Classes of complete dark photon models constrained by Z-Physics

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This talk is based on M.P. Bento, H.E. Haber, and J.P. Silva, Phys. Lett. B **850**, 138501 (2024). See also M.P. Bento, H.E. Haber, and J.P. Silva, JHEP **10** (2023) 083.

Gauge boson kinetic mixing as a portal to the dark sector

Dark matter may reside in a "dark sector" that communicates with the Standard Model (SM) via gauge boson kinetic mixing of a U(1) gauge boson of the SM and a U(1)′ gauge boson called the dark photon (sometimes called the dark Z).

Adding matter that is charged under U(1)′ but neutral with respect to the SM provides a plausible model for particle dark matter.

There are many dark photon models in the literature that are based on mixing $U(1)_{\rm EM}$ with U(1)′, under the assumption that the Z boson can be integrated out and is therefore irrelevant.

However, it is dangerous to neglect the effects of the Z due to constraints from precision electroweak data.

Indeed, the precision electroweak data are in good agreement with the SM[1] and thus can be used to constrain beyond the Standard Model (BSM) physics.

In many cases, the corrections to precision electroweak observables arise mainly through gauge boson self-energy corrections, which lead to the introduction of the so-called oblique parameters (e.g., the Peskin-Takeuchi S, T, and U parameters).

[1] There are a few intriguing deviations, e.g., $g-2$ of the muon, the Tevatron W mass measurement, and a few Z-pole observables (A_{LR} and A^b_{FB}), that could potentially be evidence for BSM physics.

Oblique parameters in an SU(2)×U(1)×U(1)′ model

The current interactions of the SM electroweak gauge bosons are given by:[2]

$$\begin{aligned}\mathscr{L}_{\rm EW} =& m_W^2 W^{+\mu} W^-_\mu + \tfrac{1}{2}\widetilde{m}_Z^2 \left(1+\Delta_1\right) Z^\mu Z_\mu - \frac{g}{\sqrt{2}}\left(J^\mu_{\rm CC} W^+_\mu + \text{h.c.}\right) \\ & - eJ^\mu_{\rm em} A_\mu - \frac{g}{2c_W}\left(1+\Delta_2\right) J^\mu_{\rm NC} Z_\mu - e\Delta_3 J^\mu_{\rm em} Z_\mu\,,\end{aligned}$$

where $m_W^2 = \frac{1}{4}g^2v^2$, $\widetilde{m}_Z^2 = \frac{1}{4}(g^2+g'^{\,2})v^2$, and $e = gs_W = g'c_W$. Here, $v \simeq 246$ GeV, $s_W \equiv \sin\theta_W$, and $c_W \equiv \cos\theta_W$.

[2] H. Davoudiasl, K. Enomoto, H.-S. Lee, J. Lee, and W.J. Marciano Phys. Rev. D **108**, 115018 (2023). See also B. Holdom, Phys. Lett. B **166**, 196 (1986).

The corresponding oblique parameters are:

$$
\begin{aligned}
\alpha_{\mathrm{EM}} S &= 8 s_W^2 c_W^2 \Delta_2 - 4 s_W c_W (c_W^2 - s_W^2) \Delta_3 \,, \\
\alpha_{\mathrm{EM}} T &= -\Delta_1 + 2\Delta_2 \,, \\
\alpha_{\mathrm{EM}} U &= -8 s_W^2 c_W (c_W \Delta_2 + s_W \Delta_3) \,,
\end{aligned}
$$

where $\alpha_{\mathrm{EM}} \equiv e^2/(4\pi)$. The precision electroweak data yield:[3]

$$
\begin{aligned}
S &= -0.04 \pm 0.10 \,, \\
T &= 0.01 \pm 0.12 \,, \\
U &= -0.01 \pm 0.09 \,,
\end{aligned}
$$

where $S = T = U = 0$ corresponds to the SM.

[3] J. Erler and A. Freitas, in S. Navas et al. (Particle Data Group), to be published in Phys. Rev. D **110**, 030001 (2024).

The parameter ρ_0

In the SM, $\rho \equiv m_W^2/(m_Z^2 c_W^2) = 1$ at tree-level. Erler and Freitas (in their 2024 Review of Particle Physics review) introduce

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2 \hat{c}_Z^2 \hat{\rho}} = 1.00031 \pm 0.00019\,,$$

where $\hat{c}_Z^2 = 1 - \sin^2\hat{\theta}_W(m_Z)$ is defined in the $\overline{\rm MS}$ scheme, and $\hat{\rho} \equiv m_W^2/(m_Z^2 \hat{c}_Z^2)$ is computed assuming the validity of the SM.

That is, $\rho_0 = 1$ in the SM, and a deviation from $\rho_0 = 1$ can be interpreted as a consequence of tree-level BSM physics (under the assumption that the latter is a small perturbation that does not significantly modify the SM electroweak radiative corrections).

A generic SU(2)×U(1)×U(1)′ model

Kinetic mixing of the hypercharge gauge boson $\hat{B}$ and the U(1)′ gauge boson $\hat{X}$ is governed by the mixing parameter ϵ,

$$\mathcal{L} \supset -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{X}_{\mu\nu}\hat{X}^{\mu\nu} + \frac{\epsilon}{2c_W}\hat{X}_{\mu\nu}\hat{B}^{\mu\nu}\,.$$

To obtain a canonical form of the kinetic Lagrangian, we transform the $\hat{B}$ and $\hat{X}$ fields such that

$$\hat{X} = \eta X\,, \quad \hat{B} = B + \frac{\epsilon}{c_W}\eta X\,,$$

where

$$\eta \equiv \frac{1}{\sqrt{1-\epsilon^2/c_W^2}}\,.$$

Scalar fields and their vacuum expectation values (vevs)

Φ: SM-like complex scalar doublet with weak isospin $t_1 = \frac{1}{2}$, U(1) and U(1)′ charges $Y = \frac{1}{2}$ and $Y' = 0$, and vev $\langle \Phi^0 \rangle = v_1/\sqrt{2}$.

φ_i $(i = 2, 3, \ldots N)$: with weak isospins t_i, U(1) and U(1)′ charges y_i and y'_i, and vevs $\langle \varphi_i^0 \rangle = v_i/\sqrt{2}$.

The resulting $W^\pm$ mass is

$$m_W^2 = \frac{g^2 v^2}{4} = \frac{g^2 \left[v_1^2 + \sum_{i=2}^{N} 2(C_{R_i} - y_i^2) v_i^2 c_i \right]}{4},$$

where $C_{R_i} = t_i(t_i+1)$ for a complex [real] φ_i multiplet, with $c_i = 1$ [$c_i = 1/2$]. Scalar field multiplets are chosen such $C_{R_i} = 3y_i^2$ (to reproduce the observed value of m_W/m_Z).

The squared-mass matrix of the massive neutral gauge bosons with respect to the $\{Z^0, X\}$ basis, where $Z^0 \equiv W^3 c_W - B s_W$ is orthogonal to the photon field A, is given by

$$\mathcal{M}^2 = \begin{bmatrix} \widetilde{m}_Z^2 & (\mathcal{M}^2)_{12} \\ (\mathcal{M}^2)_{12} & (\mathcal{M}^2)_{22} \end{bmatrix} .$$

An explicit expression for the off-diagonal element of $\mathcal{M}^2$ is

$$(\mathcal{M}^2)_{12} = -\frac{\widetilde{m}_Z^2}{v^2} \left[4\eta t_W \epsilon \sum_{i=1}^{N} v_i^2 y_i^2 + 4\eta\tau c_W \sum_{i=2}^{N} v_i^2 y_i y_i' \right] ,$$

where $t_W \equiv s_W/c_W$, $\tau \equiv g_X/g$ and $\eta \equiv 1/\sqrt{1-\epsilon^2/c_W^2}$. Diagonalizing $\mathcal{M}^2$ yields the mass eigenstate fields Z and Z'.

$$\begin{pmatrix} Z^0 \\ X \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}$$

defines the mixing angle α, and $\widetilde{m}_Z^2 = m_Z^2 \cos^2\alpha + m_{Z'}^2 \sin^2\alpha$, where m_Z $[m_{Z'}]$ is the mass of the physical Z [dark Z'] boson.

Having chosen scalar multiplets such that $C_{R_i} = 3y_i^2$, it follows that

$$\frac{m_W^2}{\widetilde{m}_Z^2 c_W^2} = 1\,,$$

at tree level. Hence,

$$\rho - 1 = (r - 1)\sin^2\alpha\,,$$

with $r \equiv m_{Z'}^2/m_Z^2$, and the value of $\sin\alpha$ is controlled by $\left(\mathcal{M}^2\right)_{12}$.

In particular,

$$\sin^2 2\alpha = \frac{4\big[(\mathcal{M}^2)_{12}\big]^2}{(m_Z^2 - m_{Z'}^2)^2}\,.$$

It is useful to eliminate $\sin\alpha$ in favor of the parameter r_{12}^2,

$$r_{12}^2 \equiv \left(\frac{(\mathcal{M}^2)_{12}}{\widetilde{m}_Z^2}\right)^2 = \frac{(1-r)^2\sin^2\alpha\cos^2\alpha}{\big[1-(1-r)\sin^2\alpha\big]^2}\,.$$

As before, $r \equiv m_{Z'}^2/m_Z^2$. The end result is:

$$\rho - 1 = \frac{-1 + r - 2r_{12}^2 + \sqrt{(1-r)^2 - 4r\,r_{12}^2}}{2(1+r_{12}^2)}\,,$$

which is a monotonically decreasing function of r_{12}. Equivalently,

$$r_{12}^2 = \frac{(1-\rho)(\rho - r)}{\rho^2}\,.$$

A dark matter (DM) candidate

Consider the dark Z' model with an additional an SU(2)×U(1) singlet Dirac fermion with a nonzero U(1)′ charge, denoted by χ. Then, the dark Lagrangian is given by

$$\mathscr{L}_{\rm DM} = i\overline{\chi}\not{D}\chi - m_\chi\overline{\chi}\chi\,,$$

where the covariant derivative can be expanded as

$$D_\mu = \partial_\mu + ig_X Y'\eta\,(s_\alpha Z_\mu + c_\alpha Z'_\mu)\,,$$

with $s_\alpha \equiv \sin\alpha$ and $c_\alpha \equiv \cos\alpha$.

In the following, we assume that the DM candidate χ is in thermal equilibrium in the early Universe.

The velocity averaged cross section for $\overline{\chi}\chi$ annihilation is given by

$$\langle \sigma_{\chi\chi} v \rangle \simeq 2 \times 10^{-26}\ \mathrm{cm}^3\,\mathrm{s}^{-1} \simeq 1.7 \times 10^{-9}\ \mathrm{GeV}^{-2}\,,$$

for values of $m_\chi \gtrsim 10$ GeV, under the assumption that χ particles saturate the observed DM abundance today. As the Universe evolves and the temperature drops, a point is reached where the DM decouples from the thermal bath and it freezes out.

We consider models where $m_{Z'} < m_Z$ (or equivalently, $r < 1$), omitting the regime where r is close to 1. Two scenarios emerge:

1. $m_{Z'} > m_\chi > m_e$
2. $m_\chi > m_{Z'} > m_e$

1. The characteristic regime: $m_{Z'} > m_\chi > m_e$.[4]

The dominant annihilation mechanism is the s-channel scattering process $\overline{\chi}\chi \to Z'^* \to \bar{f} f$. It then follows that

$$\langle \sigma_{\chi\chi} v \rangle \approx \frac{m_\chi^2}{\pi m_{Z'}^4} \left(\epsilon e g_X Y'\right)^2 ,$$

under the assumption that $m_\chi \gg m_e$ and $m_{Z'} \gg m_\chi$. By assuming $Y' = 1$, we obtain the observed DM abundance with

$$\frac{1.7 \times 10^{-9}}{\mathrm{GeV}^2} \approx \frac{0.038}{\mathrm{GeV}^2} \left(\frac{m_\chi}{0.01\,\mathrm{GeV}}\right)^2 \left(\frac{0.1\,\mathrm{GeV}}{m_{Z'}}\right)^4 (\epsilon\, g_X)^2 ,$$

which, after fixing the masses, yields a value for $\epsilon\, g_X$.

[4] See E. Izaguirre, G. Krnjaic, P. Schuster, and N. Toro, Phys. Rev. D **90**, 014052 (2014), and H. Davoudiasl and W.J. Marciano, Phys. Rev. D **92** 035008 (2015).

2. The secluded regime: $m_\chi > m_{Z'} > m_e$.[5]

The dominant annihilation mechanism is $\overline{\chi}\chi \to Z'Z'$ via t-channel χ-exchange. It then follows that

$$\langle \sigma_{\chi\chi} v \rangle \approx \frac{g_X^4\, \eta^4\, c_\alpha^4\, Y'^4}{8\pi m_\chi^2}\,,$$

under the assumption that $m_\chi \gg m_{Z'}$. Assuming again that $Y' = 1$, we obtain the observed DM abundance with

$$1.7 \times 10^{-9}\,\mathrm{GeV}^{-2} \approx 0.04\, \frac{g_X^4\, \eta^4\, c_\alpha^4}{m_\chi^2}\,.$$

After fixing m_χ and $m_{Z'}$ (and determining the mixing angle α), we may constrain the values of g_X and ϵ.

[5] See M. Pospelov, A. Ritz, and M.B. Voloshin, Phys. Lett. B **662**, 53 (2008), and J.A. Evans, S. Gori and J. Shelton, Looking for the WIMP Next Door, JHEP **02** (2018) 100.

Dark matter and the electroweak ρ parameter

Example 1: An SU(2)×U(1)×U(1)′ model with scalar multiplets φ_i beyond the SM Higgs doublet that are non-inert (i.e., $v_i \neq 0$) and charged under both U(1) and U(1)′. If we assume a parameter regime where $\epsilon \ll 1$ and $r = m^2_{Z'}/m^2_Z \ll 1$, then

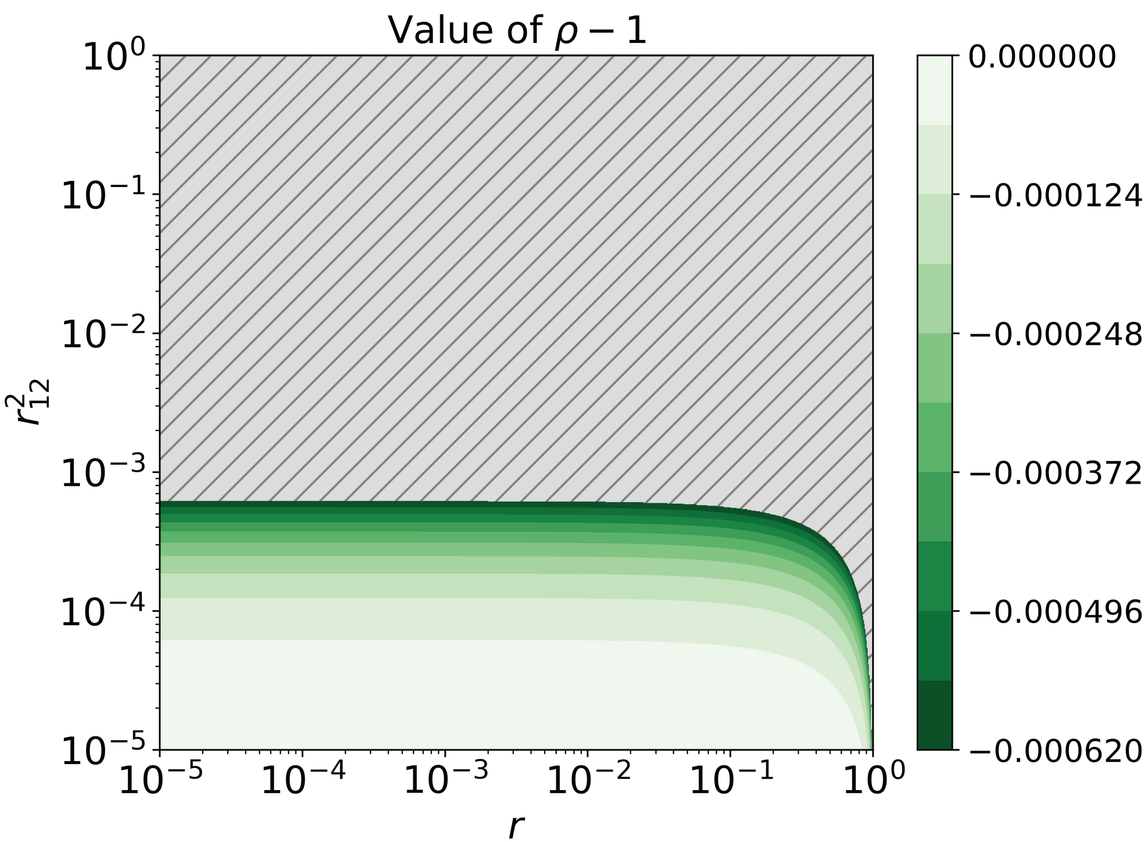
$$c^2_\alpha = \frac{1}{1 + r^2_{12}} + \mathcal{O}(r)\,.$$

where $r^2_{12} = \left[(\mathcal{M}^2)_{12}\right]^2/\widetilde{m}^4_Z \sim g^2_X/g^2 \sim 2.34 g^2_X$, assuming that

$$\frac{4c_W}{v^2}\sum_i y_i y'_i v^2_i \sim \mathcal{O}(1)\,.$$

For example, if $m_\chi = 20\,\text{GeV}$ (corresponding to the secluded regime) then $g_X \sim 0.0645$.

As g_X becomes larger, so does r_{12}^2. Because the expression obtained for $\rho - 1$ is a monotonically decreasing function with r_{12}^2, it follows that $\rho - 1$ gets more negative with larger r_{12}. Thus, a large g_X pushes towards a larger negative value of $\rho - 1$.



For $r \ll 1$, the contribution of r to $\rho - 1$ is small. Then, we may approximate

$$\rho - 1 = -\frac{r_{12}^2}{1 + r_{12}^2} + \mathcal{O}(r)\,.$$

Using $r_{12}^2 \sim g_X^2/g^2 \sim 2.34 g_X^2$, we end up with

$$\rho - 1 \sim -0.0096\,,$$

which, is inconsistent with the global electroweak fit value of $\rho_0 = 1.00031 \pm 0.00019$ quoted earlier. That is, we can assume that the deviation from $\rho = 1$, which is due to the tree-level effect exhibited above, can be constrained by the observed value of ρ_0.

Example 2: An SU(2)×U(1)×U(1)′ model with an extended Higgs sector that contains an SU(2)×U(1) singlet scalar φ (dark Higgs) with a U(1)′ charge of $y' = 1$. In this case, $r_{12}^2 = \eta^2 t_W^2 \epsilon^2$. Assuming that $|\epsilon| \ll 1$ and $1 - r \sim \mathcal{O}(1)$, it follows that

$$\rho - 1 = \frac{r_{12}^2}{r-1} + \mathcal{O}(r_{12}^4)\,.$$

We then end up with:

$$\rho - 1 = -\frac{\epsilon^2 t_W^2}{1-r} + \mathcal{O}(\epsilon^4)\,.$$

For example, assuming that the true value of ρ_0 is no more than 5σ below the central value obtained in the analysis of electroweak data, one can deduce an upper limit of $|\epsilon| \lesssim 0.05$.

Conclusions

- Models of dark matter mediated by a dark photon (or dark Z boson) cannot ignore constraints of precision electroweak data.

- The precision of the parameter ρ_0 obtained in a global fit to electroweak data imposes strong constraints on realistic models of dark matter that communicate with the SM sector via gauge boson kinetic mixing.

- Additional constraints based on the oblique parameters (or more generally, the coefficients of higher dimensional operators in SMEFT or HEFT) should also be taken into account in determining whether a particular dark matter model is viable.