Technische Universität München

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# Complete One-loop Renormalization-group Equations in the Seesaw Effective Field Theories

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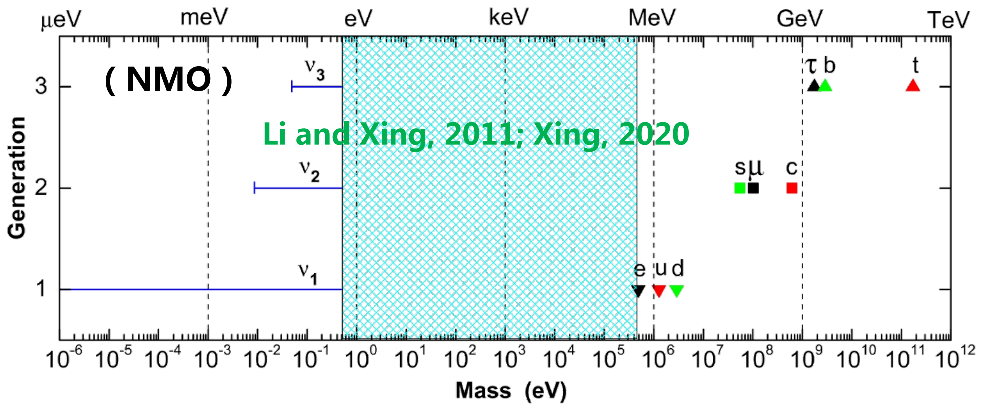
# Seesaw Mechanisms

Global-fit results for neutrino oscillation parameters:

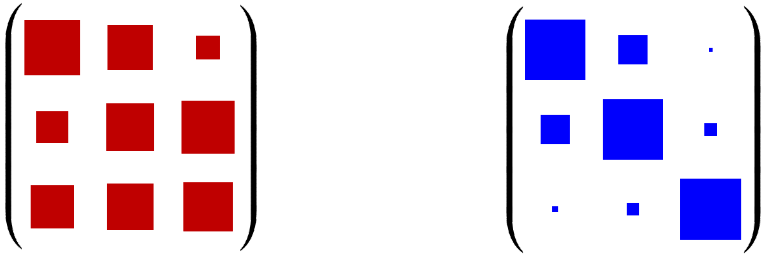
Esteban et al., 2020 NuFIT 5.3 (2024)

| | | Normal Ordering (best fit) | | Inverted Ordering ($\Delta\chi^2 = 9.1$) | |
|---|---|---|---|---|---|
| | | bfp $\pm1\sigma$ | $3\sigma$ range | bfp $\pm1\sigma$ | $3\sigma$ range |
| with SK atmospheric data | $\sin^2\theta_{12}$ | $0.307^{+0.012}_{-0.011}$ | $0.275 \to 0.344$ | $0.307^{+0.012}_{-0.011}$ | $0.275 \to 0.344$ |
| | $\theta_{12}/^\circ$ | $33.67^{+0.73}_{-0.71}$ | $31.61 \to 35.94$ | $33.67^{+0.73}_{-0.71}$ | $31.61 \to 35.94$ |
| | $\sin^2\theta_{23}$ | $0.454^{+0.019}_{-0.016}$ | $0.411 \to 0.606$ | $0.568^{+0.016}_{-0.021}$ | $0.412 \to 0.611$ |
| | $\theta_{23}/^\circ$ | $42.3^{+1.1}_{-0.9}$ | $39.9 \to 51.1$ | $48.9^{+0.9}_{-1.2}$ | $39.9 \to 51.4$ |
| | $\sin^2\theta_{13}$ | $0.02224^{+0.00056}_{-0.00057}$ | $0.02047 \to 0.02397$ | $0.02222^{+0.00069}_{-0.00057}$ | $0.02049 \to 0.02420$ |
| | $\theta_{13}/^\circ$ | $8.58^{+0.11}_{-0.11}$ | $8.23 \to 8.91$ | $8.57^{+0.13}_{-0.11}$ | $8.23 \to 8.95$ |
| | $\delta_{CP}/^\circ$ | $232^{+39}_{-25}$ | $139 \to 350$ | $273^{+24}_{-26}$ | $195 \to 342$ |
| | $\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$ | $7.41^{+0.21}_{-0.20}$ | $6.81 \to 8.03$ | $7.41^{+0.21}_{-0.20}$ | $6.81 \to 8.03$ |
| | $\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$ | $+2.505^{+0.024}_{-0.026}$ | $+2.426 \to +2.586$ | $-2.487^{+0.027}_{-0.024}$ | $-2.566 \to -2.407$ |

## Masses of fermions




## Flavor mixing of fermions



$U_{\text{PMNS}}$ $V_{\text{CKM}}$

**The origin of neutrino masses is quite different from that of charged fermions** Xing, 2020

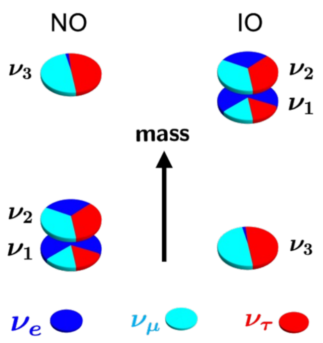
**Seesaw mechanisms** extending the Standard Model with **fermion singlets**, **scalar triplet**, or **fermion triplets**

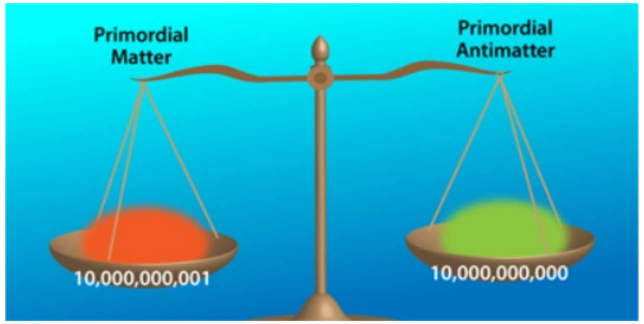
Minkowski, 1977; Yanagida, 1979; ... Konetschny and Kummer, 1977; ... Foot et al., 1989; Ma, 1998

- The **simplest** and the **most natural** ways to explain **tiny neutrino masses**
- A bonus is to account for **the matter-antimatter asymmetry of the Universe via leptogenesis** Fukugita and Yanagida, 1986

# Seesaw Effective Field Theories

$\mu$




**Seesaw Mechanisms**

Seesaw scale $\Lambda_{\rm SS}$ — **Matching** $C_i(\Lambda_{\rm SS})$ — Non-renormalizable operators

**Seesaw EFTs** — **RGE Running**

- Resummation of large logs
- Bridge among parameters at different scales

Electroweak scale $\Lambda_{\rm EW}$ — $C_i(\Lambda_{\rm EW})$ → **Mapping** → **Low-energy Observables**

To achieve the **complete** one-loop RGEs up to $\mathcal{O}(1/\Lambda_{\rm SS}^2)$ in the **seesaw EFTs** induced by **seesaw mechanisms**

# Seesaw Effective Field Theories

**The type-I seesaw mechanism** extending the SM with **three singlet right-handed neutrinos**

$$\mathcal{L}_{\rm SS} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i} \partial\!\!\!/ N_{\rm R} - \left( \frac{1}{2} \overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + {\rm h.c.} \right)$$

Minkowski, 1977; Yanagida, 1979; Gell-Mann et al., 1979; Glashow, 1980; Mohapatra, Senjanovic, 1980

Integrating out **heavy** right-handed neutrinos at the **tree level** (i.e., the tree-level matching)


The one-loop matching

DZ, Zhou, 2021a;2021b; Coy, Frigerio, 2019; Ohlsson, Pernow, 2022; Du, Li, and Yu, 2022

**The tree-level seesaw EFT up to $\mathcal{O}(1/\Lambda_{\rm SS}^2)$:** Broncano, Gavela and Jenkins, 2003a; 2003b; Abada et al, 2007

$$\mathcal{L}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \frac{1}{2} \left( C_5^{\alpha\beta} \mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.} \right) + C^{(1)\alpha\beta}_{H\ell} \mathcal{O}^{(1)\alpha\beta}_{H\ell} + C^{(3)\alpha\beta}_{H\ell} \mathcal{O}^{(3)\alpha\beta}_{H\ell}$$

- Dim-5 operator $\mathcal{O}^{(5)}_{\alpha\beta} = \overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta \rm L}$ — The Weinberg operator (S. Weinberg, 1979) → Neutrino masses

- Dim-6 operators (The Warsaw basis, Grzadkowski, 2010)

$$\mathcal{O}^{(1)}_{\alpha\beta} = \left( \overline{\ell_{\alpha \rm L}} \gamma^\mu \ell_{\beta \rm L} \right) \left( H^\dagger {\rm i} \overleftrightarrow{D}_\mu H \right)$$

$$\mathcal{O}^{(3)}_{\alpha\beta} = \left( \overline{\ell_{\alpha \rm L}} \gamma^\mu \sigma^I \ell_{\beta \rm L} \right) \left( H^\dagger {\rm i} \overleftrightarrow{D}^I_\mu H \right)$$

→ Unitarity violation of the lepton flavor mixing

Broncano, Gavela and Jenkins, 2003a; 2003b; 2005; Antusch et al., 2006; Abada et al., 2007

The corresponding **Wilson coefficients** at the matching scale $\mu_M \sim \Lambda_{SS} = O(M_R)$

$$C_5 \left( \mu_{\rm M} \right) = Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T} \qquad C^{(1)}_{H\ell} \left( \mu_{\rm M} \right) = - C^{(3)}_{H\ell} \left( \mu_{\rm M} \right) = \frac{1}{4} Y_\nu M_{\rm R}^{-2} Y_\nu^\dagger$$

# One-loop RGEs in the Seesaw EFT

**The general structure of the RGEs up to $\mathcal{O}(1/\Lambda_{\rm SS}^2)$**

Wang, DZ, and Zhou, 2023

$$16\pi^2\mu\frac{{\rm d}C_i^{(5)}}{{\rm d}\mu} = \gamma'_{ij}C_j^{(5)}$$

Chankowski, Pluciennik, 1993;
Babu, Leung, Pantaleone, 1993;
Antusch et al., 2001

$$16\pi^2\mu\frac{{\rm d}C_i^{(6)}}{{\rm d}\mu} = \gamma_{ij}C_j^{(6)} + \boxed{\widehat{\gamma}^i_{jk}C_j^{(5)}C_k^{(5)}}$$

Jenkins et al., 2013; 2014;
Alonso et al., 2014a; 2014b

Broncano, Gavela, Jenkins, 2005;
Davidson, Gorbahn, Leak, 2018

Incomplete and not fully correct!!!

**The procedure for calculations:**

$\mathcal{L}_{\rm SEFT}$ → (BFM, $R_\xi$ gauge, Off-shell scheme; FeynRules, FeynArts) → **1PI diagrams** → (Dimension regularization, $\overline{\rm MS}$ scheme; FeynCalc, Package-X) → **Counterterms in the Green's basis** (dim-5, 6) (Jiang et al., 2019; Gherardi et al., 2020) → (Reduction relations (Equations of motion)) → **Counterterms in the physical basis** → (Callan-Symanzik equation) → **RGEs**

$$\mu\frac{{\rm d}C_r}{{\rm d}\mu} = \varepsilon\left(\sum_i n'_i h_i \frac{\partial Z_r}{\partial h_i}\right)C_r$$

**Crosscheck** has been done by taking advantage of the package Matchmakereft

Carmona et al., 2022

# One-loop RGEs in the Seesaw EFT

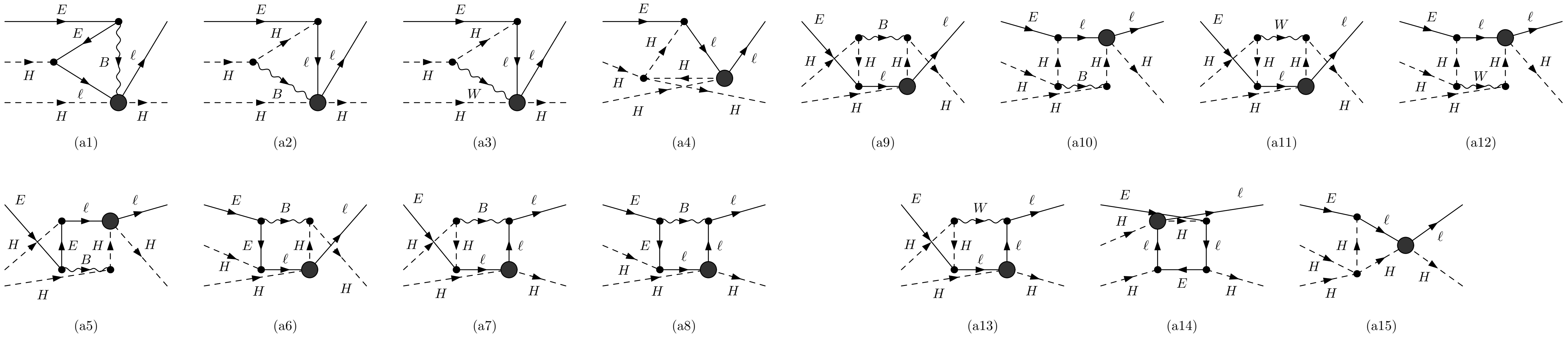
**The general structure of the RGEs up to $\mathcal{O}(1/\Lambda_{\rm SS}^2)$** Wang, DZ, and Zhou, 2023

$$16\pi^2\mu\frac{\mathrm{d}C_i^{(5)}}{\mathrm{d}\mu} = \gamma'_{ij}C_j^{(5)} \qquad 16\pi^2\mu\frac{\mathrm{d}C_i^{(6)}}{\mathrm{d}\mu} = \gamma_{ij}C_j^{(6)} + \widehat{\gamma}^i_{jk}C_j^{(5)}C_k^{(5)}$$

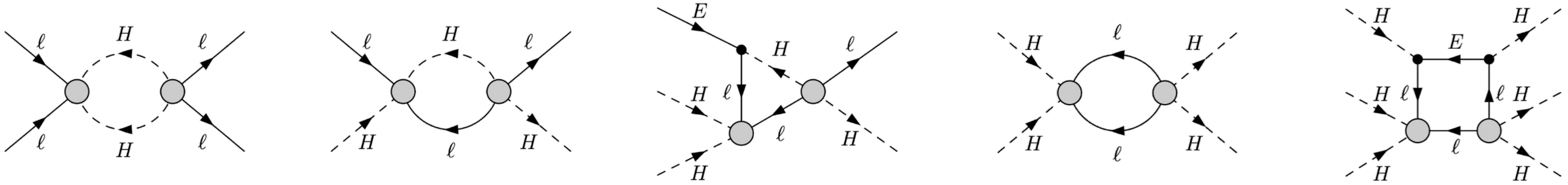
**Two types of contributions:**

- **Single insertion** of the dim-5 or dim-6 operators

An example for diagrams renormalizing $\mathcal{O}_{eH} = \overline{\ell_L} H E_R (H^\dagger H)$ vertex




- **Double insertions** of the dim-5 operator



Flavor dependent Flavor independent

**All possible diagrams**

# One-loop RGEs in the Seesaw EFT

- **The SM couplings:** Wang, DZ, and Zhou, 2023

$$16\pi^2\mu\frac{\mathrm{d}g_1}{\mathrm{d}\mu} = \frac{41}{6}g_1^3 \; ,$$

$$16\pi^2\mu\frac{\mathrm{d}g_2}{\mathrm{d}\mu} = -\frac{19}{6}g_2^3 \; , \qquad T \equiv \mathrm{tr}\left(Y_l Y_l^\dagger + 3Y_\mathrm{u} Y_\mathrm{u}^\dagger + 3Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)$$

$$16\pi^2\mu\frac{\mathrm{d}g_s}{\mathrm{d}\mu} = -7g_s^3 \; .$$

$$16\pi^2\mu\frac{\mathrm{d}Y_l}{\mathrm{d}\mu} = \left[-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + T + \frac{3}{2}Y_l Y_l^\dagger - 2m^2\left(C_{H\ell}^{(1)} + 3C_{H\ell}^{(3)}\right)\right] Y_l \; ,$$

$$16\pi^2\mu\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}\mu} = \left[-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T + \frac{3}{2}\left(Y_\mathrm{u} Y_\mathrm{u}^\dagger - Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)\right] Y_\mathrm{u} \; ,$$

$$16\pi^2\mu\frac{\mathrm{d}Y_\mathrm{d}}{\mathrm{d}\mu} = \left[-\frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_s^2 + T - \frac{3}{2}\left(Y_\mathrm{u} Y_\mathrm{u}^\dagger - Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)\right] Y_\mathrm{d} \; ,$$

$$16\pi^2\mu\frac{\mathrm{d}m^2}{\mathrm{d}\mu} = \left(-\frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2T\right) m^2$$

$$16\pi^2\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = 24\lambda^2 - 3\lambda\left(g_1^2 + 3g_2^2\right) + \frac{3}{8}\left(g_1^2 + g_2^2\right)^2 + \frac{3}{4}g_2^4 + 4\lambda T - 2\mathrm{tr}\left[\left(Y_l Y_l^\dagger\right)^2 + 3\left(Y_\mathrm{u} Y_\mathrm{u}^\dagger\right)^2 + 3\left(Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)^2\right] + m^2\mathrm{tr}\left(2C_5 C_5^\dagger - \frac{8}{3}g_2^2 C_{H\ell}^{(3)} + 8C_{H\ell}^{(3)} Y_l Y_l^\dagger\right) \; .$$

- **The Weinberg operator:**

$$16\pi^2\mu\frac{\mathrm{d}C_5}{\mathrm{d}\mu} = \left(-3g_2^2 + 4\lambda + 2T\right) C_5 - \frac{3}{2}Y_l Y_l^\dagger C_5 - \frac{3}{2}C_5\left(Y_l Y_l^\dagger\right)^T$$

- **Dim-6 operators:**

$$16\pi^2\mu\frac{\mathrm{d}C_{H\ell}^{(1)}}{\mathrm{d}\mu} = -\frac{3}{2}C_5 C_5^\dagger + \frac{2}{3}g_1^2\mathrm{tr}\left(C_{H\ell}^{(1)}\right)\mathbb{1} + \left[\frac{1}{3}g_1^2 + 2\mathrm{tr}\left(Y_l Y_l^\dagger + 3Y_\mathrm{u} Y_\mathrm{u}^\dagger + 3Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)\right] C_{H\ell}^{(1)} + \frac{1}{2}\left[\left(4C_{H\ell}^{(1)} + 9C_{H\ell}^{(3)}\right) Y_l Y_l^\dagger + Y_l Y_l^\dagger\left(4C_{H\ell}^{(1)} + 9C_{H\ell}^{(3)}\right)\right] \; ,$$

$$16\pi^2\mu\frac{\mathrm{d}C_{H\ell}^{(3)}}{\mathrm{d}\mu} = C_5 C_5^\dagger + \frac{2}{3}g_2^2\mathrm{tr}\left(C_{H\ell}^{(3)}\right)\mathbb{1} + \left[-\frac{17}{3}g_2^2 + 2\mathrm{tr}\left(Y_l Y_l^\dagger + 3Y_\mathrm{u} Y_\mathrm{u}^\dagger + 3Y_\mathrm{d} Y_\mathrm{d}^\dagger\right)\right] C_{H\ell}^{(3)} + \frac{1}{2}\left[\left(3C_{H\ell}^{(1)} + 2C_{H\ell}^{(3)}\right) Y_l Y_l^\dagger + Y_l Y_l^\dagger\left(3C_{H\ell}^{(1)} + 2C_{H\ell}^{(3)}\right)\right] \; ,$$

$$\vdots$$

# One-loop RGEs in the Seesaw EFT

- **Dim-6 operators:**

| $H^6$ and $H^4D^2$ | | $\psi^2 H^3$ | | $(\overline{\rm L}{\rm L})(\overline{\rm L}{\rm L})$ | |
|---|---|---|---|---|---|
| $\mathcal{O}_H$ | $(H^\dagger H)^3$ | $\mathcal{O}_{eH}^{\alpha\beta}$ | $(\overline{\ell_{\alpha{\rm L}}} H E_{\beta{\rm R}})(H^\dagger H)$ | $\mathcal{O}_{\ell\ell}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(\overline{\ell_{\gamma{\rm L}}}\gamma_\mu \ell_{\lambda{\rm L}})$ |
| $\mathcal{O}_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ | $\mathcal{O}_{uH}^{\alpha\beta}$ | $(\overline{Q_{\alpha{\rm L}}}\widetilde{H} U_{\beta{\rm R}})(H^\dagger H)$ | $\mathcal{O}_{\ell q}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(\overline{Q_{\gamma{\rm L}}}\gamma_\mu Q_{\lambda{\rm L}})$ |
| $\mathcal{O}_{HD}$ | $(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$ | $\mathcal{O}_{dH}^{\alpha\beta}$ | $(\overline{Q_{\alpha{\rm L}}} H D_{\beta{\rm R}})(H^\dagger H)$ | $\mathcal{O}_{\ell q}^{(3)\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu\sigma^I \ell_{\beta{\rm L}})(\overline{Q_{\gamma{\rm L}}}\gamma_\mu\sigma^I Q_{\lambda{\rm L}})$ |
| $\psi^2 H^2 D$ | | | | $(\overline{\rm L}{\rm L})(\overline{\rm R}{\rm R})$ | |
| $\mathcal{O}_{H\ell}^{(1)\alpha\beta}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H)$ | $\mathcal{O}_{Hq}^{(3)\alpha\beta}$ | $(\overline{Q_{\alpha{\rm L}}}\gamma^\mu\sigma^I Q_{\beta{\rm L}})(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H)$ | $\mathcal{O}_{\ell e}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(\overline{E_{\gamma{\rm R}}}\gamma_\mu E_{\lambda{\rm R}})$ |
| $\mathcal{O}_{H\ell}^{(3)\alpha\beta}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu\sigma^I \ell_{\beta{\rm L}})(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H)$ | $\mathcal{O}_{Hu}^{\alpha\beta}$ | $(\overline{U_{\alpha{\rm R}}}\gamma^\mu U_{\beta{\rm R}})(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H)$ | $\mathcal{O}_{\ell u}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(\overline{U_{\gamma{\rm R}}}\gamma_\mu U_{\lambda{\rm R}})$ |
| $\mathcal{O}_{He}^{\alpha\beta}$ | $(\overline{E_{\alpha{\rm R}}}\gamma^\mu E_{\beta{\rm R}})(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H)$ | $\mathcal{O}_{Hd}^{\alpha\beta}$ | $(\overline{D_{\alpha{\rm R}}}\gamma^\mu D_{\beta{\rm R}})(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H)$ | $\mathcal{O}_{\ell d}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu \ell_{\beta{\rm L}})(\overline{D_{\gamma{\rm R}}}\gamma_\mu D_{\lambda{\rm R}})$ |
| $\mathcal{O}_{Hq}^{(1)\alpha\beta}$ | $(\overline{Q_{\alpha{\rm L}}}\gamma^\mu Q_{\beta{\rm L}})(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H)$ | | | | |

- In the type-I seesaw EFT, **17** dim-6 operators can be generated via their one-loop RGEs apart from the 2 tree-level ones
- Contributions from **single insertions** of the dim-5 and dim-6 operators are consistent with results in the literature
  Antusch et al., 2001; Jenkins et al., 2013

- The contribution from **double insertions** of the dim-5 operator to the RGE of quartic Higgs coupling $\lambda$ is **new**, while those to RGEs of $\mathcal{O}_{H\Box}, \mathcal{O}_{HD}, \mathcal{O}_H, \mathcal{O}_{eH}, \mathcal{O}_{uH}, \mathcal{O}_{dH}$ missed a factor of **1/2** in **Davidson et al., 2018**, now are **corrected** and **completed**
- Contributions from **double insertions** of the dim-5 operator are **generic** and thus valid not only for the seesaw EFTs but also for the SMEFT in general
- Results for the **type-II** and **type-III** seesaw EFTs are also achieved and included in **Wang, DZ, and Zhou, 2023**

# One-loop RGEs of Flavor Mixing Parameters

**After spontaneous symmetry breaking**

$$
\begin{aligned}
\mathcal{L}^{l}_{\rm SEFT} = & -\left[\overline{l_{\alpha \rm L}}\,(M_l)_{\alpha\beta}\,l_{\beta \rm R} + \frac{1}{2}\overline{\nu_{\alpha \rm L}}\,(M_\nu)_{\alpha\beta}\,\nu^{\rm c}_{\beta \rm L} + {\rm h.c.}\right] \\
& + \left[\frac{g_2}{\sqrt{2}}\,\overline{l_{\alpha \rm L}}\gamma^\mu\left(\delta_{\alpha\beta} - \tilde{\eta}_{\alpha\beta}\right)\nu_{\beta \rm L} W^-_\mu + {\rm h.c.}\right] + \frac{g_2}{2c_{\rm W}}\,\overline{\nu_{\alpha \rm L}}\gamma^\mu\left(\delta_{\alpha\beta} - \tilde{\eta}^\prime_{\alpha\beta}\right)\nu_{\beta \rm L} Z_\mu \\
& - \frac{g_2}{2c_{\rm W}}\,\overline{l_{\alpha \rm L}}\gamma^\mu\left[\left(1-2s^2_{\rm W}\right)\delta_{\alpha\beta} + \left(\tilde{\eta}^\prime - 2\tilde{\eta}\right)_{\alpha\beta}\right] l_{\beta \rm L} Z_\mu + \frac{g_2}{c_{\rm W}} s^2_{\rm W}\overline{l_{\alpha \rm R}}\gamma^\mu l_{\alpha \rm R} Z_\mu\,, \\
= & -\left(\overline{l_{\rm L}}\,\widehat{M}_l\,l_{\rm R} + \frac{1}{2}\,\overline{\nu_{\rm L}}\,\widehat{M}_\nu\,\nu^{\rm c}_{\rm L} + {\rm h.c.}\right) \\
& + \left(\frac{g_2}{\sqrt{2}}\,\overline{l_{\rm L}}\gamma^\mu V \nu_{\rm L} W^-_\mu + {\rm h.c.}\right) + \frac{g_2}{2c_{\rm W}}\,\overline{\nu_{\rm L}}\gamma^\mu N^\dagger N \nu_{\rm L} Z_\mu \\
& - \frac{g_2}{2c_{\rm W}}\,\overline{l_{\rm L}}\gamma^\mu\left[\left(1-2s^2_{\rm W}\right) + \left(\eta^\prime - 2\eta\right)\right] l_{\rm L} Z_\mu + \frac{g_2}{c_{\rm W}} s^2_{\rm W}\overline{l_{\rm R}}\gamma^\mu l_{\rm R} Z_\mu
\end{aligned}
$$

CC: Non-unitarity

NC

$$M_l = Y_l\,v/\sqrt{2} \qquad M_\nu = -C_5\,v^2/2$$

$$\tilde{\eta} \equiv -C^{(3)}_{Hl}\,v^2 \qquad \tilde{\eta}^\prime \equiv \left(C^{(1)}_{Hl} - C^{(3)}_{Hl}\right)v^2$$

$$s_{\rm W} \equiv \sin\theta_{\rm W}$$

$$U^\dagger_\nu M_\nu U^*_\nu = \widehat{M}_\nu \equiv {\rm diag}\{m_1, m_2, m_3\}$$

$$U^\dagger_l M_l U^\prime_l = \widehat{M}_l \equiv {\rm diag}\{m_e, m_\mu, m_\tau\}$$

$V^\prime \equiv U^\dagger_l U_\nu$ Definition

$V^\prime \equiv P\cdot U\cdot Q$ Parametrization

$$P \equiv {\rm diag}\{e^{{\rm i}\phi_e}, e^{{\rm i}\phi_\mu}, e^{{\rm i}\phi_\tau}\}$$

$$Q \equiv {\rm diag}\{e^{{\rm i}\rho}, e^{{\rm i}\sigma}, 1\}$$

$$V \equiv (\mathbb{1} - \eta)\cdot U\cdot Q$$ Non-unitary Pontecorvo-Maki-Nakagawa-Sakata matrix

$$N \equiv (\mathbb{1} - \eta^\prime/2)\cdot U\cdot Q$$ Flavor-changing NC

$$\eta^\prime\left(\mu_M\right) = 2\eta\left(\mu_M\right)$$

$$\eta \equiv P^\dagger U^\dagger_l \tilde{\eta}\, U_l P = \begin{pmatrix} \eta_{ee} & |\eta_{e\mu}|\,{\rm e}^{+{\rm i}\phi_{e\mu}} & |\eta_{e\tau}|\,{\rm e}^{+{\rm i}\phi_{e\tau}} \\ |\eta_{e\mu}|\,{\rm e}^{-{\rm i}\phi_{e\mu}} & \eta_{\mu\mu} & |\eta_{\mu\tau}|\,{\rm e}^{+{\rm i}\phi_{\mu\tau}} \\ |\eta_{e\tau}|\,{\rm e}^{-{\rm i}\phi_{e\tau}} & |\eta_{\mu\tau}|\,{\rm e}^{-{\rm i}\phi_{\mu\tau}} & \eta_{\tau\tau} \end{pmatrix} \qquad \eta^\prime \equiv P^\dagger U^\dagger_l \tilde{\eta}^\prime U_l P = \begin{pmatrix} \eta^\prime_{ee} & |\eta^\prime_{e\mu}|\,{\rm e}^{+{\rm i}\phi^\prime_{e\mu}} & |\eta^\prime_{e\tau}|\,{\rm e}^{+{\rm i}\phi^\prime_{e\tau}} \\ |\eta^\prime_{e\mu}|\,{\rm e}^{-{\rm i}\phi^\prime_{e\mu}} & \eta^\prime_{\mu\mu} & |\eta^\prime_{\mu\tau}|\,{\rm e}^{+{\rm i}\phi^\prime_{\mu\tau}} \\ |\eta^\prime_{e\tau}|\,{\rm e}^{-{\rm i}\phi^\prime_{e\tau}} & |\eta^\prime_{\mu\tau}|\,{\rm e}^{-{\rm i}\phi^\prime_{\mu\tau}} & \eta^\prime_{\tau\tau} \end{pmatrix}$$

# One-loop RGEs of Flavor Mixing Parameters

RGEs for **eigenvalues**:

$$\dot{y}_\alpha = \left[\left(-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 + T\right) + \frac{3}{2}y_\alpha^2 - 2\frac{m^2}{v^2}\left(\eta' - 4\eta\right)_{\alpha\alpha}\right] y_\alpha$$

$$\dot{\kappa}_i = \left[\left(-3g_2^2 + 4\lambda + 2T\right) - 3\mathrm{Re}\mathcal{S}_{ii}\right]\kappa_i$$

$\kappa \equiv C_5$

$U_l^\dagger Y_l U_l' = \mathrm{diag}\{y_e, y_\mu, y_\tau\}$

$U_\nu^\dagger \kappa U_\nu^* = \mathrm{diag}\{\kappa_1, \kappa_2, \kappa_3\}$

$\mathcal{S} \equiv V'^\dagger \widehat{Y}_l^2 V'$

RGEs for lepton flavor **mixing parameters** and those in the **NC** interactions:

$$\dot{V}'_{\alpha i} = \sum_\beta \left(\dot{U}_l^\dagger U_l\right)_{\alpha\beta} V'_{\beta i} + \sum_j V'_{\alpha j}\left(U_\nu^\dagger \dot{U}_\nu\right)_{ji}$$

$$= \sum_{\beta\neq\alpha} 2\frac{m^2}{v^2} y_{\alpha\beta}\left(\eta' - 4\eta\right)_{\alpha\beta} e^{i(\phi_\alpha - \phi_\beta)} V'_{\beta i} - \sum_{j\neq i}\frac{3}{2}V'_{\alpha j}\frac{1}{\kappa_i^2 - \kappa_j^2}\left[\left(\kappa_i^2 + \kappa_j^2\right)\mathcal{S}_{ji} + 2\kappa_i\kappa_j\mathcal{S}_{ji}^*\right]$$

$$\dot{\eta}_{\alpha\beta} = i\left(\dot{\phi}_\beta - \dot{\phi}_\alpha\right)\eta_{\alpha\beta} + \sum_{\gamma\neq\alpha} 2\frac{m^2}{v^2}y_{\alpha\gamma}\left(\eta' - 4\eta\right)_{\alpha\gamma}\eta_{\gamma\beta} + \sum_{\varrho\neq\beta} 2\frac{m^2}{v^2}y_{\beta\varrho}\eta_{\alpha\varrho}\left(\eta' - 4\eta\right)_{\varrho\beta}$$

$$- \sum_i \kappa_i^2 v^2 U_{\alpha i}U_{\beta i}^* + \frac{2}{3}g_2^2 \mathrm{tr}\left(\eta\right)\delta_{\alpha\beta} + \left(-\frac{17}{3}g_2^2 + 2T\right)\eta_{\alpha\beta} + \frac{1}{2}\left(y_\alpha^2 + y_\beta^2\right)\left(5\eta_{\alpha\beta} - 3\eta'_{\alpha\beta}\right)$$

$$\dot{\eta}'_{\alpha\beta} = i\left(\dot{\phi}_\beta - \dot{\phi}_\alpha\right)\eta'_{\alpha\beta} + \sum_{\gamma\neq\alpha} 2\frac{m^2}{v^2}y_{\alpha\gamma}\left(\eta' - 4\eta\right)_{\alpha\gamma}\eta'_{\gamma\beta} + \sum_{\varrho\neq\beta} 2\frac{m^2}{v^2}y_{\beta\varrho}\eta'_{\alpha\varrho}\left(\eta' - 4\eta\right)_{\varrho\beta}$$

$$+ \frac{2}{3}\left(g_2^2 - g_1^2\right)\mathrm{tr}\left(\eta\right)\delta_{\alpha\beta} + \frac{2}{3}g_1^2 \mathrm{tr}\left(\eta'\right)\delta_{\alpha\beta} - \frac{1}{3}\left(17g_2^2 + g_1^2\right)\eta_{\alpha\beta} + \left(\frac{1}{3}g_1^2 + 2T\right)\eta'_{\alpha\beta}$$

$$- \frac{5}{2}\sum_i \kappa_i^2 v^2 U_{\alpha i}U_{\beta i}^* + \frac{1}{2}\left(y_\alpha^2 + y_\beta^2\right)\left(\eta'_{\alpha\beta} - 8\eta_{\alpha\beta}\right) ,$$

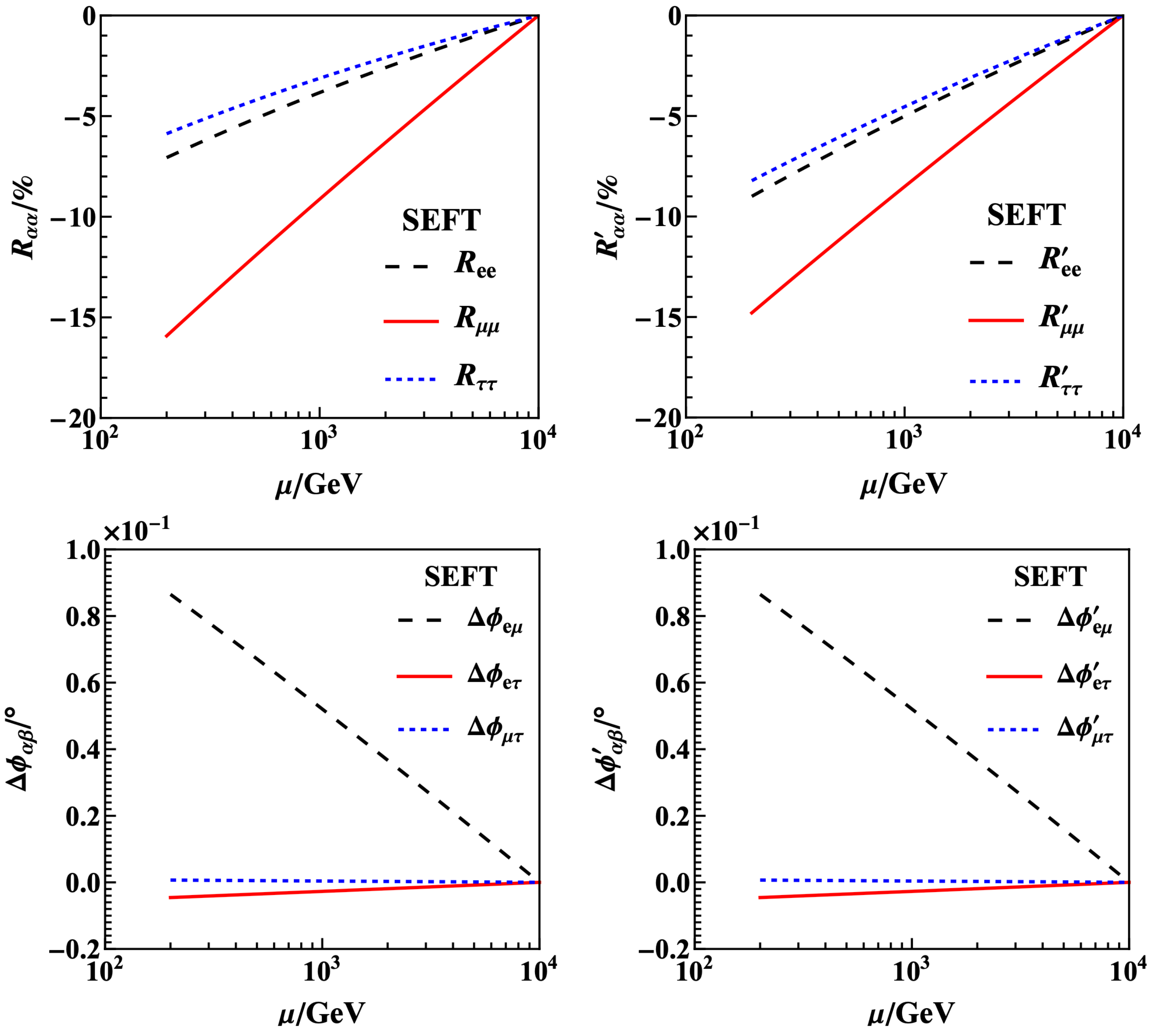
**The non-unitary PMNS matrix**

$$V \equiv (\mathbb{1} - \eta) \cdot U \cdot Q$$

- Lepton masses
- Mixing angles and phases in $V'$
- Unitarity-violating and FCNC parameters, i.e., magnitude and arguments of $\eta$ and $\eta'$

See, **Wang, DZ, and Zhou, 2023**

# Numerical Analysis




$$R^{(\prime)}_{\alpha\beta} \equiv [|\eta^{(\prime)}_{\alpha\beta}(\mu)| - |\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})|]/|\eta^{(\prime)}_{\alpha\beta}(\mu_{\rm M})| \times 100\%$$

$$R_{\alpha\alpha} \sim \Delta t \left[\frac{2}{3} g_2^2 \frac{{\rm tr}(\eta)}{\eta_{\alpha\alpha}} - \frac{17}{3} g_2^2 + 6 y_t^2\right] ,$$

$$R'_{\alpha\alpha} \sim \Delta t \left[\frac{1}{3}\left(g_1^2 + g_2^2\right) \frac{{\rm tr}(\eta)}{\eta_{\alpha\alpha}} + \frac{1}{6}\left(g_1^2 - 17 g_2^2\right) + 6 y_t^2\right] ,$$

$$\Delta t = \ln\left(\mu_{\rm B}/\mu_{\rm M}\right)/(16\pi^2) < 0$$

$$0 < \eta_{\mu\mu} < \eta_{ee} < \eta_{\tau\tau} \Rightarrow R^{(\prime)}_{\mu\mu} < R^{(\prime)}_{ee} < R^{(\prime)}_{\tau\tau} < 0$$

$$\Delta\phi^{(\prime)}_{\alpha\beta} \equiv \phi^{(\prime)}_{\alpha\beta}(\mu) - \phi^{(\prime)}_{\alpha\beta}(\mu_{\rm M})$$

$$\Delta\phi^{(\prime)}_{e\mu} \sim \left[8 \frac{m^2}{v^2} \left|\frac{\eta_{e\tau}\eta_{\mu\tau}}{\eta_{e\mu}}\right| s_{e\mu} + \frac{3}{4} \zeta_{12}^{-1} y_\tau^2 s_{23}^2 s_{2(\rho-\sigma)}\right] \Delta t ,$$

$$\Delta\phi^{(\prime)}_{e\tau} \sim \frac{3}{4} \zeta_{12}^{-1} y_\tau^2 s_{23}^2 s_{2(\rho-\sigma)} \Delta t ,$$

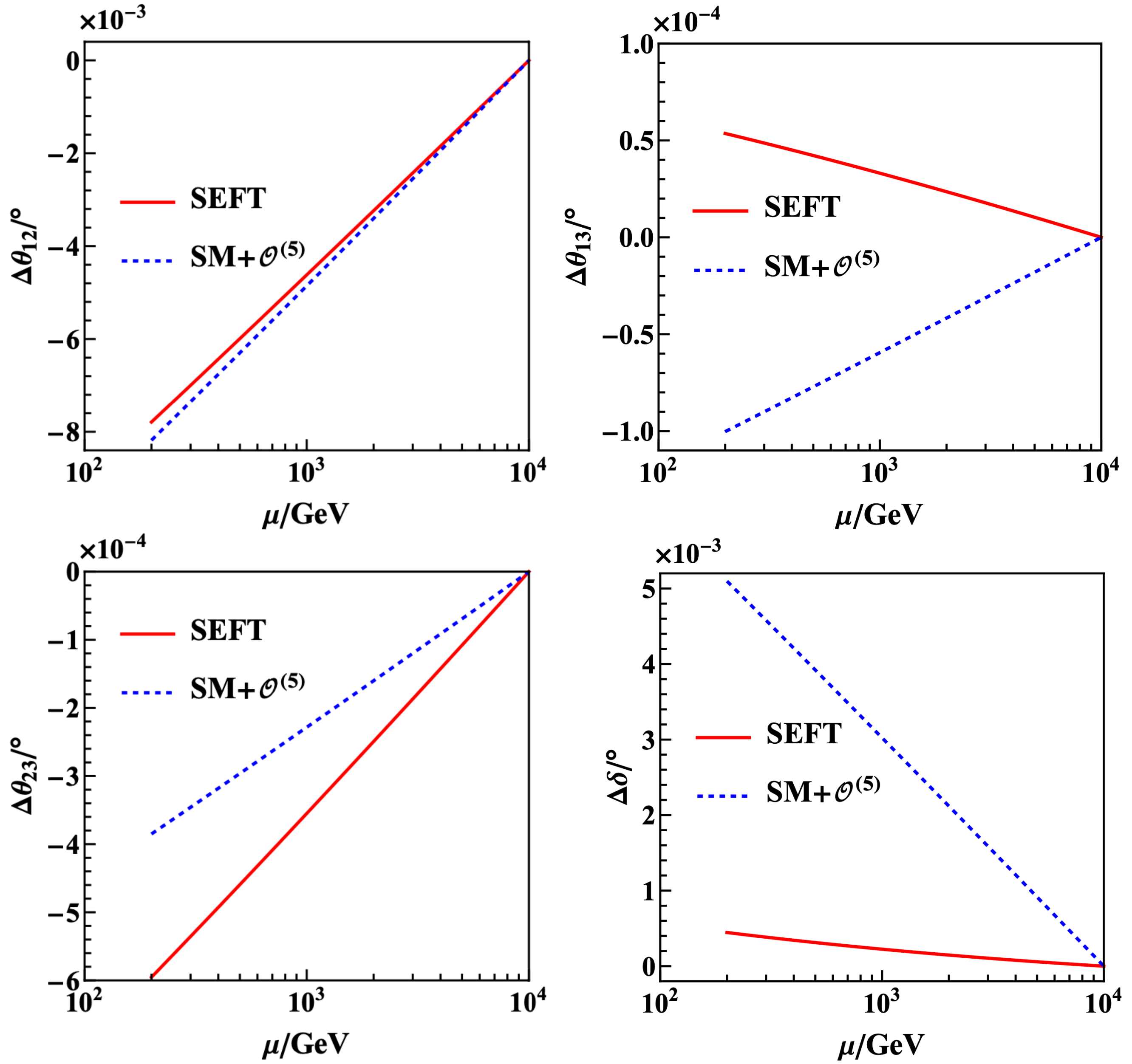
$$\left|\Delta\phi^{(\prime)}_{\mu\tau}\right| \sim \mathcal{O}\left(|\eta|, \zeta_{23}^{-1} y_\tau^2\right) |\Delta t| \sim 10^{-4} |\Delta t| ,$$

$$\zeta_{ij} \equiv (\kappa_i - \kappa_j)/(\kappa_i + \kappa_j)$$

$$\Downarrow$$

$$\Delta\phi_{\alpha\beta} \approx \Delta\phi'_{\alpha\beta}$$

# Numerical Analysis




$$\Delta\theta_{13} \sim -\Delta t \left[ 4\frac{m^2}{v^2} |\eta_{e\tau}| c_{23} c_{e\tau+\delta} + \frac{3}{8}\zeta_{23}^{-1} y_\tau^2 \sin 2\theta_{12} \sin 2\theta_{23} s_{\rho-\sigma} s_{\delta+\rho+\sigma} \right]$$

- Two terms have opposite signs and the absolute value of the first one is slightly larger than that of the second one, thus opposite running directions
- But depends on the initial inputs

- Approximately and analytically, the running behaviors of all these parameters can be well-understood
- The non-unitary parameters may significantly affect the running of leptonic flavor mixing parameters

# Summary

- We derived the **complete** set of **one-loop RGEs** for the SM couplings and Wilson coefficients of operators up to dim-6 and $\mathcal{O}(1/\Lambda_{\rm SS}^2)$ in seesaw EFTs

- Besides two tree-level-generated dim-6 operators, **17** dim-6 operators can be generated by the **one-loop** RGEs in the **type-I** seesaw EFT

- We gave the **explicit expressions** of the RGEs of all the **parameters** involved in the charged- and neutral-current interactions of leptons

- Together with the one-loop matching results at $\Lambda_{\rm SS}$, these one-loop RGEs establish a **self-consistent** EFT framework to investigate **low-energy phenomena** of seesaw models up to $\mathcal{O}(1/\Lambda_{\rm SS}^2)$ at the **one-loop level**

THANKS FOR YOUR ATTENTION

GRACIAS / DANKE / 谢谢

# Backup

## The Greens basis in the SMEFT

**Jiang et al., 2019; Gherardi et al., 2020; Carmona et al., 2022**

| $\psi^2 D^3$ | | $\psi^2 XD$ | | $\psi^2 DH^2$ | |
|---|---|---|---|---|---|
| $\mathcal{R}_{qD}^{\alpha\beta}$ | $\frac{i}{2}\overline{Q}_{\alpha L}\{D_\mu D^\mu, \not{D}\} Q_{\beta L}$ | $\mathcal{R}_{Gq}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} T^A \gamma^\mu Q_{\beta L}) D^\nu G_{\mu\nu}^A$ | $\mathcal{O}_{Hq}^{(1)\alpha\beta}$ | $(\overline{Q}_{\alpha L}\gamma^\mu Q_{\beta L})(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{R}_{uD}^{\alpha\beta}$ | $\frac{i}{2}\overline{U}_{\alpha R}\{D_\mu D^\mu, \not{D}\} U_{\beta R}$ | $\mathcal{R}_{Gq}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L} T^A \gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) G_{\mu\nu}^A$ | $\mathcal{R}_{Hq}^{\prime(1)\alpha\beta}$ | $\left(\overline{Q}_{\alpha L} i\overleftrightarrow{\not{D}} Q_{\beta L}\right)(H^\dagger H)$ |
| $\mathcal{R}_{dD}^{\alpha\beta}$ | $\frac{i}{2}\overline{D}_{\alpha R}\{D_\mu D^\mu, \not{D}\} D_{\beta R}$ | $\mathcal{R}_{\widetilde{G}q}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L} T^A \gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) \widetilde{G}_{\mu\nu}^A$ | $\mathcal{R}_{Hq}^{\prime\prime(1)\alpha\beta}$ | $(\overline{Q}_{\alpha L}\gamma^\mu Q_{\beta L})\partial_\mu(H^\dagger H)$ |
| $\mathcal{R}_{\ell D}^{\alpha\beta}$ | $\frac{i}{2}\overline{\ell}_{\alpha L}\{D_\mu D^\mu, \not{D}\} \ell_{\beta L}$ | $\mathcal{R}_{Wq}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^I\gamma^\mu Q_{\beta L}) D^\nu W_{\mu\nu}^I$ | $\mathcal{O}_{Hq}^{(3)\alpha\beta}$ | $(\overline{Q}_{\alpha L}\gamma^\mu\sigma^I Q_{\beta L})(H^\dagger i\overleftrightarrow{D}_\mu^I H)$ |
| $\mathcal{R}_{eD}^{\alpha\beta}$ | $\frac{i}{2}\overline{E}_{\alpha R}\{D_\mu D^\mu, \not{D}\} E_{\beta R}$ | $\mathcal{R}_{Wq}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) W_{\mu\nu}^I$ | $\mathcal{R}_{Hq}^{\prime(3)\alpha\beta}$ | $\left(\overline{Q}_{\alpha L} i\overleftrightarrow{\not{D}}^I Q_{\beta L}\right)(H^\dagger\sigma^I H)$ |
| $\psi^2 HD^2$ | | $\mathcal{R}_{\widetilde{W}q}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) \widetilde{W}_{\mu\nu}^I$ | $\mathcal{R}_{Hq}^{\prime\prime(3)\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^I\gamma^\mu Q_{\beta L}) D_\mu(H^\dagger\sigma^I H)$ |
| $\mathcal{R}_{uHD1}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} U_{\beta R}) D_\mu D^\mu \widetilde{H}$ | $\mathcal{R}_{Bq}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\gamma^\mu Q_{\beta L})\partial^\nu B_{\mu\nu}$ | $\mathcal{O}_{Hu}^{\alpha\beta}$ | $(\overline{U}_{\alpha R}\gamma^\mu U_{\beta R})(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{R}_{uHD2}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} i\sigma_{\mu\nu} D^\mu U_{\beta R}) D^\nu \widetilde{H}$ | $\mathcal{R}_{Bq}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L}\gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) B_{\mu\nu}$ | $\mathcal{R}_{Hu}^{\prime\alpha\beta}$ | $\left(\overline{U}_{\alpha R} i\overleftrightarrow{\not{D}} U_{\beta R}\right)(H^\dagger H)$ |
| $\mathcal{R}_{uHD3}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} D_\mu D^\mu U_{\beta R}) \widetilde{H}$ | $\mathcal{R}_{\widetilde{B}q}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{Q}_{\alpha L}\gamma^\mu i\overleftrightarrow{D}^\nu Q_{\beta L}\right) \widetilde{B}_{\mu\nu}$ | $\mathcal{R}_{Hu}^{\prime\prime\alpha\beta}$ | $(\overline{U}_{\alpha R}\gamma^\mu U_{\beta R})\partial_\mu(H^\dagger H)$ |
| $\mathcal{R}_{uHD4}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} D_\mu U_{\beta R}) D^\mu \widetilde{H}$ | $\mathcal{R}_{Gu}^{\alpha\beta}$ | $(\overline{U}_{\alpha R} T^A\gamma^\mu U_{\beta R}) D^\nu G_{\mu\nu}^A$ | $\mathcal{O}_{Hd}^{\alpha\beta}$ | $(\overline{D}_{\alpha R}\gamma^\mu D_{\beta R})(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{R}_{dHD1}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} D_{\beta R}) D_\mu D^\mu H$ | $\mathcal{R}_{Gu}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{U}_{\alpha R} T^A\gamma^\mu i\overleftrightarrow{D}^\nu U_{\beta R}\right) G_{\mu\nu}^A$ | $\mathcal{R}_{Hd}^{\prime\alpha\beta}$ | $\left(\overline{D}_{\alpha R} i\overleftrightarrow{\not{D}} D_{\beta R}\right)(H^\dagger H)$ |
| $\mathcal{R}_{dHD2}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} i\sigma_{\mu\nu} D^\mu D_{\beta R}) D^\nu H$ | $\mathcal{R}_{\widetilde{G}u}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{U}_{\alpha R} T^A\gamma^\mu i\overleftrightarrow{D}^\nu U_{\beta R}\right) \widetilde{G}_{\mu\nu}^A$ | $\mathcal{R}_{Hd}^{\prime\prime\alpha\beta}$ | $(\overline{D}_{\alpha R}\gamma^\mu D_{\beta R})\partial_\mu(H^\dagger H)$ |
| $\mathcal{R}_{dHD3}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} D_\mu D^\mu D_{\beta R}) H$ | $\mathcal{R}_{Bu}^{\alpha\beta}$ | $(\overline{U}_{\alpha R}\gamma^\mu U_{\beta R})\partial^\nu B_{\mu\nu}$ | $\mathcal{O}_{Hud}^{\alpha\beta}$ | $i(\overline{U}_{\alpha R}\gamma^\mu D_{\beta R})(\widetilde{H}^\dagger D_\mu H)$ |
| $\mathcal{R}_{dHD4}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} D_\mu D_{\beta R}) D^\mu H$ | $\mathcal{R}_{Bu}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{U}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu U_{\beta R}\right) B_{\mu\nu}$ | $\mathcal{O}_{H\ell}^{(1)\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\gamma^\mu \ell_{\beta L})(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{R}_{eHD1}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L} E_{\beta R}) D_\mu D^\mu H$ | $\mathcal{R}_{\widetilde{B}u}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{U}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu U_{\beta R}\right) \widetilde{B}_{\mu\nu}$ | $\mathcal{R}_{H\ell}^{\prime(1)\alpha\beta}$ | $\left(\overline{\ell}_{\alpha L} i\overleftrightarrow{\not{D}} \ell_{\beta L}\right)(H^\dagger H)$ |
| $\mathcal{R}_{eHD2}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L} i\sigma_{\mu\nu} D^\mu E_{\beta R}) D^\nu H$ | $\mathcal{R}_{Gd}^{\alpha\beta}$ | $(\overline{D}_{\alpha R} T^A\gamma^\mu D_{\beta R}) D^\nu G_{\mu\nu}^A$ | $\mathcal{R}_{H\ell}^{\prime\prime(1)\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\gamma^\mu \ell_{\beta L})\partial_\mu(H^\dagger H)$ |
| $\mathcal{R}_{eHD3}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L} D_\mu D^\mu E_{\beta R}) H$ | $\mathcal{R}_{Gd}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{D}_{\alpha R} T^A\gamma^\mu i\overleftrightarrow{D}^\nu D_{\beta R}\right) G_{\mu\nu}^A$ | $\mathcal{O}_{H\ell}^{(3)\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\gamma^\mu\sigma^I \ell_{\beta L})(H^\dagger i\overleftrightarrow{D}_\mu^I H)$ |
| $\mathcal{R}_{eHD4}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L} D_\mu E_{\beta R}) D^\mu H$ | $\mathcal{R}_{\widetilde{G}d}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{D}_{\alpha R} T^A\gamma^\mu i\overleftrightarrow{D}^\nu D_{\beta R}\right) \widetilde{G}_{\mu\nu}^A$ | $\mathcal{R}_{H\ell}^{\prime(3)\alpha\beta}$ | $\left(\overline{\ell}_{\alpha L} i\overleftrightarrow{\not{D}}^I \ell_{\beta L}\right)(H^\dagger\sigma^I H)$ |
| $\psi^2 XH$ | | $\mathcal{R}_{Bd}^{\alpha\beta}$ | $(\overline{D}_{\alpha R}\gamma^\mu D_{\beta R})\partial^\nu B_{\mu\nu}$ | $\mathcal{R}_{H\ell}^{\prime\prime(3)\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\sigma^I\gamma^\mu \ell_{\beta L}) D_\mu(H^\dagger\sigma^I H)$ |
| $\mathcal{O}_{uG}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu}T^A U_{\beta R})\widetilde{H} G_{\mu\nu}^A$ | $\mathcal{R}_{Bd}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{D}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu D_{\beta R}\right) B_{\mu\nu}$ | $\mathcal{O}_{He}^{\alpha\beta}$ | $(\overline{E}_{\alpha R}\gamma^\mu E_{\beta R})(H^\dagger i\overleftrightarrow{D}_\mu H)$ |
| $\mathcal{O}_{uW}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu} U_{\beta R})\sigma^I \widetilde{H} W_{\mu\nu}^I$ | $\mathcal{R}_{\widetilde{B}d}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{D}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu D_{\beta R}\right) \widetilde{B}_{\mu\nu}$ | $\mathcal{R}_{He}^{\prime\alpha\beta}$ | $\left(\overline{E}_{\alpha R} i\overleftrightarrow{\not{D}} E_{\beta R}\right)(H^\dagger H)$ |
| $\mathcal{O}_{uB}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu} U_{\beta R})\widetilde{H} B_{\mu\nu}$ | $\mathcal{R}_{W\ell}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\sigma^I\gamma^\mu \ell_{\beta L}) D^\nu W_{\mu\nu}^I$ | $\mathcal{R}_{He}^{\prime\prime\alpha\beta}$ | $(\overline{E}_{\alpha R}\gamma^\mu E_{\beta R})\partial_\mu(H^\dagger H)$ |
| $\mathcal{O}_{dG}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu}T^A D_{\beta R}) H G_{\mu\nu}^A$ | $\mathcal{R}_{W\ell}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{\ell}_{\alpha L}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell_{\beta L}\right) W_{\mu\nu}^I$ | $\psi^2 H^3$ | |
| $\mathcal{O}_{dW}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu} D_{\beta R})\sigma^I H W_{\mu\nu}^I$ | $\mathcal{R}_{\widetilde{W}\ell}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{\ell}_{\alpha L}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell_{\beta L}\right) \widetilde{W}_{\mu\nu}^I$ | $\mathcal{O}_{uH}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\widetilde{H} U_{\beta R})(H^\dagger H)$ |
| $\mathcal{O}_{dB}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L}\sigma^{\mu\nu} D_{\beta R}) H B_{\mu\nu}$ | $\mathcal{R}_{B\ell}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\gamma^\mu \ell_{\beta L})\partial^\nu B_{\mu\nu}$ | $\mathcal{O}_{dH}^{\alpha\beta}$ | $(\overline{Q}_{\alpha L} H D_{\beta R})(H^\dagger H)$ |
| $\mathcal{O}_{eW}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\sigma^{\mu\nu} E_{\beta R})\sigma^I H W_{\mu\nu}^I$ | $\mathcal{R}_{B\ell}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{\ell}_{\alpha L}\gamma^\mu i\overleftrightarrow{D}^\nu \ell_{\beta L}\right) B_{\mu\nu}$ | $\mathcal{O}_{eH}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L} H E_{\beta R})(H^\dagger H)$ |
| $\mathcal{O}_{eB}^{\alpha\beta}$ | $(\overline{\ell}_{\alpha L}\sigma^{\mu\nu} E_{\beta R}) H B_{\mu\nu}$ | $\mathcal{R}_{\widetilde{B}\ell}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{\ell}_{\alpha L}\gamma^\mu i\overleftrightarrow{D}^\nu \ell_{\beta L}\right) \widetilde{B}_{\mu\nu}$ | | |
| | | $\mathcal{R}_{Be}^{\alpha\beta}$ | $(\overline{E}_{\alpha R}\gamma^\mu E_{\beta R})\partial^\nu B_{\mu\nu}$ | | |
| | | $\mathcal{R}_{Be}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{E}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu E_{\beta R}\right) B_{\mu\nu}$ | | |
| | | $\mathcal{R}_{\widetilde{B}e}^{\prime\alpha\beta}$ | $\frac{1}{2}\left(\overline{E}_{\alpha R}\gamma^\mu i\overleftrightarrow{D}^\nu E_{\beta R}\right) \widetilde{B}_{\mu\nu}$ | | |

Table A.1: Two-fermion operators in the Green's basis in the SMEFT.

| $X^3$ | | $X^2H^2$ | | $H^2D^4$ | |
|---|---|---|---|---|---|
| $\mathcal{O}_{3G}$ | $f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $\mathcal{O}_{HG}$ | $G_{\mu\nu}^A G^{A\mu\nu} H^\dagger H$ | $\mathcal{R}_{DH}$ | $(D_\mu D^\mu H)^\dagger (D_\nu D^\nu H)$ |
| $\mathcal{O}_{\widetilde{3G}}$ | $f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$ | $\mathcal{O}_{H\widetilde{G}}$ | $\widetilde{G}_{\mu\nu}^A G^{A\mu\nu} H^\dagger H$ | $H^4D^2$ | |
| $\mathcal{O}_{3W}$ | $\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $\mathcal{O}_{HW}$ | $W_{\mu\nu}^I W^{I\mu\nu} H^\dagger H$ | $\mathcal{O}_{H\Box}$ | $(H^\dagger H)\Box(H^\dagger H)$ |
| $\mathcal{O}_{\widetilde{3W}}$ | $\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$ | $\mathcal{O}_{H\widetilde{W}}$ | $\widetilde{W}_{\mu\nu}^I W^{I\mu\nu} H^\dagger H$ | $\mathcal{O}_{HD}$ | $(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$ |
| $X^2D^2$ | | $\mathcal{O}_{HB}$ | $B_{\mu\nu}B^{\mu\nu} H^\dagger H$ | $\mathcal{R}'_{HD}$ | $(H^\dagger H)(D_\mu H)^\dagger (D^\mu H)$ |
| $\mathcal{R}_{2G}$ | $-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G_{\rho\nu}^A)$ | $\mathcal{O}_{H\widetilde{B}}$ | $\widetilde{B}_{\mu\nu}B^{\mu\nu} H^\dagger H$ | $\mathcal{R}''_{HD}$ | $(H^\dagger H) D_\mu \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)$ |
| $\mathcal{R}_{2W}$ | $-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W_{\rho\nu}^I)$ | $\mathcal{O}_{HWB}$ | $W_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$ | $H^6$ | |
| $\mathcal{R}_{2B}$ | $-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$ | $\mathcal{O}_{H\widetilde{W}B}$ | $\widetilde{W}_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$ | $\mathcal{O}_H$ | $(H^\dagger H)^3$ |
| | | $H^2XD^2$ | | | |
| | | $\mathcal{R}_{WDH}$ | $D_\nu W^{I\mu\nu}\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)$ | | |
| | | $\mathcal{R}_{BDH}$ | $\partial_\nu B^{\mu\nu}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)$ | | |

Table A.2: Bosonic operators in the Green's basis in the SMEFT.

# Backup

The Greens basis in the SMEFT  Jiang et al., 2019; Gherardi et al., 2020; Carmona et al., 2022

| Four-quark | | Four-lepton | | Semileptonic | |
|---|---|---|---|---|---|
| $\mathcal{O}_{qq}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu Q_{\beta L})(\overline{Q_{\gamma L}}\gamma_\mu Q_{\lambda L})$ | $\mathcal{O}_{\ell\ell}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \ell_{\beta L})(\overline{\ell_{\gamma L}}\gamma_\mu \ell_{\lambda L})$ | $\mathcal{O}_{\ell q}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \ell_{\beta L})(\overline{Q_{\gamma L}}\gamma_\mu Q_{\lambda L})$ |
| $\mathcal{O}_{qq}^{(3)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu \sigma^I Q_{\beta L})(\overline{Q_{\gamma L}}\gamma_\mu \sigma^I Q_{\lambda L})$ | $\mathcal{O}_{ee}^{\alpha\beta\gamma\lambda}$ | $(\overline{E_{\alpha R}}\gamma^\mu E_{\beta R})(\overline{E_{\gamma R}}\gamma_\mu E_{\lambda R})$ | $\mathcal{O}_{\ell q}^{(3)\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \sigma^I \ell_{\beta L})(\overline{Q_{\gamma L}}\gamma_\mu \sigma^I Q_{\lambda L})$ |
| $\mathcal{O}_{uu}^{\alpha\beta\gamma\lambda}$ | $(\overline{U_{\alpha R}}\gamma^\mu U_{\beta R})(\overline{U_{\gamma R}}\gamma_\mu U_{\lambda R})$ | $\mathcal{O}_{\ell e}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \ell_{\beta L})(\overline{E_{\gamma R}}\gamma_\mu E_{\lambda R})$ | $\mathcal{O}_{eu}^{\alpha\beta\gamma\lambda}$ | $(\overline{E_{\alpha R}}\gamma^\mu E_{\beta R})(\overline{U_{\gamma R}}\gamma_\mu U_{\lambda R})$ |
| $\mathcal{O}_{dd}^{\alpha\beta\gamma\lambda}$ | $(\overline{D_{\alpha R}}\gamma^\mu D_{\beta R})(\overline{D_{\gamma R}}\gamma_\mu D_{\lambda R})$ | | | $\mathcal{O}_{ed}^{\alpha\beta\gamma\lambda}$ | $(\overline{E_{\alpha R}}\gamma^\mu E_{\beta R})(\overline{D_{\gamma R}}\gamma_\mu D_{\lambda R})$ |
| $\mathcal{O}_{ud}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{U_{\alpha R}}\gamma^\mu U_{\beta R})(\overline{D_{\gamma R}}\gamma_\mu D_{\lambda R})$ | | | $\mathcal{O}_{qe}^{\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu Q_{\beta L})(\overline{E_{\gamma R}}\gamma_\mu E_{\lambda R})$ |
| $\mathcal{O}_{ud}^{(8)\alpha\beta\gamma\lambda}$ | $(\overline{U_{\alpha R}}\gamma^\mu T^A U_{\beta R})(\overline{D_{\gamma R}}\gamma_\mu T^A D_{\lambda R})$ | | | $\mathcal{O}_{\ell u}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \ell_{\beta L})(\overline{U_{\gamma R}}\gamma_\mu U_{\lambda R})$ |
| $\mathcal{O}_{qu}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu Q_{\beta L})(\overline{U_{\gamma R}}\gamma_\mu U_{\lambda R})$ | | | $\mathcal{O}_{\ell d}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}}\gamma^\mu \ell_{\beta L})(\overline{D_{\gamma R}}\gamma_\mu D_{\lambda R})$ |
| $\mathcal{O}_{qu}^{(8)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu T^A Q_{\beta L})(\overline{U_{\gamma R}}\gamma_\mu T^A U_{\lambda R})$ | | | $\mathcal{O}_{\ell edq}^{\alpha\beta\gamma\lambda}$ | $(\overline{\ell_{\alpha L}} E_{\beta R})(\overline{D_{\gamma R}} Q_{\lambda L})$ |
| $\mathcal{O}_{qd}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu Q_{\beta L})(\overline{D_{\gamma R}}\gamma_\mu D_{\lambda R})$ | | | $\mathcal{O}_{\ell equ}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{\ell^a_{\alpha L}} E_{\beta R})\,\epsilon^{ab}\,(\overline{Q^b_{\gamma L}} U_{\lambda R})$ |
| $\mathcal{O}_{qd}^{(8)\alpha\beta\gamma\lambda}$ | $(\overline{Q_{\alpha L}}\gamma^\mu T^A Q_{\beta L})(\overline{D_{\gamma R}}\gamma_\mu T^A D_{\lambda R})$ | | | $\mathcal{O}_{\ell equ}^{(3)\alpha\beta\gamma\lambda}$ | $(\overline{\ell^a_{\alpha L}}\sigma^{\mu\nu} E_{\beta R})\,\epsilon^{ab}\,(\overline{Q^b_{\gamma L}}\sigma_{\mu\nu} U_{\lambda R})$ |
| $\mathcal{O}_{quqd}^{(1)\alpha\beta\gamma\lambda}$ | $(\overline{Q^a_{\alpha L}} U_{\beta R})\,\epsilon^{ab}\,(\overline{Q^b_{\gamma L}} D_{\lambda R})$ | | | | |
| $\mathcal{O}_{quqd}^{(8)\alpha\beta\gamma\lambda}$ | $(\overline{Q^a_{\alpha L}} T^A U_{\beta R})\,\epsilon^{ab}\,(\overline{Q^b_{\gamma L}} T^A D_{\lambda R})$ | | | | |

| $B$- and $L$-number violating | |
|---|---|
| $\mathcal{O}_{duq}^{\alpha\beta\gamma\lambda}$ | $\epsilon^{ABC}\epsilon^{ab}\left[(D^A_{\alpha R})^T\, C U^B_{\beta R}\right]\left[(Q^{Ca}_{\gamma L})^T\, C \ell^b_{\lambda L}\right]$ |
| $\mathcal{O}_{qqu}^{\alpha\beta\gamma\lambda}$ | $\epsilon^{ABC}\epsilon^{ab}\left[(Q^{Aa}_{\alpha L})^T\, C Q^{Bb}_{\beta L}\right]\left[(U^C_{\gamma R})^T\, C E_{\lambda R}\right]$ |
| $\mathcal{O}_{qqq}^{\alpha\beta\gamma\lambda}$ | $\epsilon^{ABC}\epsilon^{ad}\varepsilon^{be}\left[(Q^{Aa}_{\alpha L})^T\, C Q^{Bb}_{\beta L}\right]\left[(Q^{Ce}_{\gamma L})^T\, C \ell^d_{\lambda L}\right]$ |
| $\mathcal{O}_{duu}^{\alpha\beta\gamma\lambda}$ | $\epsilon^{ABC}\left[(D^A_{\alpha R})^T\, C U^B_{\beta R}\right]\left[(U^C_{\gamma R})^T\, C E_{\lambda R}\right]$ |

Table A.3: Baryon and lepton number conserving four-fermion operators in the Green's basis (and also in the Warsaw basis) in the SMEFT.

# Backup

The tree-level Lagrange up to dimension-six for three seesaw mechanisms

$$\mathcal{L}^{\rm tree}_{\rm I,EFT} = \mathcal{L}_{\rm SM} + \left[\frac{1}{2}\left(Y_\nu M^{-1} Y_\nu^{\rm T}\right)_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta \rm L} + {\rm h.c.}\right] + \frac{1}{4}\left(Y_\nu M^{-2} Y_\nu^\dagger\right)_{\alpha\beta} \times \left[\left(\overline{\ell_{\alpha \rm L}} \gamma^\mu \ell_{\beta \rm L}\right)\left(H^\dagger {\rm i} \overleftrightarrow{D}_\mu H\right) - \left(\overline{\ell_{\alpha \rm L}} \gamma^\mu \tau^I \ell_{\beta \rm L}\right)\left(H^\dagger {\rm i} \overleftrightarrow{D}^I_\mu H\right)\right] , \quad (3.58)$$

$$\mathcal{L}^{\rm tree}_{\rm II,EFT} = \mathcal{L}_{\rm SM} + 2\lambda_\Delta^2 \left(1 + \frac{2m^2}{M_\Delta^2}\right)\left(H^\dagger H\right)^2 - \left[\frac{\lambda_\Delta \left(Y_\Delta\right)_{\alpha\beta}}{M_\Delta} \overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta \rm L} + {\rm h.c.}\right] + \frac{\left(Y_\Delta\right)_{\alpha\gamma}\left(Y_\Delta^\dagger\right)_{\beta\delta}}{4M_\Delta^2}\left(\overline{\ell_{\alpha \rm L}} \gamma^\mu \ell_{\beta \rm L}\right)\left(\overline{\ell_{\gamma \rm L}} \gamma_\mu \ell_{\delta \rm L}\right) + \frac{2\left(4\lambda - \lambda_3 + \lambda_4 - 8\lambda_\Delta^2\right)\lambda_\Delta^2}{M_\Delta^2} \times \left(H^\dagger H\right)^3 + \frac{2\lambda_\Delta^2}{M_\Delta^2}\left[\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + 2\left(H^\dagger D^\mu H\right)^\dagger\left(H^\dagger D_\mu H\right)\right] + \frac{2\lambda_\Delta^2}{M_\Delta^2}\left[\left(\overline{\ell_{\rm L}} Y_l H E_{\rm R} + \overline{Q_{\rm L}} Y_{\rm u} \widetilde{H} U_{\rm R} + \overline{Q_{\rm L}} Y_{\rm d} H D_{\rm R}\right)\left(H^\dagger H\right) + {\rm h.c.}\right] , \quad (3.59)$$

$$\mathcal{L}^{\rm tree}_{\rm III,EFT} = \mathcal{L}_{\rm SM} + \left[\frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^{\rm T}\right)_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^{\rm T} \ell^{\rm c}_{\beta \rm L} + {\rm h.c.}\right] + \frac{1}{4}\left(Y_\Sigma M_\Sigma^{-2} Y_\Sigma^\dagger\right)_{\alpha\beta} \times \left[3\left(\overline{\ell_{\alpha \rm L}} \gamma^\mu \ell_{\beta \rm L}\right)\left(H^\dagger {\rm i} \overleftrightarrow{D}_\mu H\right) + \left(\overline{\ell_{\alpha \rm L}} \gamma^\mu \tau^I \ell_{\beta \rm L}\right)\left(H^\dagger {\rm i} \overleftrightarrow{D}^I_\mu H\right)\right] + \left[\overline{\ell_{\rm L}} Y_\Sigma M_\Sigma^{-2} Y_\Sigma^\dagger Y_l H E_{\rm R}\left(H^\dagger H\right) + {\rm h.c.}\right] . \quad (3.60)$$

# Backup

Examples for RGEs of parameters

$$\dot{\eta}_{\alpha\alpha} = \sum_{\gamma\neq\alpha} 4\frac{m^2}{v^2} y_{\alpha\gamma}\left[\left|\eta'_{\alpha\gamma}\right|\cos\left(\phi'_{\alpha\gamma}-\phi_{\alpha\gamma}\right) - 4\left|\eta_{\alpha\gamma}\right|\right]\left|\eta_{\alpha\gamma}\right| + \frac{2}{3}g_2^2\,\mathrm{tr}\left(\eta\right) - \sum_i \kappa_i^2 v^2\left|U_{\alpha i}\right|^2 + y_\alpha^2\left(5\eta_{\alpha\alpha} - 3\eta'_{\alpha\alpha}\right) + \left(-\frac{17}{3}g_2^2 + 2T\right)\eta_{\alpha\alpha}\,,$$

$$\dot{\eta}'_{\alpha\alpha} = \sum_{\gamma\neq\alpha} 4\frac{m^2}{v^2} y_{\alpha\gamma}\left[\left|\eta'_{\alpha\gamma}\right| - 4\left|\eta_{\alpha\gamma}\right|\cos\left(\phi_{\alpha\gamma}-\phi'_{\alpha\gamma}\right)\right]\left|\eta'_{\alpha\gamma}\right| + \frac{2}{3}g_1^2\mathrm{tr}\left(\eta'\right) + \frac{2}{3}\left(g_2^2 - g_1^2\right)\mathrm{tr}\left(\eta\right) - \frac{5}{2}\sum_i \kappa_i^2 v^2\left|U_{\alpha i}\right|^2 + y_\alpha^2\left(\eta'_{\alpha\alpha} - 8\eta_{\alpha\alpha}\right) + \left(\frac{1}{3}g_1^2 + 2T\right)\eta'_{\alpha\alpha} - \frac{1}{3}\left(17g_2^2 + g_1^2\right)\eta_{\alpha\alpha}\,,$$

$$\begin{aligned}
\dot{\phi}_{\mu\tau} = & +\kappa_1^2 v^2\left|\eta_{\mu\tau}\right|^{-1}\left\{\left[s_{23}c_{23}\left(c_{12}^2 s_{13}^2 - s_{12}^2\right) + s_{12}c_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right)c_\delta\right]s_{\mu\tau} + s_{12}c_{12}s_{13}s_\delta c_{\mu\tau}\right\} \\
& +\kappa_2^2 v^2\left|\eta_{\mu\tau}\right|^{-1}\left\{\left[s_{23}c_{23}\left(s_{12}^2 s_{13}^2 - c_{12}^2\right) - s_{12}c_{12}s_{13}\left(c_{23}^2 - s_{23}^2\right)c_\delta\right]s_{\mu\tau} - s_{12}c_{12}s_{13}s_\delta c_{\mu\tau}\right\} \\
& +\kappa_3^2\,v^2\left|\eta_{\mu\tau}\right|^{-1} c_{13}^2 s_{23}c_{23}s_{\mu\tau} + \frac{3}{2}\left(y_\mu^2 + y_\tau^2\right)\left|\eta'_{\mu\tau}\right|\left|\eta_{\mu\tau}\right|^{-1}\sin\left(\phi_{\mu\tau} - \phi'_{\mu\tau}\right) \\
& +2\frac{m^2}{v^2}\left|\eta_{\mu\tau}\right|^{-1}\left\{y_{\mu\tau}\left(\eta_{\tau\tau} - \eta_{\mu\mu}\right)\left|\eta'_{\mu\tau}\right|\sin\left(\phi'_{\mu\tau} - \phi_{\mu\tau}\right)\right. \\
& - y_{e\mu}\left|\eta_{e\tau}\right|\left[\left|\eta'_{e\mu}\right|\sin\left(\phi_{e\tau} - \phi'_{e\mu} - \phi_{\mu\tau}\right) - 4\left|\eta_{e\mu}\right|\sin\left(\phi_{e\tau} - \phi_{e\mu} - \phi_{\mu\tau}\right)\right] \\
& \left.- y_{e\tau}\left|\eta_{e\mu}\right|\left[\left|\eta'_{e\tau}\right|\sin\left(\phi'_{e\tau} - \phi_{e\mu} - \phi_{\mu\tau}\right) - 4\left|\eta_{e\tau}\right|\sin\left(\phi_{e\tau} - \phi_{e\mu} - \phi_{\mu\tau}\right)\right]\right\} \\
& -2\frac{m^2}{v^2}s_{13}c_{13}^{-1}\left[y_{e\mu}s_{23}^{-1}\left(\left|\eta'_{e\mu}\right|s'_{e\mu+\delta} - 4\left|\eta_{e\mu}\right|s_{e\mu+\delta}\right) - y_{e\tau}c_{23}^{-1}\left(\left|\eta'_{e\tau}\right|s'_{e\tau+\delta} - 4\left|\eta_{e\tau}\right|s_{e\tau+\delta}\right)\right] \\
& -2\frac{m^2}{v^2}y_{\mu\tau}s_{23}^{-1}c_{23}^{-1}\left(c_{23}^2 - s_{23}^2\right)\left(\left|\eta'_{\mu\tau}\right|s'_{\mu\tau} - 4\left|\eta_{\mu\tau}\right|s_{\mu\tau}\right) \\
& +C_\kappa\zeta_{13}^{-1}s_{12}\left[s_{12}\left(y_\mu^2 - y_\tau^2\right)c_\rho - c_{12}s_{13}s_{23}^{-1}c_{23}^{-1}\left(y_e^2 - s_{23}^2 y_\mu^2 - c_{23}^2 y_\tau^2\right)c_{\rho+\delta}\right]s_\rho \\
& -C_\kappa\zeta_{13}\,s_{12}\left[s_{12}\left(y_\mu^2 - y_\tau^2\right)s_\rho - c_{12}s_{13}s_{23}^{-1}c_{23}^{-1}\left(y_e^2 - s_{23}^2 y_\mu^2 - c_{23}^2 y_\tau^2\right)s_{\rho+\delta}\right]c_\rho \\
& +C_\kappa\zeta_{23}^{-1}c_{12}\left[c_{12}\left(y_\mu^2 - y_\tau^2\right)c_\sigma + s_{12}s_{13}s_{23}^{-1}c_{23}^{-1}\left(y_e^2 - s_{23}^2 y_\mu^2 - c_{23}^2 y_\tau^2\right)c_{\sigma+\delta}\right]s_\sigma \\
& -C_\kappa\zeta_{23}\,c_{12}\left[c_{12}\left(y_\mu^2 - y_\tau^2\right)s_\sigma + s_{12}s_{13}s_{23}^{-1}c_{23}^{-1}\left(y_e^2 - s_{23}^2 y_\mu^2 - c_{23}^2 y_\tau^2\right)s_{\sigma+\delta}\right]c_\sigma\,,
\end{aligned}$$

# Backup

The standard parametrization of the mixing matrix Workman et al. [Particle Data Group], 2022

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & +c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

Inputs for parameters

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $m_u$/MeV | 1.2504 | $m_e$/MeV | 0.5239 | $g_1$ | 0.3589 | $\eta_{ee}$ | $1.25 \times 10^{-3}$ |
| $m_d$/MeV | 2.7176 | $m_\mu$/GeV | 0.1104 | $g_2$ | 0.6468 | $\eta_{\mu\mu}$ | $2.21 \times 10^{-4}$ |
| $m_s$/MeV | 54.120 | $m_\tau$/GeV | 1.8748 | $g_s$ | 1.1525 | $\eta_{\tau\tau}$ | $2.81 \times 10^{-3}$ |
| $m_c$/GeV | 0.6299 | $m_1$/eV | 0.05 | $\lambda$ | 0.1235 | $\lvert\eta_{e\mu}\rvert$ | $1.20 \times 10^{-5}$ |
| $m_b$/GeV | 2.8731 | $m_2$/eV | 0.05074 | $m^2/\mathrm{GeV}^2$ | −8672.61 | $\lvert\eta_{e\tau}\rvert$ | $1.35 \times 10^{-3}$ |
| $m_t$/GeV | 173.075 | $m_3$/eV | 0.07079 | $\delta^q$ | 1.144 | $\lvert\eta_{\mu\tau}\rvert$ | $6.13 \times 10^{-4}$ |
| $\sin\theta^q_{12}$ | 0.2250 | $\sin\theta_{12}$ | 0.5505 | $\delta$ | 3.438 | $\phi_{e\mu}$ | $\pi/3$ |
| $\sin\theta^q_{23}$ | 0.04182 | $\sin\theta_{23}$ | 0.7563 | $\rho$ | $\pi/6$ | $\phi_{e\tau}$ | $\pi/3$ |
| $\sin\theta^q_{13}$ | 0.00369 | $\sin\theta_{13}$ | 0.1484 | $\sigma$ | $\pi/4$ | $\phi_{\mu\tau}$ | $\pi/3$ |

Table 2: Summary of the input values of all the relevant parameters at the benchmark energy scale $\mu_B = 200$ GeV. See the main text for further explanations.

Alam and Martin, 2023

Workman et al, 2022

Esteban et al, 2020

Fernandez-Martinez, Hernandez-Garcia and Lopez-Pavon, 2016

$$\eta'(\mu_M) = 2\eta(\mu_M)$$

$$v = \sqrt{-m^2/\lambda}$$

$\mu_M = O(M_R) = 10^4$ GeV but $O(Y_\nu) \sim 1$ → Sizable $\eta$ and $\eta'$ ← Underlying symmetries

See, e.g., Kersten and Smirnov, 2007; Abada et al., 2007