Intermediate Charge-Breaking Phases in the 2-Higgs-Doublet Model

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based on JHEP02(2024)232 (arXiv:2308.04141)

with Mayumi Aoki, Lisa Biermann, Igor P. Ivanov, Margarete Mühlleitner, Hiroto Shibuya

SUSY 2024

Theory meets Experiment

The 31th International Conference on Supersymmetry and Unification of Fundamental Interactions

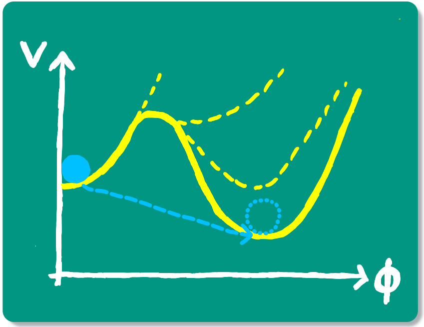
Madrid, 14 June 2024

Outline

1. Motivation
   - Early Universe
   - The CP-conserving 2HDM and its phases
2. Temperature corrections
3. Scanning for intermediate CB phases using the full $V_{\mathrm{eff}}(T)$
   - Setup of scans
   - Benchmark points
4. Summary

Evolution of the Universe around the electroweak epoch

How did the hot early Universe evolve around the electroweak epoch?



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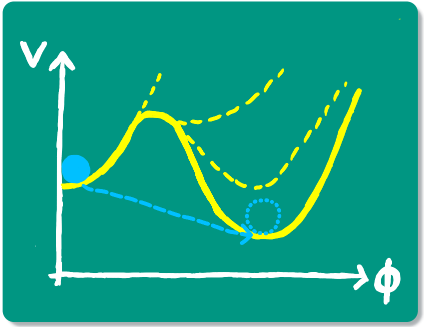

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  EW-symmetric (high T) $\longrightarrow$ neutral
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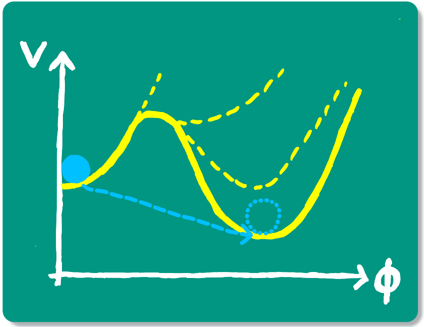

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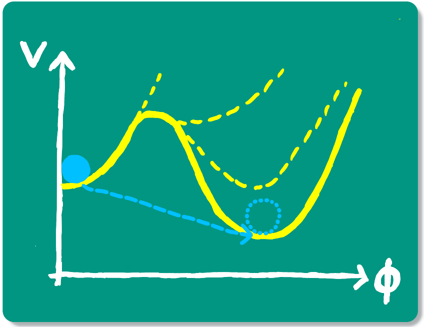

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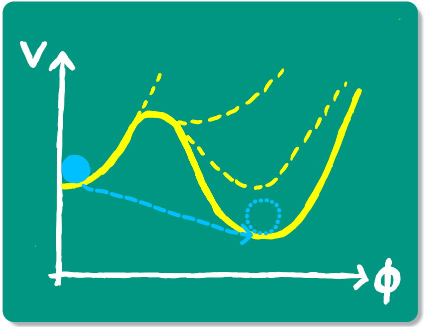

Excellent testbed for BSM physics with extended scalar sectors

The CP-conserving 2HDM (type I) with softly broken $\mathbb{Z}_2$ symmetry

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c.\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

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with

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \bar{\omega}_1 + i\psi_1 \end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_2 + \bar{\omega}_{\text{CB}} + i\eta_2 \\ \zeta_2 + \bar{\omega}_2 + i\left(\psi_2 + \bar{\omega}_{\text{CP}}\right) \end{pmatrix}$$

and real fields ρ_i, η_i, ζ_i, ψ_i ($i = 1, 2$), and VEVs $\bar{\omega}_j$ ($j = 1, 2, \text{CP}, \text{CB}$)

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- Present-day EW-breaking vacuum at zero temperature $T = 0$ (with $v_j \equiv \bar{\omega}_j|_{T=0}$):

$$v_{\text{CB}} = v_{\text{CP}} = 0 \quad \text{and} \quad v^2 \equiv v_1^2 + v_2^2 = (246.22\ \text{GeV})^2 \quad \text{and} \quad \tan\beta \equiv v_2/v_1$$

Phases in the 2HDM

Type of vacuum	$\sqrt{2}\,\langle\Phi_1\rangle$	$\sqrt{2}\,\langle\Phi_2\rangle$
Neutral EW-symmetric	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
Neutral EW-breaking	$\begin{pmatrix}0\\\bar{\omega}_1\end{pmatrix}$	$\begin{pmatrix}0\\\bar{\omega}_2\end{pmatrix}$
CP-breaking	$\begin{pmatrix}0\\\bar{\omega}_1\end{pmatrix}$	$\begin{pmatrix}0\\\bar{\omega}_2+i\bar{\omega}_{CP}\end{pmatrix}$
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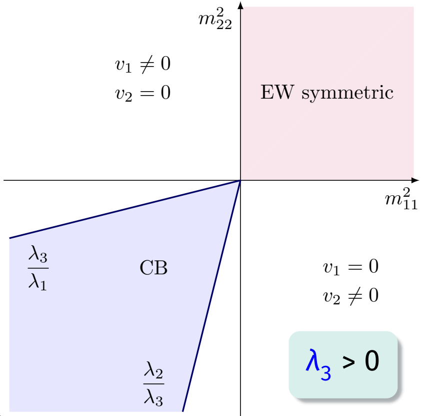
Toy model with $m_{12}^2 = 0$:



(derive e.g. with geometric methods *[Ivanov '08]*)

Phases in the 2HDM

- **Bounded-from-below conditions:**

$$\lambda_{1,2} > 0\,, \quad \sqrt{\lambda_1 \lambda_2} + \lambda_3 > 0\,,$$
$$\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 - \lambda_5 > 0$$

- **Conditions for a CB vacuum:**

$$\sqrt{\lambda_1 \lambda_2} - \lambda_3 > 0\,, \quad \lambda_4 > |\lambda_5|$$

and

$$m_{11}^2 \sqrt{\lambda_2} + m_{22}^2 \sqrt{\lambda_1} < 0\,,$$
$$m_{11}^2 < m_{22}^2 \frac{\lambda_3}{\lambda_2}\,, \quad m_{22}^2 < m_{11}^2 \frac{\lambda_3}{\lambda_1}$$

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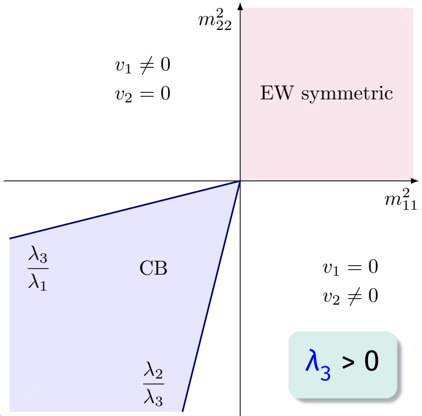

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Thermal evolution of the effective potential

Full one-loop effective potential including thermal corrections:

$$V_{\text{eff}}(T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{CT}} + V_T(T)$$

CW: Coleman-Weinberg potential
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for m_{11}^2 and m_{22}^2

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$$c_i = \frac{1}{12}\left(3\lambda_i + 2\lambda_3 + \lambda_4\right) + \frac{1}{16}\left(3g^2 + g'^2\right) + \delta_{i2}\frac{1}{12}\left(y_\tau^2 + 3y_b^2 + 3y_t^2\right)$$

including gauge and Yukawa couplings

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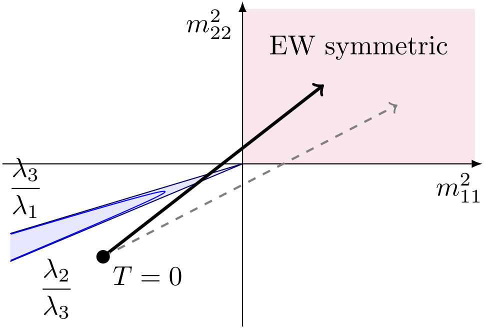


(blue hyperbola: $m_{12}^2 \neq 0$)

Thermal evolution of the effective potential

Full one-loop effective potential including thermal corrections:

$V_{\text{eff}}(T) = V$ …

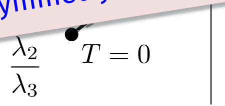
In hi…

f…

Follow-up questions

So far already known [Ivanov ’08; Ginzburg, Ivanov, Kanishev ’09], but:

- Existence of CB phases using full one-loop-corrected effective 2HDM potential (beyond high-T limit)?
- Intermediate CB phases vs. collider constraints?
- Sequences of phase transitions? EW-symmetry restoration at high T?



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$\Rightarrow$ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

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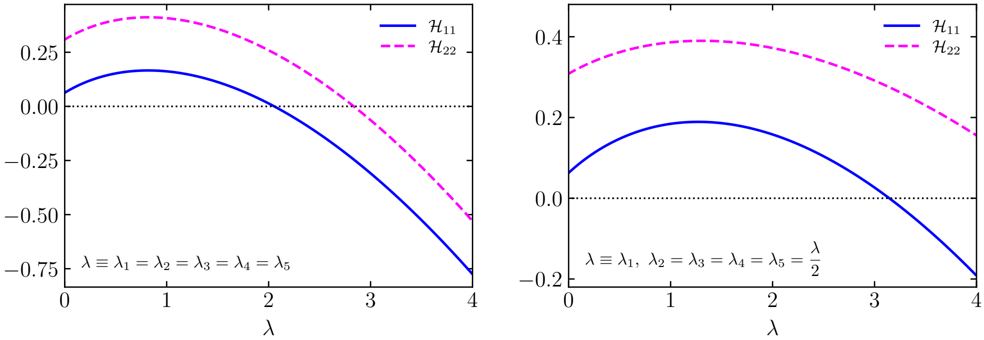

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Scans of the 2HDM parameter space

(1) Generate *seed points* at $T = 0$ and scan over parameter space around them
  = Points with a suitable trajectory for an intermediate CB phase in high-T limit
  - SM VEV and Higgs mass $v = 246.22$ GeV and $m_h = 125.09$ GeV fixed at $T = 0$

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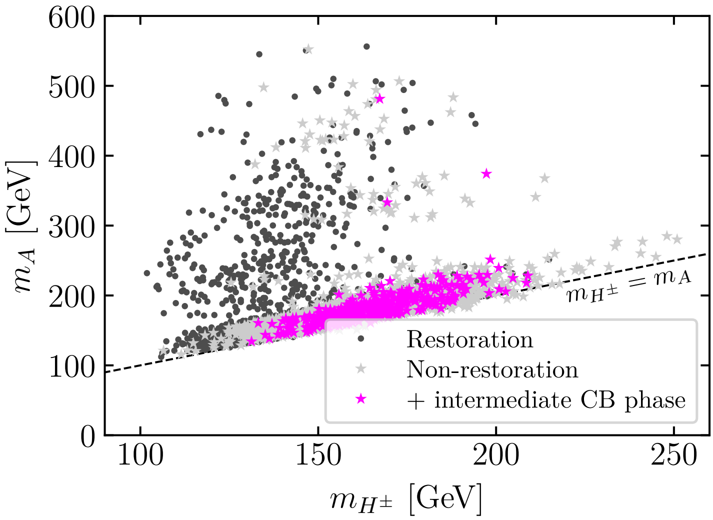
(3) Use ScannerS *[Coimbra et al. '13–'20]* to apply constraints to selected points:

Theoretical constraints:
bounded-from-below, perturbativity, perturbative unitarity *[Akeroyd, Arhrib, Naimi '00]*, absolute stability *[Barroso, Ferreira, Ivanov, Santos '13]*

Experimental constraints:
flavour physics, Higgs searches at colliders, STU-parameters *[Peskin, Takeuchi '92]*

Results of scan



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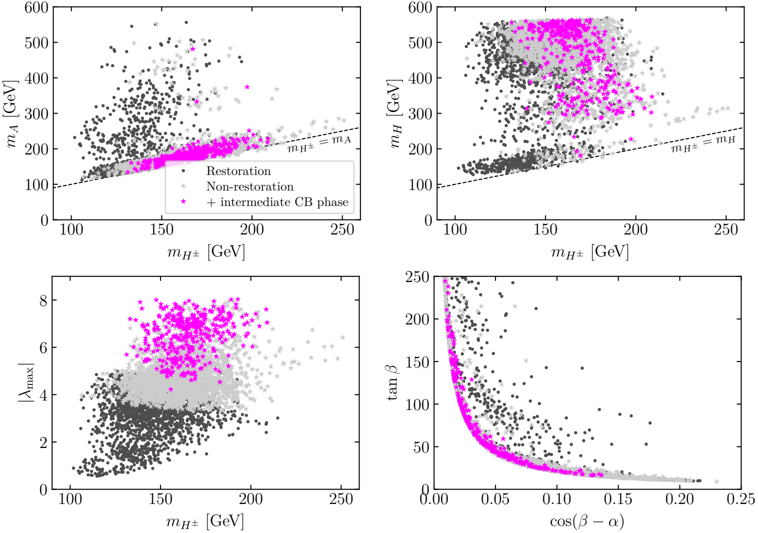

Intermediate CB phase:

- $100 \lesssim \frac{m_{H^\pm}}{\text{GeV}} \lesssim 210$
- $m_{H^\pm} \approx m_A$ or $m_{H^\pm} \approx m_H$
- $|\lambda_{\text{max}}| \gtrsim 4$

$\Rightarrow$ Possibility for $H \to AZ$ and $H \to H^\pm W^\mp$ decays

EW symmetry restoration at high T:

- $|\lambda_{\text{max}}| < 5$

↯ CB phase + constraints

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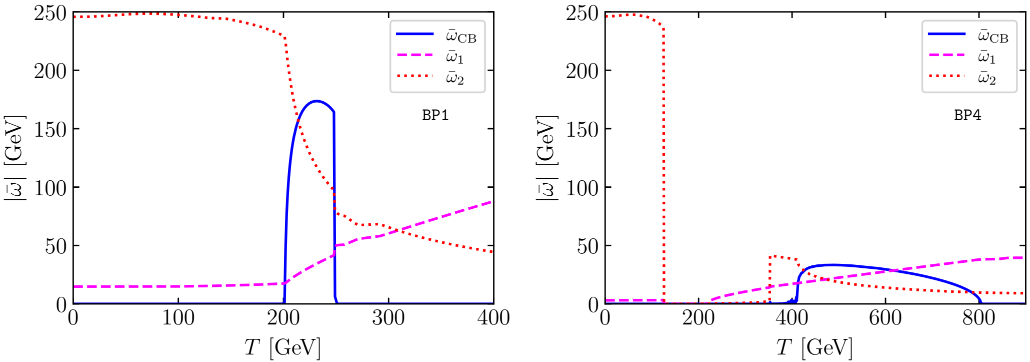
Benchmark points



	m_H (GeV)	m_A (GeV)	$m_{H^\pm}$ (GeV)	$\tan\beta$	$\cos(\beta-\alpha)$	m_{12}^2 (GeV^2)
BP1	562.84	168.56	164.51	16.58	0.128	18933.44
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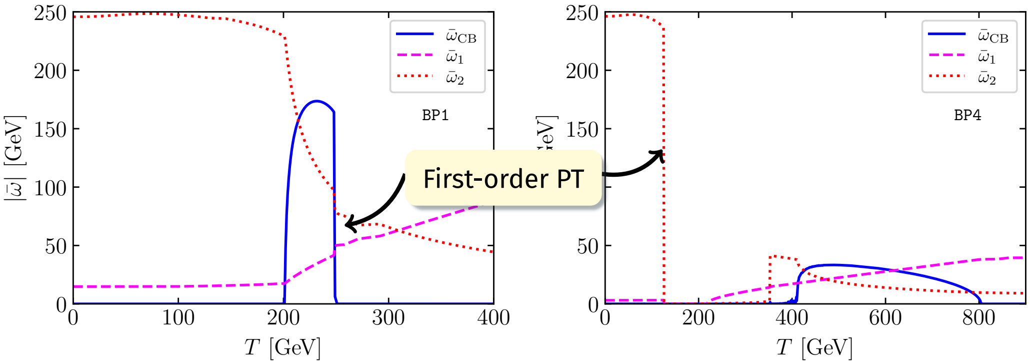

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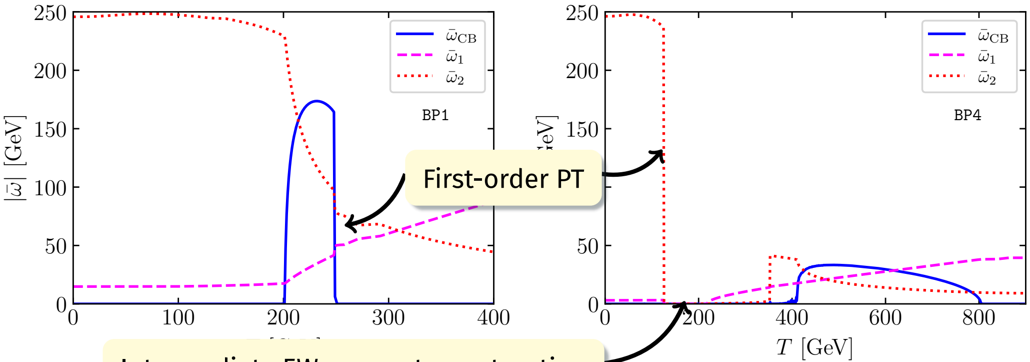

First-order PT

Intermediate EW-symmetry restoration

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Phase transitions including intermediate charge-breaking phases in the 2HDM

- Intermediate CB phases can occur in the CP-conserving 2HDM **with full one-loop thermal corrections**
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See JHEP02(2024)232 for more details!

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THANK YOU FOR YOUR ATTENTION! ☺

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Backup

$\mathbb{Z}_2$ symmetry

To avoid dangerously large flavour-changing neutral currents at tree level:

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	u-type	d-type	leptons
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-Specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

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- m_{12}^2 term in V breaks $\mathbb{Z}_2$ symmetry softly

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$$\Phi_1 \longrightarrow \Phi_1, \quad \Phi_2 \longrightarrow -\Phi_2$$

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	u-type	d-type	leptons
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-Specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

- m_{12}^2 term in V breaks $\mathbb{Z}_2$ symmetry softly

Types of vacua in the 2HDM

Type of vacuum	$\sqrt{2}\,\langle\Phi_1\rangle$	$\sqrt{2}\,\langle\Phi_2\rangle$
Neutral EW-symmetric	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
Neutral EW-breaking	$\begin{pmatrix}0\\v_1\end{pmatrix}$	$\begin{pmatrix}0\\v_2\end{pmatrix}$
CP-breaking	$\begin{pmatrix}0\\\bar{v}_1\end{pmatrix}$	$\begin{pmatrix}0\\\bar{v}_2\,e^{i\theta}\end{pmatrix}$
Charge-breaking (CB)	$\begin{pmatrix}0\\v_1'\end{pmatrix}$	$\begin{pmatrix}\alpha\\v_2'\end{pmatrix}$

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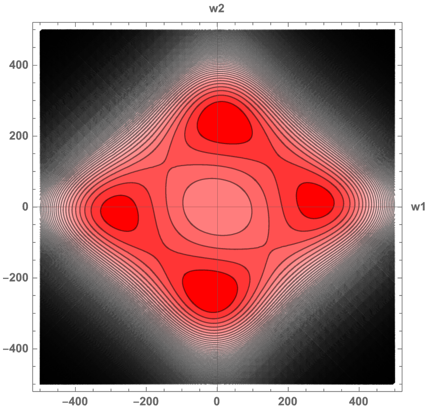

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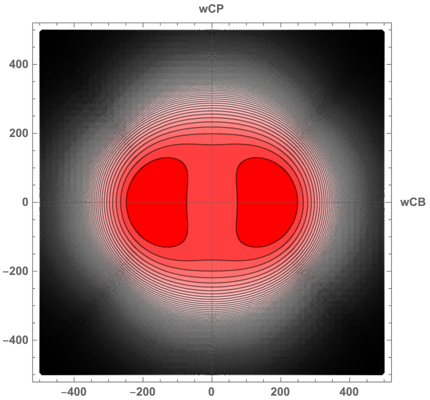


Phase diagram in (m_{11}^2, m_{22}^2) plane

Start with toy model with $m_{12}^2 = 0$; derive e.g. with geometric methods *[Ivanov '08]*:

- **Bounded-from-below conditions:**

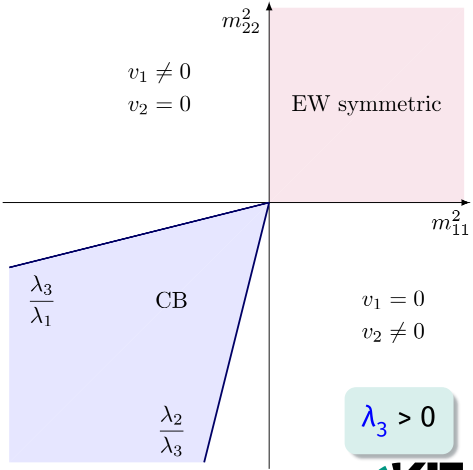
$$\lambda_{1,2} > 0\,, \quad \sqrt{\lambda_1 \lambda_2} + \lambda_3 > 0\,,$$
$$\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 - \lambda_5 > 0$$

- **Conditions for a CB vacuum:**

$$\sqrt{\lambda_1 \lambda_2} - \lambda_3 > 0\,, \quad \lambda_4 > |\lambda_5|$$

and

$$m_{11}^2 \sqrt{\lambda_2} + m_{22}^2 \sqrt{\lambda_1} < 0\,,$$
$$m_{11}^2 < m_{22}^2 \frac{\lambda_3}{\lambda_2}\,, \quad m_{22}^2 < m_{11}^2 \frac{\lambda_3}{\lambda_1}$$



$\lambda_3 > 0$

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Can we classify the different vacua geometrically?

Tilde basis

Introduce rescaled fields with $k^4 \equiv \sqrt{\lambda_2/\lambda_1}$:

$$\Phi_1 = k\tilde{\Phi}_1, \quad \Phi_2 = k^{-1}\tilde{\Phi}_2 \qquad \Leftrightarrow \qquad \tilde{\Phi}_1 = k^{-1}\Phi_1, \quad \tilde{\Phi}_2 = k\Phi_2$$

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Rescaled terms in V:

$$\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 = \lambda_1 k^4\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right)^2 + \lambda_2 k^{-4}\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^2 = \tilde{\lambda}\left[\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right)^2 + \left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^2\right]$$

$$m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) = \tilde{m}_{11}^2\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1\right) + \tilde{m}_{22}^2\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)$$

with $\quad \tilde{\lambda} \equiv \sqrt{\lambda_1\lambda_2}, \quad \tilde{m}_{11}^2 \equiv k^2 m_{11}^2, \quad \tilde{m}_{22}^2 \equiv k^{-2} m_{22}^2$

- Other quartic terms and m_{12}^2 term remain unchanged

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Potential in the tilde basis

$$V = \tilde{m}_{11}^2 \tilde{\Phi}_1^\dagger \tilde{\Phi}_1 + \tilde{m}_{22}^2 \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - m_{12}^2 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2 + h.c.\right) + \frac{\tilde{\lambda}}{2}\left[\left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_1\right)^2 + \left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_2\right)^2\right]$$
$$+ \lambda_3 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_1\right)\left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_2\right) + \lambda_4 \left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2\right)\left(\tilde{\Phi}_2^\dagger \tilde{\Phi}_1\right) + \frac{\lambda_5}{2}\left[\left(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2\right)^2 + \mathrm{h.c.}\right]$$

Bilinear form

Introduce vector $r^\mu = \left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1 + \tilde{\Phi}_2^\dagger\tilde{\Phi}_2,\ 2\,\mathrm{Re}\,\tilde{\Phi}_1^\dagger\tilde{\Phi}_2,\ 2\,\mathrm{Im}\,\tilde{\Phi}_1^\dagger\tilde{\Phi}_2,\ \tilde{\Phi}_1^\dagger\tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger\tilde{\Phi}_2\right)^T$

⇒ **2HDM potential in bilinear form** (following *[Ivanov '06-'08]*):

$$V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu$$

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where ($\Lambda_{\mu\nu}$ diagonal in tilde basis!)

$$M_\mu = (M_0, M_1, M_2, M_3)^T = \left(-\frac{\tilde m_{11}^2 + \tilde m_{22}^2}{2},\ -\mathrm{Re}\, m_{12}^2,\ \mathrm{Im}\, m_{12}^2,\ \frac{\tilde m_{11}^2 - \tilde m_{22}^2}{2}\right)^T$$

$$\Lambda_{\mu\nu} = \mathrm{diag}\left(\Lambda_0, -\Lambda_1, -\Lambda_2, -\Lambda_3\right) = \mathrm{diag}\left(\frac{\tilde\lambda + \lambda_3}{2}, \frac{-\lambda_4 - \lambda_5}{2}, \frac{-\lambda_4 + \lambda_5}{2}, \frac{-\tilde\lambda + \lambda_3}{2}\right)$$

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- 'Bounded-from-below' (BFB): $\Lambda_0 > 0\,, \quad \Lambda_0 > \Lambda_i \quad$ for $i = 1, 2, 3$ (Λ_i can be < 0)

Physical configurations

It follows from the definition of r^μ (Schwarz inequality):

$$r_0 \geq 0, \quad r_\mu r^\mu = r_0^2 - \sum_i r_i^2 \geq 0$$

$\Rightarrow$ Physically realisable configurations inside/on the future lightcone

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(ii) Neutral EW-breaking: $\langle r_\mu \rangle \neq 0$ and $\langle r_\mu \rangle \langle r^\mu \rangle = 0$ (on the surface of the cone)

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Evaluation of minima: (ii) neutral EW-breaking vacuum

Add constraint to potential (*to* surface of cone, $r_\mu r^\mu = r_0^2 - \sum_i r_i^2 = 0$):

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- Equation has up to 6 solutions for ζ/extrema ⇒ extract minimum numerically

Evaluation of minima: (iii) charge-breaking vacuum

From $\frac{dV}{dr^\mu} \stackrel{!}{=} 0$, it follows:

$$\Lambda_0 r_0 = M_0 , \quad \Lambda_i r_i = M_i$$

(minimum: $\Lambda_i < 0$)

Evaluation of minima: (iii) charge-breaking vacuum

From $\frac{dV}{dr^\mu} \overset{!}{=} 0$, it follows:

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(minimum: $\Lambda_i < 0$)

- Plugging solution into inequalities $r_0 > 0$ and $r_0^2 - \sum_i r_i^2 > 0$:

$$M_0 > 0 \qquad \text{and} \qquad \frac{M_0^2}{\Lambda_0^2} > \sum_i \frac{M_i^2}{\Lambda_i^2}$$

or, slightly rewritten,

$$\tilde{m}_{11}^2 + \tilde{m}_{22}^2 < 0 \qquad \text{and} \qquad \frac{\mu_1^2}{a_1^2} + \frac{\mu_2^2}{a_2^2} + \frac{\mu_3^2}{a_3^2} < 1$$

with $\mu_i^2 = \frac{M_i^2}{M_0^2}$, $a_i^2 = \frac{\Lambda_i^2}{\Lambda_0^2}$

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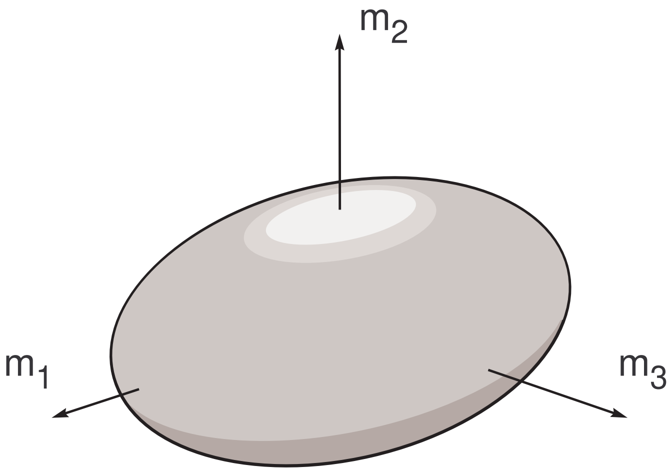
⇒ Points that lie inside an ellipsoid with semi-axes $a_{1,2,3}$

Evaluation of minima: (iii) charge-breaking vacuum: ellipsoid



[Ivanov '08]

Toy model: $\mathbb{Z}_2$-symmetric 2HDM with $m_{12}^2 = 0$

For a simpler graphical representation, discuss **toy model: $\mathbb{Z}_2$-symmetric 2HDM**

$$V_{\text{toy}} = V\big|_{m_{12}^2=0}$$

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Constraints

- Bounded-from-below:

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- CB minimum:

$$\sqrt{\lambda_1\lambda_2} - \lambda_3 > 0\,, \quad \lambda_4 > |\lambda_5|$$

and

$$m_{11}^2\sqrt{\lambda_2} + m_{22}^2\sqrt{\lambda_1} < 0\,, \quad m_{11}^2 < m_{22}^2\frac{\lambda_3}{\lambda_2}\,, \quad m_{22}^2 < m_{11}^2\frac{\lambda_3}{\lambda_1}$$

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Effective potential at finite temperatures

Full one-loop and thermally corrected effective potential:

$$V_{1L} = V + V_{\text{CW}} + V_{\text{CT}} + V_T$$

with

- V_{CW}: T-independent one-loop Coleman-Weinberg potential
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Perturbative expansion becomes unreliable at high T

- Resum ‘Daisy‘ diagrams (‘Arnold-Espinosa‘ method) to recover perturbativity

$\Rightarrow$ Certain mass eigenvalues obtain T-dependent contributions

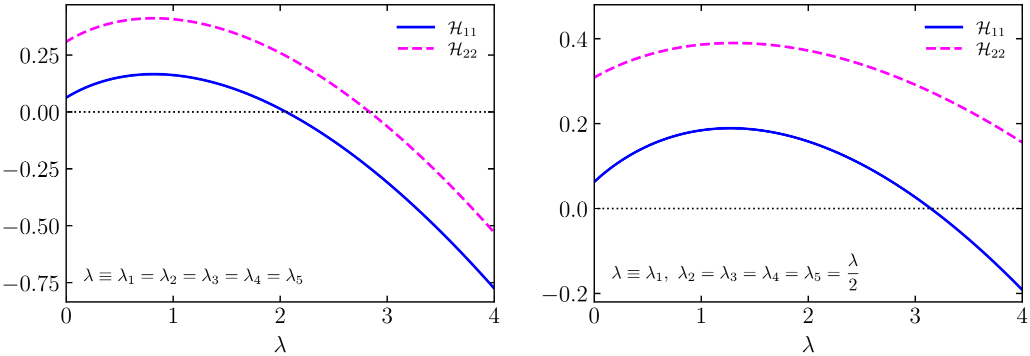
One-loop thermal corrections for $T \to \infty$

$$V_T \overset{T\to\infty}{\approx} -\sum_k n_k \begin{cases} \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}\overline{m}_k^3 T & k = H^\pm, h, H, A, W_L, Z_L, \gamma_L \\ \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}m_k^3 T & k = W_T, Z_T \\ \frac{7\pi^2}{720}T^4 - \frac{1}{48}m_k^2T^2 & k = t, b, \tau \end{cases}$$

with

- n_k: number of d.o.f.s of field k
- $\overline{m}_k$ (m_k): mass eigenvalue for field k including (excluding) thermal Debye corrections from Daisy resummation

Electroweak symmetry (non-)restoration in the 2HDM



⇒ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

Scans of the 2HDM parameter space

Parameter	Scan range
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	$[0, 4\pi]$
λ_5	$[-4\pi, 4\pi]$
m_{11}^2, m_{22}^2	$[-10^6, 0]$ GeV^2
m_{12}^2	$[0, 10^6]$ GeV^2

- Random "smart" scan over parameter space
  → Get VEV v_0 and light CP-even Higgs mass $m_{h,0}$
- Rescale to get $v = 246.22$ GeV, $m_h = 125.09$ GeV:

$$m_{ij}^2 \to m_{ij}^2 \frac{m_h^2}{m_{h,0}^2}, \quad \lambda_k \to \lambda_k \frac{m_h^2}{m_{h,0}^2} \frac{v_0^2}{v^2}$$

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  - Perturbative unitarity *[Akeroyd, Arhrib, Naimi '00]*
  - Absolute stability *[Barroso, Ferreira, Ivanov, Santos '13]*

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⇒ **Seed points**