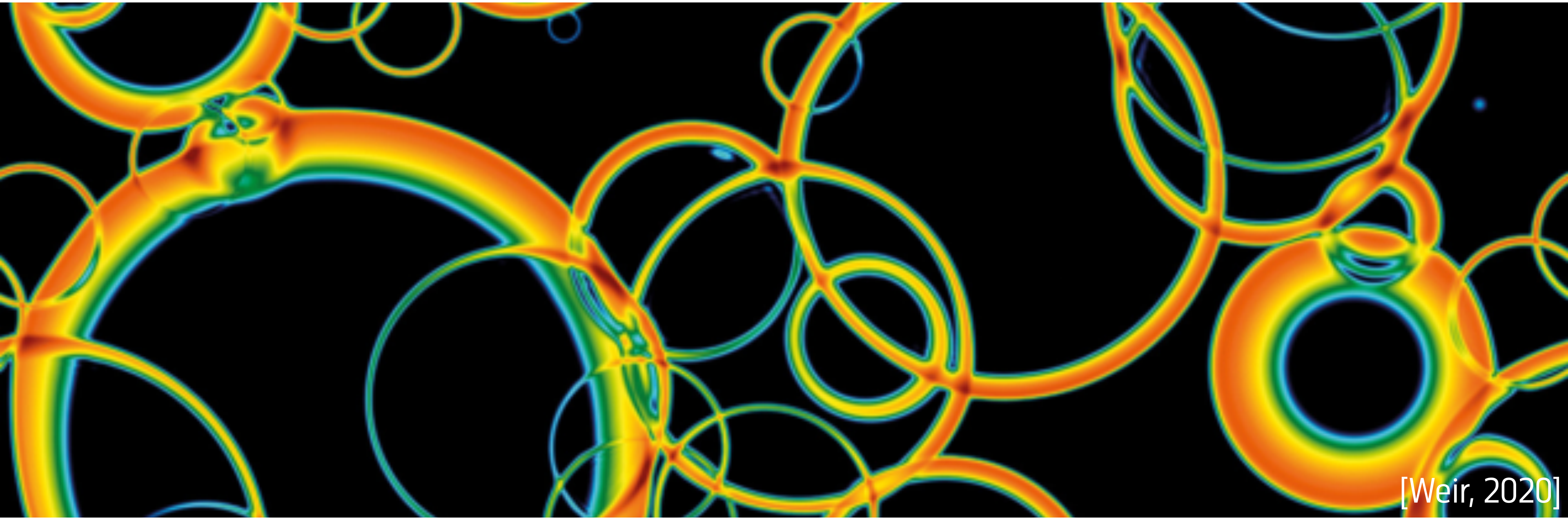
“Precision Calculations of Effective Potentials and Electroweak Phase Transitions in the Early Universe”



[Weir, 2020]

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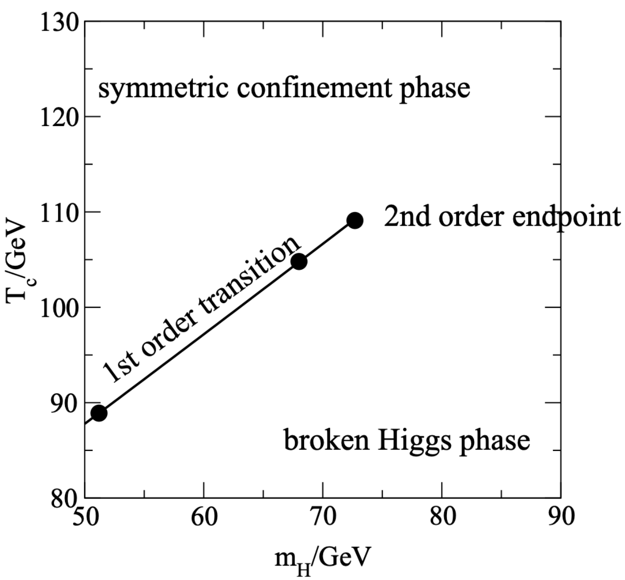
SUSY 2024, Madrid, 14.06.2024

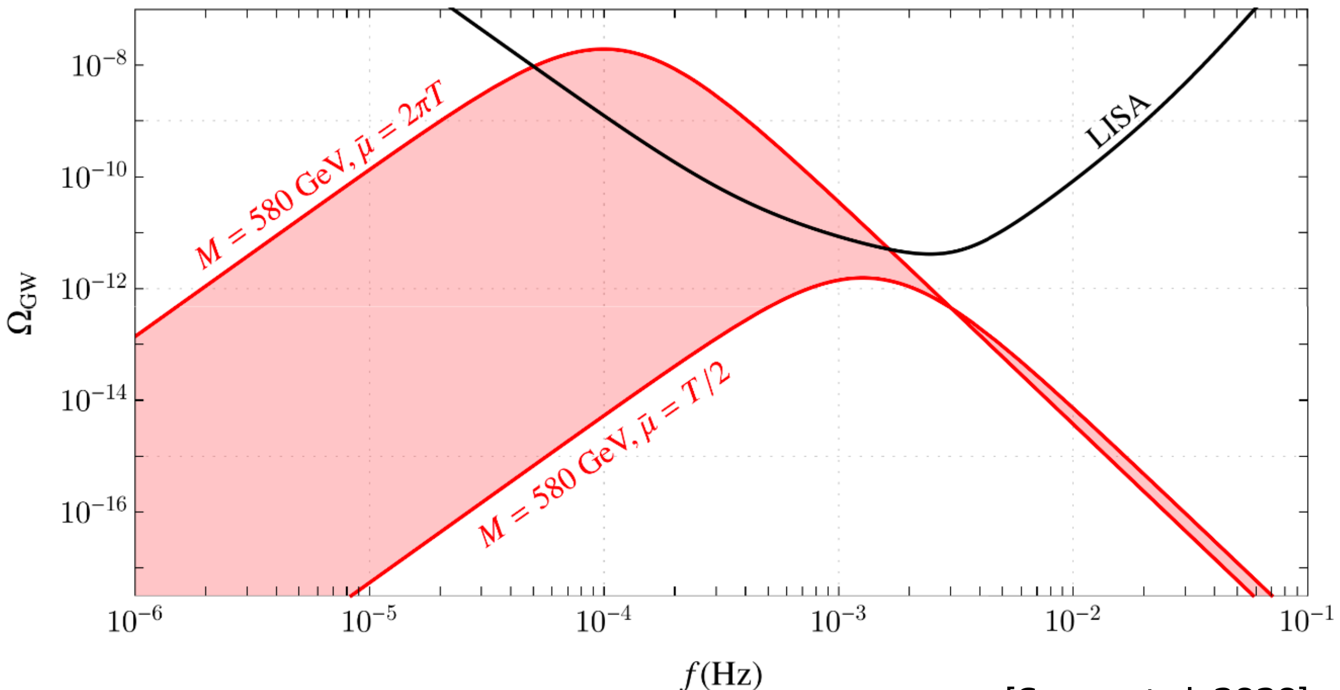
Motivation: First Order Phase Transition in the Early Universe

- Mechanism to satisfy Sakharov's conditions for generation matter-antimatter asymmetry
- Naturally occurring in most of extensions of Higgs sectors
- Perfect candidate for gravitation wave detection in close future experiments (LISA, etc)



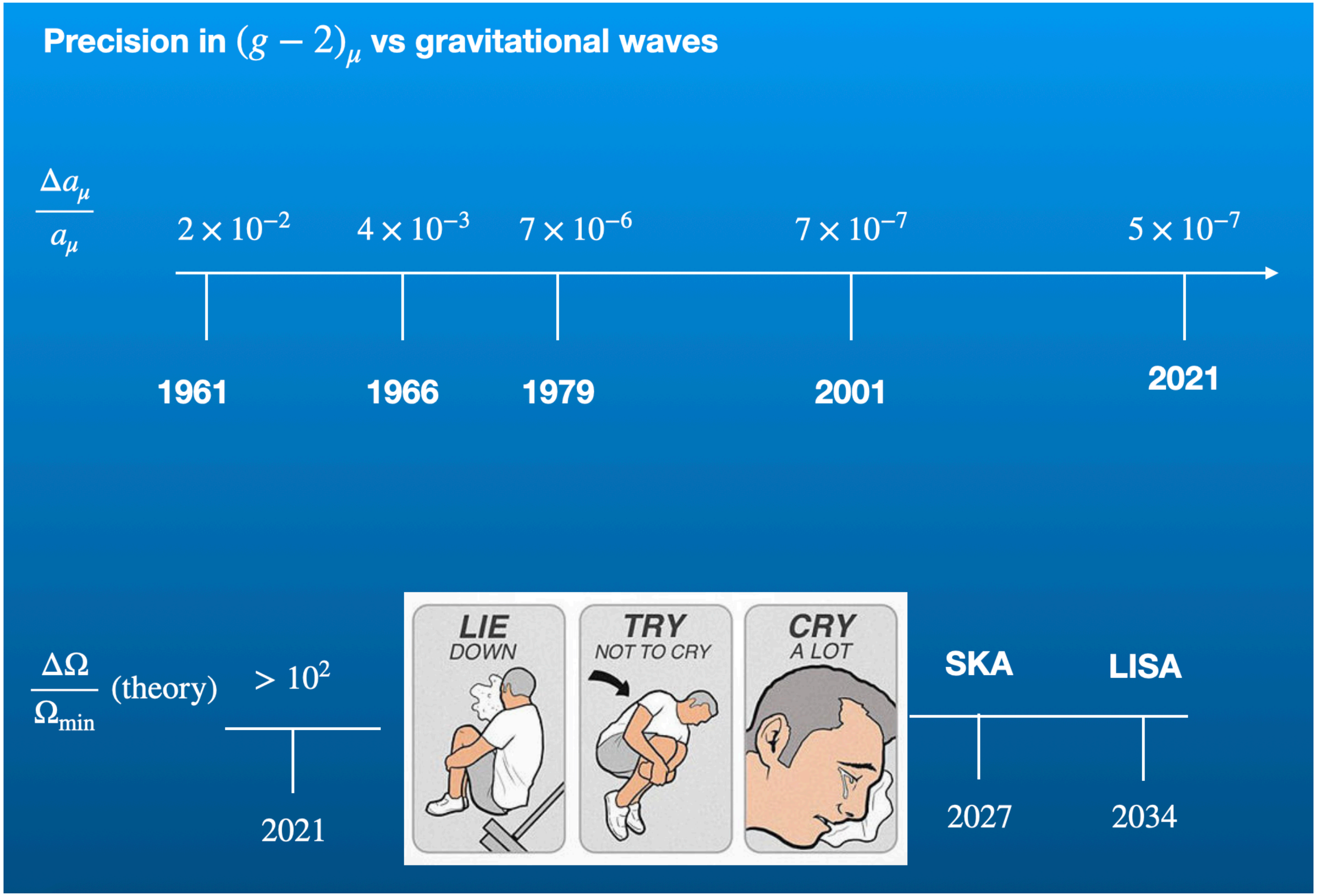
[Laine, 2000]

Motivation: uncertainties



[Croon et al, 2020]

Motivation: uncertainties



[White, 2024]

Effective action/potential

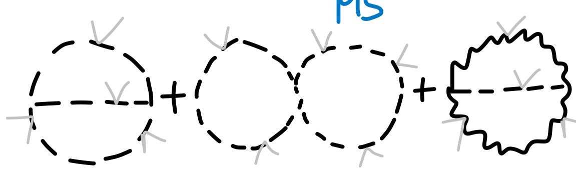
$$\Gamma[\phi] = W[J] - \int d^4x\, J(x)\phi(x),$$

$$V_{eff}[\phi] = \frac{\Gamma[\phi]}{(vol)}$$

$$V^{eff} = V_{tree} + V_{1\text{-}loop} + V_{2\text{-}loop} + \ldots$$

$$V^{T=0}_{1\text{-}loop} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 - m_i^2\right) = \sum_p \frac{n_i \cdot m_i^4}{64\pi^2}\left(\log\left(\frac{m_i^2}{\mu^2}\right) - c_i\right)$$

$\overline{MS}$

$$V^{T=0}_{2\text{-}loop} \supset$$  $+ \ldots$

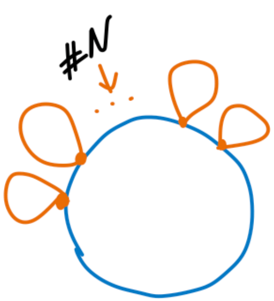
* full 2-loop result in arbitrary gauge is known

Renormalization scale & Daisy resummation

- Thermal loop corrections, which are dominant during the phase transition, introduce large renormalization scale dependence.

$$V_{therm} = \underbrace{V_{tree} + V^{T=0}_{1\text{-loop}}}_{\mu\text{-inv}} + \underbrace{V^{T\neq 0}_{1\text{-loop}}}_{\mu\text{-non-inv!}}$$

- The presence of hierarchy between hard ($\sim T$) and soft ($\sim gT$) scales requires resummation of the hard modes, which messes up with the loop order



$$\sim m^3 T \left(\frac{gT}{m}\right)^{2N}$$

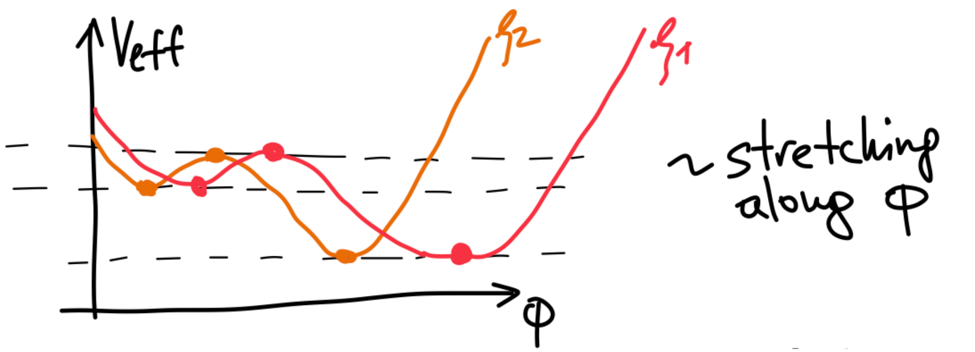
Gauge dependency

- The effective action itself is an intrinsically gauge dependent quantity, as it's defined for the non-zero source term, and the system has a non-dynamical background change.

- But, it's gauge dependent according to Nielsen identity:

$$\frac{\partial V_{eff}}{\partial \xi} = C_i(\varphi_i, \xi)\frac{\partial V_{eff}}{\partial \varphi_i}$$

- V_{eff} is gauge invariant at stationary point (extremums)



- Gauge invariant results can be obtained by systematic $\hbar$-expansion

[Nielsen, 1975]

[Patel, et.al., 2011]

High-T EFT

Separating logarithms

- Large μ - dependence & need for resummations indicate large separation of scales

- Relevant scales:

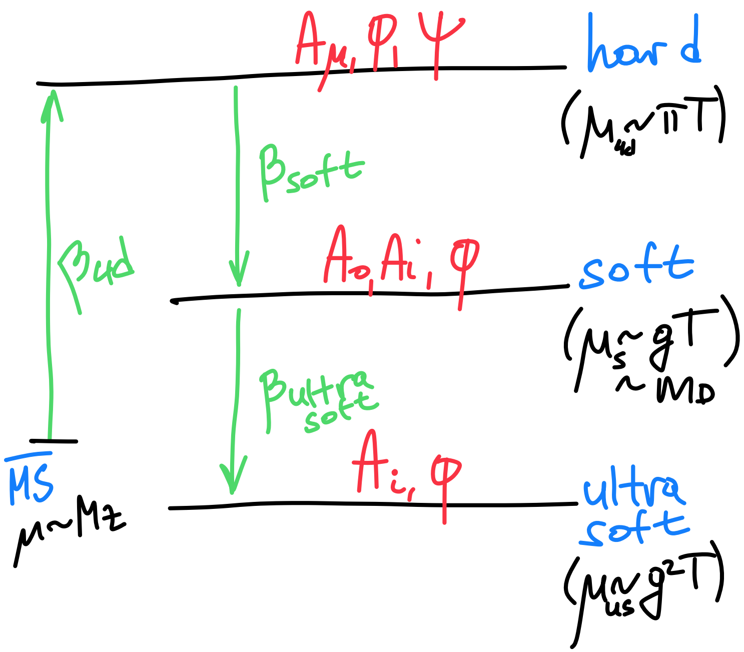
  - hard mode $\sim \pi T$
  - soft $\sim gT \sim m_D$
  - ultrasoft $\sim g^2 T$

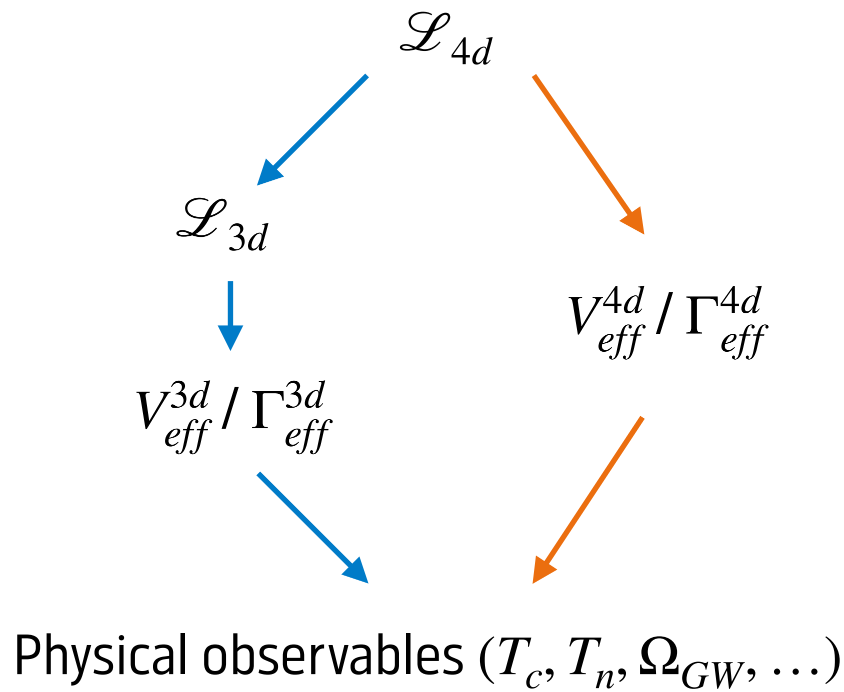
- Basically, we have too many logarithms:

$$\log\left(\frac{\pi T}{\mu}\right), \log\left(\frac{gT}{\mu}\right), \log\left(\frac{g^2 T}{\mu}\right)$$

[Kajantie et al, 1994]

High-T EFT





High-T EFT lives in 3D:

- Simpler integrals
- Superrenormalizable theory

Automated matching tool:

DRalgo [Ekstedt, et.al. 2022]

Temperature is integrated out

- Use $T = 0$ QFT framework
- Resummations are already included
- Gauge invariance is straightforward

Lagrange parameters determination

Another possible source of uncertainties

Physical inputs $(M_h, M_W, M_Z, G_F, \ldots)$

V_{eff}^{4d}

n-loop $\overline{MS}$ relations

$$M_h = m_h + \Pi_h(p^2 = M_h^2)$$

OS-like renormalization

$$\partial_h V_{tree} = \partial_h V_{eff} + \partial_h V_{c.t.}$$

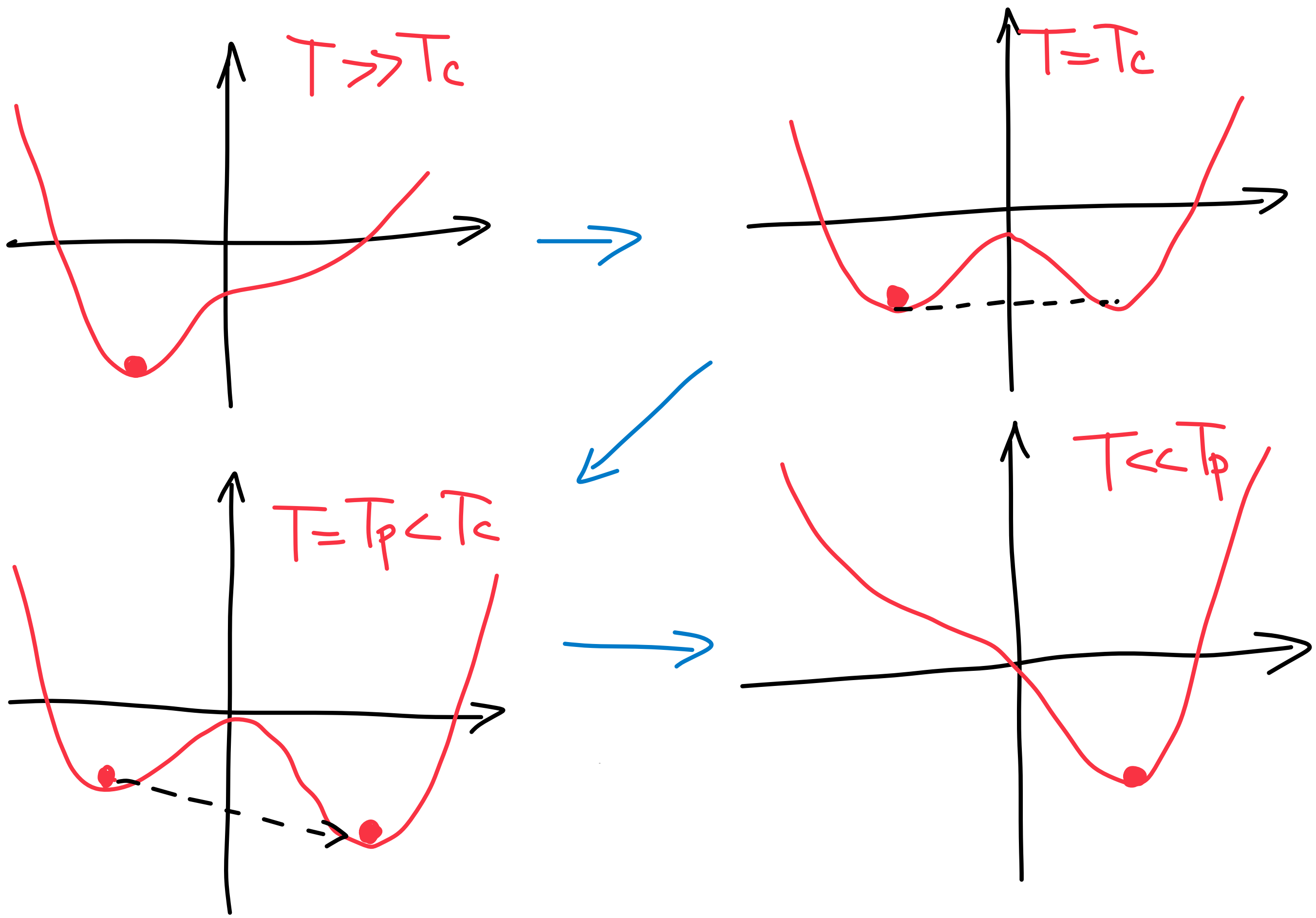
$$\partial_h^2 V_{tree} = \partial_h^2 V_{eff} + \partial_h^2 V_{c.t.}$$

$$(\sim \Pi_h(p^2 = 0))$$

Physical observables $(\mu, \lambda, g, \ldots)$

- Missing momentum contribution
- Gauge dependent
- Taking derivatives must be handled carefully*
- Only scalar potential couplings are renormalised

Thermodynamics of the Phase Transition

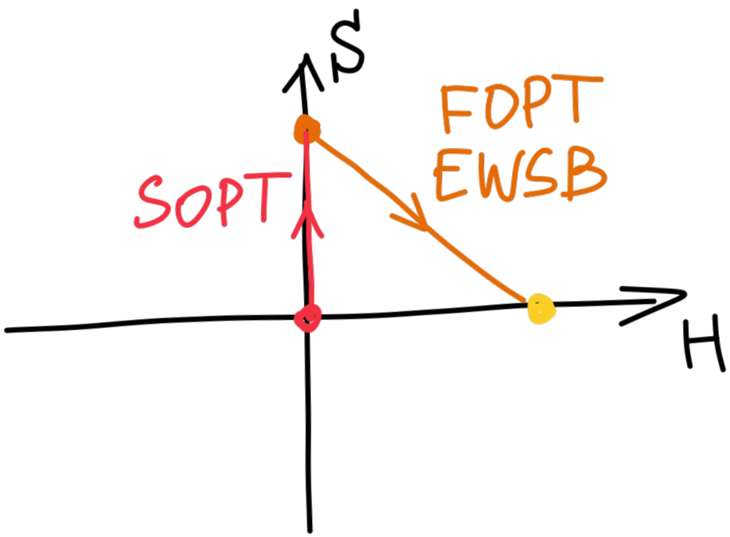


*Tunnelling and Thermal Fluctuations over the barrier

Benchmark model: cxSM

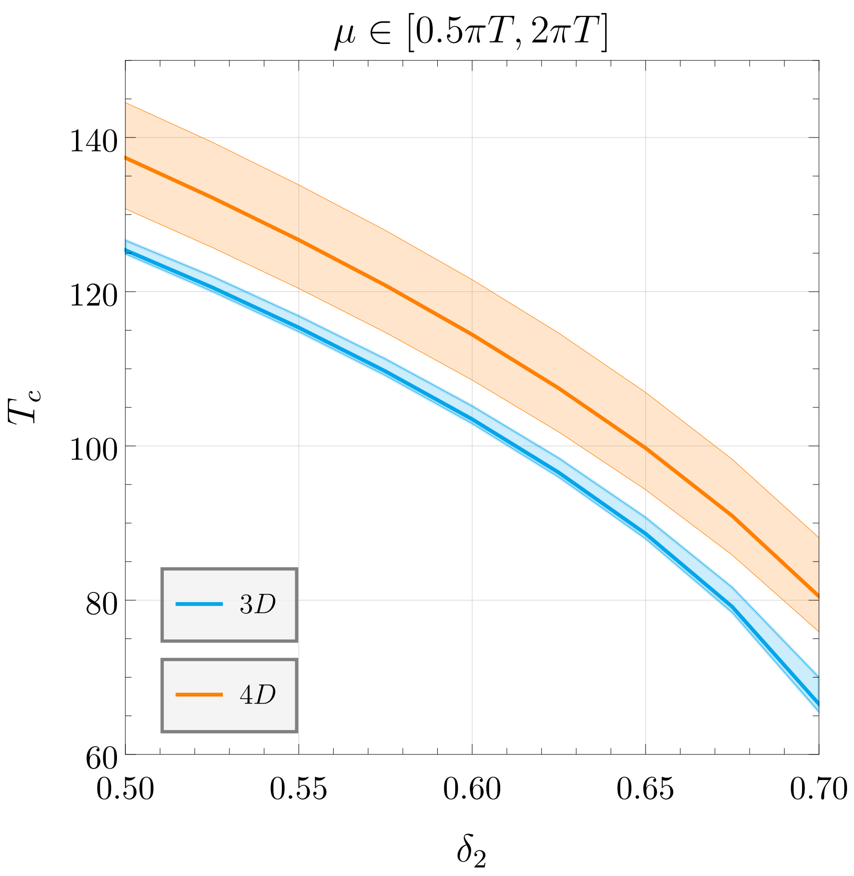
$$\mathscr{L}_{cxSM} \supset \partial^\mu H^\dagger \partial^\mu H + \mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2$$

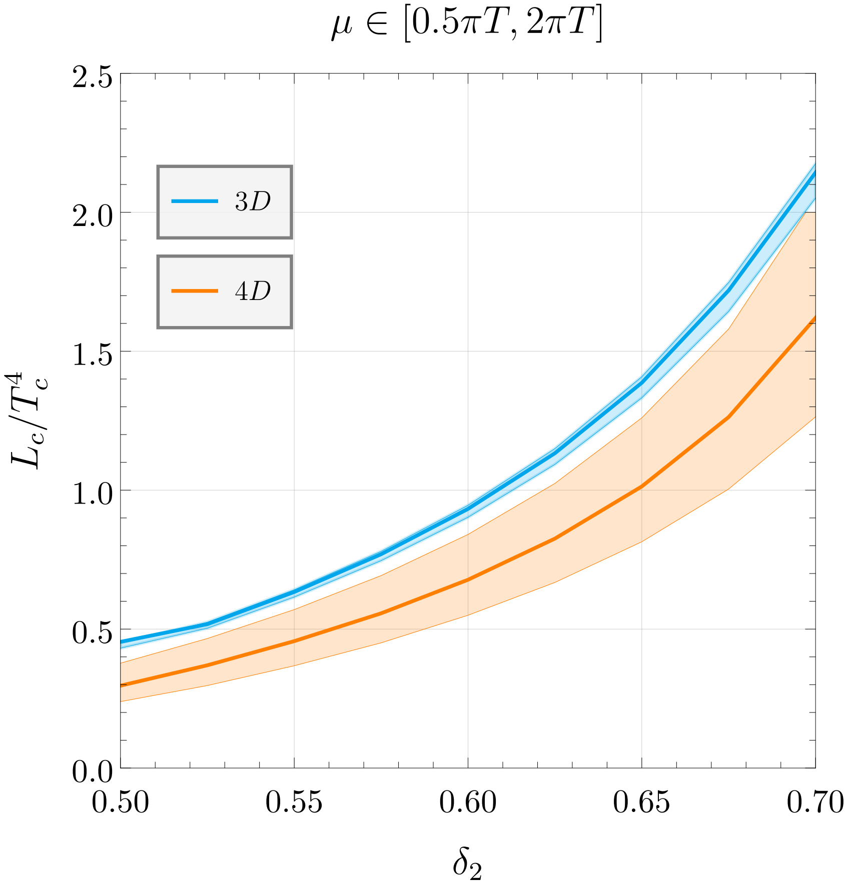
$$+\frac{1}{2}|\partial^\mu S|^2 + \frac{1}{2} b_2 |S|^2 + d_2 |S|^4 + \frac{1}{2}\delta_2 |S|^2 H^\dagger H$$



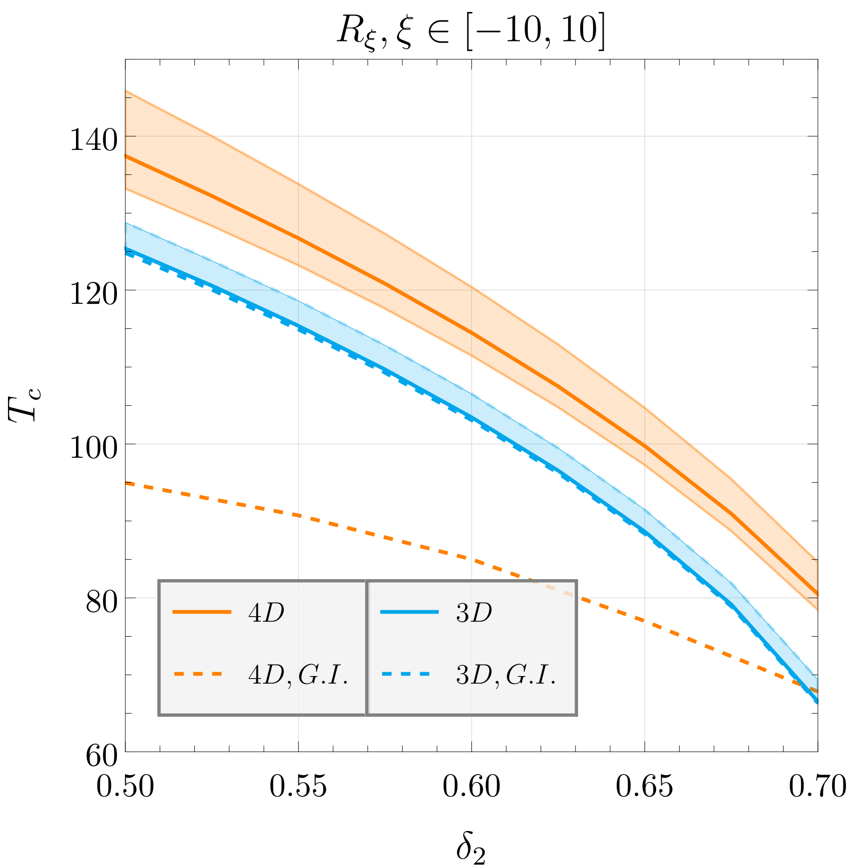
$$m_{H_2} = m_A = 62.5 \text{ GeV},$$
$$d_2 = 0.5, \delta_2 = 0.55.$$

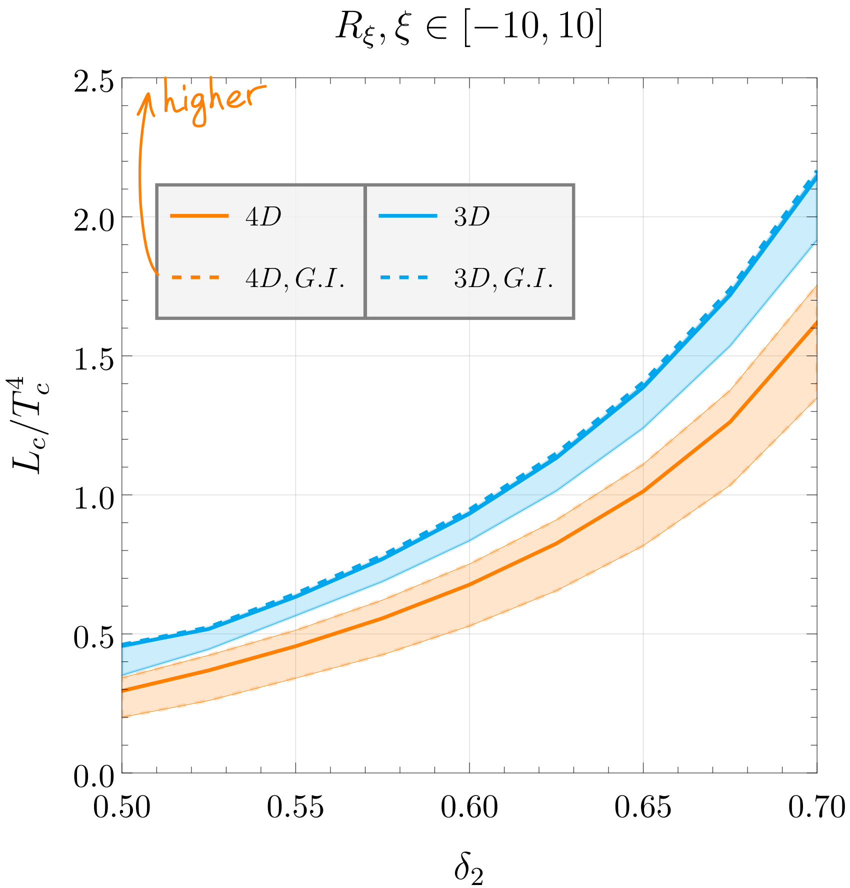
Results: μ scale dependence



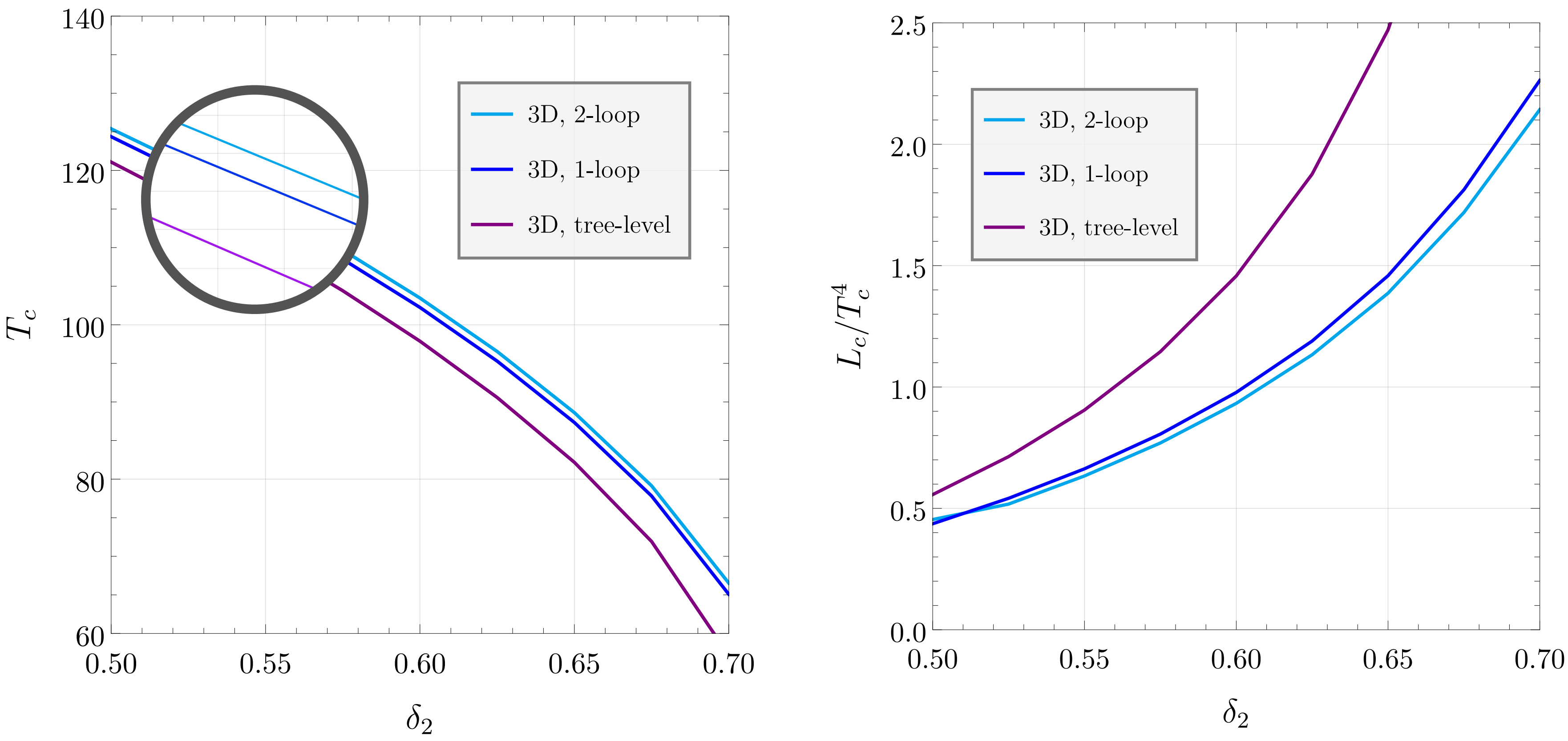


Results: gauge (in)variance





Results: Loop convergence in EFT approach

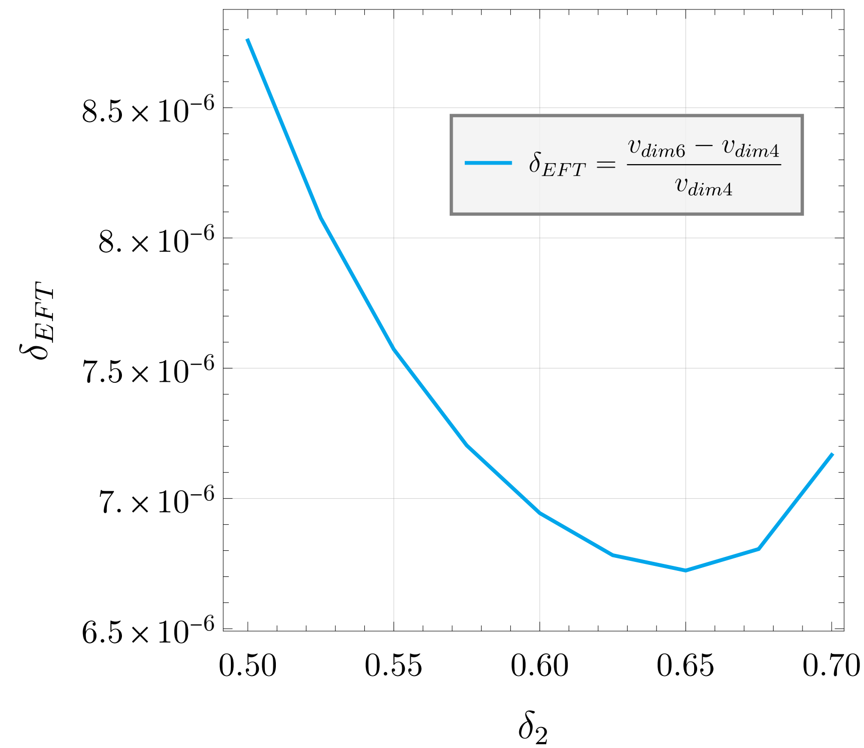


Results: EFT validity

$$\mathscr{L}_{4d}$$

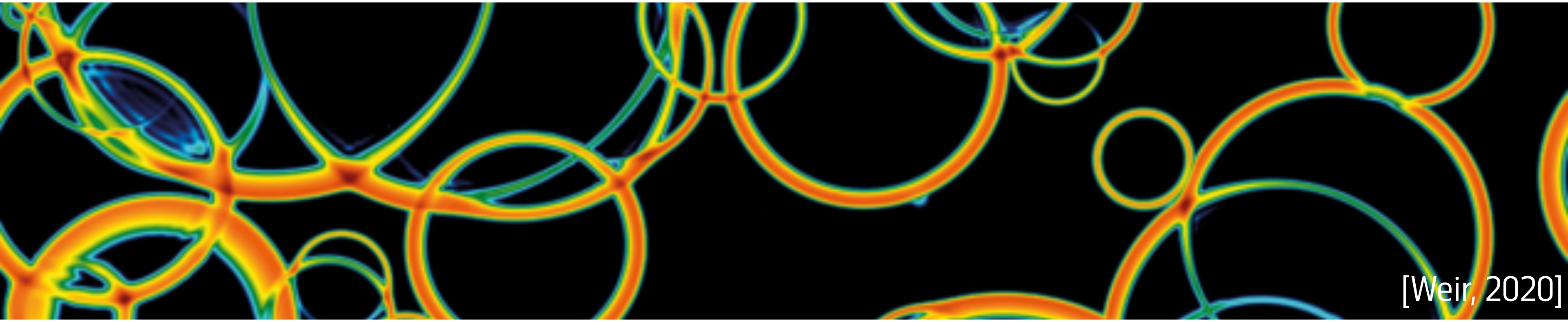
↓

$$\mathscr{L}_{3d}$$



Conclusions.

- Thermally driven phase transitions come with uncertainties, which are essential to be accounted for.
- Large scale separations and related resummations can be rigorously taken into account with the help of EFT techniques.
- Phenomenological studies of phase transitions have to go together with the systematic studies of uncertainties, which could shine light on our understanding on physics behind it.



Background vs standard R_ξ gauges.

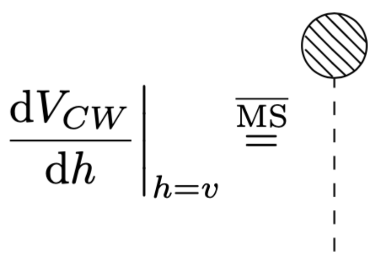
$$\mathscr{L}_{g.f.} = -\frac{1}{2\xi}(\partial^\mu A^a_\mu + i\xi g_a t^a_{ik}\tilde{\phi}_i \phi_k)^2$$

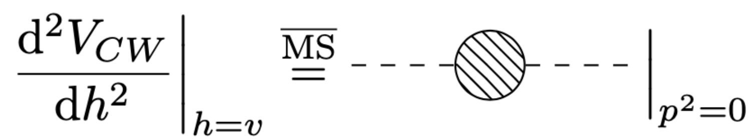
$\tilde{\phi}_i = h_i$ - background field

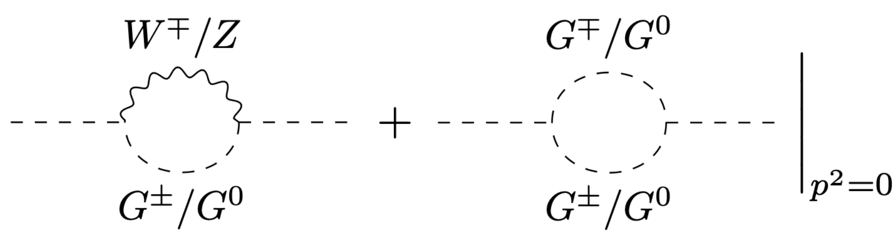
(!wrong derivatives*!)

$\tilde{\phi}_i = h_{i,min}$ - minimal field configuration

(complicated mixings)







Lagrange parameters determination

$'OS'$ vs $\overline{MS}$