Optimal extraction of New Physics

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Precision frontier and lepton collider

- Complimentarity to LHC search at the high energy frontier.
- Lepton colliders give us information of initial state.
- Free from PDF uncertainties.
- QCD-dominated backgrounds are absent.
- Advantage of initial beam polarization, which leads to background suppression and also helps one isolate polarization-sensitive new physics.

Our goal

- Most pertinent question : The maximum precision that can be achieved, for a particular measurement, crucial information for future collider design.
- In our work, we have utilized an elegant statistical principle OOT, making it far more powerful, beyond being just a theoretical set-up.
- We have proposed a way of closely connecting it to the actual realistic experimental scenarios, including SM backgrounds and imposition of kinematical cuts.
- We demonstrate for lepton colliders, but equally applicable to LHC as well.

Optimal Observable Technique

$\mathcal{O}(\phi)$ is an observable and $f(\phi)$ are known functions of phase-space variables ϕ and c_i's are model-dependent co-efficients.

$$\mathcal{O}(\phi) = c_i f_i(\phi)$$

With suitable weight factors $w_i(\phi)$, one can isolate c_i's.

$$\int w_i(\phi)\mathcal{O}(\phi)d\phi = c_i$$

The unique choice for $w_i(\phi)$ in order to minimize the *statistical* error in determining c_i, equivalently the covariance matrix V_{ij} is a stationary point in terms of varying the functional form of $w_i(\phi)$, while maintaining $\int w_i(\phi)f_j(\phi)d(\phi) = \delta_{ij}$.

$$V_{ij} \propto \int w_i(\phi)w_j(\phi)\mathcal{O}(\phi)d(\phi)$$

The conditions:

$$\delta V_{ij} \propto \int [w_i(\phi) w_j(\phi)] \mathcal{O}(\phi) d(\phi) = 0$$

$$\int \delta w_i(\phi) f_j(\phi) d(\phi) = 0$$

Rao-Cramer-Frechet bound for minimum variance of unbiased estimator

The weighting functions that satisfy the conditions are

$$w_i(\phi) = \frac{\sum_j M_{ij}^{-1} f_j(\phi)}{\mathcal{O}(\phi)}$$

$$M_{ij} = \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}(\phi)} d\phi$$

Then

$$c_i = \sum_k M_{ik}^{-1} \int f_k(\phi) d\phi, \quad V_{ij} = \frac{M_{ij}^{-1}}{\mathcal{L}}$$

Optimal NP estimation in presence of SM background

OOT was initially proposed in measuring $t\bar{t}$ and ZZ coupling of Higgs boson of arbitrary CP nature. *Gunion, Grzadkowski, He(Phys. Rev. Lett. 77 (1996) 5172)*

Shortcomings of previous analyses:

- Previous works did not really consider the non-interfering SM background effects in detail. The impact of phase space cuts have been studied but merely as an efficiency factor.
- It was concluded that phase-space cuts essentially deteriorates optimal sensitivity.
- The full reconstruction of the final states was necessary for doing the OOT analysis, making it difficult to use especially in situations with multiple sources of missing energy.

General framework

Including background contributions, the observable cross sections of interest take the general form,

$$\frac{d\sigma}{d\phi} = \mathcal{O}(\phi) = \frac{d\sigma_{\mathtt{sig}}}{d\phi} + \frac{d\sigma_{\mathtt{bkg}}}{d\phi} = \sum_i g_i f_i(\phi)$$

If NP has two parameters a and b and amplitude is linear in them then

$$\mathcal{O} = f_0 + af_1 + bf_2 + a^2 f_3 + abf_4 + b^2 f_5 = \sum_{i=0}^{5} g_i f_i$$

We divide the phase space integration region into $\mathtt{R}$ bins $\mathtt{s}_1,\ \mathtt{s}_2, \ldots, \mathtt{s}_\mathtt{R}$

$$\Delta_\mathtt{r} = \int_{\mathtt{s}_\mathtt{r}} d\phi, \qquad n_{\mathtt{r}i} = \mathfrak{L}_{\mathtt{int}} \int_{\mathtt{s}_\mathtt{r}} d\phi\ f_i$$

Then total number of events in $\mathtt{s}_\mathtt{r}$ is

$$N_\mathtt{r} = \mathfrak{L}_{\mathtt{int}} \int_{\mathtt{s}_\mathtt{r}} d\phi\ \mathcal{O} = \sum_i g_i n_{\mathtt{r}i}$$

$$f_i|_{s_\mathrm{r}} \simeq \frac{1}{\Delta_\mathrm{r}} \int_{s_\mathrm{r}} d\phi f_i = \frac{1}{\Delta_\mathrm{r}} \frac{n_{\mathrm{r}i}}{\mathfrak{L}_\mathtt{int}}$$

$$\mathcal{O}|_{s_\mathrm{r}} \simeq \frac{1}{\Delta_\mathrm{r}} \int_{s_\mathrm{r}} d\phi\ \mathcal{O} = \frac{1}{\Delta_\mathrm{r}} \frac{N_\mathrm{r}}{\mathfrak{L}_\mathtt{int}}$$

Therefore,

$$M_{ij} = \sum_r \int_{s_\mathrm{r}} d\phi \frac{f_i f_j}{\mathcal{O}} \simeq \sum_r \frac{1}{\mathfrak{L}_\mathtt{int} \Delta_\mathrm{r}} \frac{n_{\mathrm{r}i} n_{\mathrm{r}j}}{N_\mathrm{r}}$$

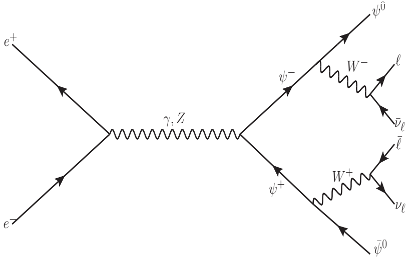
$$\begin{aligned}
n_{\mathrm{r}0} =& N_\mathrm{r}(0,0) \\
n_{\mathrm{r}1} =& \frac{1}{2}[-3N_\mathrm{r}(0,0) + 4N_\mathrm{r}(1,0) - N_\mathrm{r}(2,0)]\,, \\
n_{\mathrm{r}2} =& \frac{1}{2}[-3N_\mathrm{r}(0,0) + 4N_\mathrm{r}(0,1) - N_\mathrm{r}(0,2)]\,, \\
n_{\mathrm{r}3} =& \frac{1}{2}[N_\mathrm{r}(0,0) - 2N_\mathrm{r}(1,0) + N_\mathrm{r}(2,0)]\,, \\
n_{\mathrm{r}4} =& N_\mathrm{r}(0,0) - 2N_\mathrm{r}(0,1) - N_\mathrm{r}(1,0) + N_\mathrm{r}(1,1)\,, \\
n_{\mathrm{r}5} =& \frac{1}{2}[N_\mathrm{r}(0,0) - 2N_\mathrm{r}(0,1) + N_\mathrm{r}(0,2)]\,.
\end{aligned}$$

$N_{\mathtt{r}}(a,b)$ and M_{ij} can be obtained with all experimental cuts and optimal χ^2 can be obtained as

$$\chi^2_{\mathtt{t}} = \sum_{i,j}(g_i - g_i^0)(g_j - g_j^0)V_{ij}^{-1}; \quad V = \frac{1}{\mathfrak{L}_{\mathtt{int}}}M^{-1}$$

- In this method the application of cuts play a non-trivial role, beyond a multiplicative factor.
- The accuracy of this method will increase with increasing number of bins.

Signal

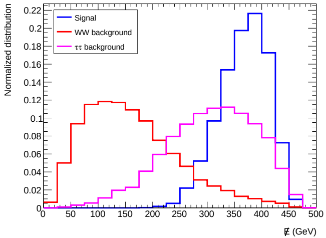


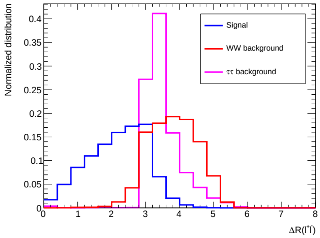
The final state involves Opposite-sign di-lepton(OSD) and $\not{E}$.

$$\psi^+\psi^- Z : -\frac{ie_0}{2s_w c_w}\gamma^\mu\left(a + b\gamma^5\right)\ ,\ \psi^+\psi^-\gamma : -\ ie_0\gamma^\mu$$

Background

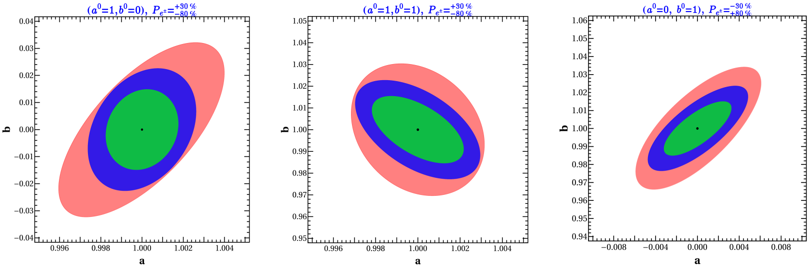
- $W(\ell\nu)W(\ell\nu)$
- $\tau\tau/\mu\mu$ with τ/μ decaying leptonically
- $Z(\nu\nu)Z(\ell\ell)$
- $\nu\nu Z(\ell\ell)$
- $W(\ell\nu)W(\ell\nu)Z(\nu\nu)$





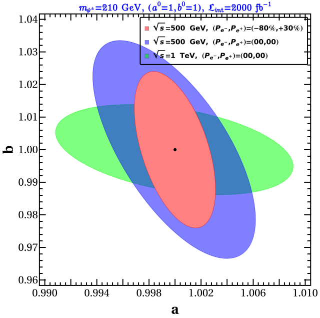
- $m_{\psi^\pm} = 210$ GeV and $m_{\psi_0} = 60$ GeV, $\sqrt{s} = 500$ GeV.
- $\mathcal{C}_1$: $p_T^\ell > 10$ GeV, $N_{\mathtt{lep}} = 2$, $|m_{\ell\ell} - m_Z| > 15$ GeV and $\Delta R_{\ell\ell} < 3.0$,
- $\mathcal{C}_2$: $\not{E} > 325$ GeV
- Initial beam polarization also plays a crucial role in signal background separation.

Results at 2000 fb^{-1}



- Pink contour for signal + background, blue contour for signal + background after applying $\mathcal{C}_1 + \mathcal{C}_2$ and green contour for signal only scenario.
- For different signal hypotheses, different polarization choices result in best sensitivity.

Effect of CM energy and beam polarization



- The covariance matrix can be strongly dependent on $\sqrt{s}$.
- Uncertainty in b drops, while that in a enhances with larger $\sqrt{s}$.
- The overall uncertainty can be significantly reduced by an appropriate choice of polarization.

Summary

- OOT guides us to extract NP couplings and to predict the optimal statistical uncertainty in their measurement.
- It is also possible to distinguish between various NP models(hypothesis) via OOT.
- We further extended OOT framework such that it can be used for even non-reconstructible final state, and also in the presence of non-negligible SM background, non-trivial impact of kinematical cuts.
- In all cases we study the impact of beam polarization.
- Comparison of OOT with collider sensitivity is also presented.

Back-up : Comparison with standard collider analysis

Standard χ^2(bin-wise) is defined as

$$\chi_e^2 = \sum_k \left(\frac{N_k^{\text{exp}} - N_k^{\text{theo}}(a,b)}{\sqrt{N_k^{\text{exp}}}} \right)^2$$

N^{exp} is obtained using simulation with a^0 and b^0.