Semi-inclusive DIS at small-x: CGC, TMDs and Sudakov factor

Cyrille Marquet

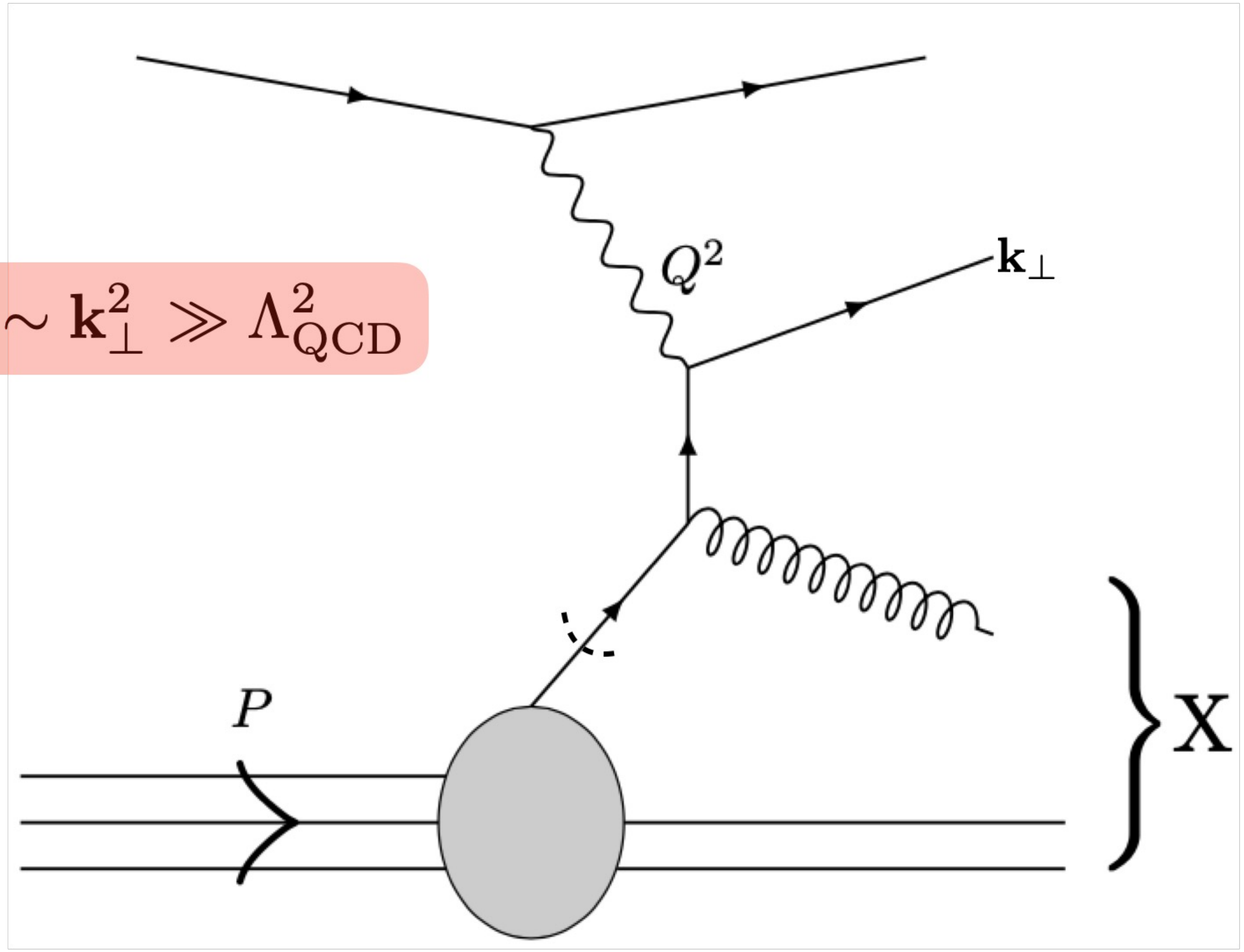
Centre de Physique Théorique
Ecole Polytechnique & CNRS

based on: C.M., B.-W. Xiao and F. Yuan, Phys. Lett. B682 (2009) 207
T. Altinoluk, J. Jalilian-Marian and C.M., arXiv:2406.08277 and work in progress

Introduction

Collinear factorization

$s \sim Q^2 \sim \mathbf{k}_\perp^2 \gg \Lambda_{\rm QCD}^2$

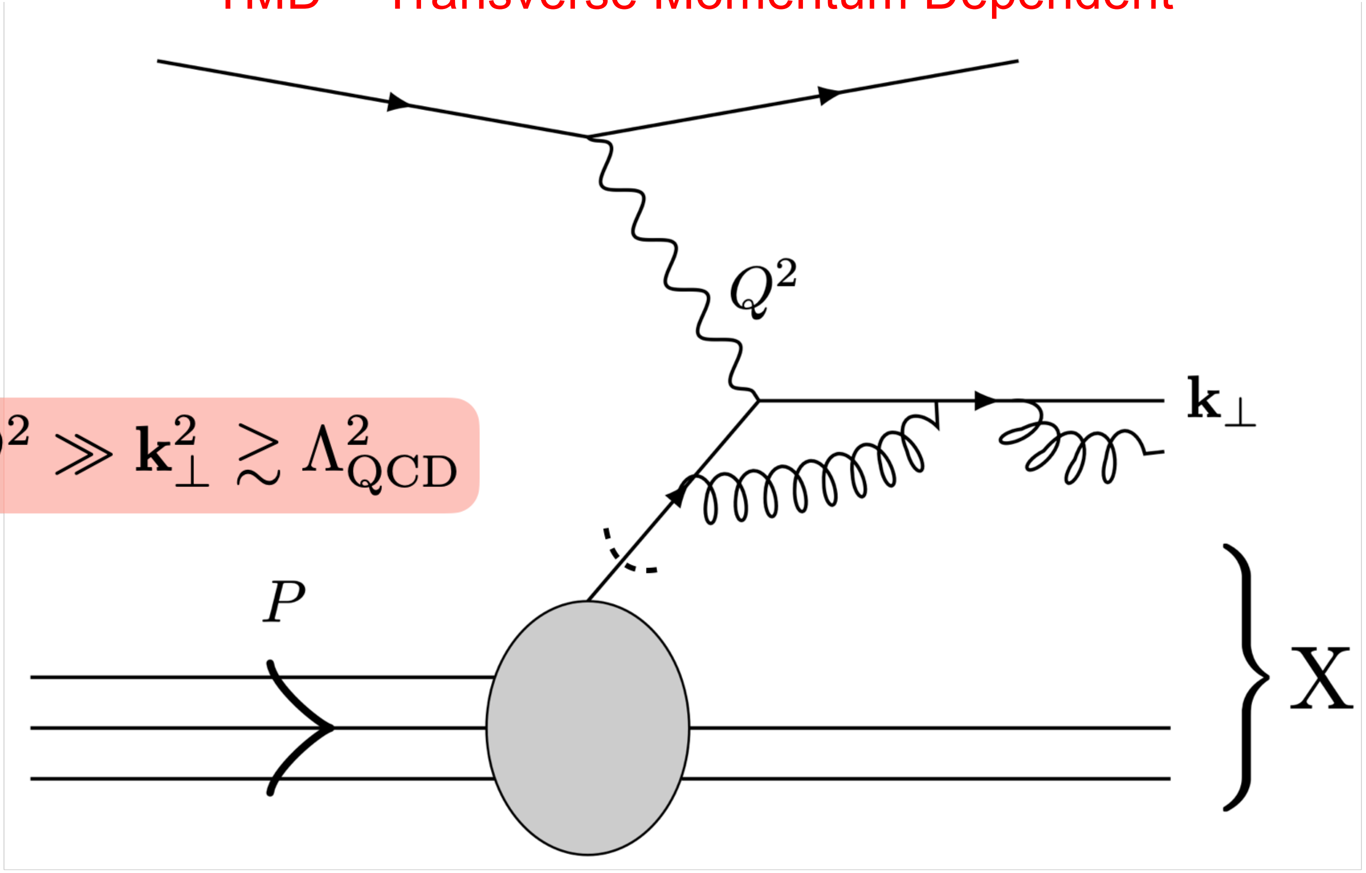


$$\sigma_{\rm coll} = \hat{\sigma}(Q^2) \otimes f(x, Q^2) + \mathcal{O}(\Lambda_{\rm QCD}/Q)^n$$

Large logarithms $\ln(Q^2/\Lambda_{\rm QCD}^2)$ resummed using DGLAP

TMD factorization

TMD = Transverse Momentum Dependent



$$s \sim Q^2 \gg \mathbf{k}_\perp^2 \gtrsim \Lambda_{\rm QCD}^2$$

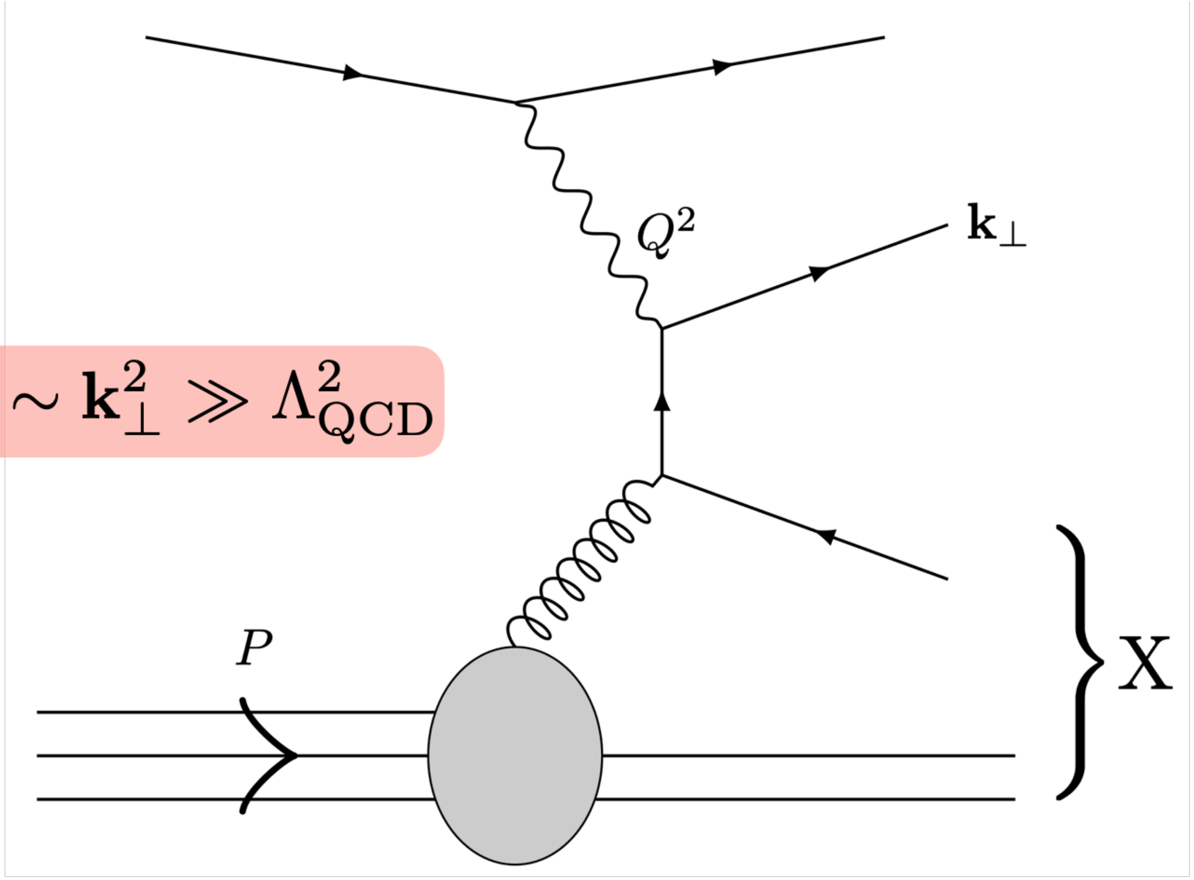
$$\sigma_{\rm TMD} = \hat{\sigma}(Q^2) \otimes f(x, \mathbf{k}_\perp, Q^2) \otimes + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}, \frac{\mathbf{k}_\perp}{Q}\right)^n$$

Additional Sudakov logarithms $\ln(Q^2/\mathbf{k}_\perp^2)$
resummed using CSS

Collins, Soper, Sterman ('85-'89);
Ji, Ma, Yuan (2005); Collins (2011);
Echevarria, Idilbi, Scimemi (2012)

High-energy factorization

$$s \gg Q^2 \sim \mathbf{k}_\perp^2 \gg \Lambda_{\mathrm{QCD}}^2$$



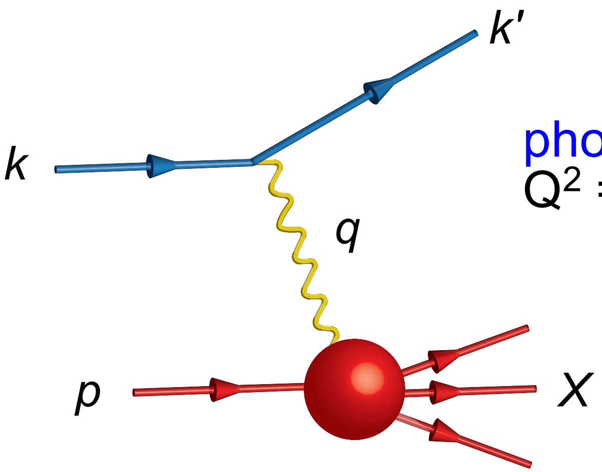
$$\sigma_{\mathrm{HEF}} = \hat{\sigma}\big(\mathbf{k}_\perp^2, Q^2\big) \otimes \mathcal{G}(x, \mathbf{k}_\perp, Q^2) + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}}{Q}\Big)^n$$

Additional logarithms $\ln(s/Q^2) \sim \ln(1/x)$ resummed using BFKL

Catani, Ciafaloni, Hautmann ('90-'94)

SIDIS in the small-x limit

The dipole factorization in DIS



photon virtuality
$Q^2 = -(k-k')^2 > 0$

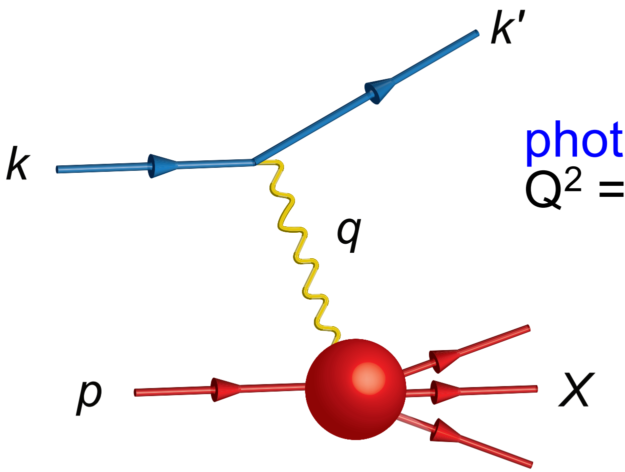
ep center-of-mass energy
$S = (k+P)^2$

$\gamma^* p$ center-of-mass energy
$W^2 = (k-k'+P)^2$

$$x_B = \frac{Q^2}{2P.(k-k')} = \frac{Q^2}{W^2 - M_h^2 + Q^2}$$

$$y = \frac{P.(k-k')}{P.k} = \frac{Q^2/x_B}{S - M_h^2}$$

The dipole factorization in DIS



photon virtuality
$Q^2 = -(k-k')^2 > 0$

ep center-of-mass energy
$S = (k+P)^2$
$\gamma^* p$ center-of-mass energy
$W^2 = (k-k'+P)^2$

$$x_B = \frac{Q^2}{2P.(k-k')} = \frac{Q^2}{W^2 - M_h^2 + Q^2}$$

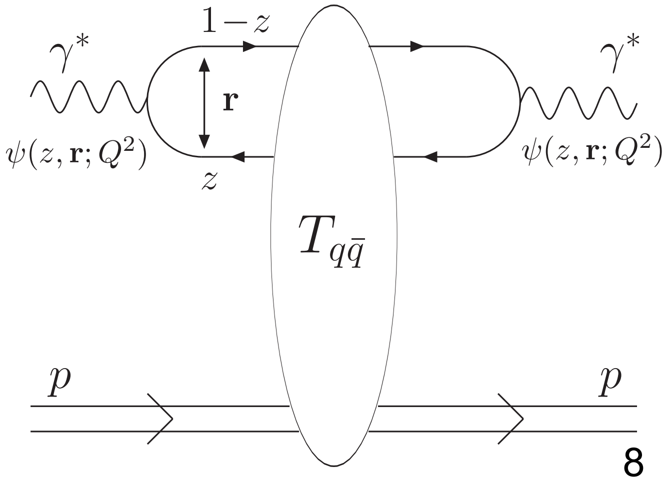
$$y = \frac{P.(k-k')}{P.k} = \frac{Q^2/x_B}{S - M_h^2}$$

- the cross section at small x

Mueller (1990), Nikolaev and Zakharov (1991)

$$\sigma_{T,L}^{\gamma^* p \to X} = 2 \int d^2r\, dz\, |\psi_{T,L}(z, \mathbf{r}; Q^2)|^2 \underbrace{\int d^2b\, T_{q\bar{q}}(\mathbf{r}, \mathbf{b}, x_B)}_{\text{dipole-hadron cross-section}}$$

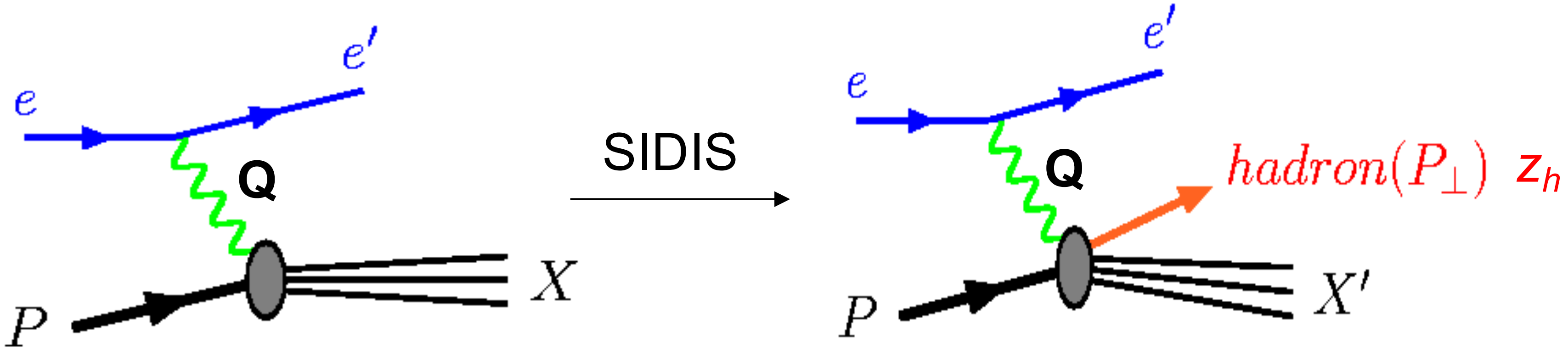
overlap of $\gamma^* \to q\bar{q}$
splitting functions



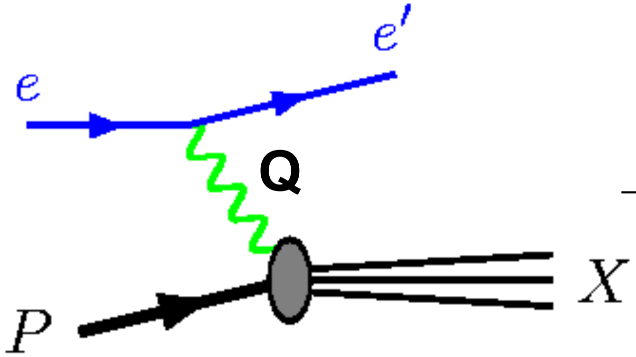
link to the unintegrated gluon distribution

$$F(q_\perp, x_B) = \int \frac{d^2r}{(2\pi)^2}\, e^{-iq_\perp \cdot \mathbf{r}} [1 - T_{q\bar{q}}(\mathbf{r}, x_B)]$$

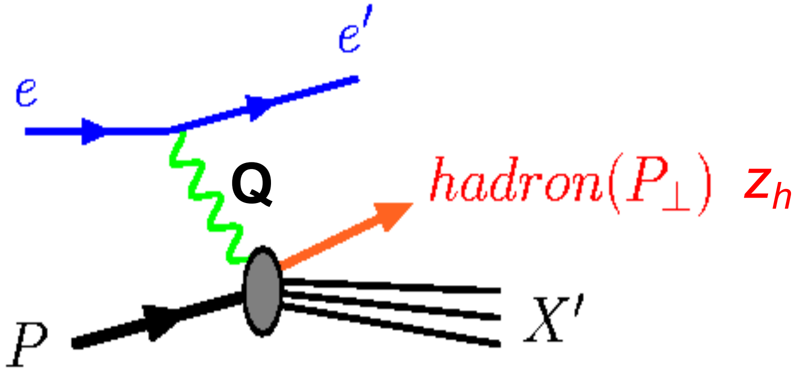
The dipole factorization in SIDIS



The dipole factorization in SIDIS



SIDIS →

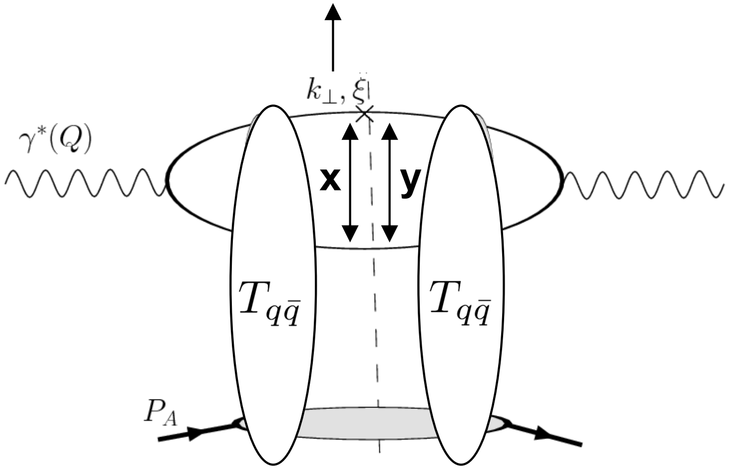


- the cross section at small x

$$\Phi(\xi, \mathbf{x}, \mathbf{y}; Q^2) = \psi(\xi, \underset{\uparrow}{\mathbf{x}}; Q^2)\psi^*(\xi, \underset{\uparrow}{\mathbf{y}}; Q^2)$$

dipoles in amplitude / conj. amplitude

fragmentation into hadron



$$\frac{d\sigma^{\gamma^* p\to hX}}{dz_h d^2P_\perp} = \frac{d\sigma_{T,L}^{\gamma^* p\to qX}}{d\xi d^2k_\perp}\left(k_\perp = \frac{\xi}{z_h}P_\perp\right) \otimes D_{h/q}(z_h/\xi)$$

$$\frac{d\sigma_{T,L}^{\gamma^* p\to qX}}{d\xi d^2k_\perp} = \int \frac{d^2x}{2\pi}\frac{d^2y}{2\pi}\, e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})}\Phi_{T,L}(\xi, \mathbf{x}, \mathbf{y}; Q^2) \int d^2b\, [T_{q\bar{q}}(\mathbf{x}, x_B) + T_{q\bar{q}}(\mathbf{y}, x_B) - T_{q\bar{q}}(\mathbf{x}-\mathbf{y}, x_B)]$$

McLerran and Venugopalan, Mueller, Kovchegov and McLerran (1999)

Cross section in momentum space

- the lepto-production cross section

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}} = \frac{\alpha_{em}^2 N_c}{2\pi^3 x_B Q^2}\sum_f e_f^2 \int_{z_h}\frac{dz}{z}\frac{D(z)}{z^2}\int \overbrace{d^2b d^2q_\perp F(q_\perp, x_B)\,\mathcal{H}\left(\xi=\frac{z_h}{z}, k_\perp=\frac{P_\perp}{z}\right)}^{\text{k}_\text{T}\text{ factorization}}$$

$$\downarrow$$

phase space $d\mathcal{P} = dx_B dQ^2 dz_h dP_\perp^2$

Cross section in momentum space

- the lepto-production cross section

k$_T$ factorization

$$\frac{d\sigma(ep \to e'hX)}{d\mathcal{P}} = \frac{\alpha_{em}^2 N_c}{2\pi^3 x_B Q^2} \sum_f e_f^2 \int_{z_h} \frac{dz}{z} \frac{D(z)}{z^2} \int \overbrace{d^2 b d^2 q_\perp F(q_\perp, x_B)\, \mathcal{H}\left(\xi = \frac{z_h}{z}, k_\perp = \frac{P_\perp}{z}\right)}$$

phase space $d\mathcal{P} = dx_B dQ^2 dz_h dP_\perp^2$

the unintegrated gluon distribution

$$F(q_\perp, x_B) = \int \frac{d^2 r}{(2\pi)^2}\, e^{-iq_\perp \cdot \mathbf{r}} [1 - T_{q\bar{q}}(\mathbf{r}, x_B)]$$

Cross section in momentum space

- the lepto-production cross section

$$\frac{d\sigma(ep \to e'hX)}{d\mathcal{P}} = \frac{\alpha_{em}^2 N_c}{2\pi^3 x_B Q^2} \sum_f e_f^2 \int_{z_h} \frac{dz}{z} \frac{D(z)}{z^2} \int \overbrace{d^2b d^2q_\perp F(q_\perp, x_B)\, \mathcal{H}\left(\xi = \frac{z_h}{z}, k_\perp = \frac{P_\perp}{z}\right)}^{\text{k}_\text{T} \text{ factorization}}$$

phase space $d\mathcal{P} = dx_B dQ^2 dz_h dP_\perp^2$

F.T. of photon wave function

the unintegrated gluon distribution

$$F(q_\perp, x_B) = \int \frac{d^2r}{(2\pi)^2}\ e^{-iq_\perp \cdot \mathbf{r}}[1 - T_{q\bar{q}}(\mathbf{r}, x_B)]$$

$\epsilon_f^2 = \xi(1-\xi)Q^2$

massless quarks

$$\mathcal{H}(\xi, k_\perp) \quad = \quad \left(1 - y + \frac{y^2}{2}\right)(\xi^2 + (1-\xi)^2)\left|\frac{k_\perp}{k_\perp^2 + \epsilon_f^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + \epsilon_f^2}\right|^2$$

photon T

$$+(1-y)4\xi^2(1-\xi)^2Q^2\left(\frac{1}{k_\perp^2 + \epsilon_f^2} - \frac{1}{(k_\perp - q_\perp)^2 + \epsilon_f^2}\right)^2$$

photon L

Large-Q^2 limit of small-x result

- keeping the leading 1/Q^2 term:

CM, Xiao and Yuan (2009)

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}|_{P_\perp^2\ll Q^2}=\frac{\alpha_{em}^2N_c}{2\pi^3Q^4x_B}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\int d^2bd^2q_\perp F(q_\perp,x_B)A(q_\perp,k_\perp=P_\perp/z_h)$$

only transverse photons

simple function

$$A(q_\perp,k_\perp)=\int d\xi\left|\frac{k_\perp|k_\perp-q_\perp|}{(1-\xi)k_\perp^2+\xi(k_\perp-q_\perp)^2}-\frac{k_\perp-q_\perp}{|k_\perp-q_\perp|}\right|^2$$

Large-Q^2 limit of small-x result

- keeping the leading $1/Q^2$ term:

CM, Xiao and Yuan (2009)

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}|_{P_\perp^2\ll Q^2} = \frac{\alpha_{em}^2 N_c}{2\pi^3 Q^4 x_B}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\int d^2b d^2q_\perp F(q_\perp,x_B)A(q_\perp,k_\perp=P_\perp/z_h)$$

simple function

only transverse photons

$$A(q_\perp,k_\perp)=\int d\xi\left|\frac{k_\perp|k_\perp-q_\perp|}{(1-\xi)k_\perp^2+\xi(k_\perp-q_\perp)^2}-\frac{k_\perp-q_\perp}{|k_\perp-q_\perp|}\right|^2$$

- the saturation regime can still be probed

the cross section above has contributions to all orders in $Q_s^2/P_\perp^2$

even if Q^2 is much bigger than Q_s^2, the saturation regime will be important when $P_\perp^2\sim Q_s^2$

in fact, thanks to the existence of Q_s, the limit $|P_\perp|\to 0$ is finite, and computable with weak-coupling techniques ($Q_s\gg\Lambda_{QCD}$)

eventually true at small x

SIDIS in the large-Q^2 limit

TMD factorization

- the cross section can be factorized in 4 pieces

Collins and Soper (1981), Collins, Soper and Sterman (1985), Ji, Ma and Yuan (2005)

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}} = \frac{4\pi\alpha_{em}^2}{Q^2}\left(1-y+\frac{y^2}{2}\right)\int d^2k_\perp d^2p_{1\perp}d^2\lambda_\perp$$

$$q(x_B,k_\perp;x_B\zeta)D(z_h,p_{1\perp};\hat{\zeta}/z_h)$$

TMD ff

TMD quark distribution

$$S(\lambda_\perp;\rho)H(Q^2,x_B,z_h;\rho)\delta^{(2)}(z_hk_\perp+p_{1\perp}+\lambda_\perp-p_\perp)$$

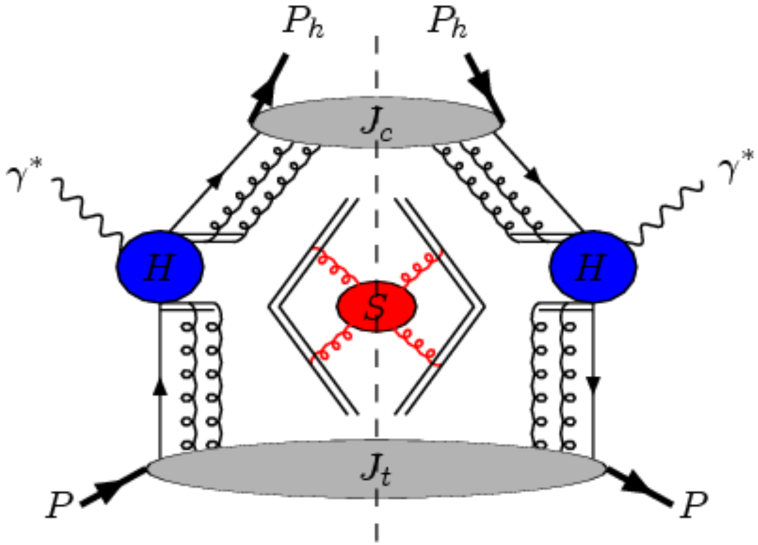
soft factor

hard part

valid to leading power in 1/Q^2 and to all orders in α_s

(the gluon TMD piece is power-suppressed)

however we shall only discuss the leading α_s order



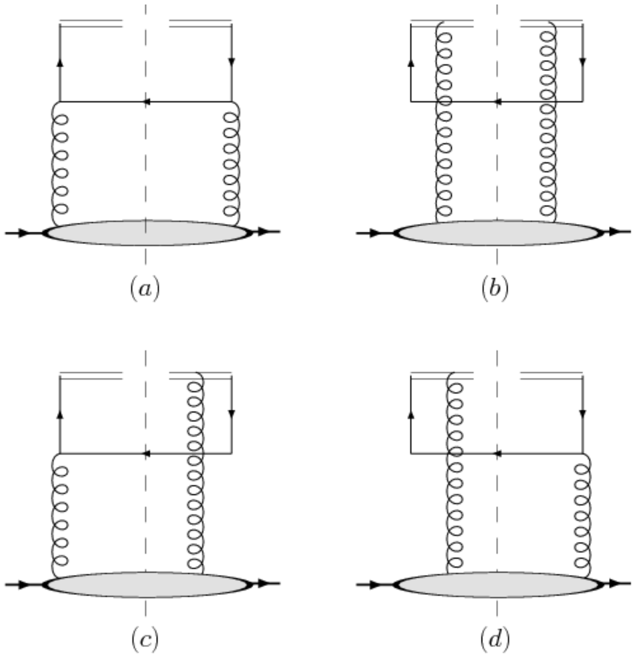
Small-x limit of large-Q^2 result

CM, Xiao and Yuan (2009)

- at small-x, the leading contribution reads:

$$\frac{d\sigma(ep \to e'hX)}{d\mathcal{P}}|_{x_B \ll 1} = \frac{4\pi\alpha_{em}^2}{Q^4}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\, q(x_B, P_\perp/z_h)$$

- and the TMD quark distribution comes from gluon splitting



$$xq(x,k_\perp) = \frac{N_c}{8\pi^4}\int d^2b d^2q_\perp F(q_\perp,x)A(q_\perp,k_\perp)$$

gluon distribution gluon to quark splitting

saturation/multiple scatterings are included in this TMD formula, simply by calculating $F(q_\perp,x)$ to all orders in $Q_s^2/q_\perp^2$

TMD-pdf / u-pdf relation

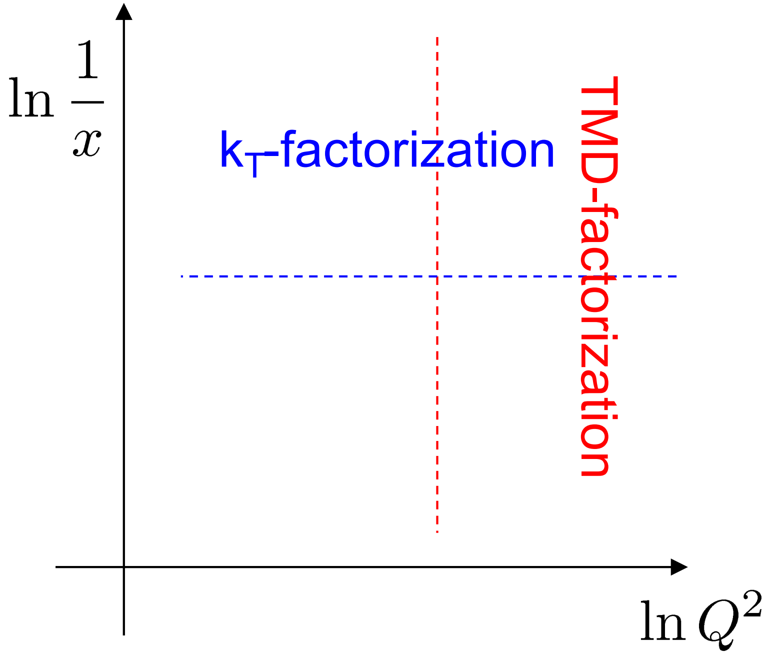
- at small *x* and large Q^2

CM, Xiao and Yuan (2009)

the two results for the SIDIS cross section are identical, with

$$\underset{\text{quark TMD}}{\underbrace{xq(x,k_\perp)}} = \frac{N_c}{4\pi^4}\int d^2b d^2q_\perp \underset{\text{gluon TMD}}{\underbrace{F(q_\perp,x)}} \left[1 - \frac{k_\perp\cdot(k_\perp-q_\perp)}{k_\perp^2-(k_\perp-q_\perp)^2}\ln\left(\frac{k_\perp^2}{(k_\perp-q_\perp)^2}\right)\right]$$

in the overlaping domain of validity,
TMD & kT factorization are consistent



TMD-pdf / u-pdf relation

- at small *x* and large Q^2

CM, Xiao and Yuan (2009)

the two results for the SIDIS cross section are identical, with

$$xq(x,k_\perp) = \frac{N_c}{4\pi^4}\int d^2b d^2q_\perp F(q_\perp,x)\left[1-\frac{k_\perp\cdot(k_\perp-q_\perp)}{k_\perp^2-(k_\perp-q_\perp)^2}\ln\left(\frac{k_\perp^2}{(k_\perp-q_\perp)^2}\right)\right]$$

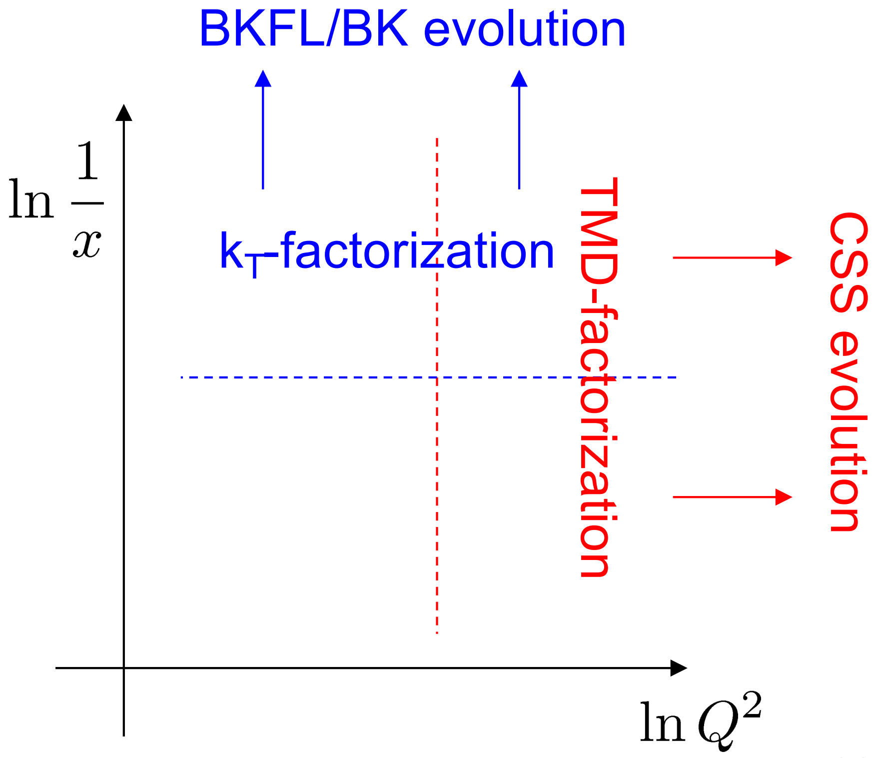
quark TMD

gluon TMD

in the overlaping domain of validity,
TMD & kT factorization are consistent

- next step

can one consistently re-sum both
types of large logarithms ?



NLO corrections
and
QCD evolution

Re-summing large logarithms

Simultaneous resummation of high-energy $\ln(1/x)$ and Sudakov $\ln(Q^2/\mathbf{k}_\perp^2)$ logarithms?

Longstanding problem, studied using many different approaches, including recently:

SW: Balitsky, Tarasov (2015)
RO: Balitsky (2021-2023)
HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)
BFKL: Nefedov (2021)
PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)
CGC: Mueller, Xiao, Yuan (2011); Hatta, Xiao, Yuan, Zhou (2017-2021); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022-2023)

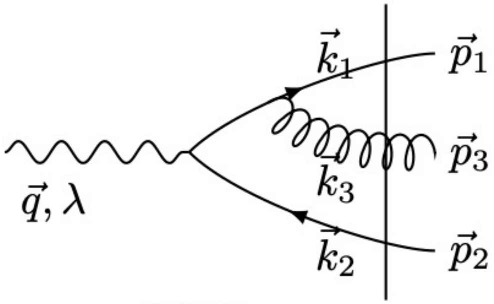
Related talks at the workshop:

S. Mukherjee, Tuesday 18:30:
TMD factorization Bridging large and small x

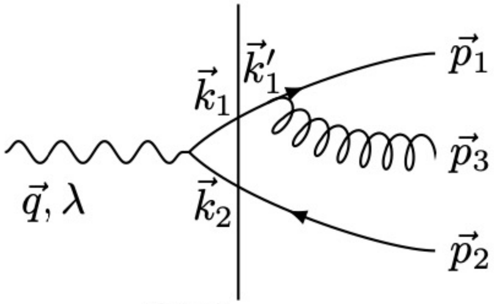
P. Caucal, Thursday 9:20:
NLO calculations for inclusive back-to-back dijet in DIS in the saturation regime

Real emission diagrams

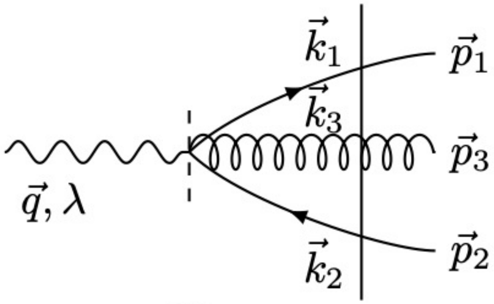
Altinoluk, Boussarie, CM and Taels (2020)



QSW



QFS



RI

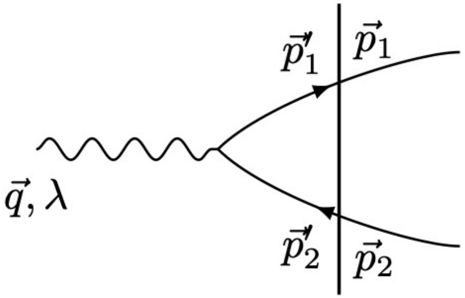
linearly-polarized gluon TMD involved at NLO, even for photo-production

see also

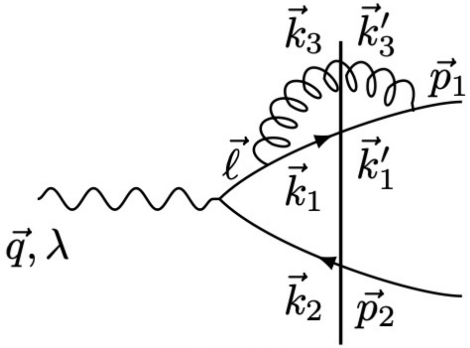
Caucal, Salazar and Venugopalan (2021)
Bergabo and Jalilian-Marian (2022)
Iancu and Mulian (2023)

Virtual diagrams

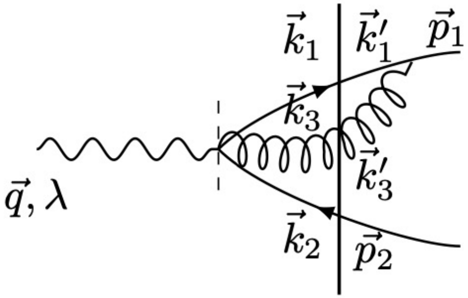
Caucal, Salazar and Venugopalan (2021)



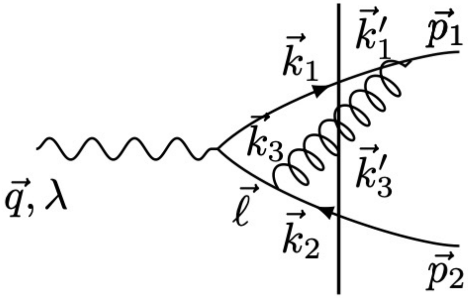
LO



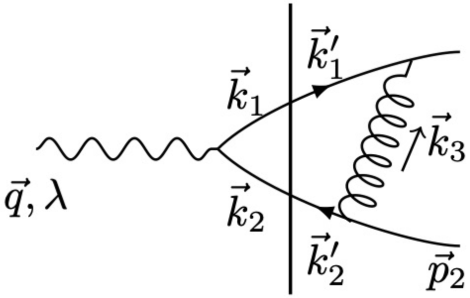
SESW



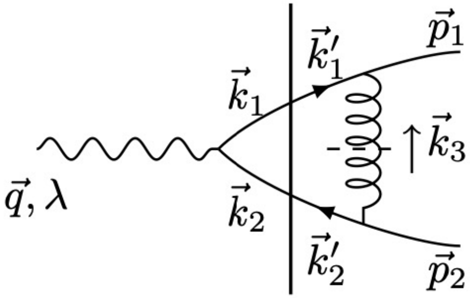
ISW



GESW



GEFS



IFS

full NLO CGC is UV and soft finite
collinear divergences give DGLAP evolution of the fragmentation function
rapidity divergences give Baltisky-Kovchegov evolution

see also Taels, Altinoluk, Beuf and CM (2022)
Bergabo and Jalilian-Marian (2022)

Sudakov double logs in SIDIS

large-Q^2 (TMD) limit   Altinoluk, Jalilian-Marian and CM (2024)

the standard rapidity subtraction of the small-x logarithms, which leads to BK/JIMWLK equations, is not compatible with TMD evolution

Sudakov and small-x logs aren't completely separated in phase space!

Sudakov double logs in SIDIS

large-Q^2 (TMD) limit Altinoluk, Jalilian-Marian and CM (2024)

the standard rapidity subtraction of the small-x logarithms, which leads to BK/JIMWLK equations, is not compatible with TMD evolution

Sudakov and small-x logs aren't completely separated in phase space!

To obtain $d\sigma^{\mathrm{NLO}}_{\mathrm{TMD}}$ " = " $d\sigma^{\mathrm{LO}}_{\mathrm{TMD}} \times \left(-\frac{\alpha_s C_F}{2\pi}\right) \ln^2(Q^2|\mathbf{x}-\mathbf{y}|^2)$

and then write

$$q(x,k_\perp;Q^2) = \int \frac{d^2(\mathbf{x}-\mathbf{y})}{(2\pi)^2}\, e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})} e^{-S_{sud}(\mathbf{Q},\mathbf{x}-\mathbf{y})} \int d^2k'_\perp\ e^{ik'_\perp\cdot(\mathbf{x}-\mathbf{y})} q(x,k'_\perp)$$

the rapidity subtraction must be altered

This leads to a kinematically-constrained small-x evolution

Sudakov double logs in SIDIS

large-Q^2 (TMD) limit Altinoluk, Jalilian-Marian and CM (2024)

the standard rapidity subtraction of the small-x logarithms, which leads to BK/JIMWLK equations, is not compatible with TMD evolution

Sudakov and small-x logs aren't completely separated in phase space!

To obtain $d\sigma^{\mathrm{NLO}}_{\mathrm{TMD}} \;\text{"}=\text{"}\; d\sigma^{\mathrm{LO}}_{\mathrm{TMD}} \times \left(-\frac{\alpha_s C_F}{2\pi}\right) \ln^2(Q^2|\mathbf{x}-\mathbf{y}|^2)$

and then write

$$q(x,k_\perp;Q^2) = \int \frac{d^2(\mathbf{x}-\mathbf{y})}{(2\pi)^2}\, e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})} e^{-S_{sud}(\mathbf{Q},\mathbf{x}-\mathbf{y})} \int d^2k'_\perp\, e^{ik'_\perp\cdot(\mathbf{x}-\mathbf{y})} q(x,k'_\perp)$$

the rapidity subtraction must be altered

This leads to a kinematically-constrained small-x evolution

→ in the small-x evolved LO contribution, the kernel of the JIMWLK equation now contains an extra theta term $\theta\left[(k_g^+/k_f^+)Q^2 - \mathbf{k}_g^2\right]$

study of single logs and the associated scheme dependence in progress

Conclusions

- to match collinear physics and small-x physics in the linear BFKL regime, the necessity of a kinematical constraint in the small-x evolution was recognized a long time ago (led to CCFM equation)

  Ciafaloni ('88); Andersson, Gustafson, Samuelsson ('96);
  Kwiecinski, Martin, Sutton ('96); Salam ('98)

- more recently, that necessity also emerged in CGC calculations, often in connection with the issue of negative NLO cross sections

  Beuf (2014); Hatta, Iancu (2016);
  Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2019)

- now it also appears in the context of two-scale processes and TMD physics

- semi-inclusive DIS provides a good testing ground for these theoretical developments → key process at the EIC