

# The Muon ( $g - 2$ ) in the Standard Model

Gilberto Colangelo

*u*<sup>b</sup>

---

<sup>b</sup>  
UNIVERSITÄT  
BERN

AEC  
ALBERT EINSTEIN CENTER  
FOR FUNDAMENTAL PHYSICS

SUSY 2024, Madrid, June 14, 2024

# Outline

Introduction:  $(g - 2)_\mu$  in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

The new CMD3 measurement of  $e^+e^- \rightarrow \pi^+\pi^-$

Hadronic light-by-light

Conclusions and Outlook

# Outline

Introduction:  $(g - 2)_\mu$  in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

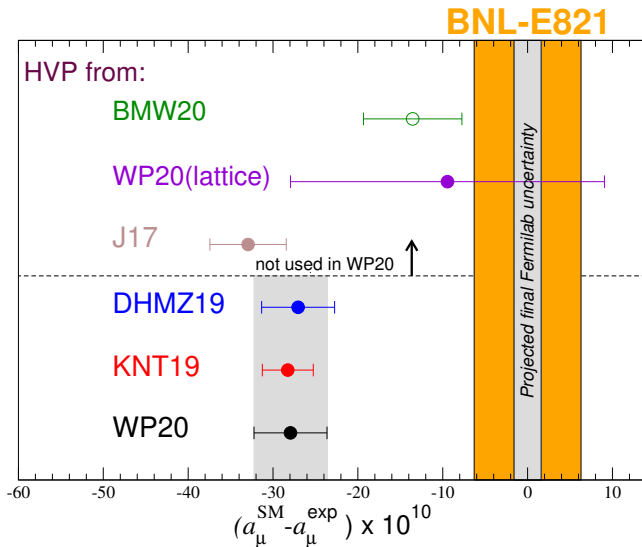
The new CMD3 measurement of  $e^+e^- \rightarrow \pi^+\pi^-$

Hadronic light-by-light

Conclusions and Outlook

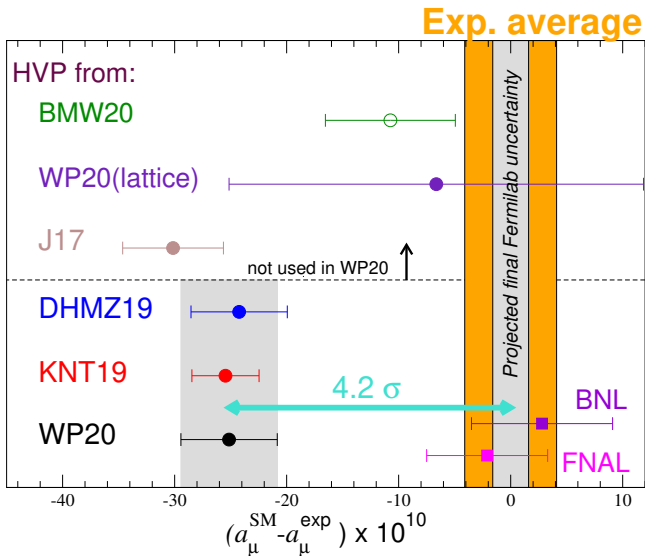
# Present status of $(g - 2)_\mu$ : experiment vs SM

Before



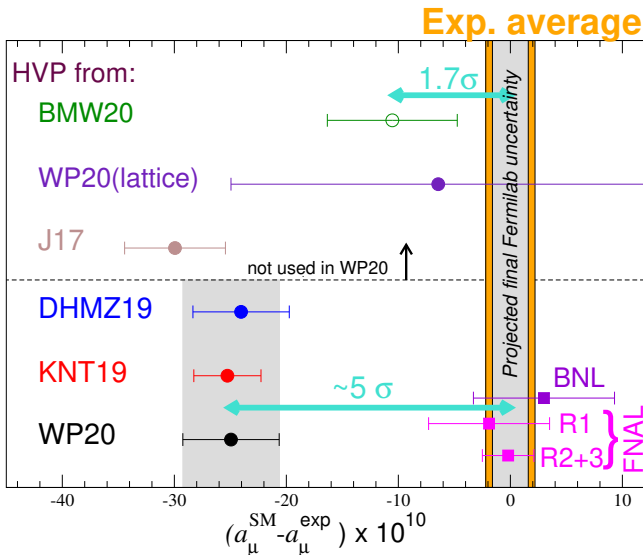
# Present status of $(g - 2)_\mu$ : experiment vs SM

After the 2021 Fermilab result



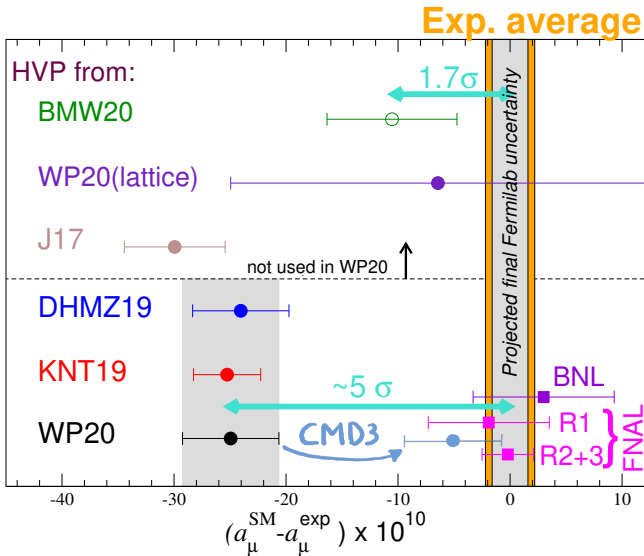
# Present status of $(g - 2)_\mu$ : experiment vs SM

After the 2023 Fermilab result



# Present status of $(g - 2)_\mu$ : experiment vs SM

After the 2023 Fermilab result and  $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$  from CMD3



White Paper (2020):  $(g - 2)_\mu$ , experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO ( $e^+e^-$ )	6931(40)
HVP NLO ( $e^+e^-$ )	-98.3(7)
HVP NNLO ( $e^+e^-$ )	12.4(1)
HVP LO (lattice, $udsc$ )	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, $uds$ )	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP ( $e^+e^-$ , LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)



White Paper (2020):  $(g - 2)_\mu$ , experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO ( $e^+ e^-$ )	6931(40)
HVP NLO ( $e^+ e^-$ )	-98.3(7)
HVP NNLO ( $e^+ e^-$ )	12.4(1)
HVP LO (lattice, $udsc$ ) $\rightarrow$ BMW(20)	7075(55)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, $uds$ )	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP ( $e^+ e^-$ , LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

# White Paper (2020): $(g - 2)_\mu$ , experiment vs SM

## White Paper:

T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

## Muon $g - 2$ Theory Initiative

### Steering Committee:

GC

Michel Davier (vice-chair)

Aida El-Khadra (chair)

Martin Hoferichter

Laurent Lellouch

Christoph Lehner (vice-chair)

Tsutomu Mibe (J-PARC E34 experiment)

Lee Roberts (Fermilab E989 experiment)

Thomas Teubner

Hartmut Wittig

# White Paper (2020): $(g - 2)_\mu$ , experiment vs SM

## White Paper:

T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

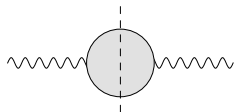
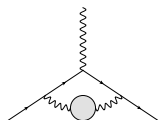
## Muon $g - 2$ Theory Initiative

### Plenary Workshops:

- ▶ 1<sup>st</sup>, Q-Center (Fermilab), 3-6 June 2017
- ▶ 2<sup>nd</sup>, Mainz, 18-22 June 2018
- ▶ 3<sup>rd</sup>, Seattle, 9-13 September 2019
- ▶ 4<sup>th</sup>, KEK (virtual), 28 June-02 July 2021
- ▶ 5<sup>th</sup>, Higgs Center Edinburgh, 5-9 Sept. 2022
- ▶ 6<sup>th</sup>, Bern, 4-8 Sept. 2023
- ▶ 7<sup>th</sup>, KEK, 9-13 Sept. 2024

# Theory uncertainty comes from hadronic physics

- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is  $\mathcal{O}(\alpha^2)$ , dominates the total uncertainty, despite being known to  $< 1\%$

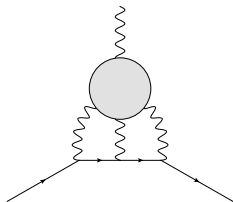


- ▶ unitarity and analyticity  $\Rightarrow$  dispersive approach
- ▶  $\Rightarrow$  direct relation to experiment:  $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$
- ▶  $e^+e^-$  Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- ▶ **alternative approach**: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

# Theory uncertainty comes from hadronic physics

- ▶ Hadronic contributions responsible for most of the theory uncertainty
- ▶ Hadronic vacuum polarization (HVP) is  $\mathcal{O}(\alpha^2)$ , dominates the total uncertainty, despite being known to  $< 1\%$
- ▶ Hadronic light-by-light (HLbL) is  $\mathcal{O}(\alpha^3)$ , known to  $\sim 20\%$ , second largest uncertainty (now subdominant)



- ▶ **earlier**: model-based—uncertainties difficult to quantify
- ▶ **recently**: dispersive approach  $\Rightarrow$  data-driven, systematic treatment
- ▶ lattice QCD is competitive

(Mainz, RBC/UKQCD)

# Outline

Introduction:  $(g - 2)_\mu$  in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

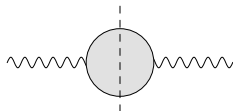
The new CMD3 measurement of  $e^+e^- \rightarrow \pi^+\pi^-$

Hadronic light-by-light

Conclusions and Outlook

# HVP contribution: Master Formula

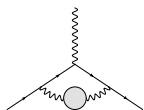
Unitarity relation: **simple**, same for all intermediate states



$$\text{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \rightarrow \text{hadrons}) = \sigma(e^+e^- \rightarrow \mu^+\mu^-)R(q^2)$$

Analyticity  $\left[ \bar{\Pi}(q^2) = \frac{q^2}{\pi} \int ds \frac{\text{Im}\bar{\Pi}(s)}{s(s-q^2)} \right] \Rightarrow$  **Master formula for HVP**

Bouchiat, Michel (61)

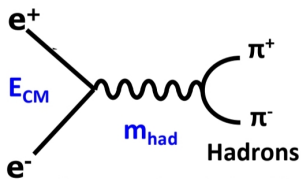


$\Leftrightarrow$

$$a_{\mu}^{\text{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s)R(s)$$

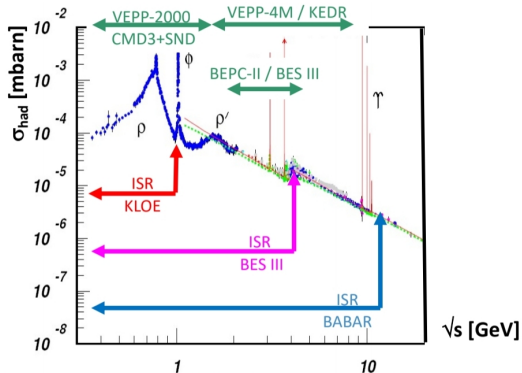
$K(s)$  known, depends on  $m_{\mu}$  and  $K(s) \sim \frac{1}{s}$  for large  $s$

# HVP contribution: Master Formula



- No systematic variation of  $E_{beam}$
- High statistics thanks to high luminosity
- Radiative corrections ( $H_{rad}$ )

PHOKHARA event generator





# Comparison between DHMZ19 and KNT19

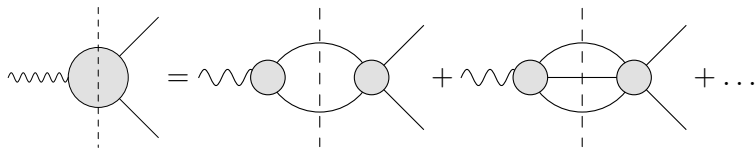
	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	-0.12
$K^+K^-$	23.08(0.44)	23.00(22)	0.08
$K_S K_L$	12.82(0.24)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	-0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$ )	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, $\infty$ ) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

# Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	<b>3.62</b>
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	-0.12
$K^+K^-$	23.08(0.44)	23.00(22)	0.08
$K_S K_L$	12.82(0.24)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	-0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$ )	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, $\infty$ ) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

For the dominant  $\pi\pi$  channel more theory input can be used

# Omnès representation including isospin breaking

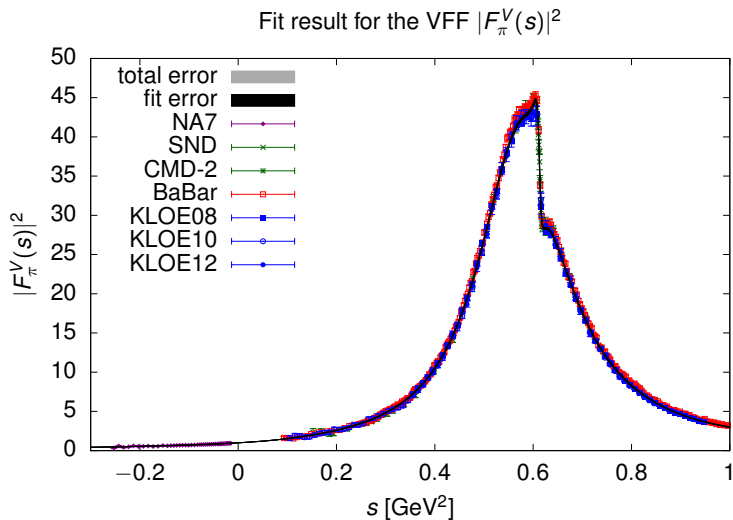


$$F_V(s) = \Omega_{\pi\pi}(s) \cdot G_\omega(s) \cdot \Omega_{\text{in}}(s)$$

main contribution  $\Omega_{\pi\pi}(s)$ : 2 parameters

# Fit results

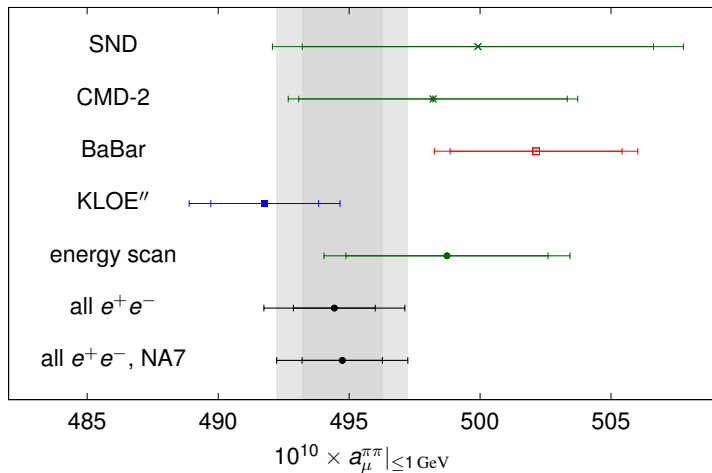
GC, Hoferichter, Stoffer (18)



# Fit results

GC, Hoferichter, Stoffer (18)

Result for  $a_{\mu}^{\pi\pi}|_{\leq 1 \text{ GeV}}$  from the VFF fits to single experiments and combinations



## $2\pi$ : comparison with the dispersive approach

$2\pi$  channel described dispersively  $\Rightarrow$  more theory constraints

Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18) WP(20)

Energy range	CHS18	DHMZ19	KNT19
$\leq 0.6$ GeV	110.1(9)	110.4(4)(5)	108.7(9)
$\leq 0.7$ GeV	214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq 0.8$ GeV	413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq 0.9$ GeV	479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq 1.0$ GeV	495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV	104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV	198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV	66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV	15.3(1)	15.5(1)(2)	15.3(1)
$\leq 0.63$ GeV	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV	369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV	490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

## Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 ( $2\pi$ ) and HHK19 ( $3\pi$ ), have been so combined:

- ▶ central values are obtained by simple averages (for each channel and mass range)
- ▶ the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- ▶ 1/2 difference DHMZ–KNT (or BABAR–KLOE in the  $2\pi$  channel, if larger) is added to the uncertainty

Final result:

$$\begin{aligned}
 a_{\mu}^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\
 &= 693.1(4.0) \times 10^{-10}
 \end{aligned}$$

# The BMW result

Borsanyi et al. Nature 2021

State-of-the-art lattice calculation of  $a_{\mu}^{\text{HVP, LO}}$  based on

- ▶ current-current correlator, summed over all distances, integrated in time with appropriate kernel function (TMR)
- ▶ using staggered fermions on an  $L \sim 6$  fm lattice ( $L \sim 11$  fm used for finite volume corrections)
- ▶ at (and around) physical quark masses
- ▶ including isospin-breaking effects



# The BMW result

Borsanyi et al. Nature 2021

## Isospin-symmetric



Connected light

$$633.7(2.1)_{\text{stat}}(4.2)_{\text{sys}}$$



Connected strange

$$53.393(89)_{\text{stat}}(68)_{\text{sys}}$$



Connected charm

$$14.6(0)_{\text{stat}}(1)_{\text{sys}}$$



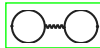
Disconnected

$$-13.36(1.18)_{\text{stat}}(1.36)_{\text{sys}}$$

## QED isospin breaking: valence



$$\text{Connected } -1.23(40)_{\text{stat}}(31)_{\text{sys}}$$



$$\text{Disconnected } -0.55(15)_{\text{stat}}(10)_{\text{sys}}$$

## Strong-isospin breaking



Connected

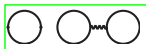
$$6.60(63)_{\text{stat}}(53)_{\text{sys}}$$



Disconnected

$$-4.67(54)_{\text{stat}}(69)_{\text{sys}}$$

## QED isospin breaking: sea



$$\text{Connected } 0.37(21)_{\text{stat}}(24)_{\text{sys}}$$



$$\text{Disconnected } -0.040(33)_{\text{stat}}(21)_{\text{sys}}$$

## Other

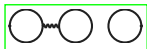
Bottom; higher-order;  
perturbative

$$0.11(4)_{\text{tot}}$$

## QED isospin breaking: mixed



$$\text{Connected } -0.0093(86)_{\text{stat}}(95)_{\text{sys}}$$



$$\text{Disconnected } 0.011(24)_{\text{stat}}(14)_{\text{sys}}$$

## Finite-size effects

Isospin-symmetric

$$18.7(2.5)_{\text{tot}}$$

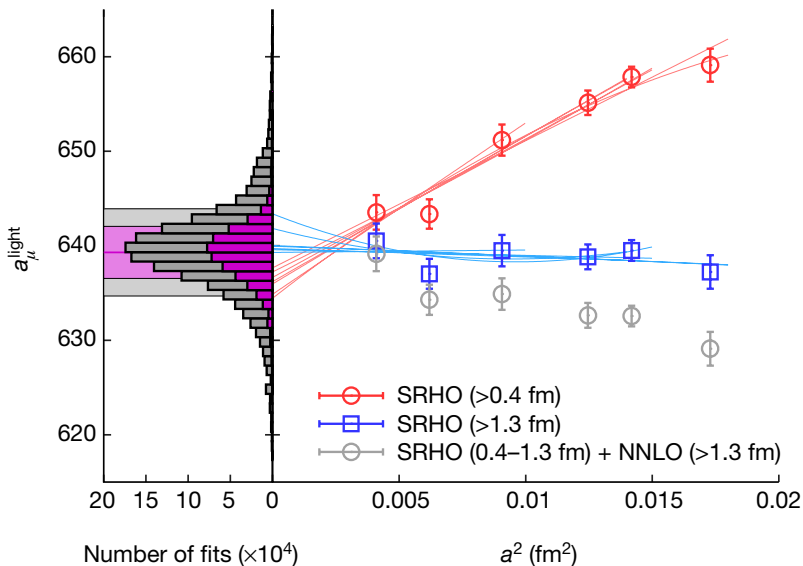
Isospin-breaking

$$0.0(0.1)_{\text{tot}}$$

$$a_{\mu}^{\text{LO-HVP}} (\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}(5.5)_{\text{tot}}$$

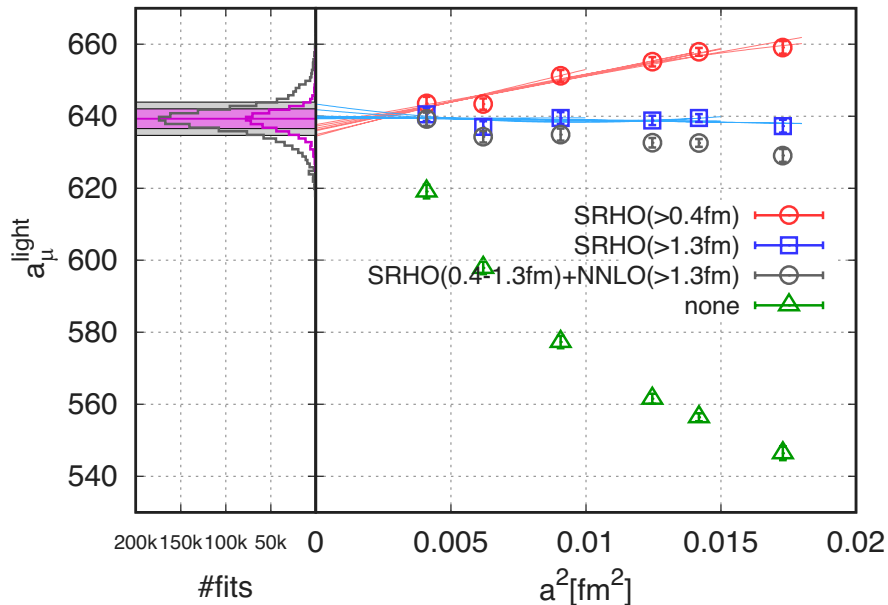
## The BMW result

Borsanyi et al. Nature 2021



## The BMW result

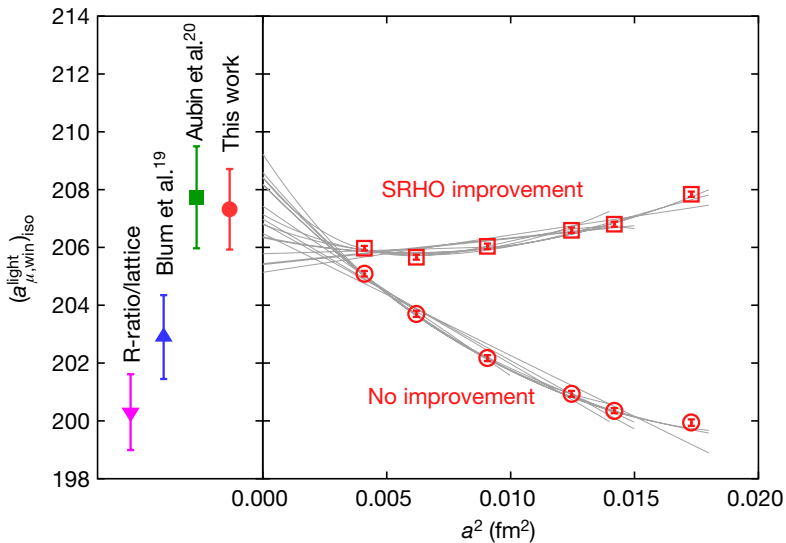
Borsanyi et al. Nature 2021



## The BMW result

Borsanyi et al. Nature 2021

## Article

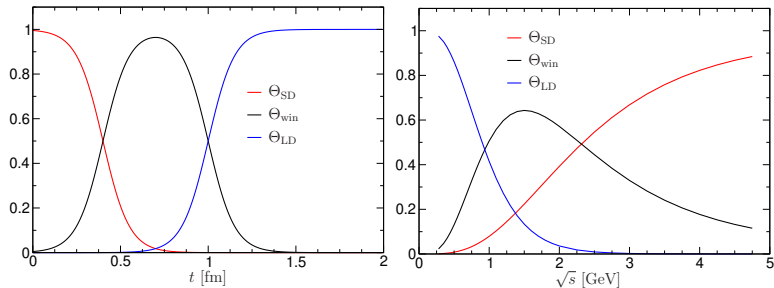


# The BMW result

Borsanyi et al. Nature 2021

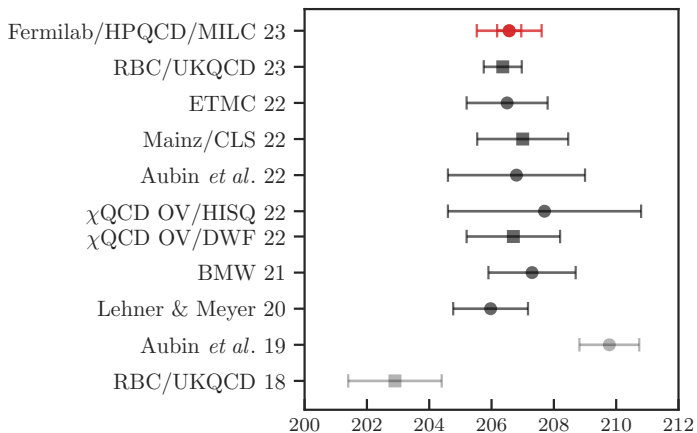
## Weight functions for window quantities

RBC/UKQCD (18)



# Present status of the window quantities

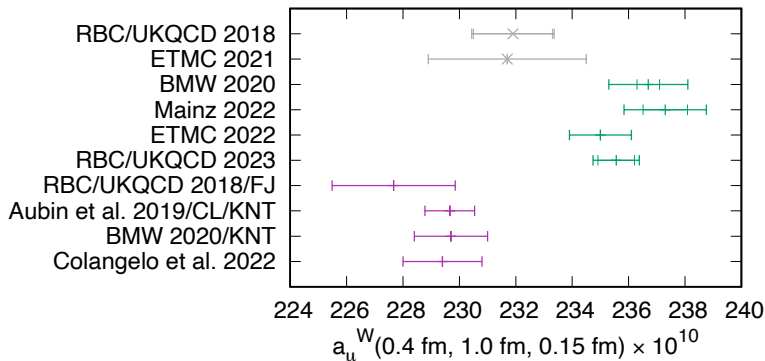
Several lattice calculations have confirmed BMW's result



arXiv:2301.08274, [Fermilab Lattice-HPQCD-MILC \(23\)](#)

# Present status of the window quantities

Several lattice calculations have confirmed BMW's result



arXiv:2301.08696 [RBC/UKQCD \(23\)](#)

# Individual-channel contributions to $a_\mu^{\text{win}}$

Channel	total	window
$\pi^+\pi^-$	504.23(1.90)	144.08(49)
$\pi^+\pi^-\pi^0$	46.63(94)	18.63(35)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19)	8.88(12)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74)	11.20(46)
$K^+K^-$	23.00(22)	12.29(12)
$K_S^0K_L^0$	13.04(19)	6.81(10)
$\pi^0\gamma$	4.58(10)	1.58(4)
Sum of the above	623.62(2.27)	203.47(78)
[1.8, 3.7] GeV (without $c\bar{c}$ )	34.45(56)	15.93(26)
$J/\psi, \psi(2S)$	7.84(19)	2.27(6)
[3.7, $\infty$ ) GeV	16.95(19)	1.56(2)
WP(20) / GC, El-Khadra <i>et al.</i> (22)	693.1(4.0)	229.4(1.4)
BMWc	707.5(5.5)	236.7(1.4)
Mainz/CLS		237.3(1.5)
ETMc		235.0(1.1)
RBC/UKQCD		235.6(0.8)

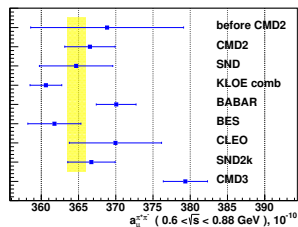
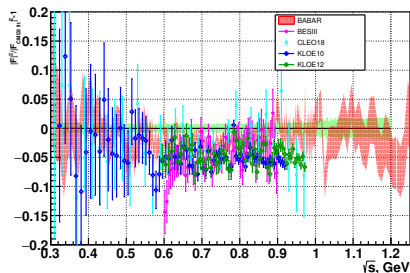
Numbers for the channels refer to KNT19 — thanks to Alex Keshavarzi for providing them

$$\Delta a_\mu^{\text{HVP, LO}} = 14.4(6.8) (2.1\sigma), \quad \Delta a_\mu^{\text{win}} \sim 6.5(1.5) (\sim 4.3\sigma)$$



# CMD-3 measurement of $e^+e^- \rightarrow \pi^+\pi^-$

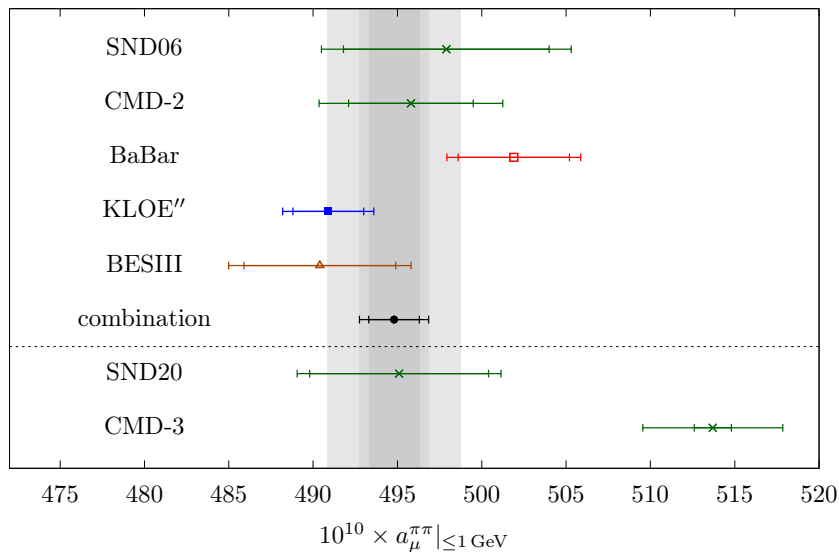
F. Ignatov et al., CMD-3, arXiv:2302.08834



The comparison of pion form factor measured in this work with the most recent ISR experiments (BABAR [21], KLOE [18, 19], BES [22]) is shown in Fig. 34. The comparison with the most precise previous energy scan experiments (CMD-2 [12, 13, 14, 15], SND [16] at the VEPP-2M and SND [23] at the VEPP-2000) is shown in Fig. 35. **The new result generally shows larger pion form factor in the whole energy range under discussion.** The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of  $\rho$ -meson ( $\sqrt{s} = 0.6 - 0.75$  GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. **The source of this difference is unknown at the moment.**

# Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, [arXiv:2308.04217](https://arxiv.org/abs/2308.04217) (thanks for providing the plot)



# Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

$10^{10} \times$	$a_{\mu}^{\pi\pi}  _{\leq 1\text{GeV}}$	$a_{\mu}^{\pi\pi, \text{win}}  _{\leq 1\text{GeV}}$	$\chi^2/\text{dof}$
SND06	497.9(6.1)(4.2)	139.6(1.8)(1.0)	1.09
CMD-2	495.8(3.7)(4.0)	139.4(1.0)(0.8)	1.01
BaBar	501.9(3.3)(2.2)	140.6(1.0)(0.7)	1.17
KLOE''	490.9(2.1)(1.7)	137.1(0.6)(0.4)	1.13
BESIII	490.4(4.5)(3.0)	137.8(1.3)(0.4)	1.01
SND20	495.1(5.3)(2.9)	139.2(1.5)(0.4)	1.88
CMD-3	513.7(1.1)(4.0)	144.0(0.3)(1.1)	1.09
Combination	494.8(1.5)(1.4)(3.4)	138.3(0.4)(0.3)(1.1)	1.21

Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1), \quad \Delta a_{\mu}^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_{\mu}^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \quad \Delta a_{\mu}^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

# Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

Discrepancy	$a_\mu^{\pi\pi} \Big _{[0.60,0.88] \text{ GeV}}$	$a_\mu^{\pi\pi} \Big _{\leq 1 \text{ GeV}}$	int window
SND06	$1.8\sigma$	$1.7\sigma$	$1.7\sigma$
CMD-2	$2.3\sigma$	$2.0\sigma$	$2.1\sigma$
BaBar	$3.3\sigma$	$2.9\sigma$	$3.1\sigma$
KLOE''	$5.6\sigma$	$4.8\sigma$	$5.4\sigma$
BESIII	$3.0\sigma$	$2.8\sigma$	$3.1\sigma$
SND20	$2.2\sigma$	$2.1\sigma$	$2.2\sigma$
Combination	$4.2\sigma$ [6.1 $\sigma$ ]	$3.7\sigma$ [5.0 $\sigma$ ]	$3.8\sigma$ [5.7 $\sigma$ ]

Combination: NA7 + all data sets other than SND20 and CMD-3

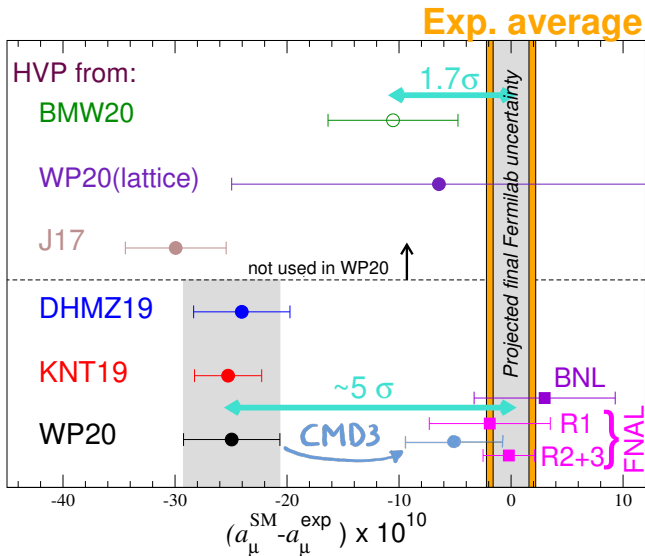
$$\Delta a_\mu^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1),$$

$$\Delta a_\mu^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_\mu^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8),$$

$$\Delta a_\mu^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

# Present status of $(g - 2)_\mu$ : experiment vs SM



# Outline

Introduction:  $(g - 2)_\mu$  in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

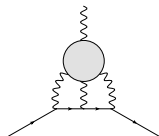
Lattice vs data-driven: the window quantity

The new CMD3 measurement of  $e^+e^- \rightarrow \pi^+\pi^-$

**Hadronic light-by-light**

Conclusions and Outlook

# HLbL contribution: Master Formula



$$a_{\mu}^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^{\infty} dQ_1 \int_0^{\infty} dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

$Q_i^{\mu}$  are the **Wick-rotated** four-momenta and  $\tau$  the four-dimensional angle between Euclidean momenta:  $Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$

The integration variables  $Q_1 := |Q_1|$ ,  $Q_2 := |Q_2|$ .

GC, Hoferichter, Procura, Stoffer (15)

- ▶  $T_i$ : known kernel functions
- ▶  $\bar{\Pi}_i$  are amenable to a dispersive treatment:  
imaginary parts are related to measurable subprocesses

# Improvements obtained with the dispersive approach

Contribution	PdRV(09) <i>Glasgow consensus</i>	N/JN(09)	J(17)	WP(20)
$\pi^0, \eta, \eta'$ -poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
$\pi, K$ -loops/boxes	-19(19)	-19(13)	-20(5)	-16.4(2)
S-wave $\pi\pi$ rescattering	-7(7)	-7(2)	-5.98(1.20)	-8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	-	-	-	} - 1(3)
tensors	-	-	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	
$u, d, s$ -loops / short-distance	-	21(3)	20(4)	15(10)
c-loop	2.3	-	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

► significant reduction of uncertainties in the first three rows

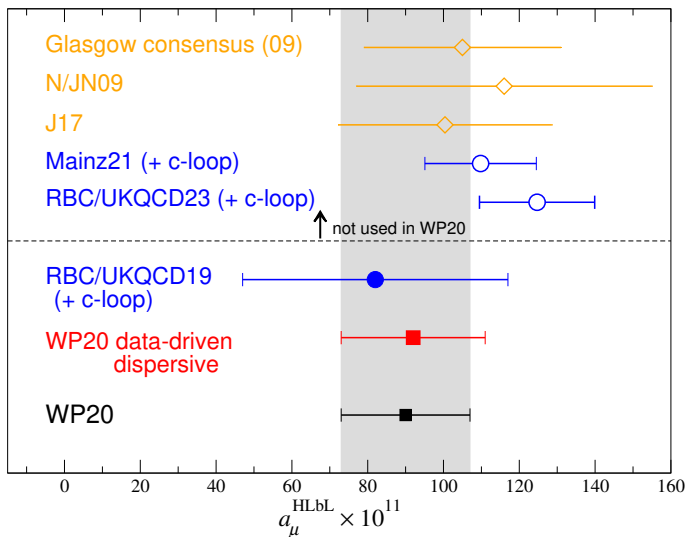
CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid et al. (18), Gerardin, Meyer, Nyffeler (19)

► resonances and short-distance constraints need to be improved

Danilkin, Hoferichter, Stoffer (21), Lüdtke, Procura, Stoffer (23), Melnikov, Vainshtein (04), Nyffeler (09), Bijmans et al. (20,21), Capiello et al. (20), Leutgeb, Rebhan (19,21)



# Situation for HLbL



# Outline

Introduction:  $(g - 2)_\mu$  in the Standard Model

Hadronic Vacuum Polarization contribution

White Paper: data-driven approach

Lattice: the BMW result

Lattice vs data-driven: the window quantity

The new CMD3 measurement of  $e^+e^- \rightarrow \pi^+\pi^-$

Hadronic light-by-light

Conclusions and Outlook

# Conclusions

- ▶ Data-driven evaluation of the HVP contribution (WP20):  
0.6% error  $\Rightarrow$  **dominates the theory uncertainty**
- ▶ Dominant contribution to HVP:  $\pi\pi$  (<1 GeV). WP20 based on:  
CMD-2, SND06, BaBar, KLOE, BES-III  
**New puzzle: measurement by CMD-3 significantly higher!**
- ▶ Recent lattice calculation [BMW(20)] has reached a similar precision  
but **differs from the dispersive one** (=from  $e^+e^-$  data).  
If confirmed  $\Rightarrow$  discrepancy with experiment  $\searrow$  **below  $2\sigma$**
- ▶ **Intermediate window** of BMW has been confirmed by other lattice  
collaborations (Aubin et al., Mainz, ETMc, RBC/UKQCD, Fermilab-HPQCD-MILC)  
and disagrees with data-driven [other than CMD-3, which would agree]
- ▶ Evaluation of the HLbL contribution based on the dispersive  
approach: **20% accuracy**. Two recent lattice calculations  
[RBC/UKQCD(23), Mainz(21)] agree with it

# Outlook

- ▶ The Fermilab experiment aims to reduce the BNL uncertainty by a **factor four**  $\Rightarrow$  potential  **$7\sigma$**  discrepancy
- ▶ Improvements on the SM theory/data side:
  - ▶ Situation for HVP data-driven **urgently needs to be clarified**:
    - New **CMD-3** result—after thorough scrutiny—is a puzzle
    - Forthcoming measur./analyses: **BaBar**, **Belle II**, **BESIII**, **KLOE**, **SND**
    - Model-independent evaluation of **RadCorr** underway (but unlikely the culprit)
    - Monte Carlo codes used by experiments: **what is their role?**
    - **MuonE** will provide an alternative way to measure HVP
- ▶ HVP lattice: calculations w/ precision  $\sim$  **BMW** for  $a_{\mu}^{\text{HVP, LO}}$  expected soon
- ▶ HLbL: goal of  $\sim$  **10% uncertainty** within reach (both data-driven and lattice)

# Future: Muon $g - 2$ /EDM experiment @ J-PARC

