# Entanglement and Bell Inequality Violation in Semi-Leptonic Top Decays

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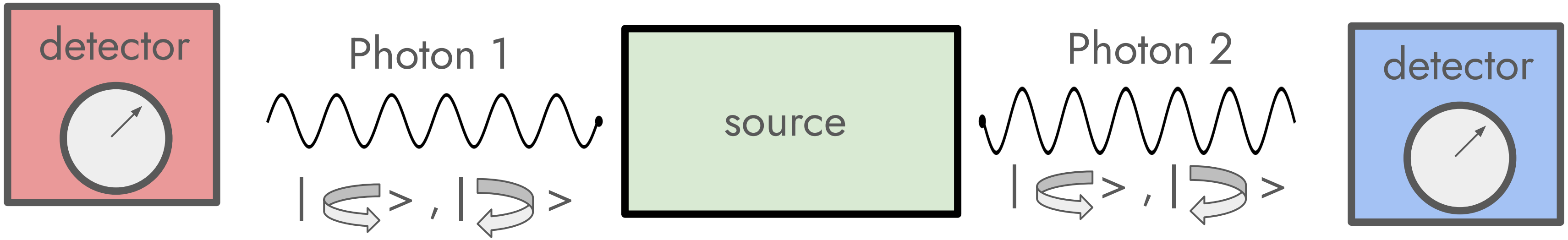
arXiv:2310.17696

arXiv:2311.09166

with Kun Cheng, Tao Han, Arthur Wu

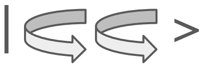
# Entanglement

- Einstein's "spooky action at a distance"
- Can only describe subsystem A with knowledge of subsystem B
- Usually measured with photons or electrons




- Examples:

*Not* entangled

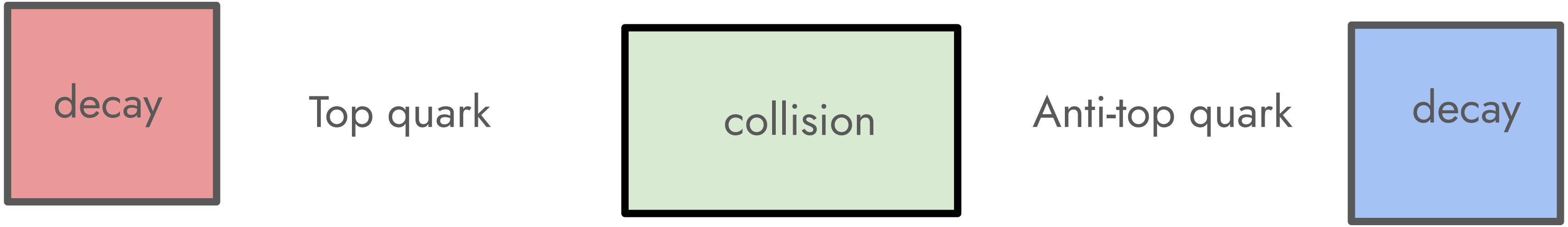


*Entangled*

# Entanglement

See talks by Alan Barr and James Ferrando

- Let the spin of the *top* be a qubit and let the spin of the *anti-top* be a qubit




- Parametrize the tt system by: scattering angle $\theta$ and invariant mass $m_{\bar{t}t}$
- Each point $(\theta, m_{\bar{t}t})$ is quantum state $\rho(\theta, m_{\bar{t}t})$
- Integrate to obtain *total quantum state*

Afik, de Nova 2003.02280

- With the same procedure, can test Bell Inequalities

Fabbrichesi, Floreanini, Panizzo 2102.11883
Barr 2106.01377

See also Abel, Dittmar, Dreiner 1992

# Measuring the Density Matrix

- Quantum state described by density matrix $\rho = \sum |\psi\rangle\langle\psi|$
- Can be decomposed in Pauli basis

$$\rho = \frac{1}{4}\left(\mathbf{I}_4 + B_i^+ \sigma_i \otimes \mathbf{I}_2 + B_i^- \mathbf{I}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j\right)$$

$B_i^+$ Polarization vector of qubit A

$B_i^-$ Polarization vector of qubit B

$C_{ij}$ Spin correlation matrix

- Reconstruct density matrix by measuring spins
- At a colliders, measuring spins *statistically* by angles of decay products
- Example:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \rightarrow \quad B_i^+ = \begin{pmatrix}0\\0\\0\end{pmatrix} \quad B_i^- = \begin{pmatrix}0\\0\\0\end{pmatrix} \quad C_{ij} = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

# Measuring the Density Matrix

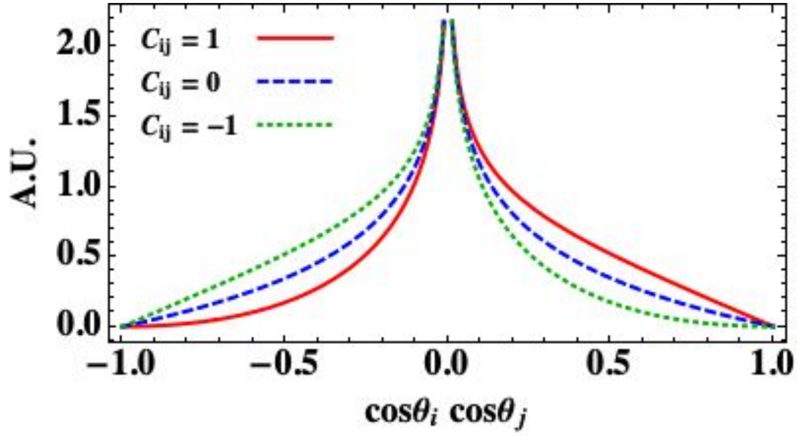
- *Differential* cross section (with respect to decay angles)

$$\frac{1}{\sigma}\frac{d^4\sigma}{d^2\Omega^{\mathcal{A}}d^2\Omega^{\mathcal{B}}} = \frac{1}{(4\pi)^2}\left(1 + \sum_i \left(B_i^{\mathcal{A}}\Omega_i^{\mathcal{A}} + B_i^{\mathcal{B}}\Omega_i^{\mathcal{B}}\right) + \sum_{i,j}\Omega_i^{\mathcal{A}}C_{ij}\Omega_j^{\mathcal{B}}\right)$$

- Partially integrate to isolate parameters

$$\frac{1}{\sigma}\frac{d\sigma}{d(\cos\theta_i^{\mathcal{A}}\cos\theta_j^{\mathcal{B}})} = -\frac{1}{2}\left(1 + C_{ij}\cos\theta_i^{\mathcal{A}}\cos\theta_j^{\mathcal{B}}\right)\log\left|\cos\theta_i^{\mathcal{A}}\cos\theta_j^{\mathcal{B}}\right|$$

- Measure angles to extract parameters



# Testing Entanglement and Bell Inequalities

- **Entanglement**: Peres-Horodecki criterion (Positive Partial Transpose)
  - Apply transpose to subsystem B
  - If the resulting matrix corresponds to a state (→ separable), if not (→ entangled)
  - Inequality using elements of spin correlation matrix

$$C_{11} + C_{22} + C_{33} < -1$$

(tt near threshold)

- **Bell Inequality Violation**: Clauser-Horne-Shimony-Holt inequality
  - Make 4 measurements $\langle A_1 B_1 \rangle, \langle A_1 B_2 \rangle, \ldots$
  - Test the inequality $|\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle| \leq 2$
  - Inequality using elements of spin correlation matrix

$$\sqrt{2}|C_{11} - C_{22}| \leq 2$$

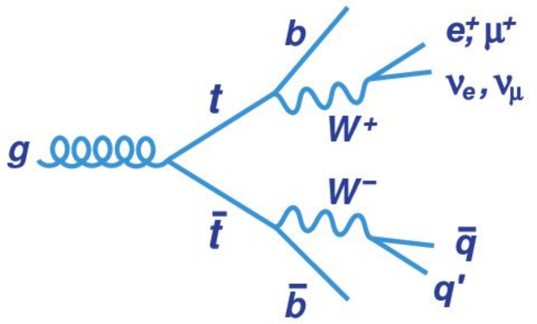
# Top-Antitop Final States

- Fully Leptonic Top Decays
  - Lepton carries maximal spin information of the top
  - "Clean" final state
  - Existing spin correlation measurements by CMS (prior to present entanglement work)

Fabbrichesi, Floreanini, Panizzo 2102.11883
Severi et al 2110.10112
Afik, de Nova 2203.05582
Aguilar-Saavedra, Casas 2205.00542

- Semi-leptonic Top Decays
  - Lose some spin information from the hadronic top
  - Larger branching ratio
  - Only one invisible particle in event

← **Our work**



# Back of the Envelope

- Sensitivity estimated by

$$\frac{\mathcal{O}_{\text{measured}} - \mathcal{O}_{\text{separable}}}{\Delta\mathcal{O}}$$

- Measurement *degraded* by using angle of jets (instead of lepton)

$$\mathcal{O}_{\text{measured}} \to (0.64)\mathcal{O}_{\text{measured}}$$

- Branching ratio *improves* uncertainty by sqrt(num events)

$$\Delta\mathcal{O} \to \left(\frac{1}{2.5}\right)\Delta\mathcal{O}$$

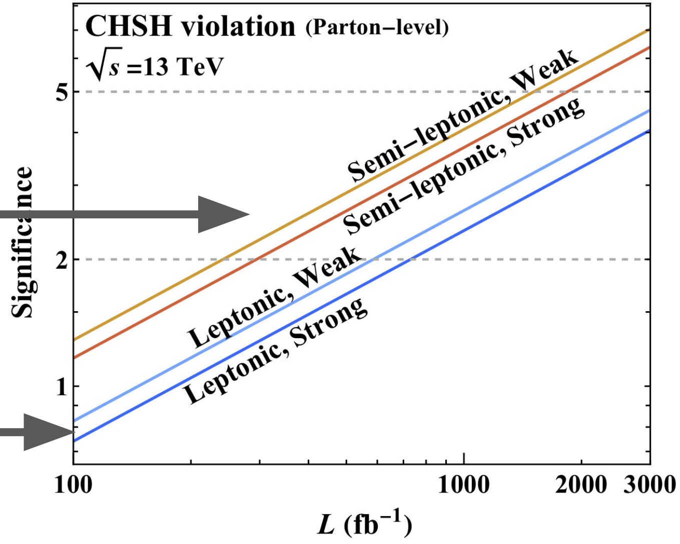
- Sensitivity naively expected to ***improve*** by a factor **1.6**

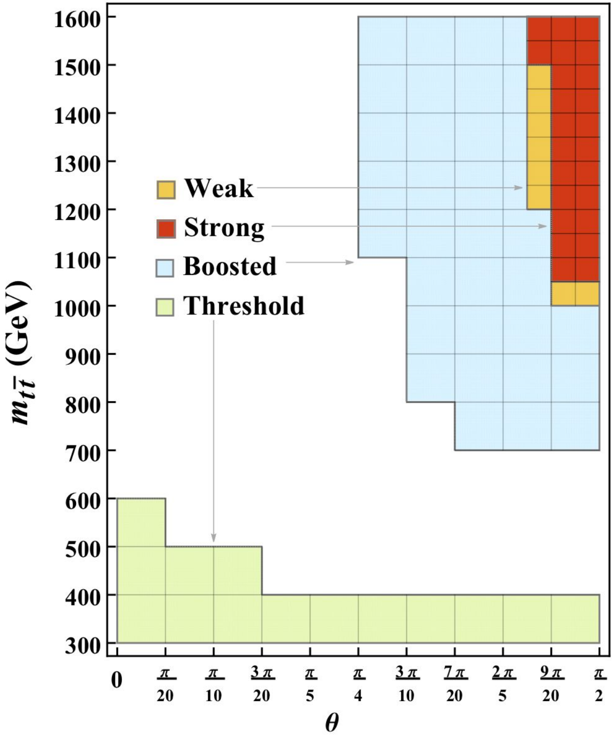
# Simulation

- Step one: parton-level study
  - Madgraph, Madspin, no event selection
  - Signal region in $(\theta, m_{\bar{t}t})$ space
  - Extract spin correlation coefficients
  - Scale results down by reconstruction efficiency

New semi-leptonic channel

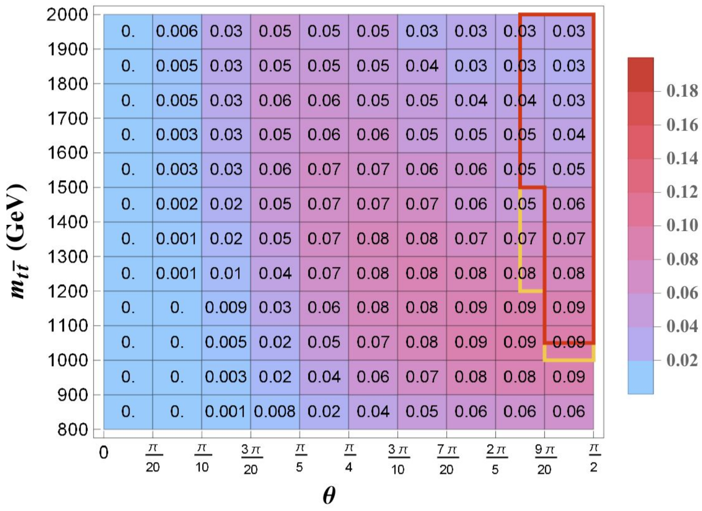
Previously studied channel



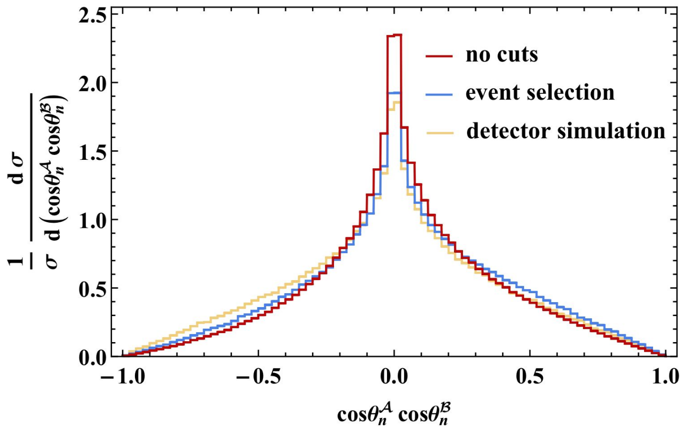



# Simulation

- Step two: detector-level study
  - Pythia, Delphes, event selection
  - Signal region in $(\theta, m_{\bar{t}t})$ space
  - Calculate reconstruction efficiency
  - Apply parametric fitting (i.e. unfolding)

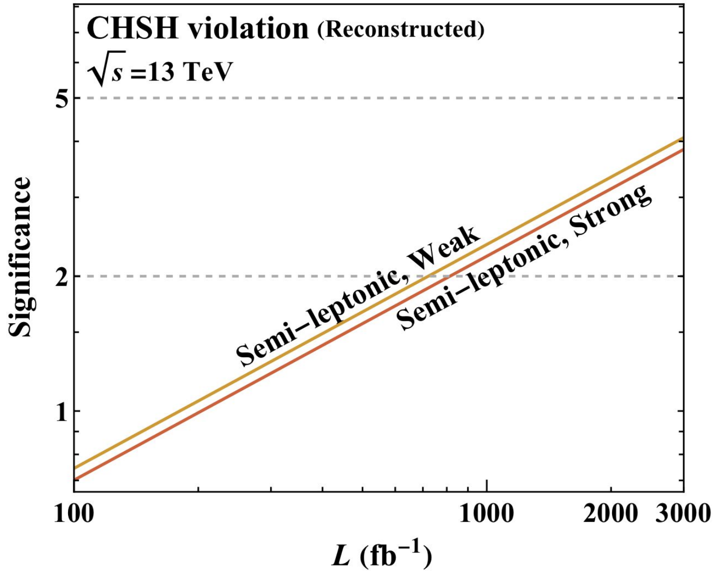



Parametric fitting is essential and inflates the uncertainty



# Results

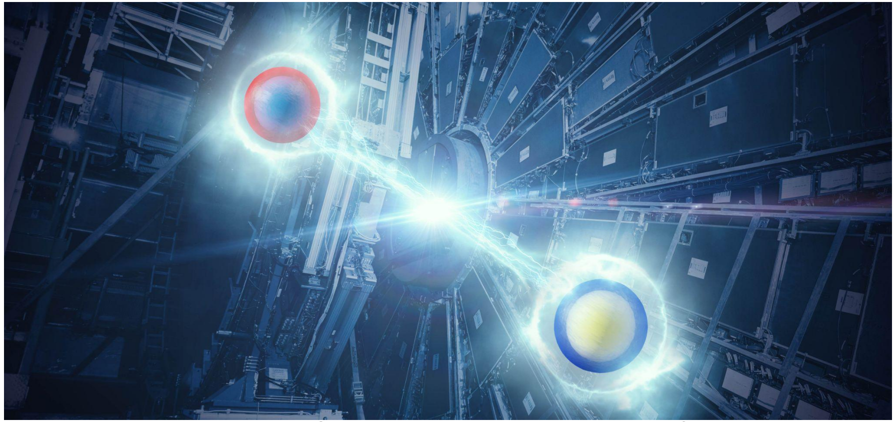
- Bell inequality violation
  - Degraded slightly from parton-level
  - Needs additional improvements to be discoverable at HL-LHC
  - Factor of 3x more sensitive than leptonic channel




| Reconstructed | $N_{\text{detected}}$ (300 fb$^{-1}$) | $B - \sqrt{2}$ (Individual) | $B - \sqrt{2}$ (Direct) | Significance (300 fb$^{-1}$) | Significance (3000 fb$^{-1}$) |
|---|---|---|---|---|---|
| Weak | 6280 | $0.23 \pm 0.18$ | $0.22 \pm 0.22$ | $1.3\sigma$ | $4.1\sigma$ |
| Strong | 4127 | $0.27 \pm 0.22$ | $0.25 \pm 0.28$ | $1.2\sigma$ | $3.8\sigma$ |

# Conclusions

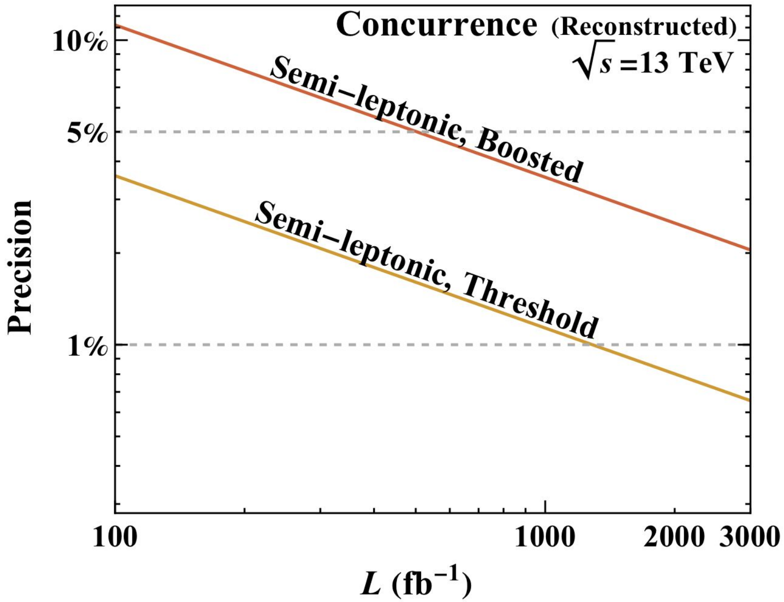
- Treating the top and anti-top as qubits yields a quantum system
- Can measure entanglement and Bell inequality in this system
- ATLAS and CMS have measured entanglement in the leptonic channel
- Semi-leptonic channel may have higher sensitivity (>60%)



Source: https://atlas.cern/Updates/Briefing/Top-Entanglement

# Back-Up: Entanglement

- Results: Entanglement
  - Degraded slightly from parton-level
  - Well above 5σ
  - Measured by ATLAS and CMS in leptonic channel



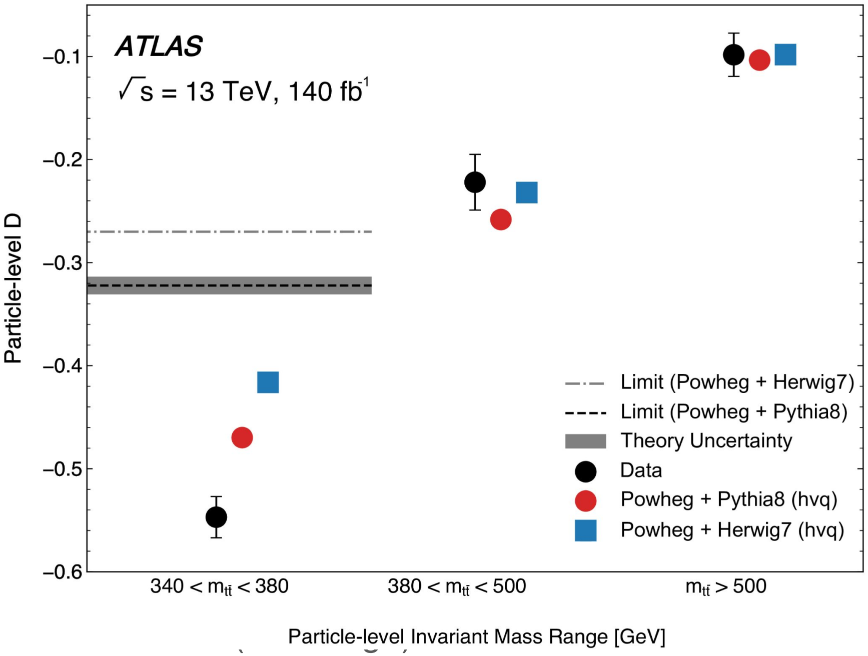

| Reconstructed | $N_{\text{detected}}$ (139 fb$^{-1}$) | $2\mathcal{C}(\rho)$ (Individual) | $2\mathcal{C}(\rho)$ (Direct) | Precision |
|---|---|---|---|---|
| Threshold | $1.26 \times 10^6$ | $0.523 \pm 0.033$ | $0.522 \pm 0.016$ | 3.0% |
| Boosted | $1.15 \times 10^5$ | $0.549 \pm 0.084$ | $0.552 \pm 0.052$ | 9.5% |

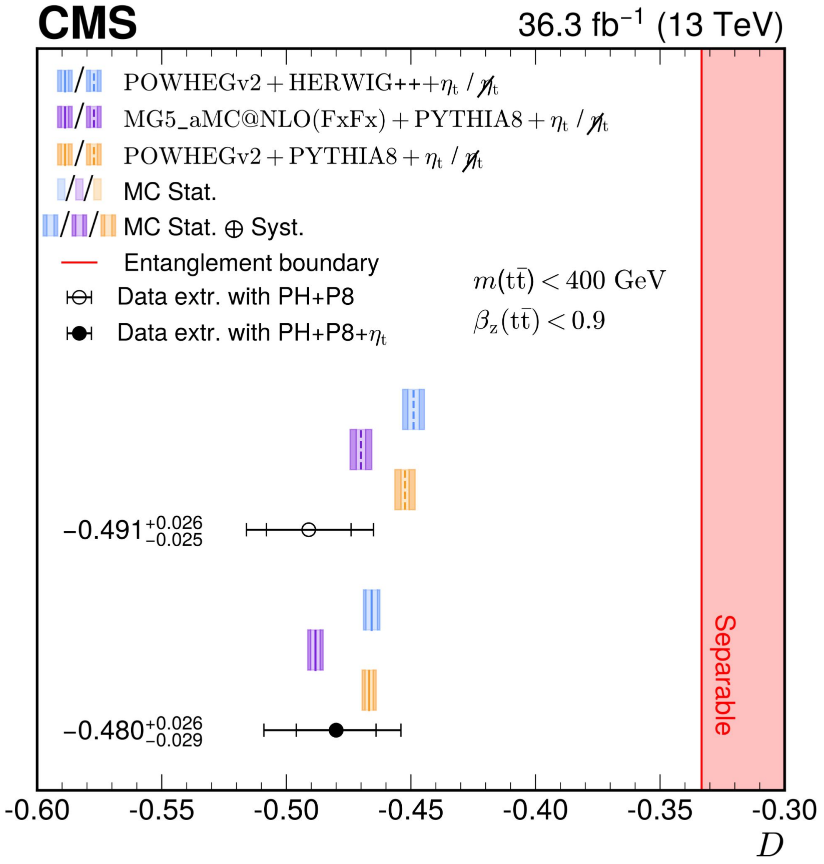
# Back-Up: ATLAS and CMS

CMS [2406.03976](https://arxiv.org/abs/2406.03976)

- Measurements

ATLAS [2311.07288](https://arxiv.org/abs/2311.07288)



# Back-Up: Unfolding

- Unfolding

$$\vec{x}_{\text{truth}} \xrightarrow{\text{folding}} \vec{x}_{\text{detected}} = R \cdot \vec{x}_{\text{truth}},$$
$$\vec{x}_{\text{unfolded}} = R^{-1} \cdot \vec{x}_{\text{detected}}.$$

- Parametric/template fitting

$$\vec{x}_{\text{truth}}(\Theta) \xrightarrow{\text{folding}} \vec{x}_{\text{predicted}}(\Theta) = R \cdot \vec{x}_{\text{truth}}(\Theta).$$