Multiphonon Processes in Spin-Dependent Dark-Matter Scattering

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IN COLLABORATION WITH P. MUNBODH, S. KNAPEN, S. GORI, & T. LIN

Outline

1. Why phonons + DM
2. Derivation of Multiphonon scattering
3. Results!

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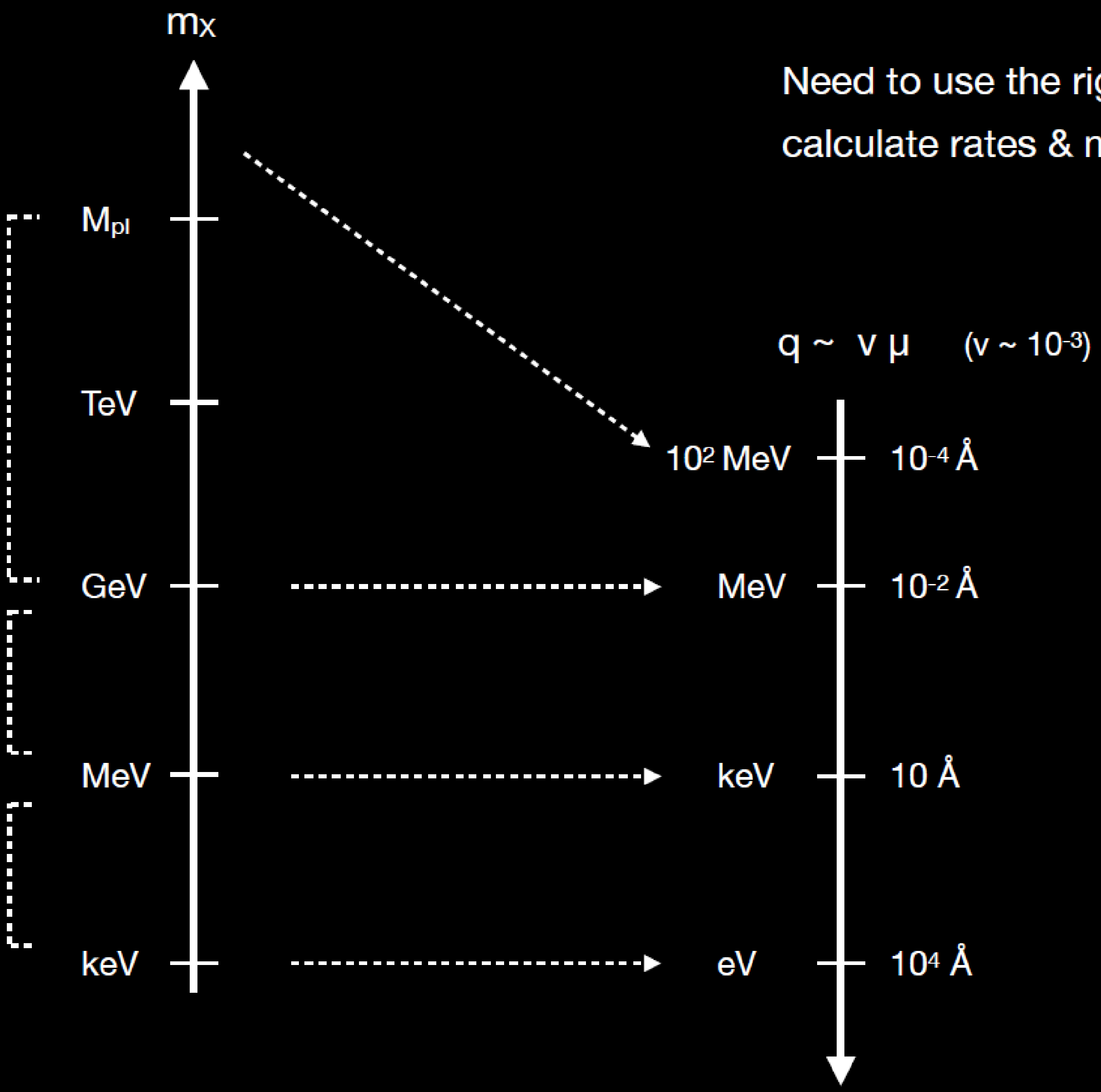
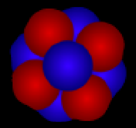
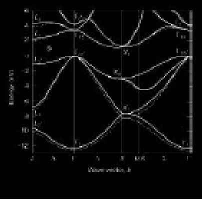
The need for theory

Need to use the right effective theory to calculate rates & match it to the UV theory

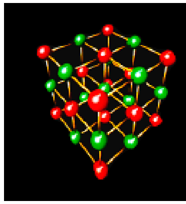




Nuclear recoil



Electronic structure



Phonon excitations

Slide by S. Knapen

Phonons

Very complicated system of coupled harmonic oscillators

Acoustic Phonons

- Coherent motion of the lattice atoms
- Wavelength → 0 corresponds to displacement of the crystal

Optical Phonons

- Out of phase motion of the lattice atoms
- If material is polar, couples to EM field

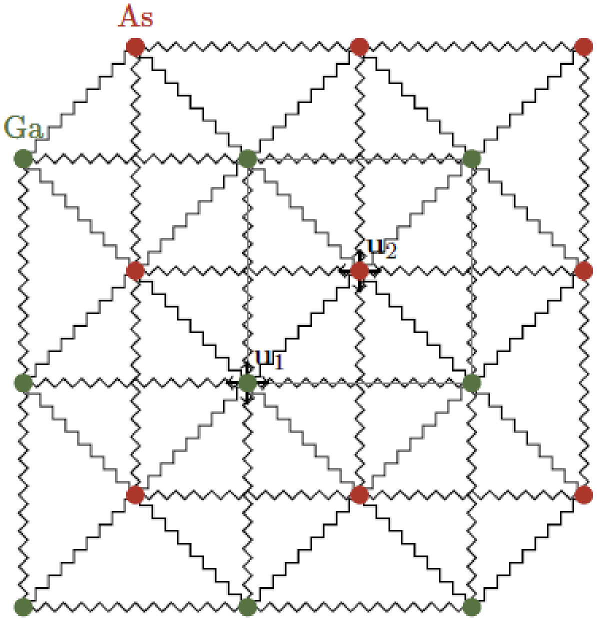


Figure from S. Knapen

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1. Why phonons + DM
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DM-Multiphonon Expansion

Nuclear Recoil:

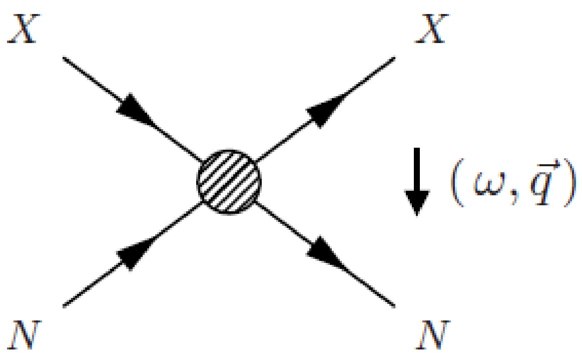
$$\omega = \frac{q^2}{2m_N}$$

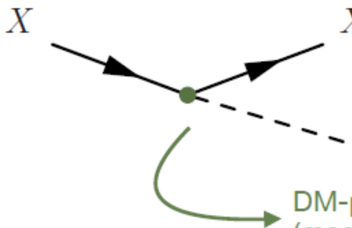
Phonon Regime:

$$q \ll \sqrt{2m_N\omega}$$

- Momentum is a good expansion parameter
- Single phonon vs multiphonon

S. Knapen, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291
B. Campbell-Deem, P. Cox, S. Knapen, T. Lin, T. Melia: 1911.0348
B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

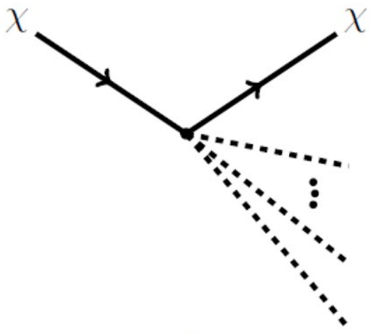




DM-phonon vertex
(model dependent)

$\mathcal{O}(q)$, $\mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

(Depends on DM model & phonon branch)



$\mathcal{O}(q^{2n})$

Multiphonon Rate

Fermi's Golden Rule

λ_i & λ_f denote initial & final spin & phonon states

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_{i,f} \left| \sum_{d}^{N} \langle \lambda_f | \mathcal{O}(q)\, e^{iq\cdot r_d} | \lambda_i \rangle \right|^2 \delta(E_f - \omega)$$

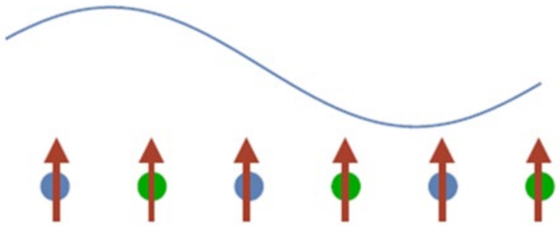
d labels each atom in the crystal

$\mathcal{O}(q)$ is some spin dependent operator

r_d is the position of each atom

$\mathcal{O}(q)$ = 1 is spin independent (already been done: 2205.0225)

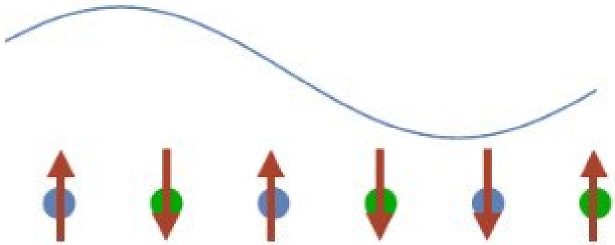
Spin Dependent Interactions



Nuclear Spin Polarized

Similar to spin independent case, with a form factor correction

(see T. Trickle et. al. 2009.13534)



Nuclear Spin Unpolarized

Cross section doesn't average away:

$d = d'$ (incoherent) terms contribute $\sim \langle S_N^2 \rangle$

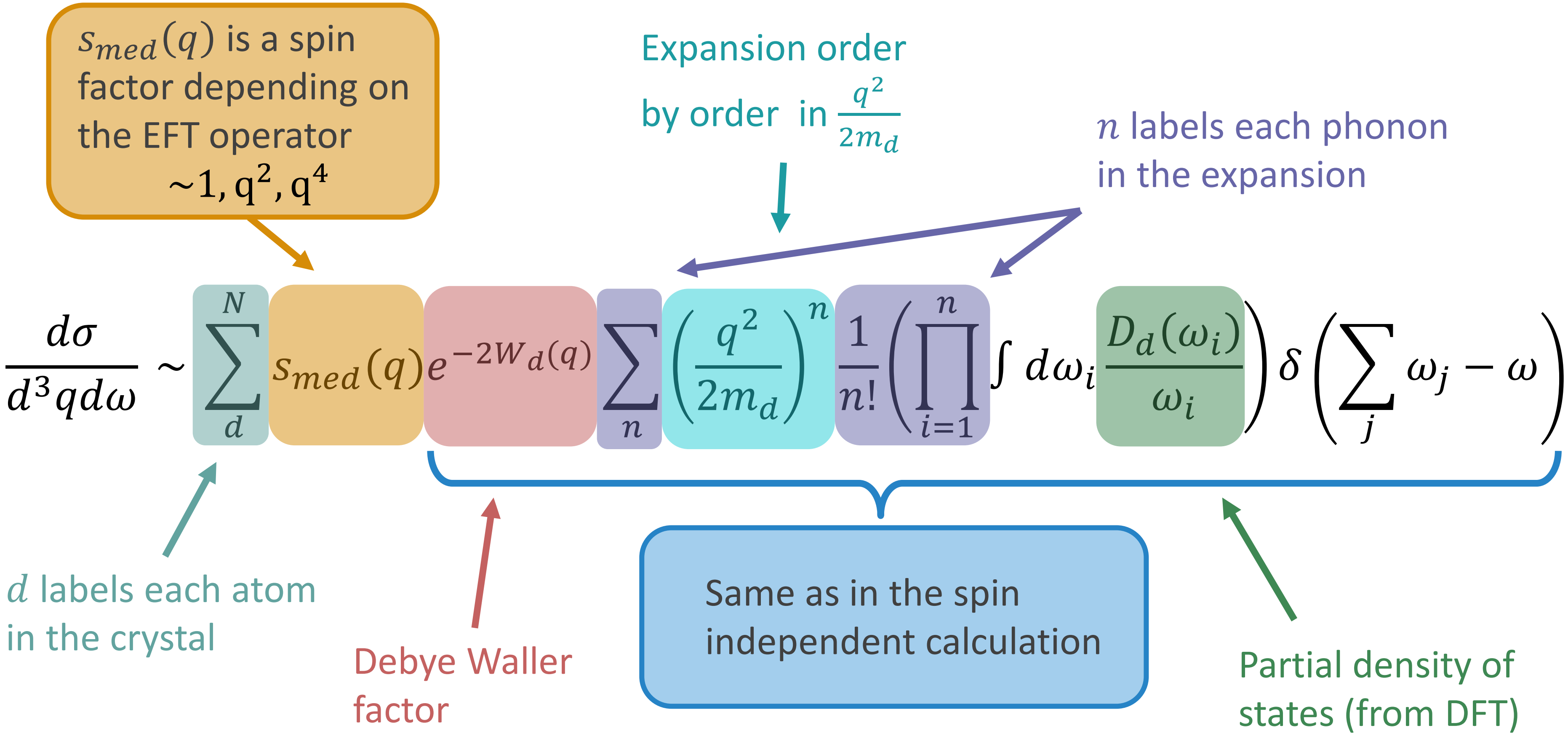
Modify spin independent calculation!

Figures by S. Knapen

Approximations

- Harmonic Approximation:
  - Decompose into a sum of harmonic oscillators weighted by the phonon density of states
  - Good for crystals with few anharmonicities
- Isotropic crystal
- Nuclei spins are distributed spherically symmetrically

Multiphonon Rate



$s_{med}(q)$ is a spin factor depending on the EFT operator
$\sim 1, \mathrm{q}^2, \mathrm{q}^4$

Expansion order by order in $\frac{q^2}{2m_d}$

n labels each phonon in the expansion

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_{d}^{N} s_{med}(q)\, e^{-2W_d(q)} \sum_{n} \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!}\left(\prod_{i=1}^{n} \int d\omega_i \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$

d labels each atom in the crystal

Debye Waller factor

Same as in the spin independent calculation

Partial density of states (from DFT)

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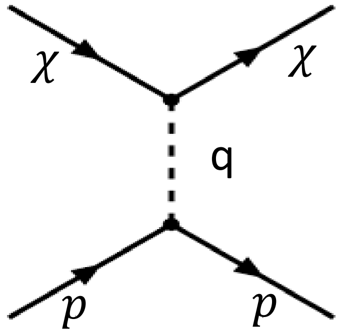
Model Dependence: EFT operators



Scalar mediator ϕ

$$\mathcal{L} \supset g_\chi \phi \bar{\chi}\chi + g_p \phi \bar{N}\gamma^5 N$$

$$\mathcal{L}_{NR} \sim (q \cdot S_N)$$

→

$$s_\phi \sim \frac{q^2}{m_N^2}\left(\overline{f_d^2\langle S_d^2\rangle}\right)$$

Pseudoscalar mediator a

$$\mathcal{L} \supset g_\chi a \bar{\chi}\gamma^5\chi + g_p a \bar{N}\gamma^5 N$$

$$\mathcal{L}_{NR} \sim (q \cdot S_N)(q \cdot S_\chi)$$

→

$$s_a \sim \frac{q^4}{m_N^2 m_\chi^2}\langle S_\chi^2\rangle\left(\overline{f_d^2\langle S_d^2\rangle}\right)$$

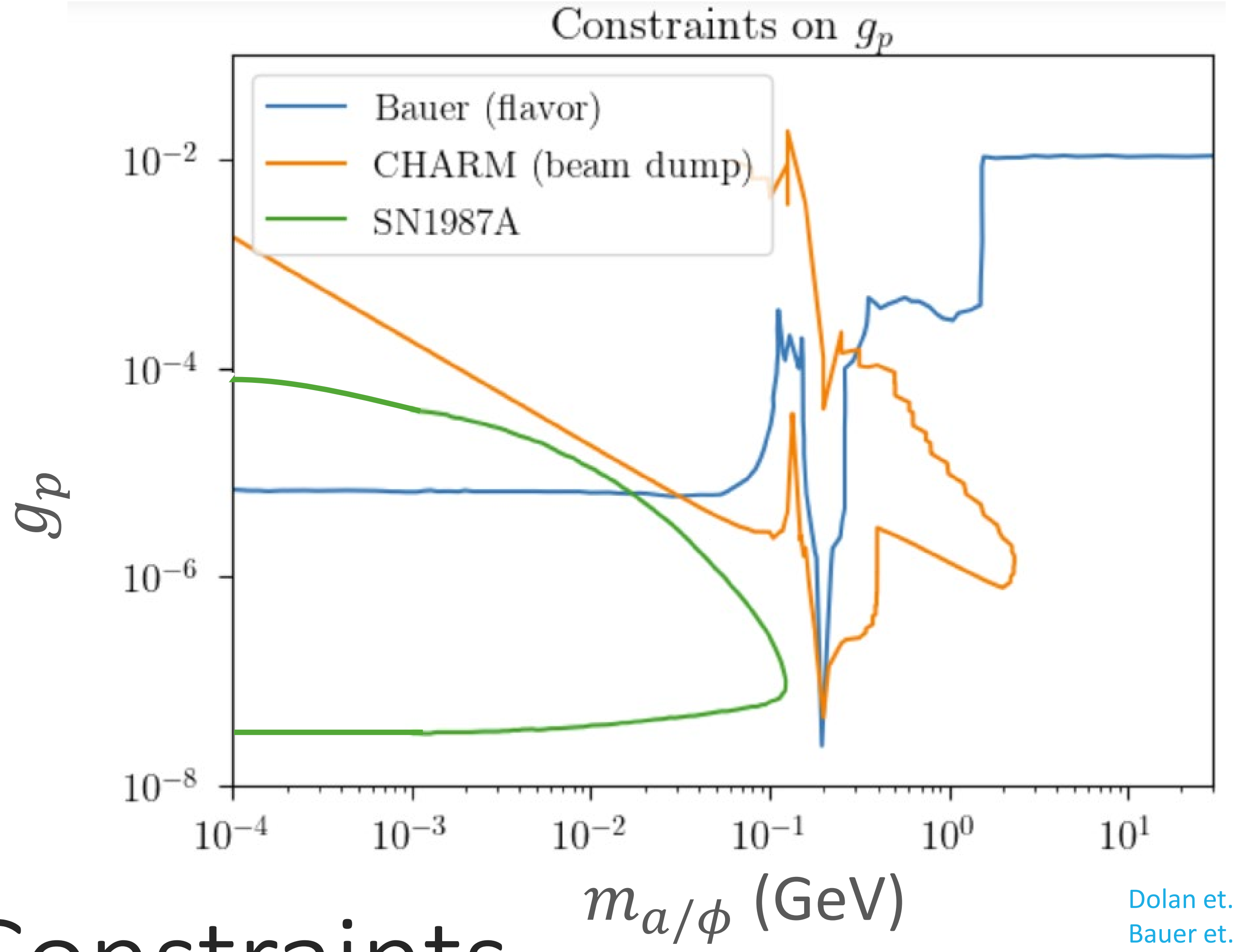
Pseudovector mediator A'_μ

$$\mathcal{L} \supset g_\chi A'_\mu \bar{\chi}\gamma^\mu\gamma^5\chi + g_p A'_\mu \bar{N}\gamma^\mu\gamma^5 N$$

$$\mathcal{L}_{NR} \sim (S_N \cdot S_\chi)$$

→

$$s_{A'_\mu} \sim \langle S_\chi^2\rangle\left(\overline{f_d^2\langle S_d^2\rangle}\right)$$

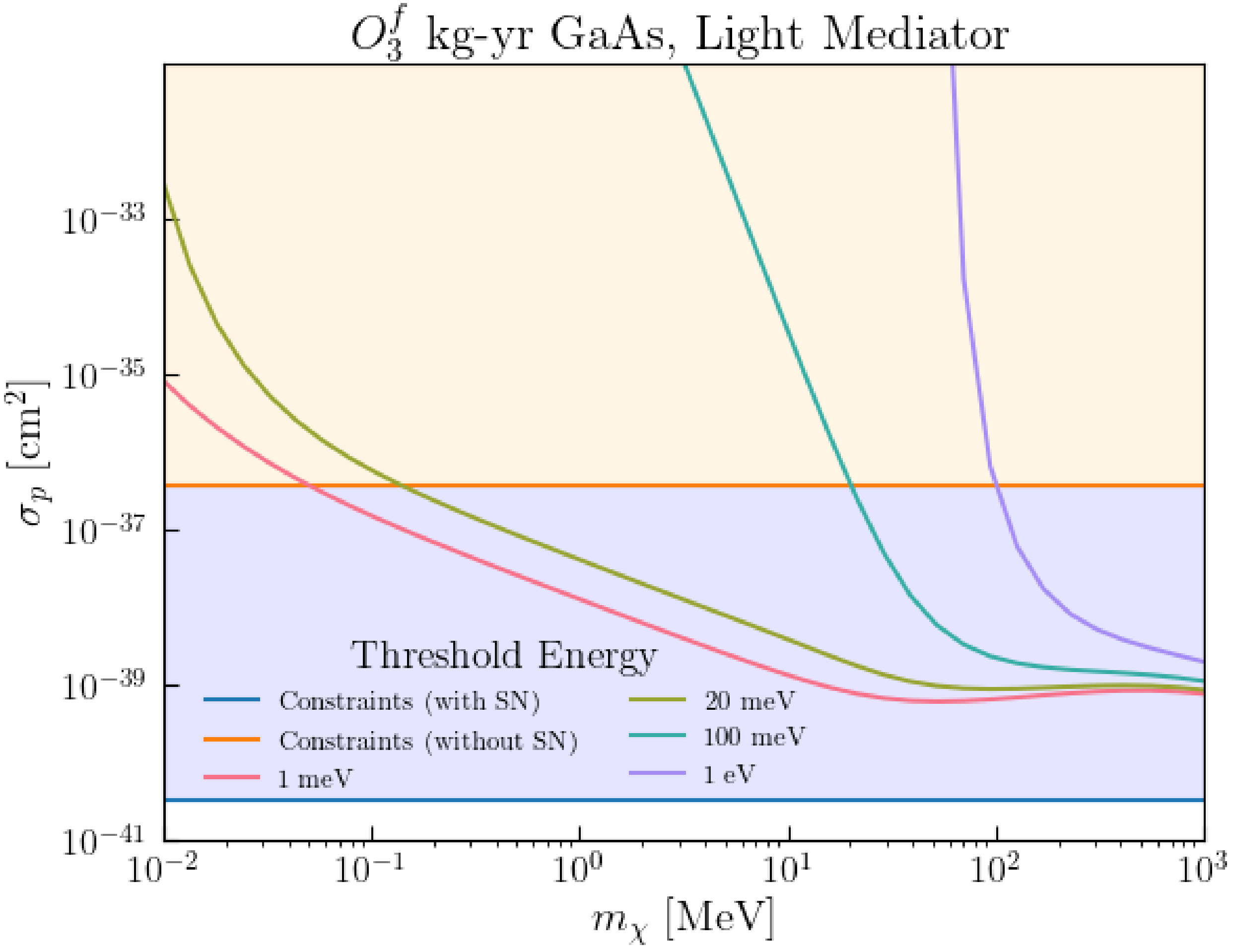


Dolan et. al: 1412.5174
Bauer et. al: 2110.10698
Chang et. al: 1803.00993

Constraints

Results

- Most optimistic reach curves: O_3^f (scalar mediator)
- Beaten by supernovae constraints at light mediator masses (see shaded in area)
-

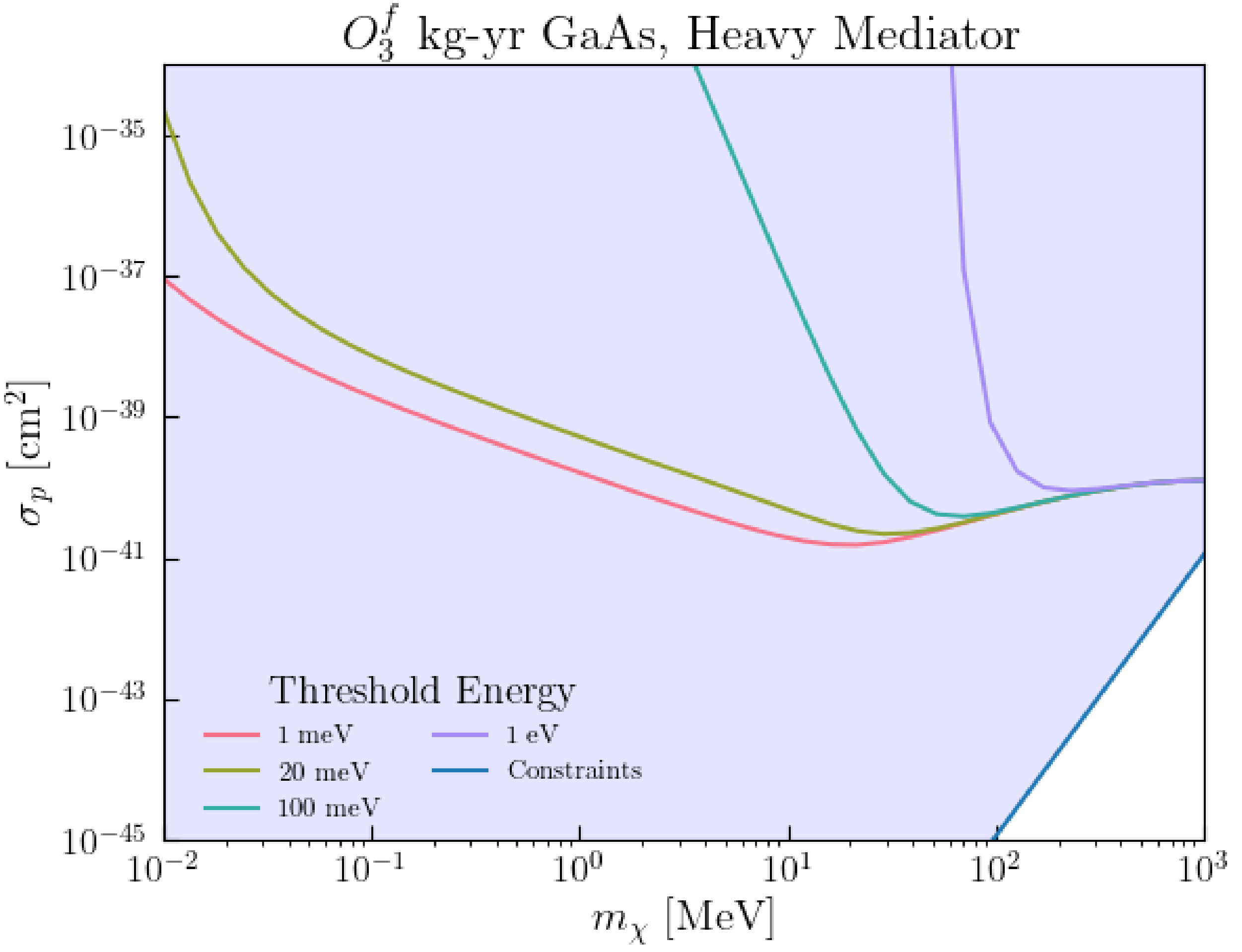


Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

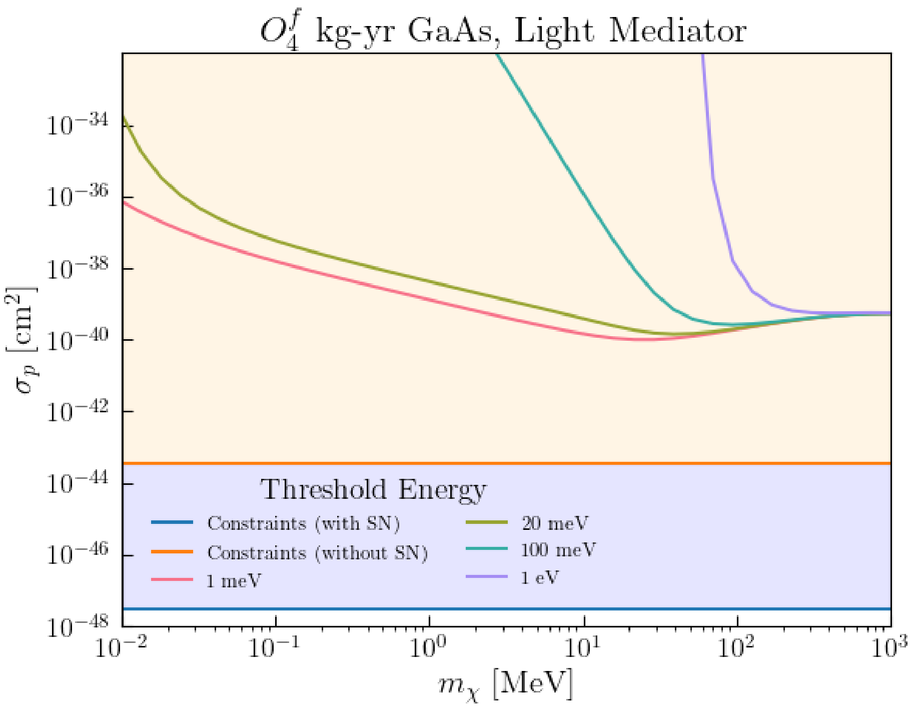
Results

- Most optimistic reach curves: O_3^f (scalar mediator)
- Beaten by supernovae constraints at light mediator masses (see shaded in area)
- Beaten by flavor constraints at heavy mediator masses

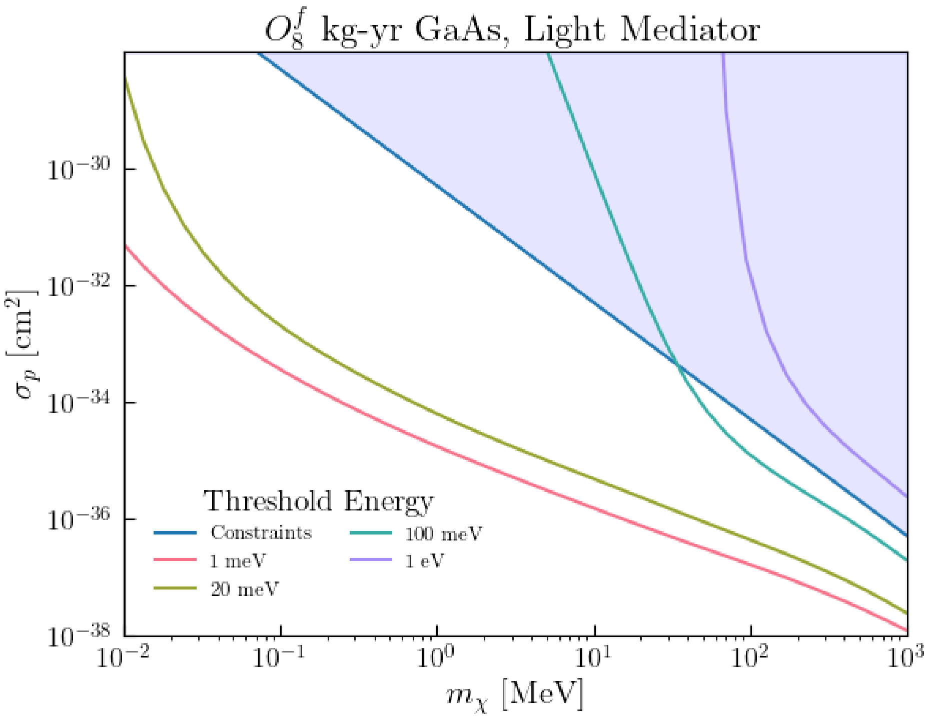
$\sigma_p \sim 10^{-61} - 10^{-41}\ cm^2$



Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies



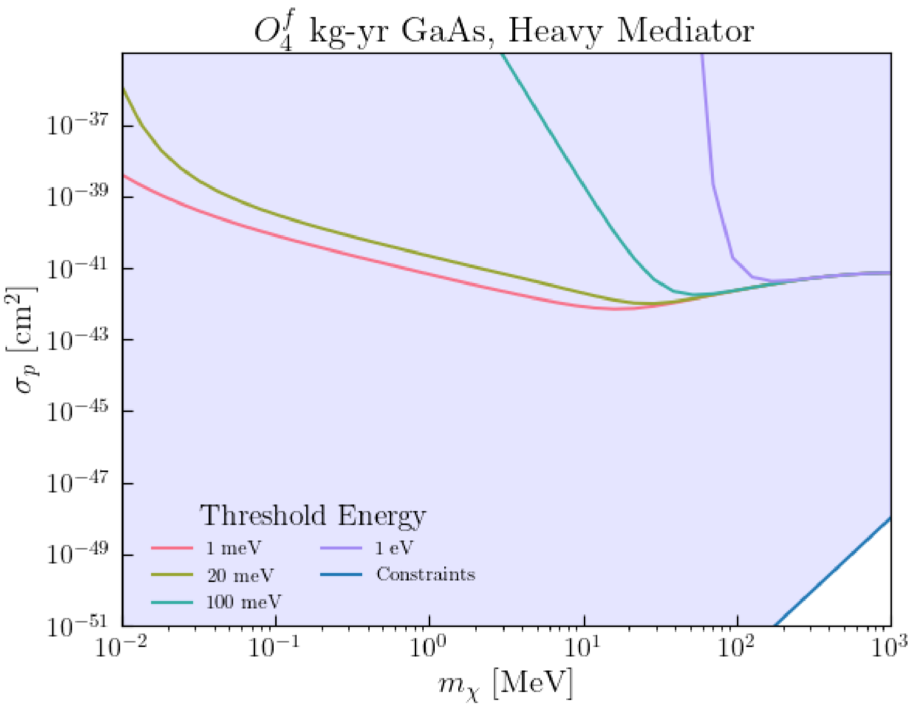
Light pseudoscalar mediator



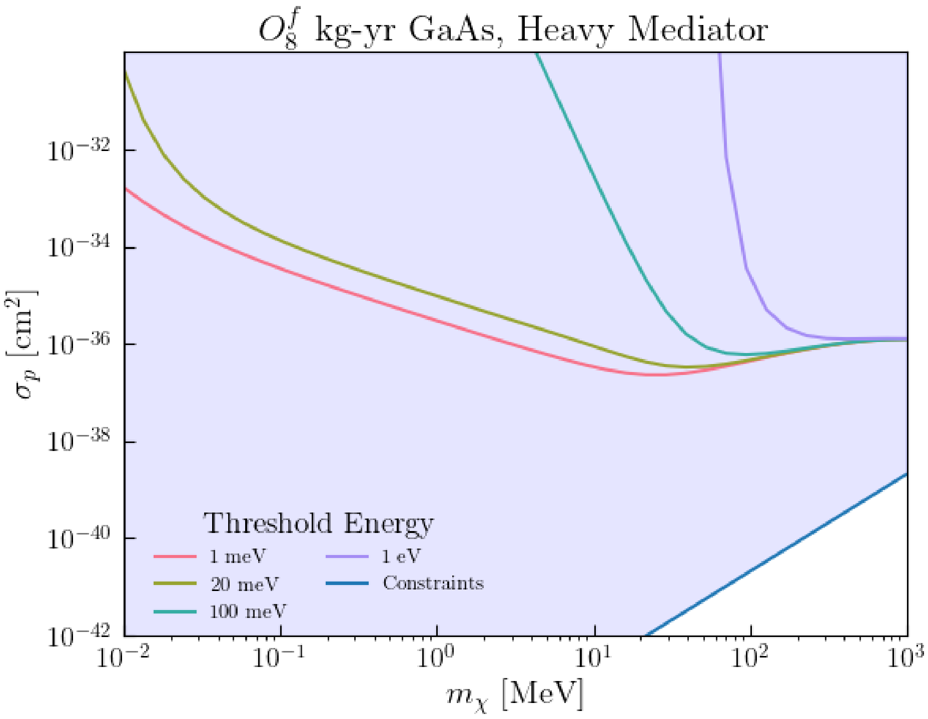
Light pseudovector mediator

Results

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies


O_4^f kg-yr GaAs, Heavy Mediator

Heavy pseudoscalar mediator


O_8^f kg-yr GaAs, Heavy Mediator

Heavy pseudovector mediator

Results

Cross sections needed for 3 events/kg-year rate for GaAs using several threshold energies

Summary

Multiphonon – DM interactions cover intermediate DM mass ranges between nuclear recoil and single phonon detection

Spin dependent multiphonon scattering utilizes similar methods as spin independent when crystal spins are randomly distributed

The reach of multiphonon experiments falls well short of current limits on spin dependent DM interactions (SN, mesons, etc), except in a few possible cases

Thus, repurposing a spin independent multiphonon experiment (like Tesseract) may not the best way to reach new spin dependent parameter space. Sorry Experimentalists!

Questions?

Backup Slides

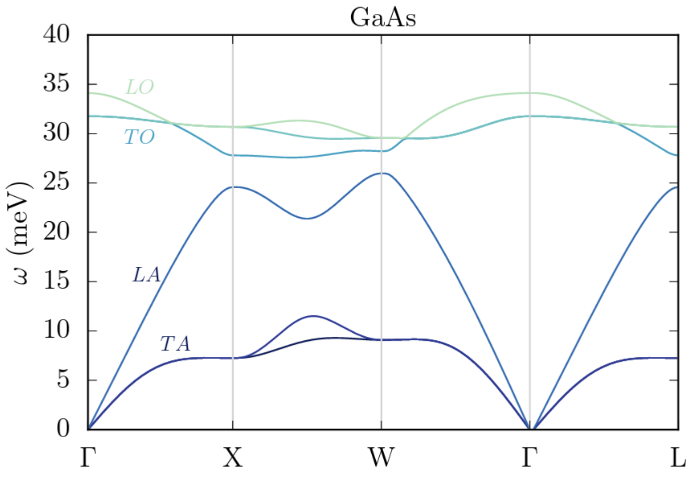
Acoustic vs Optical

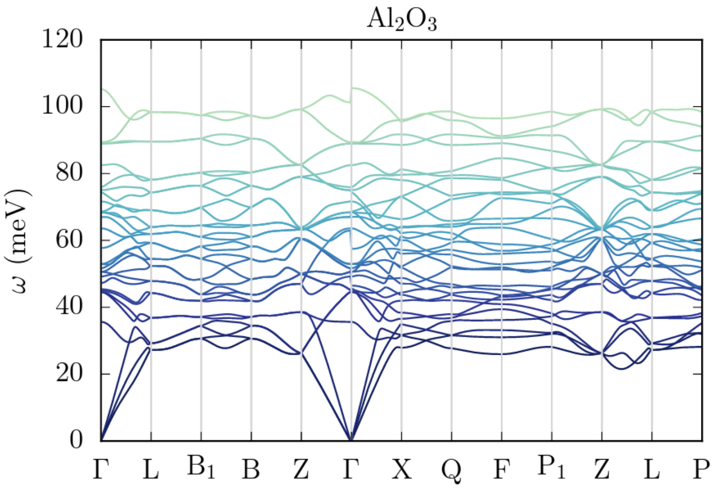
- Low momentum transfer = only excite acoustic phonon in 1st Brillouin zone

$$\omega = c_s|q| \approx 2c_s v m_X$$
$$\sim 7\,meV \times \frac{m_X}{100keV}$$

- Best threshold is in 10 – 100 meV range, so difficult or impossible to detect
- Optical modes don't have this scaling:

$$\omega \sim 30\,meV \text{ as } |q| \to 0$$



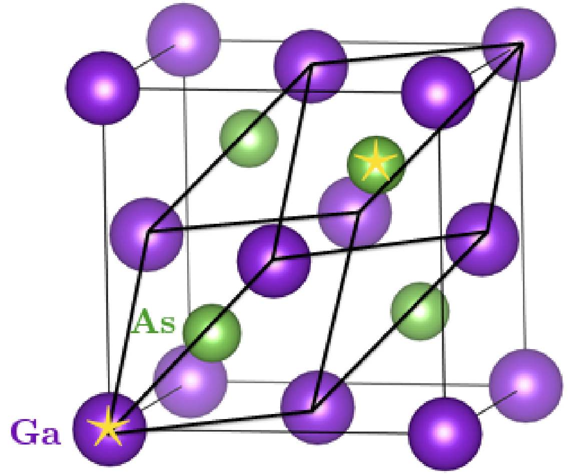


Figures from arXiv:1807.10291

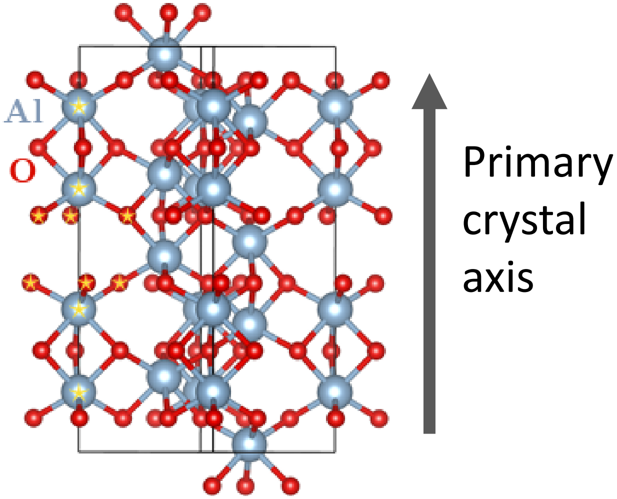
Polar Materials

- At least two different atoms with **different** effective charges
- Each unit cell forms an electric dipole
- E field or dark photon causes vibrations
  - → Optical phonons
- GaAs
  - 2 atoms in unit cell
  - 3 acoustic phonons, 3 optical phonons
- Sapphire
  - 10 atoms in unit cell
  - 3 acoustic phonons, 27 optical phonons

GaAs



Al_2O_3 (Sapphire)



Figures from arXiv:1807.10291

Benefits of Polar Materials

- Gapped dispersion of optical phonons
  - Single or multiphonon
- Anisotropic crystal structures
  - Daily modulation in rate
- Low screening
  - Required: few free electrons, high polarizability
  - Gap for electronic excitations $\sim O(1 - 10\ eV)$
  - Kinetic mixing with dark photon couples to dipole moment
- Easy to fabricate

S. Griffin, S. Knapen, T. Lin, M. Pyle, K. Zurek: 1807.10291

DM-Multiphonon Expansion

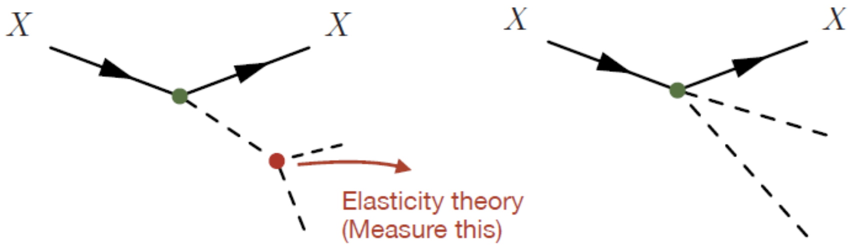
LO

DM-phonon vertex
(model dependent)

$$\mathcal{O}(q),\ \mathcal{O}(q^2)\text{ or }\mathcal{O}(q^4)$$

(Depends on DM model & phonon branch)

NLO

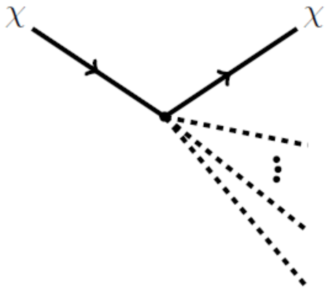


Elasticity theory
(Measure this)

$$\mathcal{O}(q^4)$$

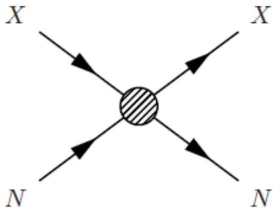
B. Campbell-Deem, P. Cox, S. Knapen, T. Lin, T. Melia: 1911.0348

N^{n}LO



$$\mathcal{O}(q^{2n})$$

N$^\infty$LO = nuclear recoil



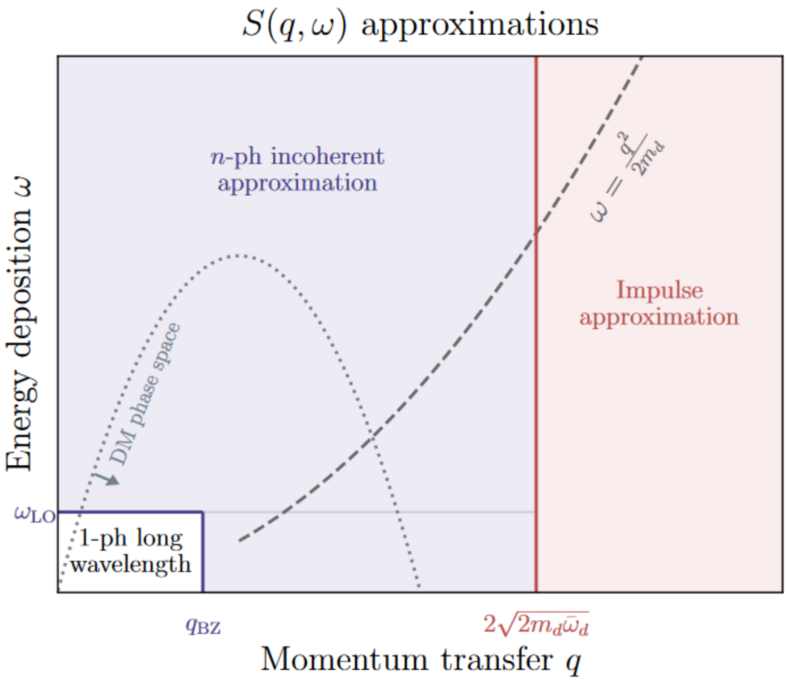
$$\sim \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

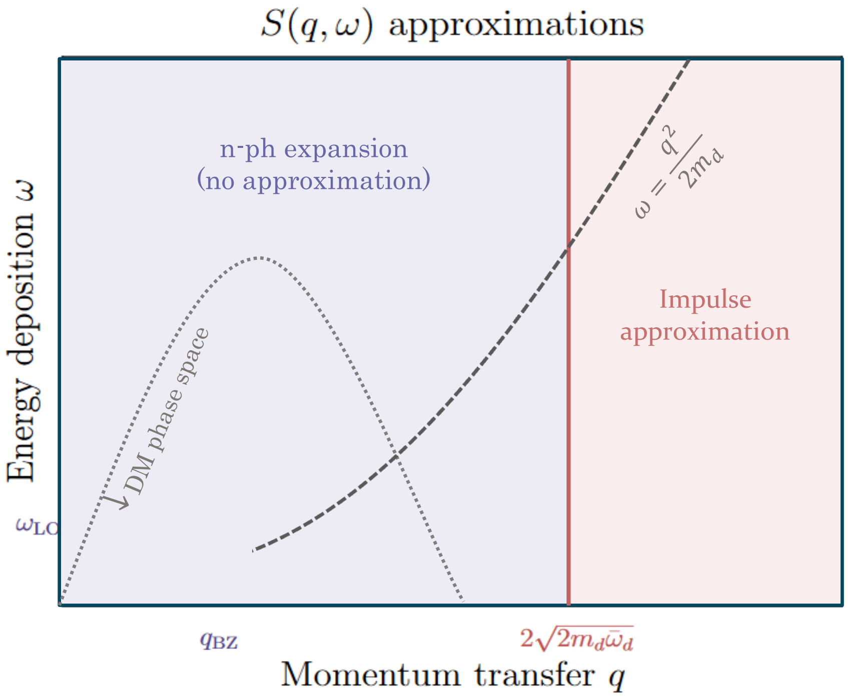
Figures from S. Knapen

Approximations

Spin Independent



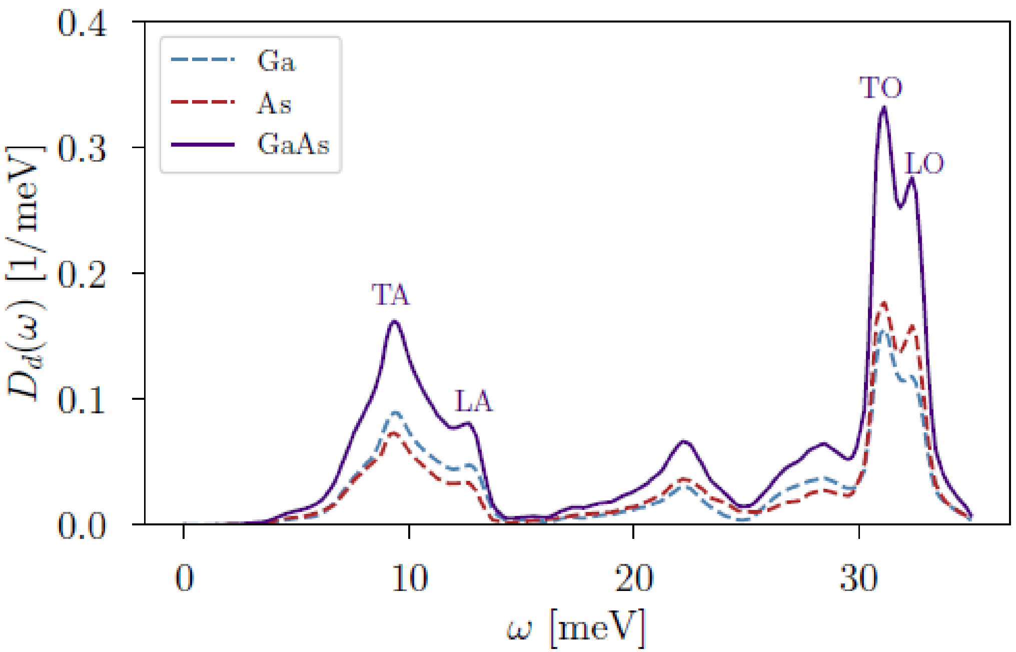
Spin Dependent



B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

Density of States

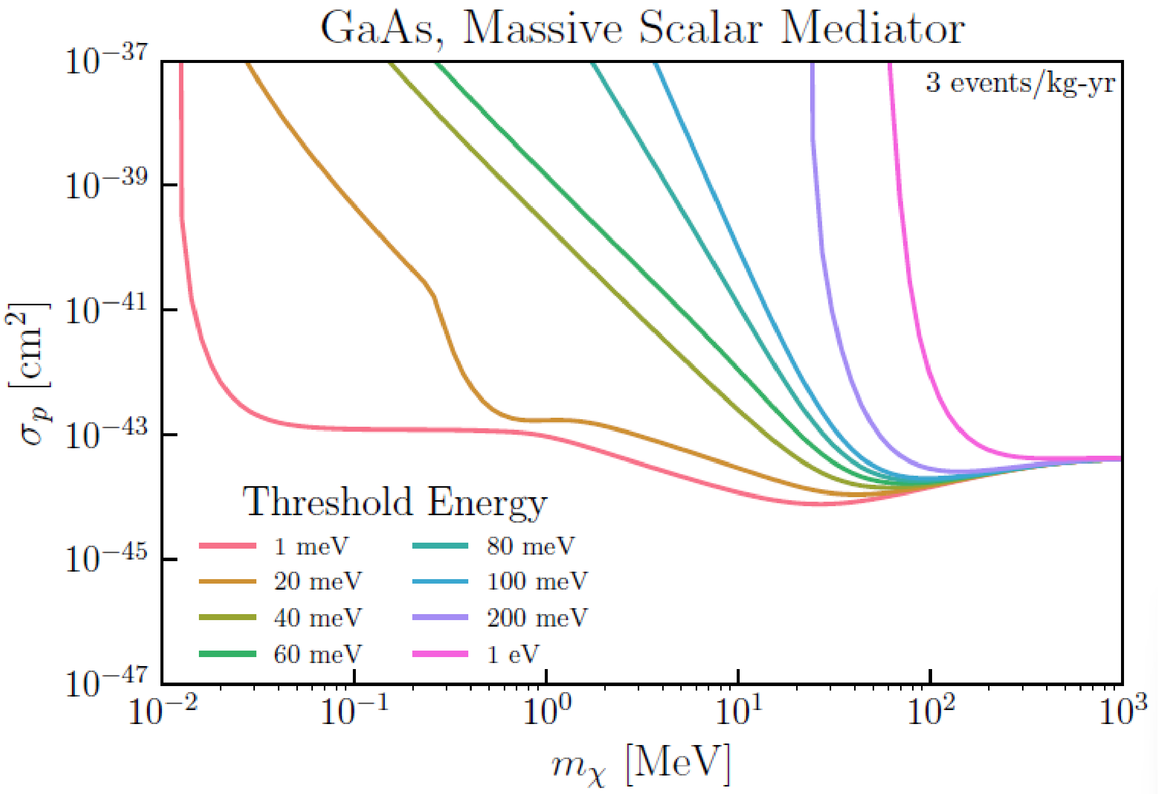
- The peaks from each phonon are visible
- The partial density of states are calculated using Density Effective Theory where data is not yet taken
- Many materials’ density of states can be taken from the Material Project data



A. Jain, Et. Al. https://doi.org/10.1063/1.4812323

Spin Independent Results

- Assumes a coupling $\sim A_d$
- Isotropic approximation
- Anharmonic corrections around $m_\chi \sim 1 - 10\ MeV$: 2309.10839



B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

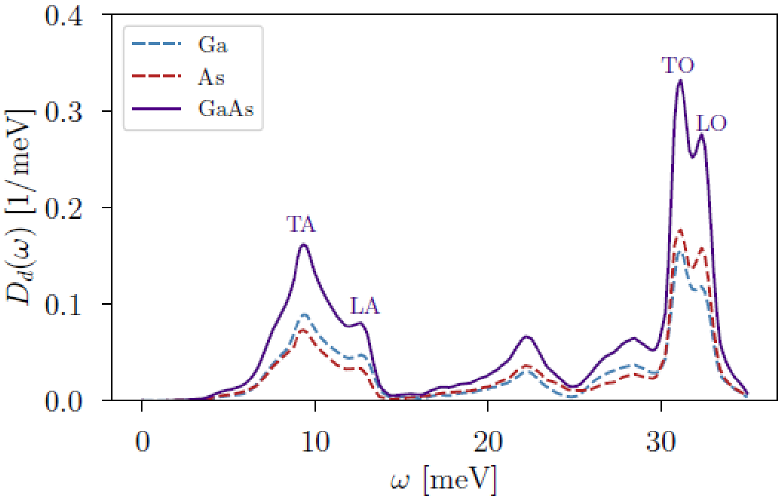
Multiphonon Rate (SI)

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2\, e^{-2W_d(q)} \sum \left(\frac{q^2}{2m_d}\right)^n \frac{1}{n!}\left(\prod_{i=1}^{n} \int d\omega_i\, \frac{D_d(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$

$$\Big\downarrow \quad q \gg \sqrt{2\omega m_d}$$

Impulse Approximation

Partial density of States



$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \sqrt{\frac{2\pi}{\Delta_d^2}} \exp\left(-\frac{\left(\omega - \frac{q^2}{2m_d}\right)^2}{2\Delta_d^2}\right)$$

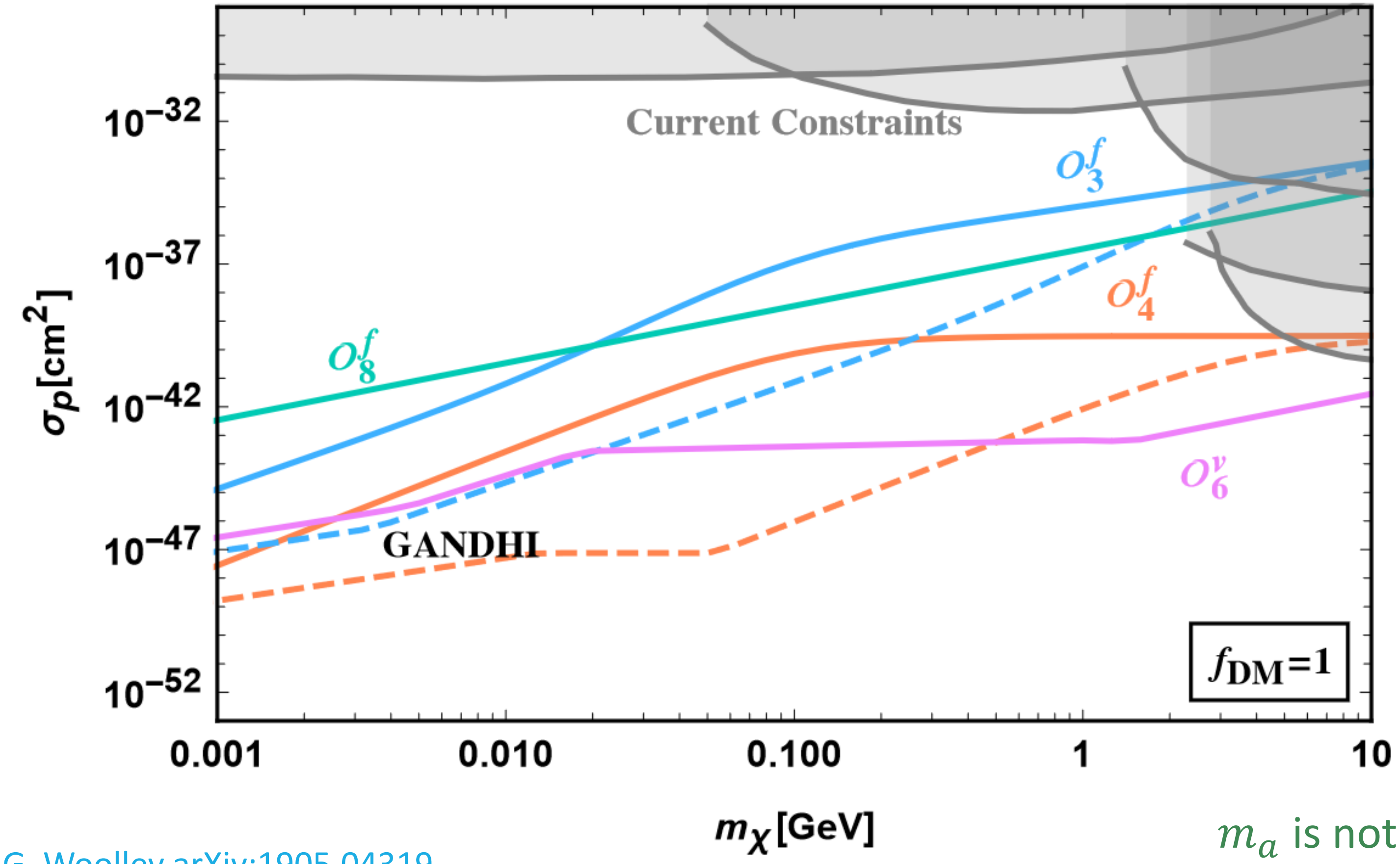
$$\Big\downarrow \quad q \ggg \sqrt{2\omega m_d}$$

$$\frac{d\sigma}{d^3q\,d\omega} \sim \sum_d^N f_d^2 \times \delta\left(\omega - \frac{q^2}{2m_d}\right)$$

Free nuclear recoil limit!

B. Campbell-Deem, S. Knapen, T. Lin, E. Villarama: 2205.02250

Spin Dependent Constraints

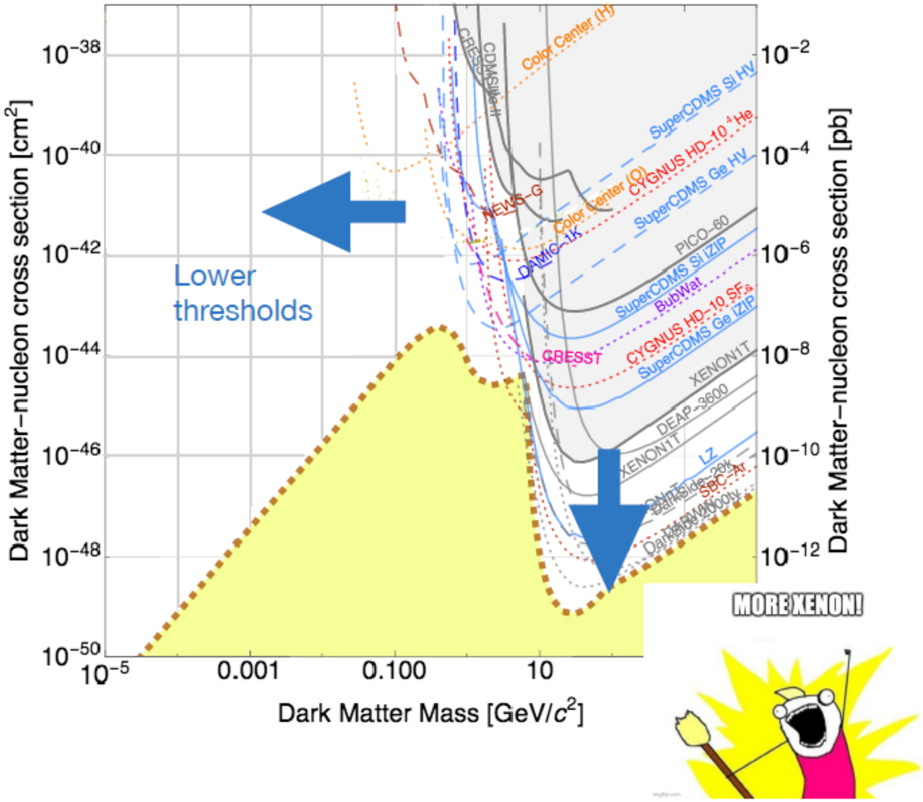


m_a is not fixed

H. Ramani, G. Woolley arXiv:1905.04319

Light Dark Matter Direct Detection

What do we need?

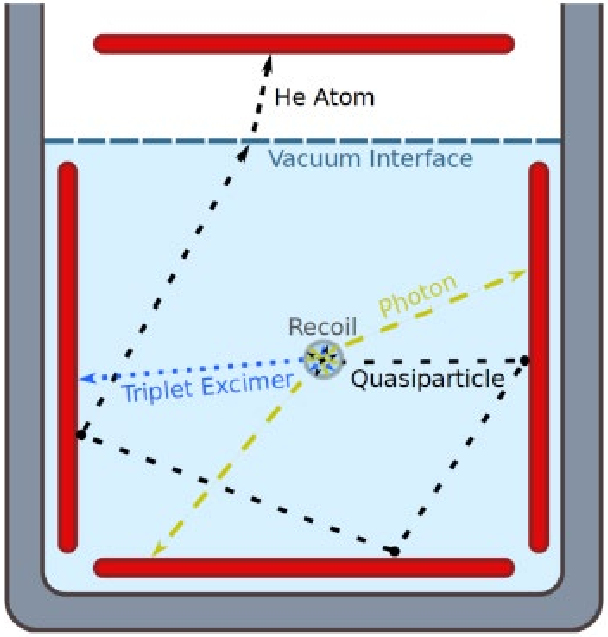


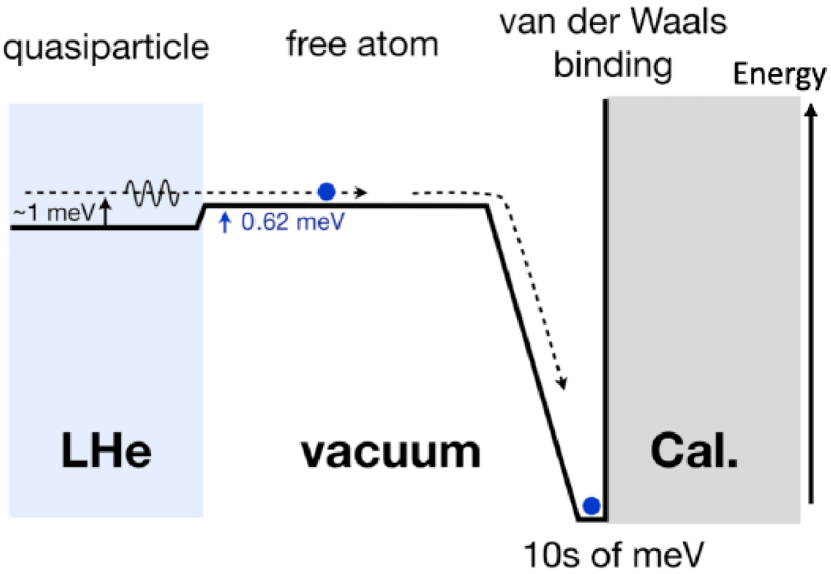
Ultra-low threshold calorimeters:
Transition Edge Sensors

M. Battaglieri: 1707.04591
Meme credit: S. Knapen

Phonon Detector: HeRALD

SPICE / HeRALD
TESSERACT



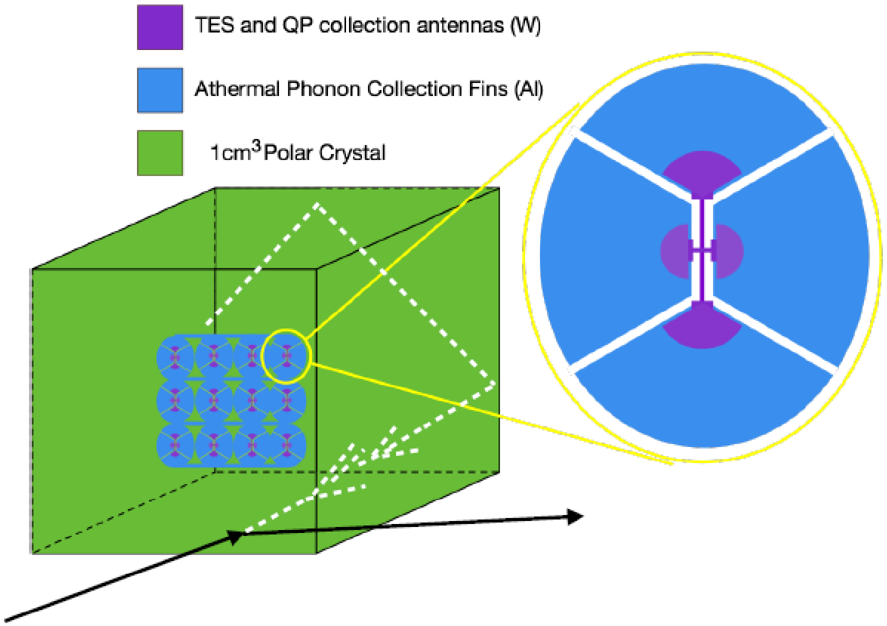


- Calorimeters with TES readout
- Quantum evaporation of He atoms

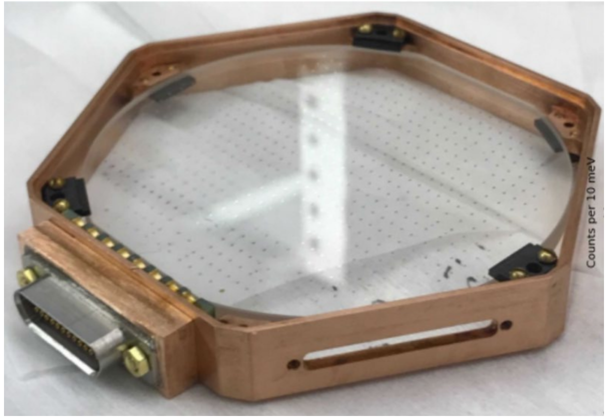
S. Hertel, A. Biekert, J.Lin, V.Velan, & D. McKinsey arXiv:1810.06283
Figures from J. Lin slides

Phonon Detector: SPICE

SPICE / HeRALD
TESSERACT



3’’ sapphire detector



- Polar Materials: GaAs or Sapphire
- Scintillation & phonons
    - Background discrimination!
- Low energy TES

~ 10 meV threshhold

Figure from M. Pyle
Picture from TESSERACT Website

Transition Edge Sensors (TES)

- Superconducting film acting at the phase "transition edge"
- Large change in resistance with tiny shifts in temperature
- Smaller band widths → lower threshold energies