

Impact factor for forward η_c meson production at NLO

Maxim Nefedov¹

Diffraction and low-x 2024
Trebia, September 9th, 2024



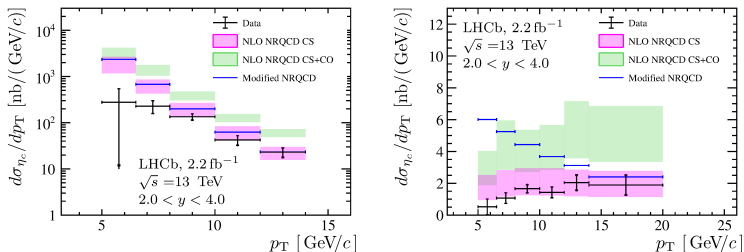
This project is supported by the European Commission's Marie Skłodowska-Curie action

"RadCor4HEF", grant agreement No. 101065263

¹Université Paris-Saclay, CNRS, IJCLab, Orsay, France

Motivation: inclusive heavy quarkonium production at moderate p_T

It is well known that quarkonium production p_T spectra at $p_T \gtrsim M$ are not described by collinear factorisation computations at NLO. Example: η_c production at LHCb [LHCb, '24] (left panel $d\sigma/dp_T$, right panel: ratio to J/ψ):

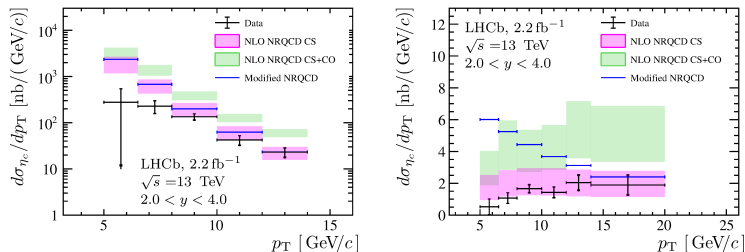


- ▶ NLO CF overshoots the data for $5 < p_T < 8$ GeV: nothing to do with TMD/Sudakov logs ($\ln p_T/M$) which contribute only at $p_T \ll M \simeq 3$ GeV
- ▶ Physical shape of the p_T -spectrum is reproduced in k_T -factorisation [Kniehl, Vasin, Saleev '06;...;MN,Saleev,Shipilova '12;...] and Saturation/CGC calculations [Kang, Ma, Venugopalan '13; ... Mantysaari et al. '24] **at LO in α_s**

The goal of the present work is to compute the NLO imact-factor for the forward production of η_c in the Colour-Singlet Model($c\bar{c}[^1S_0^{[1]}]$) in order to advance the High-Energy factorisation calculations of η_c p_T spectra beyond LLA.

Inclusive heavy quarkonium production at moderate p_T in CF

It is well known that quarkonium production p_T spectra at $p_T \gtrsim M$ are not described by collinear factorisation computations at NLO. Example: η_c production at LHCb [LHCb, '24] (left panel $d\sigma/dp_T$, right panel: ratio to J/ψ):



$p(P_1) + p(P_2) \rightarrow \eta_c(p) + X$, parton level e.g.: $g(x_1 P_1) + g(x_2 P_2) \rightarrow c\bar{c}[^1S_0^{[1]}] + X$, with $S = (P_1 + P_2)^2$ and $p^2 = M^2$. Cross section in **collinear factorisation**:

$$\frac{d\sigma}{dp_T^2 dy} = \frac{M^2}{S} \int_0^{\eta_{\max}} d\eta \int_{z_{\min}}^{z_{\max}} dz f_i \left(\frac{M_T e^y}{\sqrt{S} z}, \mu_F \right) f_j \left(\frac{M^2 z(1+\eta)}{M_T \sqrt{S}} e^{-y}, \mu_F \right) \frac{d\hat{\sigma}_{ij}(\eta, z, \mathbf{p}_T^2)}{dz d\mathbf{p}_T^2},$$

where $\eta = \hat{s}/M^2 - 1$ with $\hat{s} = Sx_1x_2$, $z = \frac{p_+}{x_1 P_1^+}$, $M_T = \sqrt{M^2 + \mathbf{p}_T^2}$.

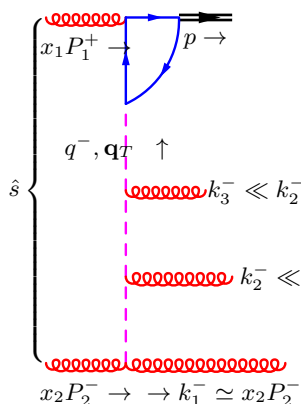
High-Energy Factorization, forward η_c hadroproduction

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann,

'91, '94]

Physical picture in the

LLA:



The LLA in $\ln(x_2 P_2^- / p^-)$:

$$\mathcal{H} \quad \hat{\sigma}_{\text{HEF}}(p_- / (x_2 P_2^-), z) \propto \int_0^\infty dq^- \int_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C} \left(\frac{q^-}{x_2 P_2^-}, \mathbf{q}_{T1}^2, \mu_F, \mu_R \right) \mathcal{H}(q^-, \mathbf{q}_{T1}^2, z),$$

The LLA in $\ln(\hat{s}/M^2)$:

$$\mathcal{C} \quad \hat{\sigma}_{\text{HEF}}(\hat{s}/M^2, z) \propto \int_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C} \left(\frac{\hat{s}}{M^2}, \mathbf{q}_{T1}^2, \mu_F, \mu_R \right) \int_0^\infty dq^- \mathcal{H}(q^-, \mathbf{q}_{T1}^2, z).$$

Two kinds of LLA are equivalent up to NLL terms because

$$\frac{\hat{s}}{M^2} = \frac{x_1 x_2 S}{M^2} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{x_2 P_2^-}{q^-} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{x_2 P_2^-}{p^-}.$$

The Leading Order

The LLA resummation formula for \mathbf{p}_T^2 and $z = p_-/q_-$ -differential partonic cross section:

$$\begin{aligned}\frac{d\hat{\sigma}_{ig}^{(\text{LLA})}}{dz d\mathbf{p}_T^2} &= \frac{1}{2M^2} \int \frac{d^2\mathbf{q}_T}{\pi} \mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{q}_T^2, \mu_F, \mu_R\right) \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2), \\ &= \frac{1}{2M^2} \mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{p}_T^2, \mu_F, \mu_R\right) \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(1-z),\end{aligned}$$

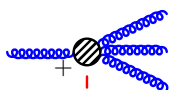
with [Kniehl, Vasin, Saleev, '06]

$$\mathcal{H}_{gg}^{(\text{LO})} = \frac{32\pi^3 \alpha_s^2(\mu_R) M^4}{N_c^2 (N_c^2 - 1) (M^2 + \mathbf{p}_T^2)^2} \frac{\langle \mathcal{O} [{}^1S_0^{[1]}] \rangle}{M^3} \delta(1-z) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2),$$

where $\langle \mathcal{O} [{}^1S_0^{[1]}] \rangle = 2N_c |R(0)|^2 / (4\pi)$.

In this talk we will compute $\mathcal{H}_{gg}^{(\text{NLO})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2)$, which includes **virtual** and **real-emission** corrections.

The Gauge-Invariant EFT for Multi-Regge processes in QCD



- ▶ Reggeized gluon fields R_{\pm} carry $(k_{\pm}, \mathbf{k}_T, k_{\mp} = 0)$: $\partial_{\mp} R_{\pm} = 0$.

- ▶ **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s} \text{tr} \left[R_+ \partial_{\perp}^2 \partial_- \left(W[A_-] - W^{\dagger}[A_-] \right) + (+ \leftrightarrow -) \right],$$

$$\text{with } W_{x_{\mp}}[x_{\pm}, \mathbf{x}_T, A_{\pm}] = P \exp \left[\frac{-ig_s}{2} \int_{-\infty}^{x_{\mp}} dx'_{\mp} A_{\pm}(x_{\pm}, x'_{\mp}, \mathbf{x}_T) \right] = (1 + ig_s \partial_{\pm}^{-1} A_{\pm})^{-1}.$$

- ▶ Expansion of the Wilson line generates **induced vertices**:

$$\begin{aligned} & \text{tr} \left[R_+ \partial_{\perp}^2 A_- + (-ig_s)(\partial_{\perp}^2 R_+)(A_- \partial_-^{-1} A_-) \right. \\ & \left. + (-ig_s)^2 (\partial_{\perp}^2 R_+)(A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + O(g_s^3) + (+ \leftrightarrow -) \right]. \end{aligned}$$

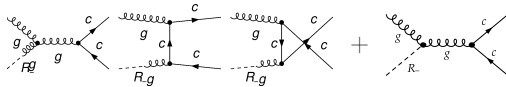
- ▶ The *Eikonal propagators* $\partial_{\pm}^{-1} \rightarrow -i/(k^{\pm})$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

$$n_{\pm}^{\mu} \rightarrow \tilde{n}_{\pm}^{\mu} = n_{\pm}^{\mu} + r n_{\mp}^{\mu}, \quad r \ll 1: \quad \tilde{k}^{\pm} = \tilde{n}^{\pm} k.$$

The terms for conversion of the result into any other regularisation scheme for RDs can be easily computed.

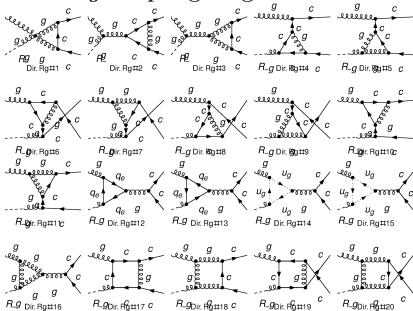
$Rg \rightarrow c\bar{c} [^1S_0^{[1]}]$ and $c\bar{c} [^3S_1^{[8]}]$ @ 1 loop

Interference with LO:

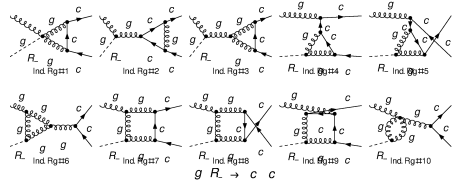


Induced Rg coupling diagrams:

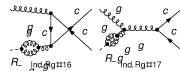
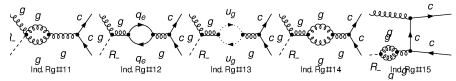
Some Rg -coupling diagrams:



$g R \rightarrow c c$



$g R \rightarrow c c$



and so on...

- ▶ Diagrams had been generated using custom `FeynArts` model-file, projector on the $c\bar{c} [^1S_0^{[1]}]$ -state is inserted
- ▶ heavy-quark momenta = $p_Q/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- ▶ IBP reduction to master integrals has been performed using `FIRE`
- ▶ Master integrals with linear and massless quadratic denominators are expanded in $r \ll 1$ using Mellin-Barnes representation. The differential equations technique is used when the integral depends on more than one scale of virtuality.
- ▶ In presence of the linear denominator the massive propagator can be converted to the massless one:

$$\frac{1}{((\tilde{n}+l) + k_+)(l^2 - m^2)} = \frac{1}{((\tilde{n}+l) + k_+)(l + \kappa\tilde{n}_+)^2} + \frac{2\kappa \left[(\tilde{n}+l) + \frac{m^2 + \tilde{n}_+^2 + \kappa^2}{2\kappa} \right]}{\cancel{((\tilde{n}+l) + k_+) (l + \kappa\tilde{n}_+)^2} (l^2 - m^2)}$$

\Rightarrow all the masses can be moved to integrals with **only quadratic propagators**.

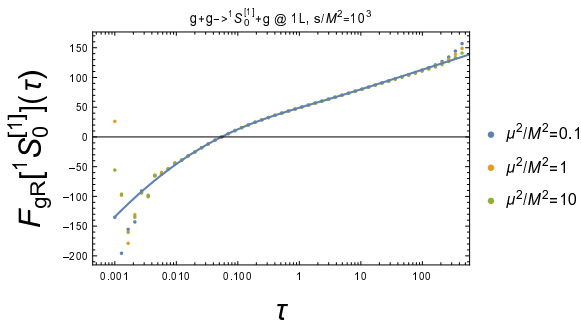
See [\[hep-ph/2408.06234\]](https://arxiv.org/abs/hep-ph/2408.06234) for details.

Result: $Rg \rightarrow c\bar{c} [^1S_0^{[1]}] @ 1 \text{ loop}$

Result [MN, '23, '24] for $2\Re \left[\frac{H_{1L} \times \text{LO}(\mathbf{q}_T) - (\text{On-shell mass CT})}{(\alpha_s/(2\pi))H_{LO}(\mathbf{q}_T)} \right]$:

$$\left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^\epsilon \left\{ -\frac{N_c}{\epsilon^2} + \frac{1}{\epsilon} \left[N_c \left(\ln \frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{25}{6} \right) - \frac{2n_F}{3} - \frac{3}{2N_c} \right] \right\} - \frac{10}{9}n_F + F_{1S_0^{[1]}}(\mathbf{q}_T^2/M^2)$$

Cross-check against the Regge limit of one-loop amplitude ($\tau = \mathbf{q}_T^2/M^2$):



Points – the function $F_{1S_0^{[1]}}(\tau)$ extracted from numerical results for interference between **exact** one-loop and tree-level QCD amplitudes of $g + g \rightarrow c\bar{c}[^1S_0^{[1]}] + g$ at $s = 10^3 M^2$. Solid line – analytic result from the EFT.

Real-emission correction

The real-emission contribution:

$$g(x_1 P_1) + R_-(q) \rightarrow c\bar{c}[^1 S_0^{[1]}](p) + g(k),$$

to the coefficient function is given by:

$$\mathcal{H}_{gg}^{(\text{NLO}, R)}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d\Omega_{2-2\epsilon}}{(2\pi)^{1-2\epsilon}} \frac{\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2},$$

where the **function** $\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)$ **is very complicated**. The following subtraction term:

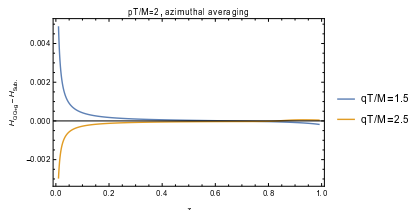
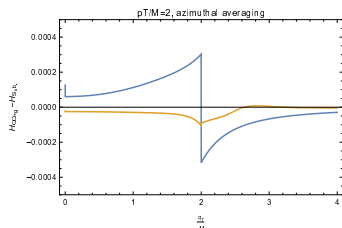
$$\mathcal{J}_{Rj}^{(\text{sub.})} = \frac{2C_A}{\mathbf{k}_T^2} \left[\frac{1-z}{(1-z)^2 + r \frac{\mathbf{k}_T^2}{q_-^2}} + z(1-z) + 2 \frac{\mathbf{k}_T^2 \mathbf{q}_T^2 - (\mathbf{k}_T \mathbf{q}_T)^2}{z \mathbf{k}_T^2 \mathbf{q}_T^2} - \frac{3\mathbf{k}_T^2 \mathbf{q}_T^2 - 2(\mathbf{k}_T \mathbf{q}_T)^2}{\mathbf{k}_T^2 \mathbf{q}_T^2} \right],$$

captures it's singular behaviour in:

- ▶ **Regge limit:** $z \rightarrow 1$, $\mathbf{k}_T = \mathbf{q}_T - \mathbf{p}_T$ - fixed,
- ▶ **Collinear limit:** $\mathbf{k}_T \rightarrow 0$, z -fixed
- ▶ **Soft limit:** $\mathbf{k}_T \rightarrow 0$, $z \rightarrow 1$

Real-emission correction, finite part

$$\mathcal{H}_{gj}^{(\text{fin.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int_0^{2\pi} \frac{d\phi}{2\pi} \left[\frac{\tilde{H}_{Rj}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2} - \mathcal{J}_{Rj}^{(\text{sub.})}(\mathbf{q}_T, \mathbf{p}_T, z, r=0) \right]$$



This contribution is finite for $\mathbf{k}_T \rightarrow 0$ and $z \rightarrow 1$ and can be safely convoluted with the resummation factor or unintegrated-PDF in \mathbf{q}_T and gluon PDF in z .

Integrated subtraction term

$$\mathcal{H}_{gj}^{(\text{int. sub.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi} \frac{\Omega_{2-2\epsilon} \mu^{2\epsilon}}{(2\pi)^{1-2\epsilon}} \int d^{2-2\epsilon} \mathbf{k}_T \mathcal{J}_{Rj}^{(\text{sub.})}(\mathbf{q}_T, \mathbf{q}_T - \mathbf{k}_T, z, r) \\ \times \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T - \mathbf{k}_T, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})}$$

$$\mathcal{H}_{gj}^{(\text{int. sub. I})} = \frac{\alpha_s(\mu_R) C_A}{\pi} \int \frac{d^2 \mathbf{k}_T}{\mathbf{k}_T^2} \left[\frac{1}{(1-z)_+} + (\dots) \right. \\ \left. - \delta(1-z) \frac{1}{2} \ln \frac{r \mathbf{k}_T^2}{q_-^2} \right] \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)$$

$$\mathcal{H}_{gj}^{(\text{int. sub. II})} = \frac{\alpha_s(\mu_R)}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{p}_T^2 - \mathbf{q}_T^2) \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{\mathbf{p}_T^2} \right)^\epsilon \\ \times \left\{ -\frac{1}{\epsilon} P_{gg}(z) + 2C_A \frac{1-z}{z} + \delta(1-z) \left[\frac{C_A}{\epsilon^2} + \frac{\beta_0}{2} \frac{1}{\epsilon} + \frac{C_A}{\epsilon} \ln \frac{r \mathbf{p}_T^2}{q_-^2} - \frac{\pi^2}{6} C_A \right] + O(\epsilon^2) \right\}.$$

Rapidity factorisation schemes

The $\ln r$ -regularisation is equivalent to the cut in rapidity, for **HEF** we need to cut in “projectile light-cone component” k_- :

$$\begin{aligned} \mathcal{H}_{gj}^{(\text{HEF-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) &= \frac{\bar{\alpha}_s}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z) \left[-\frac{1}{\epsilon} \left(\frac{\beta_0}{2} - C_A \right) + \frac{8}{3} C_A - \frac{5}{3} \beta_0 - \frac{\pi^2}{3} C_A \right] \\ &\quad - \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2 \mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \\ &\quad \times \left(-\frac{1}{2} \ln r + \ln \frac{|\mathbf{k}_T|}{\Lambda_+} \right), \end{aligned}$$

where $\Lambda_+ \simeq q_1^+ = (M^2 + \mathbf{p}_T^2)/q_-$. The **blue** terms come from R self-energy. In **BFKL** we cut in $\ln(s_{\eta_{cg}}/s_0)$:

$$\begin{aligned} \mathcal{H}_{gj}^{(\text{BFKL-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) &= \frac{\bar{\alpha}_s}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z) \left[-\frac{1}{\epsilon} \left(\frac{\beta_0}{2} - C_A \right) + \frac{8}{3} C_A - \frac{5}{3} \beta_0 \right] \\ &\quad - \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2 \mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \\ &\quad \times \left(-\frac{1}{2} \ln r + \ln \frac{q_-}{\sqrt{s_0}} \right). \end{aligned}$$

Impact factor, HEF scheme

$$\begin{aligned}
 \mathcal{H}_{gg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T) &= \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{HEF-sch.})} \\
 &= \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2 \mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \left[\frac{1}{(1-z)_+} \right. \\
 &\quad \left. + z(1-z) + 2 \frac{\mathbf{k}_T^2 \mathbf{q}_T^2 - (\mathbf{k}_T \mathbf{q}_T)^2}{z \mathbf{k}_T^2 \mathbf{q}_T^2} - \frac{3 \mathbf{k}_T^2 \mathbf{q}_T^2 - 2(\mathbf{k}_T \mathbf{q}_T)^2}{\mathbf{k}_T^2 \mathbf{q}_T^2} + \delta(1-z) \ln \left(\frac{M^2 + \mathbf{p}_T^2}{\mathbf{k}_T^2} \right) \right] \\
 &\quad + \frac{\alpha_s C_A}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ -\ln \frac{\mu_F^2}{\mathbf{p}_T^2} P_{gg}(z) + 2C_A \frac{1-z}{z} \right. \\
 &\quad \left. + \delta(1-z) \left[-\frac{\pi^2}{2} C_A + \frac{8}{3} C_A - \frac{5}{3} \beta_0 - 2C_F \left(2 + \frac{2}{3} \ln \frac{\mathbf{p}_T^2}{m_c^2} \right) + \beta_0 \ln \frac{\mu_R^2}{\mathbf{p}_T^2} + F_{1S_0^{[1]}}(\mathbf{p}_T^2/M^2) \right] \right\}
 \end{aligned}$$

This result should be added to the $\mathcal{H}_{gg}^{(\text{fin.})}$.

Impact factor, BFKL scheme

$$\begin{aligned}
 \mathcal{H}_{gg}^{(\text{NLO, analyt., BFKL})}(\mathbf{q}_T, z, \mathbf{p}_T) &= \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{BFKL-sch.})} \\
 &= \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2 \mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \left[\frac{1}{(1-z)_+} \right. \\
 &+ z(1-z) + 2 \frac{\mathbf{k}_T^2 \mathbf{q}_T^2 - (\mathbf{k}_T \mathbf{q}_T)^2}{z \mathbf{k}_T^2 \mathbf{q}_T^2} - \frac{3 \mathbf{k}_T^2 \mathbf{q}_T^2 - 2(\mathbf{k}_T \mathbf{q}_T)^2}{\mathbf{k}_T^2 \mathbf{q}_T^2} + \delta(1-z) \ln \left(\frac{\sqrt{s_0}}{|\mathbf{k}_T|} \right) \left. \right] \\
 &\quad + \frac{\alpha_s C_A}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ -\ln \frac{\mu_F^2}{\mathbf{p}_T^2} P_{gg}(z) + 2C_A \frac{1-z}{z} \right. \\
 &\quad \left. + \delta(1-z) \left[-\frac{\pi^2}{6} C_A + \frac{8}{3} C_A - \frac{5}{3} \beta_0 - 2C_F \left(2 + \frac{2}{3} \ln \frac{\mathbf{p}_T^2}{m_c^2} \right) + \beta_0 \ln \frac{\mu_R^2}{\mathbf{p}_T^2} + F_{1S_0^{[1]}}(\mathbf{p}_T^2/M^2) \right] \right\}
 \end{aligned}$$

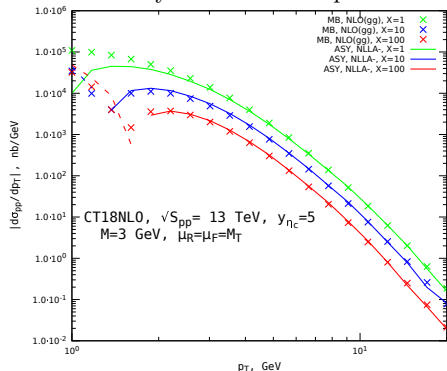
This result should be added to **the same** $\mathcal{H}_{gg}^{(\text{fin.})}$.

Numerical cross-check against NLO CF computation

Expansion of the NLL HEF result should reproduce the $\hat{s} \gg M^2$ asymptotics of the full NLO CF computation:

$$\left. \frac{d\hat{\sigma}^{(\text{LO+NLO})}}{dzd\mathbf{p}_T^2} \right|_{\hat{s} \gg M^2} = \frac{\hat{\alpha}_s}{\mathbf{p}_T^2} \left[1 + \hat{\alpha}_s \ln \frac{\hat{s}}{M_T^2} \ln \frac{\mu_F^2}{\mathbf{p}_T^2} + (\text{NLLA } C) \right] \mathcal{H}^{(\text{LO})} + \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \otimes \mathcal{H}^{(\text{NLO})}.$$

Preliminary numerical comparison:



Crosses – NLO CF results by *M. Butenschön*, solid lines – NLL HEF prediction. The NLO CF computation is done with the cut on $\hat{s} > X \hat{s}_{\min}$ with $\hat{s}_{\min} = 2M_T[M_T + |\mathbf{p}_T|] - M^2$ – kinematical lower bound of \hat{s} for given p_T .
 Green – $X = 1$, blue – $X = 10$ and red – $X = 100$.

Conclusions and outlook

- ▶ The complete NLO HEF coefficient function (impact factor) for the $g + R \rightarrow c\bar{c}[^1S_0^{[1]}]$ process is computed, including one-loop and real-emission corrections
- ▶ The computation for other NRQCD-factorisation intermediate states: $c\bar{c}[^1S_0^{[8]}, ^3S_1^{[8]}, ^3P_J^{[1,8]}]$ are in progress. The $c\bar{c}[^3S_1^{[1]}]$ is more challenging.
- ▶ The result in HEF scheme is useful for the resummation of $\ln \hat{s}/M^2$ corrections in CF coefficient function
- ▶ The result in BFKL scheme is useful for the study of double- η_c production at large rapidity separation
- ▶ The result in the “shockwave” scheme, corresponding to the cut in “projectile” light-cone component (k^+) is easy to obtain. However this is $1R$ -exchange only.
- ▶ The same computaion technology can be applied to the central production vertices $RR \rightarrow c\bar{c}[n]$.

Thank you for your attention!

Perturbative instability of quarkonium total cross sections

Inclusive η_c -hadroproduction (CSM)

[Mangano *et al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \rightarrow c\bar{c} \left[{}^1S_0^{[1]} \right] + X, \text{ LO: } g(p_1)+g(p_2) \rightarrow c\bar{c} \left[{}^1S_0^{[1]} \right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1, \mu_F) \otimes f_j(x_2, \mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1 + p_2)^2$.

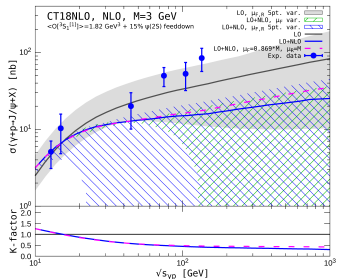
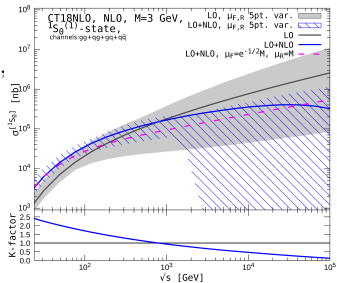
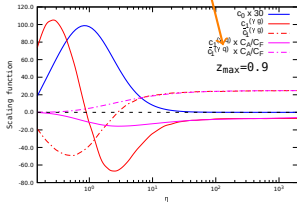
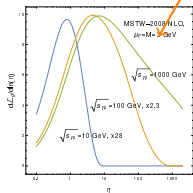
Inclusive J/ψ -photoproduction (CSM)

[Krämer, '96, ..., Colpani Serri *et al.*, '21]

$$\gamma+p \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] + X, \text{ LO: } \gamma(q)+g(p_1) \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] + g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1, \mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q + p_1)^2$, $z = \frac{pP}{qP}$.

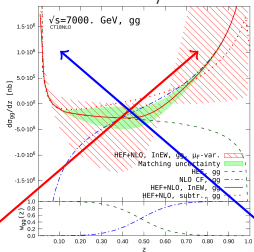


Matching with NLO

The HEF is valid in the **leading-power** in M^2/\hat{s} , so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria et al., 18'].

η_c -hadroproduction,

$$z = M^2/\hat{s}:$$

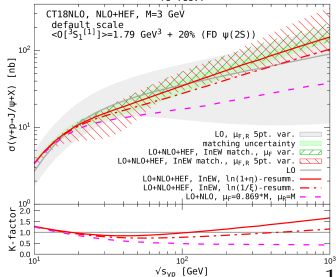
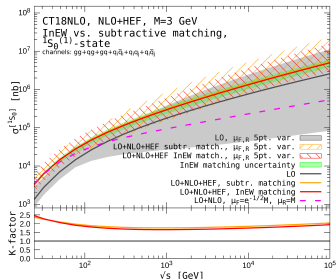
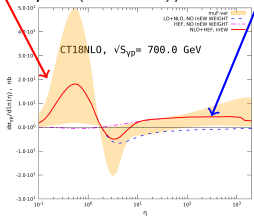


NLO

HEF

J/ψ -photoproduction,

$$\eta = (\hat{s} - M^2)/M^2:$$



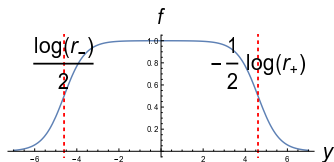
Rapidity divergences and regularization.

$$\begin{array}{c} p \downarrow \\ | \\ + \\ \text{---} \\ | \\ - \\ q \downarrow \end{array} \circlearrowleft = g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{(\mathbf{p}_T^2 (n_+ n_-))^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^- (\dots)}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2 (\dots)}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed [Lipatov, '95] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\delta_{ab} \mathbf{p}_T^2 \times C_A g_s^2 \underbrace{\int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:



$$\Gamma_{+\mu} - \Gamma_{+\nu} - P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

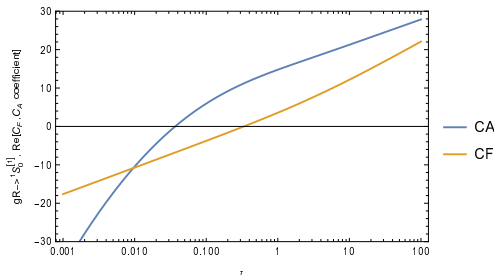
$$\leftarrow f(y) = \frac{1}{(re^{-y} + e^y)(re^y + e^{-y})},$$

$$\int_{-\infty}^{+\infty} dy f(y) = -\log r + O(r)$$

The C_F coefficient

$$C_{gR}[{}^1S_0^{[8]}, C_F] = \frac{1}{6(\tau+1)^2} \left\{ -12\tau(\tau+1)\text{Li}_2(-2\tau-1) + \frac{6L_2}{\tau}(-2L_1\tau + L_1 + 6\tau(\tau+1)) \right. \\ \left. + \frac{1}{(2\tau+1)^2} \left[(\tau+1)12\ln(2)(\tau+1)(6\tau^2+8\tau+3) - 8\tau^3(9\ln(\tau+1) + 2\pi^2 + 15) \right. \right. \\ \left. \left. - 4\tau^2(30\ln(\tau+1) + \pi^2 + 63) + 8\tau(-6\ln(\tau+1) + \pi^2 - 21) \right. \right. \\ \left. \left. + 18(\tau+1)(2\tau+1)^2\ln(\tau) + 3\pi^2 - 36 \right] \right\},$$

where $L_1 = L_1^{(+)} - L_1^{(-)} - L_2/2$ with $L_1^{(\pm)} = \sqrt{\tau(1+\tau)} \ln(\sqrt{1+\tau} \pm \sqrt{\tau})$ and $L_2 = \sqrt{\tau(1+\tau)} \ln(1+2\tau+2\sqrt{\tau(1+\tau)})$.

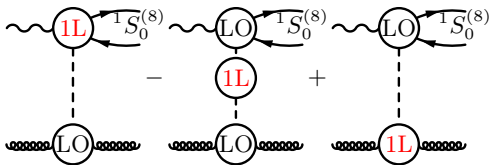


The C_A coefficient for $gR \rightarrow c\bar{c} [^1S_0^{[1]}]$

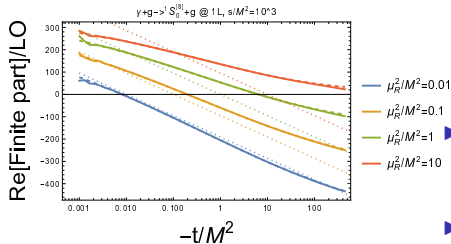
$$\begin{aligned}
 C_{gR}[^1S_0^{[1]}, C_A] &= \frac{2(\tau(\tau(\tau(7\tau + 8) + 2) - 4) - 1)}{(\tau - 1)(\tau + 1)^3} \text{Li}_2(-\tau) \\
 &- \frac{\tau(\tau(4\tau + 5) + 3)}{(\tau + 1)^3} \text{Li}_2(-2\tau - 1) - \frac{L_2^2}{2\tau(\tau + 1)^2} \\
 &+ \frac{1}{18(\tau - 1)(\tau + 1)^3} \left\{ -2(\tau^2 - 1)(18 \ln(2)(\tau - 1)\tau - 67(\tau + 2)\tau - 67) \right. \\
 &+ 18[\ln(\tau)(-2\tau^4 + (\tau(-\tau^3 + \tau + 3) + 2)\tau \ln(\tau) + 2\tau^2 + \ln(\tau)) \\
 &- (\tau - 1)^2(\tau + 1)^3 \ln^2(\tau + 1) + 2(\tau - 1)(\tau + 1)^2(\tau + (\tau + 1)^2 \ln(\tau)) \ln(\tau + 1)] \\
 &\left. + \pi^2(3\tau(\tau(\tau(15\tau + 14) - 3) - 12) - 6) \right\},
 \end{aligned}$$

$R\gamma \rightarrow c\bar{c} \left[{}^1S_0^{(8)} \right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit ($s \gg -t$) of the real part of the $2 \rightarrow 2$ 1-loop QCD amplitude:



Solid lines – QCD, dashed lines – EFT, dotted

lines – $-2C_A \ln(-t/\mu_R^2) \ln(s/M^2)$

$$\gamma + g \rightarrow c\bar{c} \left[{}^1S_0^{(8)} \right] + g.$$

- ▶ The $2 \rightarrow 2$ QCD 1-loop amplitude can be computed numerically using **FormCalc** (with some tricks, due to Coulomb divergence)
- ▶ The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- ▶ For the finite part agreement within few % is reached, need to push to higher s

Quarkonium in the potential model

Cornell potential:

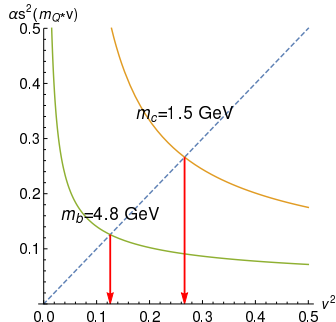
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is “small” (~ 0.3 fm) \rightarrow Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1 + m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \quad \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is carbon-copy of corresponding arguments from atomic physics (hierarchy of E-dipole/M-dipole with ΔS /M-dipole transitions):

$$\begin{aligned} |J/\psi\rangle &= O(1) \left| c\bar{c} \left[{}^3S_1^{(1)} \right] \right\rangle + O(v) \left| c\bar{c} \left[{}^3P_J^{(8)} \right] + g \right\rangle \\ &+ O(v^{3/2}) \left| c\bar{c} \left[{}^1S_0^{(8)} \right] + g \right\rangle + O(v^2) \left| c\bar{c} \left[{}^3S_1^{(8)} \right] + gg \right\rangle + \dots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$ -pair is produced in a point, by local operator:

$$\mathcal{A}_{\text{NRQCD}} = \langle J/\psi + X | \chi^\dagger(0) \kappa_n \psi(0) | 0 \rangle,$$

Different operators “couple” to different Fock states:

$$\begin{aligned} \chi^\dagger(0) \psi(0) &\leftrightarrow \left| c\bar{c} \left[{}^1S_0^{(1)} \right] \right\rangle, \quad \chi^\dagger(0) \sigma_i \psi(0) \leftrightarrow \left| c\bar{c} \left[{}^3S_1^{(1)} \right] \right\rangle, \\ \chi^\dagger(0) \sigma_i T^a \psi(0) &\leftrightarrow \left| c\bar{c} \left[{}^3S_1^{(8)} \right] \right\rangle, \quad \chi^\dagger(0) D_i \psi(0) \leftrightarrow \left| c\bar{c} \left[{}^1P_1^{(8)} \right] \right\rangle, \dots \end{aligned}$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0 | \psi^\dagger \kappa_n^\dagger \chi a_{J/\psi}^\dagger \underbrace{a_{J/\psi} \chi^\dagger \kappa_n \psi}_{\mathcal{O}_n^{J/\psi}} | 0 \rangle = \langle \mathcal{O}_n^{J/\psi} \rangle,$$

Exclusive J/ψ photoproduction

$$p(P) + \gamma(q) \rightarrow J/\psi(p) + p(P'), \quad q^2 \simeq 0,$$

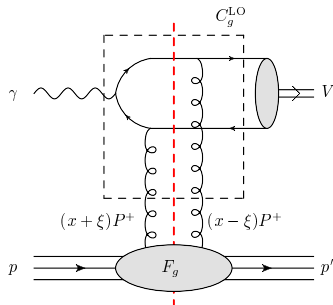
Kinematics (skewness):

$$\xi = \frac{p^+}{2P^+} = \frac{M}{4E_p} e^{y(J/\psi)},$$

Factorisation formula ($P \simeq P'$):

$$A = \int_{-1}^1 \frac{dx}{x} F_g(x, \xi, \mu_F) C_g(x),$$
$$\sigma \propto |A|^2.$$

Figure from
[hep-ph/1507.06942](https://arxiv.org/abs/hep-ph/1507.06942)



Exclusive J/ψ photoproduction at NLO

Partonic energy ($t = (P - P')^2 \simeq 0$):

$$\hat{s} = M^2 \frac{x + \xi}{2\xi} \gg M^2 \text{ if } \xi \ll x \ll 1,$$

NLO γg amplitude at $\xi \ll 1$ [Ivanov, Schaefer, Szymanowski; Gracey, Jones, Teubner] :

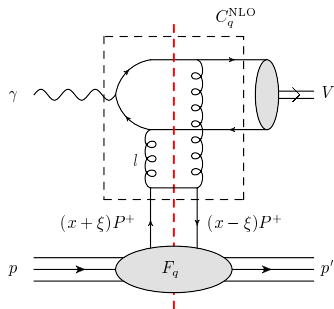
$$\text{Im}A_{\text{NLO}} \sim \hat{\alpha}_s \ln \frac{M^2}{4\mu_F^2} \int_{\xi}^1 \frac{dx}{x} F_g(x, \xi, \mu_F) + \dots$$

The GPD F_g is relatively flat as function of x
so $\int_{\xi}^1 \frac{dx}{x} \sim \ln 1/\xi$.

Also in Mellin space

$$\hat{\alpha}_s \int_{\xi}^1 \frac{dx}{x} \rightarrow \frac{\hat{\alpha}_s}{N}.$$

Figure from
[hep-ph/1507.06942](https://arxiv.org/abs/hep-ph/1507.06942)



Treatment of the instability

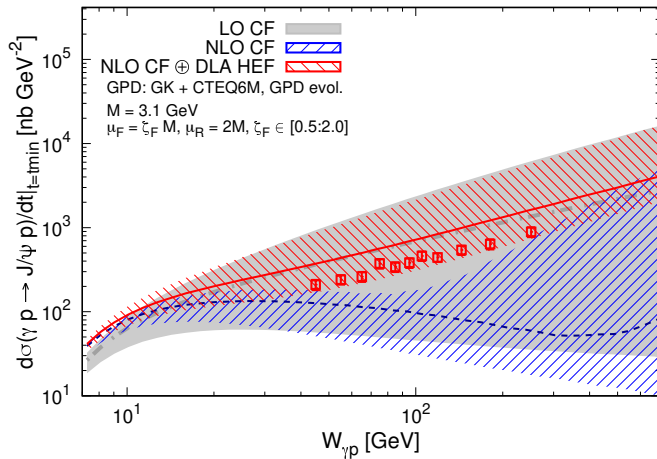
Work in progress, together with Jean-Philippe Lansberg, Chris Flett, Saad Nabebacus and Jakub Wagner.

- ▶ Simplest solution: choose $\hat{\mu}_F = \frac{M}{2}$ (+ some other less conventional tricks...) [Jones, Martin, Ryskin, Teubner, 2016; ...]
- ▶ HEF resummation of $\hat{\alpha}_s^n/N^n$ corrections [Ivanov 2007]
- ▶ **My proposal:** one have to do matching of the HEF-resummed $C(x)$ at $\xi \ll x \ll 1$ and NLO CPM at $x \sim 1$.
- ▶ The closed formula for for the coefficient function at $x \ll 1$ can be derived in DLA:

$$\frac{2C_{\perp g}^{\text{HEF}}(x)}{-i\pi\hat{\alpha}_s F_{\text{LO}}} = \frac{1}{|x|} \sqrt{\frac{L_\mu}{L_x}} \left\{ I_1 \left(2\sqrt{L_x L_\mu} \right) - 2 \sum_{k=1}^{\infty} \text{Li}_{2k}(-1) \left(\frac{L_x}{L_\mu} \right)^k I_{2k-1} \left(2\sqrt{L_x L_\mu} \right) \right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$, $L_x = \hat{\alpha}_s \ln 1/|x|$ and Bessel functions $I_n(2\sqrt{L_\mu L_x})$ turn into $J_n(2\sqrt{-L_\mu L_x})$ if $L_\mu < 0$.

Exclusive J/ψ photoproduction in CF+HEF



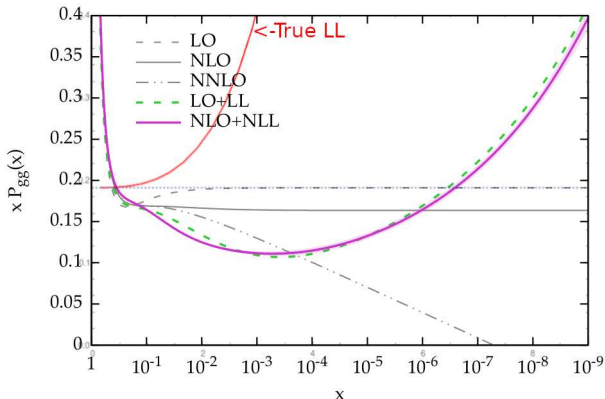
Backup: DGLAP P_{gg} at small z

$$\text{LO: } P_{gg}(z) = \frac{2CA}{z} + \dots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$$

Plot from [hep-ph/1607.02153](https://arxiv.org/abs/hep-ph/1607.02153) with my curve (in red) for the **strict LLA**:

$$\frac{\hat{\alpha}_s}{N} \chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots$$

$$\alpha_s = 0.2, n_f = 4, Q_0 \overline{\text{MS}}$$



The “LO+LL” and “NLO+NLL” curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by [Altarelli, Ball and Forte](#) which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots$
together with NLO PDF.

