GUTs – how common are they?

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Max-Planck-Institut für Kernphysik Heidelberg

The Puzzle

Experiment: SU(3) × SU(2) × U(1)

$$3\times \quad \begin{pmatrix} \nu \\ e_L \end{pmatrix} \; e_R \; \begin{pmatrix} u_{L,1} u_{L,2} u_{L,3} \\ d_{L,1} d_{L,2} d_{L,3} \end{pmatrix} \; \left(d_{R,1} d_{R,2} d_{R,3}\right) \; \left(u_{R,1} u_{R,2} u_{R,3}\right)$$

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What are the odds!?

Motivation

How common are "unifiable" fermions among "Standard Model like" theories?

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silly question

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interesting answer

- if common: no need to be surprised by grand unifiability, can stay GUT agnostic
- if rare: purely group-theoretical bottom-up indication for Grand Unification

Outline

Assumptions

- "unifiability"
- "SM-like" theories

Methods

- construct consistent theories
- determine their unifiability

Results

- fraction of theories that are unifiable
- discuss dependence on assumptions

Unifiability

observation

1. fermion sector of the SM *remarkably complete*
   - anomaly free generation-by-generation
   - no evidence for BSM fermios charged under SM
   - LHC: new fermions chiral under SM all but ruled out by Higgs measurements
2. SM *fermions unify neatly* into representation of SU(5)
   - unification of all gauge forces (simple group)
   - no additional fermions in GUT rep

$\longrightarrow$ *closure*

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→ *closure*

How common is neat unifiability of fermions among SM-like theories?

→ *we do not consider gauge coupling unification*

SM-like theories

characteristic features

- 3 gauge forces $\sim$ reductive rank-4 gauge algebra
- $D = 15$ fermions per generation
- three separately anomaly free generations
- integer* hypercharges $|Q| \leq 6$
- fermion representation is chiral

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SM-like theories (single generation)

- anomaly-free reps of SM-like gauge group
- similar number of fermions
- similar range of charges
- not just semisimple-singlets
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SM-like theories (single generation)

- anomaly-free reps of SM-like gauge group — SM (for starters)
- similar number of fermions — $D \leq 20$
- similar range of charges — $|Q| \leq 10$
- not just semisimple-singlets — number of SS-singlets $S \leq 5$
- fermions not completely vector-like

⇒ result depends on these assumptions; can be discussed

Base set of anomaly-free representations

1. find all non-anomalous reps of semisimple part ($SU(3) \times SU(2)$)
2. assign $U(1)$ charges, keep those that satisfy anomaly cancellation
3. remove equivalent representations
   - rescaling of $U(1)$ charge, conjugate reps

$\Rightarrow$ easy using Mathematica packages SuperFlocci or GroupMath.

Some subtleties in efficient $U(1)$ charge assignment.

thanks to Joseph Tooby-Smith!

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examples

$\{\mathbf{1} \otimes \mathbf{2} \otimes 0, \ \mathbf{3} \otimes \mathbf{1} \otimes -1, \ \overline{\mathbf{3}} \otimes \mathbf{1} \otimes 1\}$ smallest, $D = 8$

$\{\mathbf{3} \otimes \mathbf{2} \otimes 0, \ \overline{\mathbf{3}} \otimes \mathbf{1} \otimes -1, \ \overline{\mathbf{3}} \otimes \mathbf{1} \otimes 1\}$ smallest chiral, $D = 12$

$\{\mathbf{1} \otimes \mathbf{1} \otimes -6, \ \mathbf{1} \otimes \mathbf{2} \otimes 3, \ \overline{\mathbf{3}} \otimes \mathbf{2} \otimes -1, \ \mathbf{3} \otimes \mathbf{1} \otimes -2, \ \mathbf{3} \otimes \mathbf{1} \otimes 4\}$ you know this one, $D = 15$

SuperFlocci – checking unifiability bottom-up

https://github.com/jstoobysmith/Superfloccinaucinihilipilification [2306.16439]

Semisimple unifications of any gauge theory

Andrew Gomes,[*] Maximilian Ruhdorfer,[†] and Joseph Tooby-Smith[‡]
Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York 14853, USA

We present a *Mathematica* package that takes any reductive gauge algebra and fully-reducible fermion representation, and outputs all semisimple gauge extensions under the condition that they have no additional fermions, and are free of local anomalies. These include all simple completions, also known as grand unified theories. We additionally provide a list of all semisimple completions for 5835 fermionic extensions of the one-generation Standard Model.

$\Rightarrow$ *plug in every theory in base set and check for simple gauge extension*

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```
SuperFlocci Output
Program by: Andrew Gomes, Maximillian Ruhdorfer, and Joseph Tooby-Smith. 2022

Input algebra: su(2)⊕su(3)⊕u(1)
Input representation: (2,3,0)⊕(1,3̄,-1)⊕(1,3̄,1)
Date of generation: Sun 9 Jun 2024 11:58:36

Maximal algebras:
1) su(2)⊕su(2)⊕su(3)  (1,2,3)⊕(2,1,3̄)

Minimal algebras:
```

```
SuperFlocci Output
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Input algebra: su(2)⊕su(3)⊕u(1)
Input representation: (1,1,-6)⊕(2,1,3)⊕(2,3̄,-1)⊕(1,3,-2)⊕(1,3,4)
Date of generation: Sun 9 Jun 2024 12:00:08

Maximal algebras:
1) su(5)  (5̄)⊕(10)
```

GroupMath – top-down decoposing all candidate GUTs

https://renatofonseca.net/groupmath [R.Fonseca'2011.01764]

- candidate GUTs with non-singlet fermion rep with $D \leq D_{\text{max}}$

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{{SU2, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}}, {SU3, {1, 3, $\overline{3}$, 6, $\overline{6}$, 8, 10, $\overline{10}$, 15, $\overline{15}$, 15', $\overline{15'}$}},
{SU4, {1, 4, $\overline{4}$, 6, 10, $\overline{10}$, 15, 20, $\overline{20}$, 20', 20'', $\overline{20''}$}}, {SU5, {1, 5, $\overline{5}$, 10, $\overline{10}$, 15, $\overline{15}$}}, {SU6, {1, 6, $\overline{6}$, 15, $\overline{15}$, 20}}, {SU7, {1, 7, $\overline{7}$}},
{SU8, {1, 8, $\overline{8}$}}, {SU9, {1, 9, $\overline{9}$}}, {SU10, {1, 10, $\overline{10}$}}, {SU11, {1, 11, $\overline{11}$}}, {SU12, {1, 12, $\overline{12}$}}, {SU13, {1, 13, $\overline{13}$}}, {SU14, {1, 14, $\overline{14}$}},
{SU15, {1, 15, $\overline{15}$}}, {SU16, {1, 16, $\overline{16}$}}, {SU17, {1, 17, $\overline{17}$}}, {SU18, {1, 18, $\overline{18}$}}, {SU19, {1, 19, $\overline{19}$}}, {SU20, {1, 20, $\overline{20}$}},
{SP4, {1, 4, 5, 10, 14, 16, 20}}, {SP6, {1, 6, 14, 14'}}, {SP8, {1, 8}}, {SP10, {1, 10}}, {SP12, {1, 12}}, {SP14, {1, 14}}, {SP16, {1, 16}},
{SP18, {1, 18}}, {SP20, {1, 20}}, {SO5, {1, 4, 5, 10, 14, 16, 20}}, {SO6, {1, 4, $\overline{4}$, 6, 10, $\overline{10}$, 15, 20, $\overline{20}$, 20', 20'', $\overline{20''}$}},
{SO7, {1, 7, 8}}, {SO8, {1, 8_v, 8_s, 8_c}}, {SO9, {1, 9, 16}}, {SO10, {1, 10, 16, $\overline{16}$}}, {SO11, {1, 11}}, {SO12, {1, 12}}, {SO13, {1, 13}},
{SO14, {1, 14}}, {SO15, {1, 15}}, {SO16, {1, 16}}, {SO17, {1, 17}}, {SO18, {1, 18}}, {SO19, {1, 19}}, {SO20, {1, 20}}, {G2, {1, 7, 14}}}

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- GroupMath computes all distinct decompositions, fast!

```
In[ ]:= DecomposeRep[SO10, -16, {SU3, SU2, U1}]
```

There are 2 non-equivalent ways of embedding {SU3, SU2, U1} in SO10.

Under each of them, the representation

-16 decomposes as follows (x1 is a free real number):

Embedding	Decomposition					
#1	$3\otimes 2\otimes -1$	$\overline{3}\otimes 1\otimes (1+x1)$	$\overline{3}\otimes 1\otimes (1-x1)$	$1\otimes 2\otimes 3$	$1\otimes 1\otimes (-3+x1)$	$1\otimes 1\otimes (-3-x1)$
#2	$3\otimes 2\otimes -1$	$\overline{3}\otimes 2\otimes 1$	$1\otimes 2\otimes 3$	$1\otimes 2\otimes -3$		

- assign charges and count

First result

naive result:

$$\frac{\text{\#neatly unifiable reps}}{\text{\#all SM-like reps}} = \frac{61}{4541567} \simeq 10^{-5} \qquad (D \leq 20, |Q| \leq 10, S \leq 5)$$

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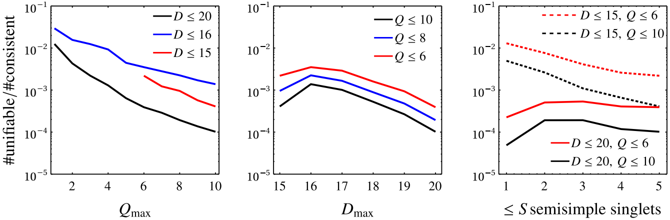

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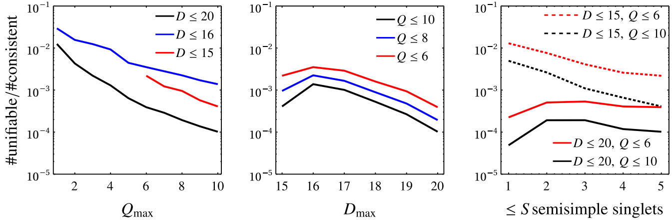


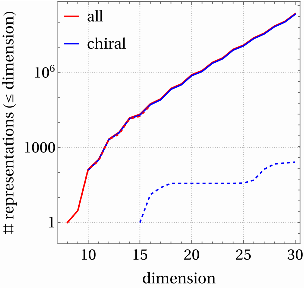
most conservative result:

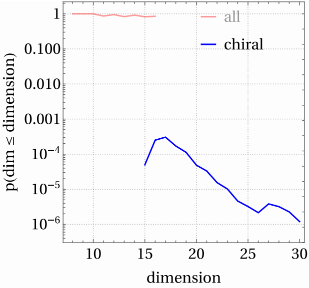
$$\frac{\#\text{neatly unifiable reps}}{\#\text{all SM-like reps}} = \frac{1}{76} \simeq \mathbf{10^{-2}} \qquad (D \leq 15, |Q| \leq 6, S \leq 1)$$

$\Rightarrow$ rare at "2σ" level.

Top-down strikes back – the role of chirality

- using GroupMath, we can extend analysis to $D \leq 30$, as well as consider VL theories





restricted to $\tilde{S} \leq 4$ same SS reps here

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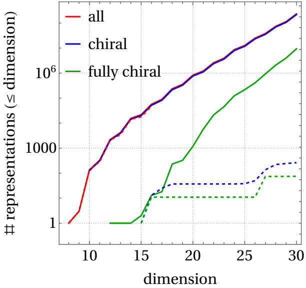
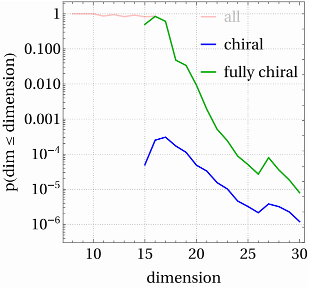
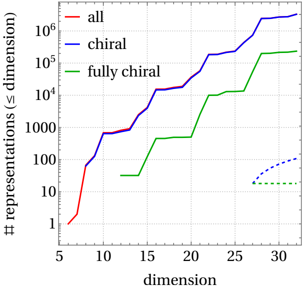




restricted to $\tilde{S} \leq 4$ same SS reps here

Is SU(3) × SU(2) × U(1) special?

$SU(3) \times U(1)$



- need to go to larger D to find unifiable chiral theory
- there, similar result as SM group

further groups → work in progress

$$3\times \quad \begin{pmatrix} \nu \\ e_L \end{pmatrix} \; e_R \; \begin{pmatrix} u_{L,1} u_{L,2} u_{L,3} \\ d_{L,1} d_{L,2} d_{L,3} \end{pmatrix} \; \begin{pmatrix} d_{R,1} d_{R,2} d_{R,3} \end{pmatrix} \; \begin{pmatrix} u_{R,1} u_{R,2} u_{R,3} \end{pmatrix}$$

It unifies. Surprised?

$$3\times \quad \begin{pmatrix} d^c_{R,1} \\ d^c_{R,1} \\ d^c_{R,1} \\ e_L \\ -\nu \end{pmatrix} \qquad \begin{pmatrix} 0 & u^c_{R,3} & -u^c_{R,2} & u_{L,1} & d_{L,1} \\ -u^c_{R,3} & 0 & u^c_{R,1} & u_{L,2} & d_{L,2} \\ u^c_{R,2} & -u^c_{R,1} & 0 & u_{L,3} & d_{L,3} \\ -u_{L,1} & -u_{L,2} & -u_{L,3} & 0 & e^c_R \\ -d_{L,1} & -d_{L,2} & -d_{L,3} & -e^c_R & 0 \end{pmatrix}$$

"Evidence"?!

Conclusions

- The SM fermions unify neatly into a representation of a simple group
- *Is that surprising?*
  - *finding an SM-like theory to be neatly unifiable is $\mathcal{O}(1/100)$ odd, but not impossibly rare*
  - among *fully chiral* fermion representations, no need to be surprised by unifiability
- Bottom-up indication for Grand Unification, relying only on group theory
  - without measure in theory space → no "evidence"
- Result comparable to a fine-tuning measure

We wish to thank Joseph Tooby-Smith for collaboration during the early stages of this project and for coding support throughout.