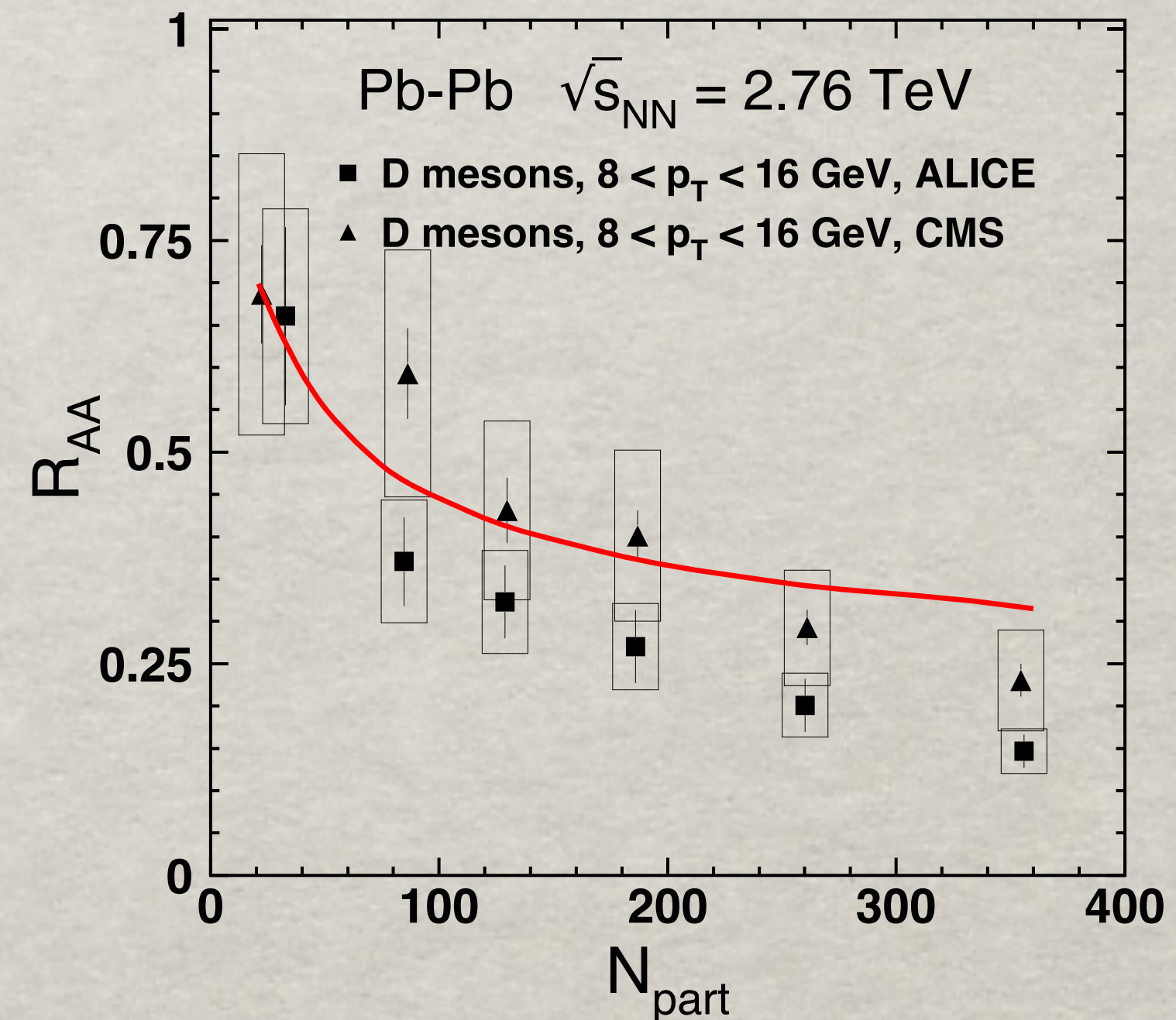
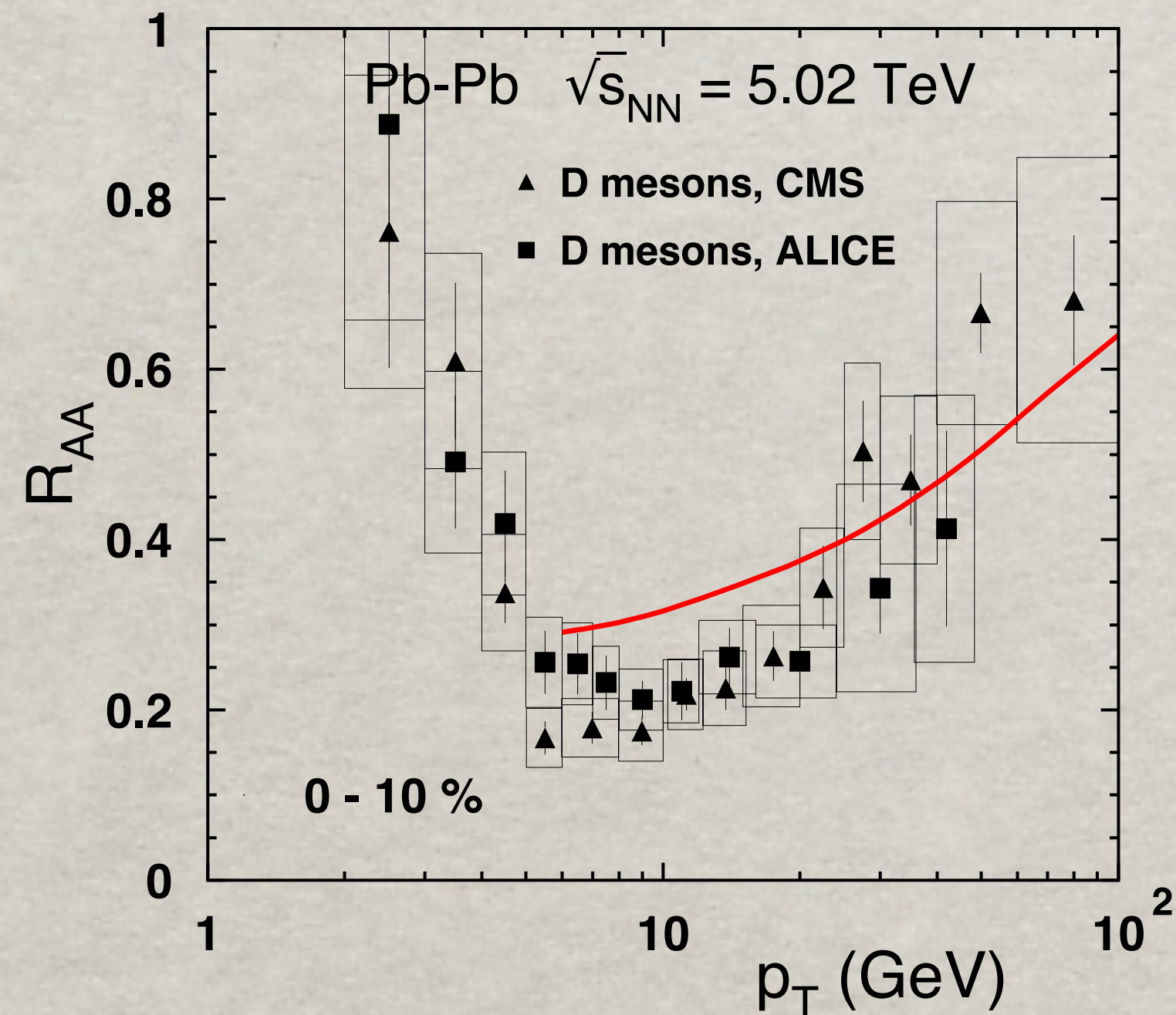


Results: D mesons

c-quarks radiate in vacuum more energy than b-quarks, while the effects of absorption of c-qbar and b-qbar dipoles in the medium are similar. Therefore D-mesons are suppressed in AA collisions more than B-mesons.

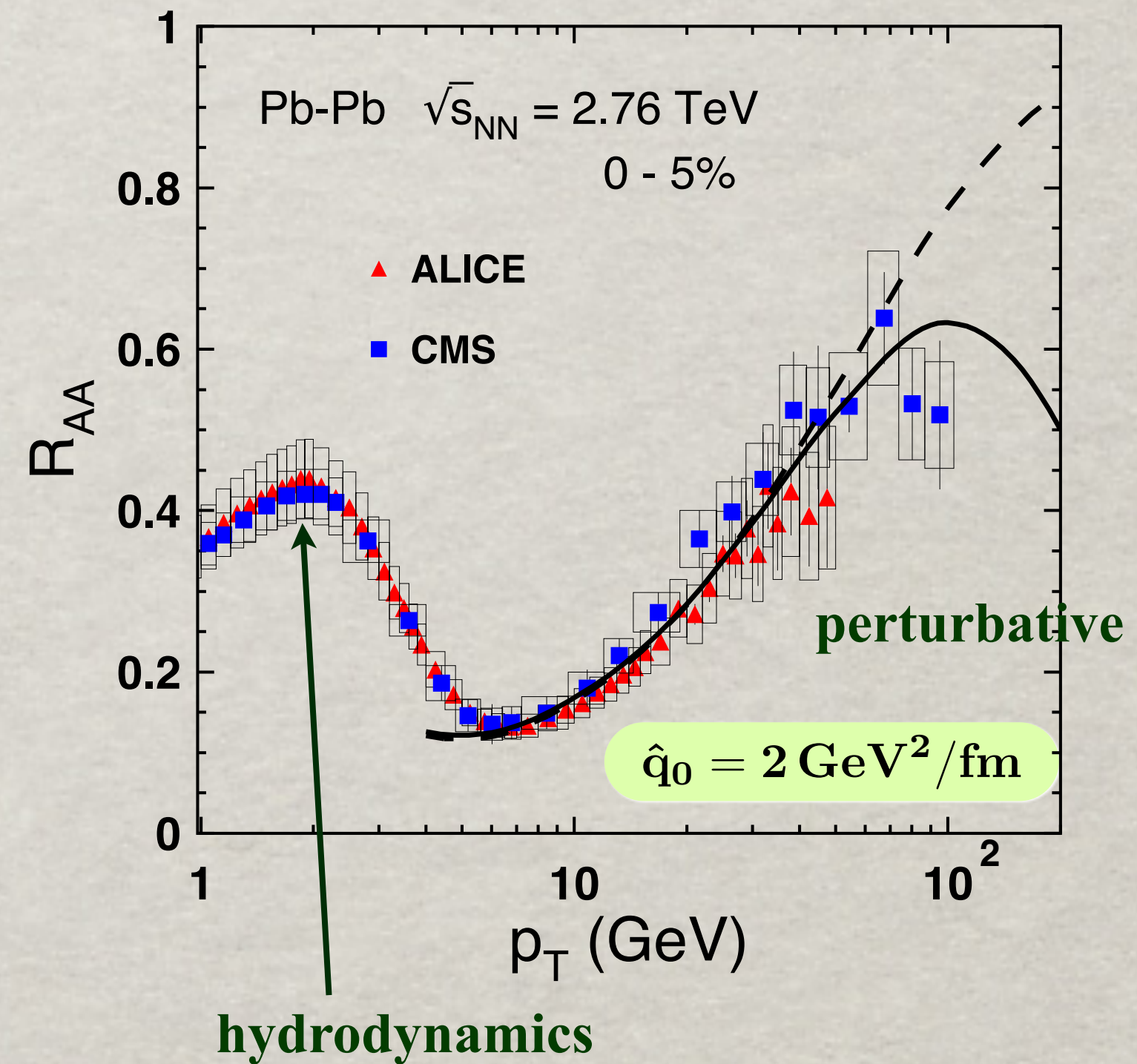
$R_{AA}(p_T)$ for D-mesons steeply rises due to color transparency.

Since $b\bar{q}$ dipoles expand much faster than $c\bar{q}$, no color transparency effects are seen in $R_{AA}(p_T)$ for B-mesons.



BK, J.Nemchik, I.Potashnikova, I.Schmidt
Phys.Rev. C86(2012)054904

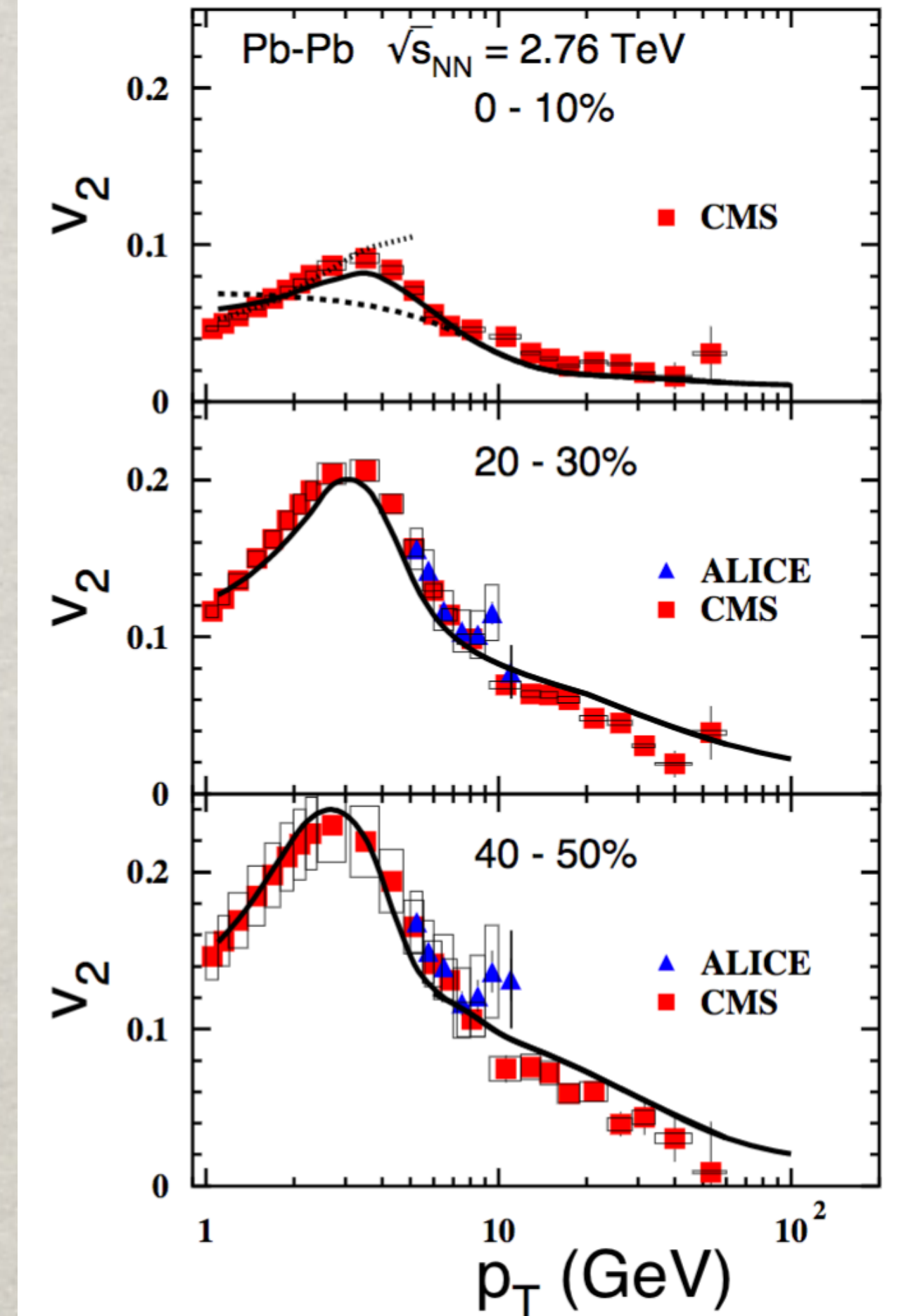
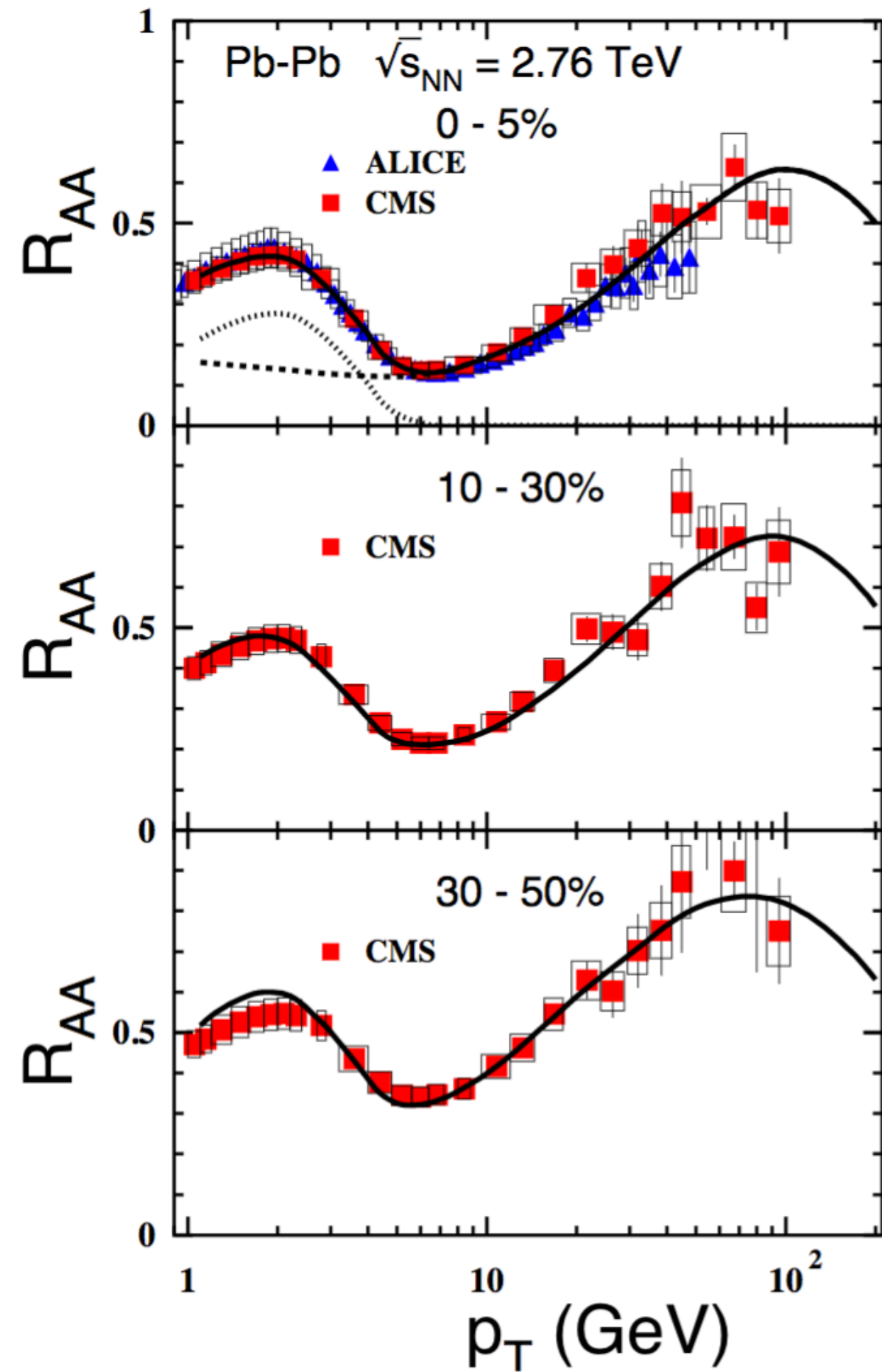
Why $R_{AA}(p_T)$
is falling at small p_T ?



BK, J.Nemchik, I.Potashnikova,
Yu.Karpenko, Yu.Sinyukov
EDS 13, Arxiv 1310.3455

HYDRO + pQCD

J.Nemchik, Yu.Karpenko, I.Potashnikova
I.Schmidt, Yu.Sinyukov & B.K.
[arXiv:1310.3455](https://arxiv.org/abs/1310.3455)



Summary

- Heavy and light quarks produced in high- p_T partonic collisions radiate differently. Heavy quarks regenerate their stripped-off color field much faster than light ones and radiate a significantly smaller fraction of the initial energy.
- This peculiar feature of heavy-quark jets leads to a specific shape of the fragmentation functions. Differently from light flavors, the heavy quark fragmentation function strongly peaks at large fractional momentum z , i.e. the produced heavy-light meson, B or D, carry the main fraction of the jet momentum. This is a clear evidence of a short production time of a heavy-light mesons.
- Contrary to the propagation of a small $q-\bar{q}$ dipole, which survives in the medium due to color transparency, a $\bar{q}-Q$ dipole promptly expands to a large size. Such a big dipole has no chance to remain intact in a hot medium. On the other hand, a breakup of such a dipole does not suppress the production rate of $\bar{q}-Q$ mesons, differently from light $q\bar{q}$ mesons.