Institute for Cosmic Ray Research
University of Tokyo

Revisiting Metastable Cosmic String Breaking

SUSY24 @ IFT, Madrid

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Based on:

JHEP 04 (2024) 068 [arXiv:2312.15662]
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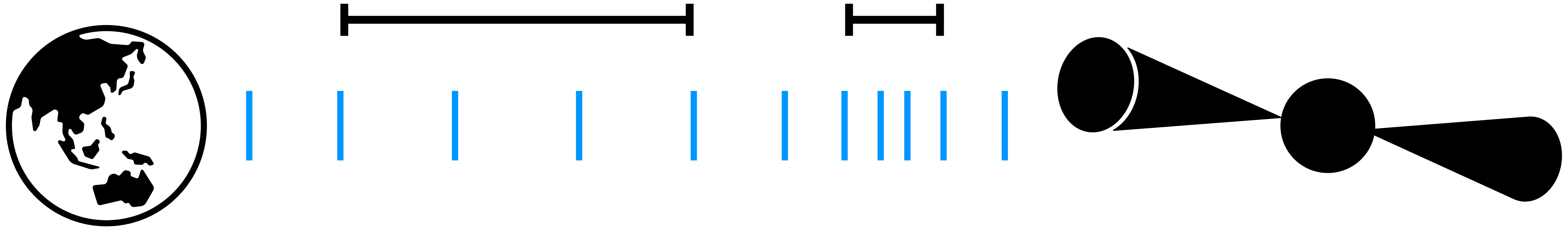
Stochastic Gravitational Wave Background

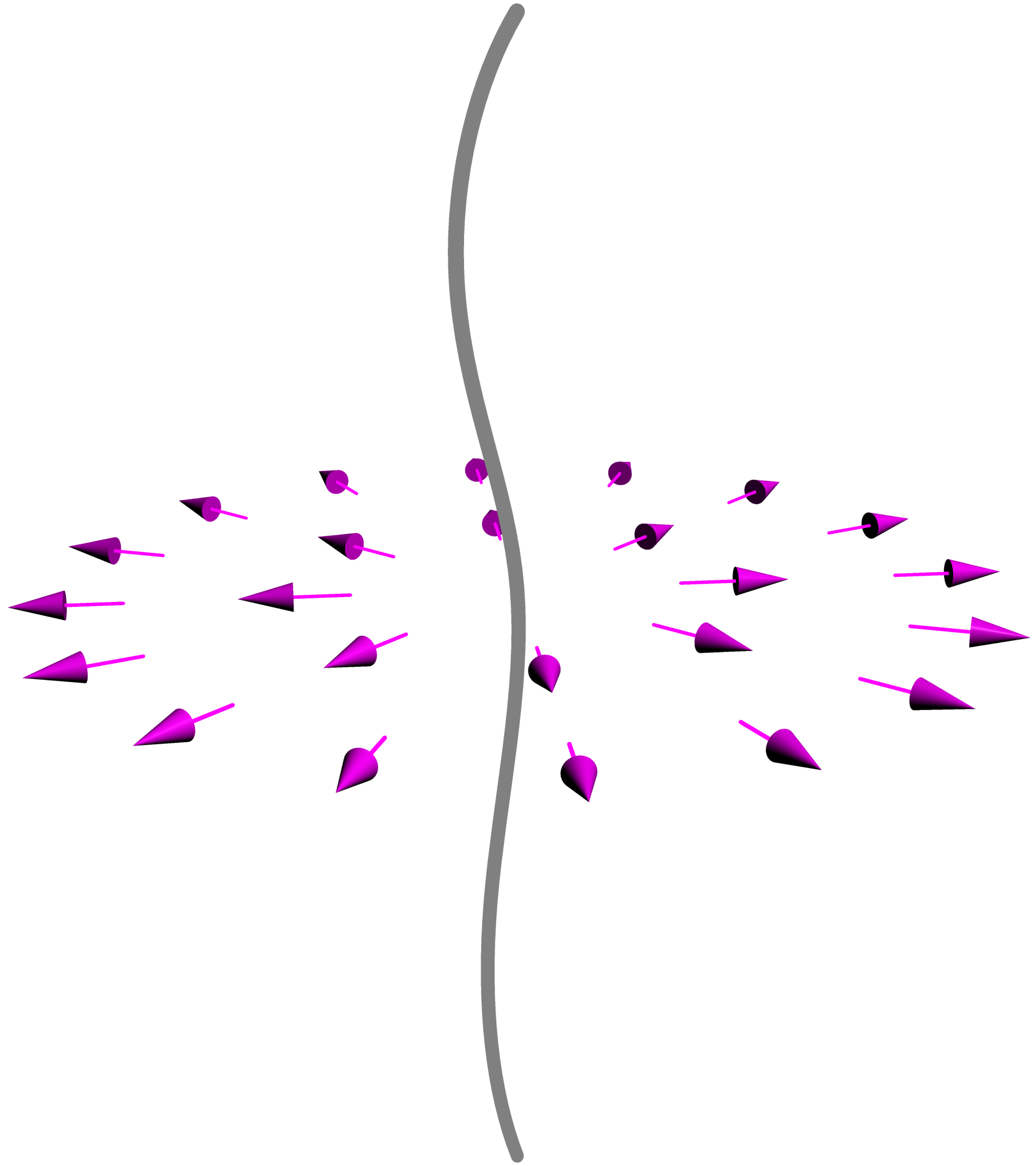
- Evidenced by PTA observations (NANOGrav, InPTA, EPTA, PPTA, CPTA)
  - Observed at nHz range
- Many possible origins
  - Black holes?
  - Phase transition?
  - Domain Walls?
  - ….



Cosmic Strings

Probing BSM with GW

- Linear solitons in QFT
- Created in the Universe by
  e.g. spontaneous U(1) breaking
- Predicted by many BSM physics e.g. GUT



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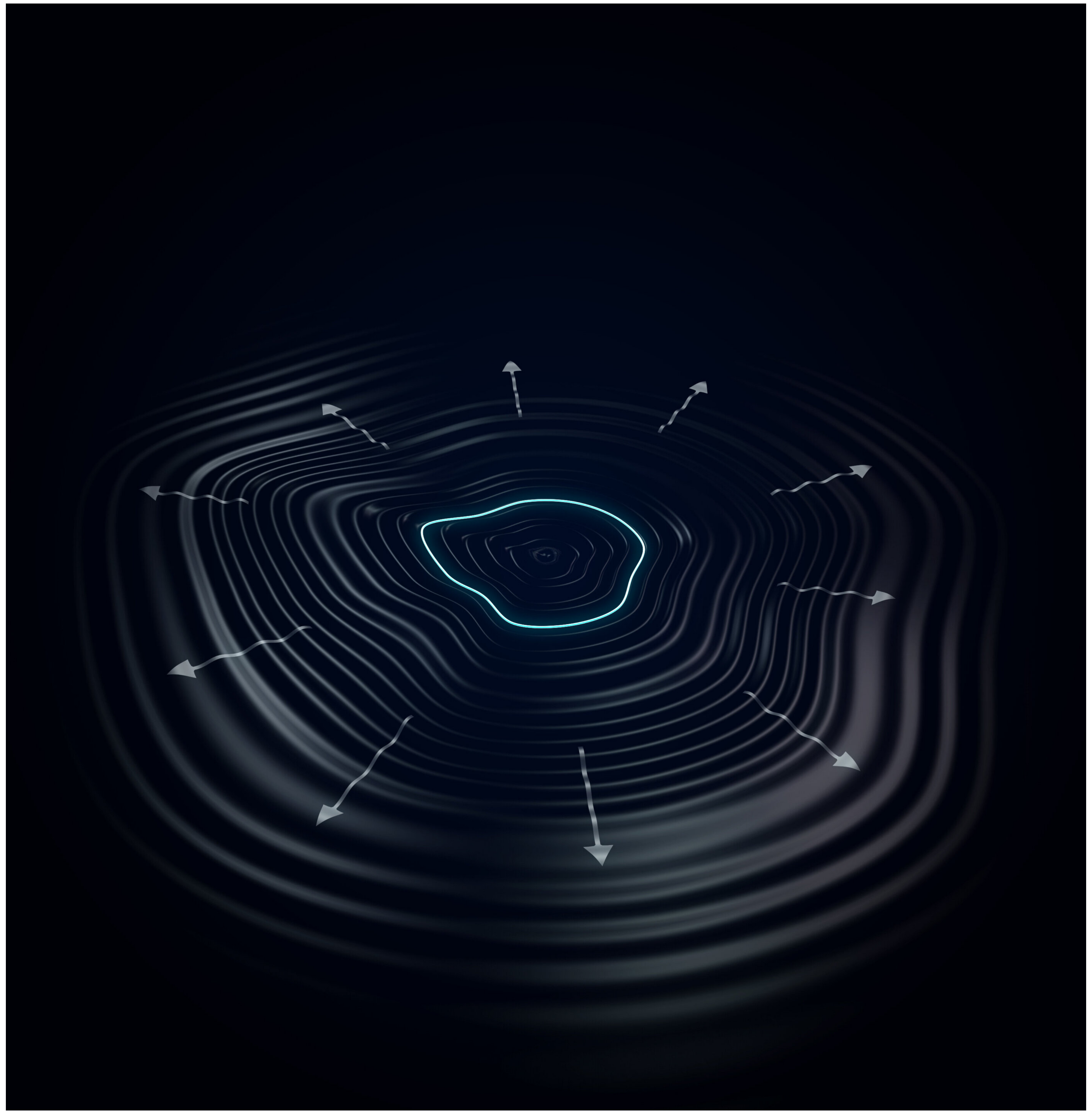
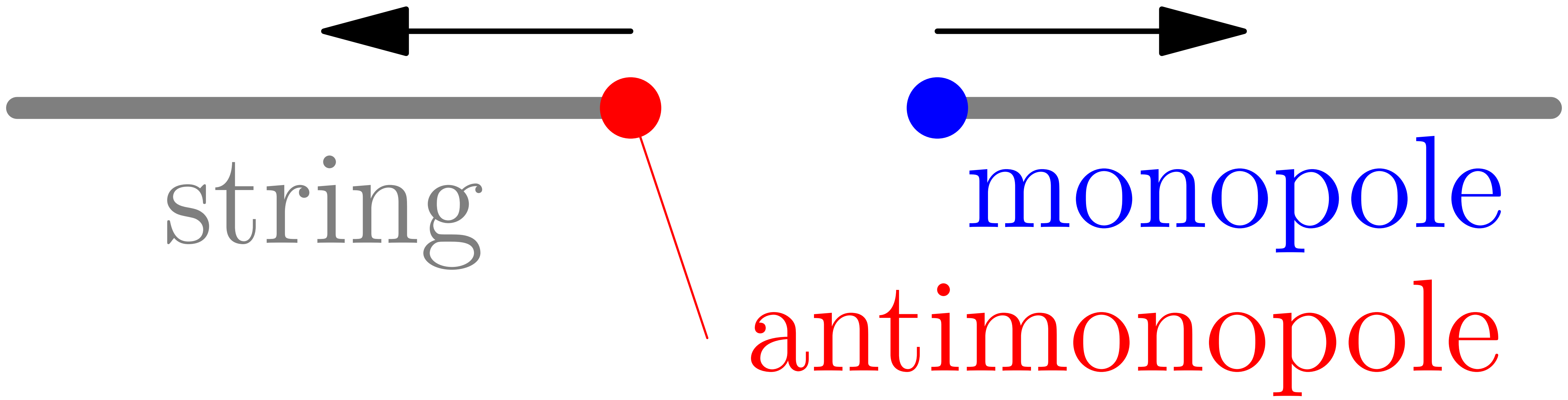


Credit: Daniel Dominguez from CERN's Education, Communications & Outreach (ECO) Department.

Metastable Cosmic Strings

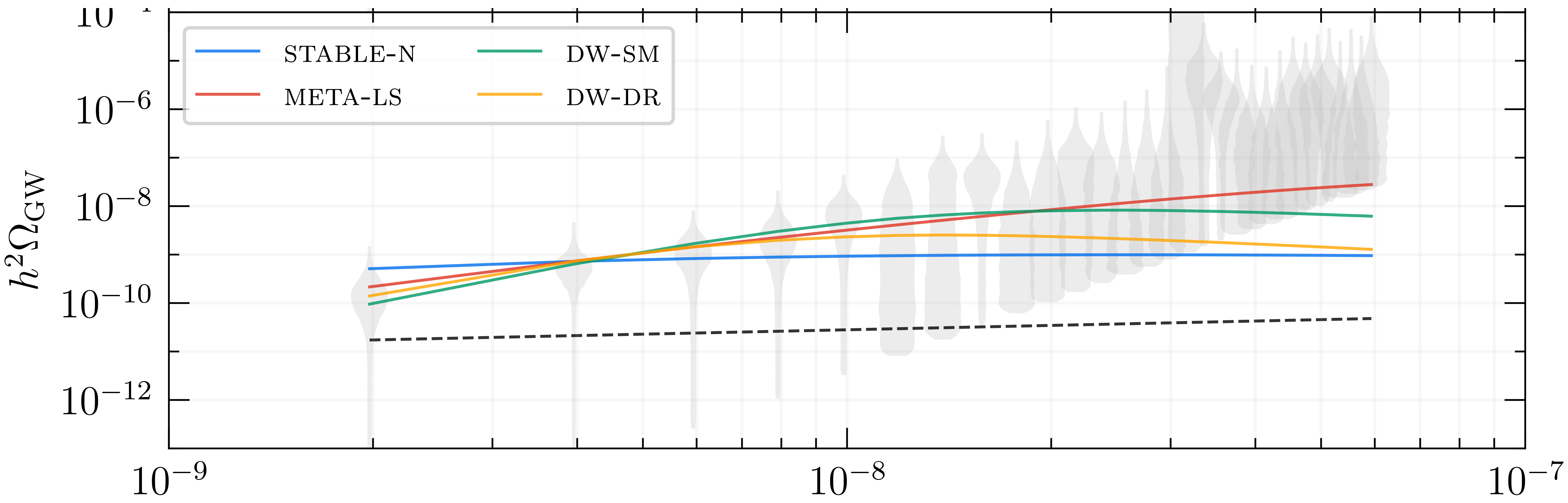
- Spontaneously cut by monopole-antimonopole pair creation
- Metastable for e.g. $G \to \mathrm{U}(1) \to 1$ with $\pi_1(G) = 0$



Metastable Cosmic Strings

NANOGrav requires metastability

- NANOGrav requires the strings to be metastable
- Precise estimate of the decay rate Γ is crucial
  - $\sqrt{\kappa} \sim 8$ for $\Gamma \sim \exp[-\pi\kappa]$



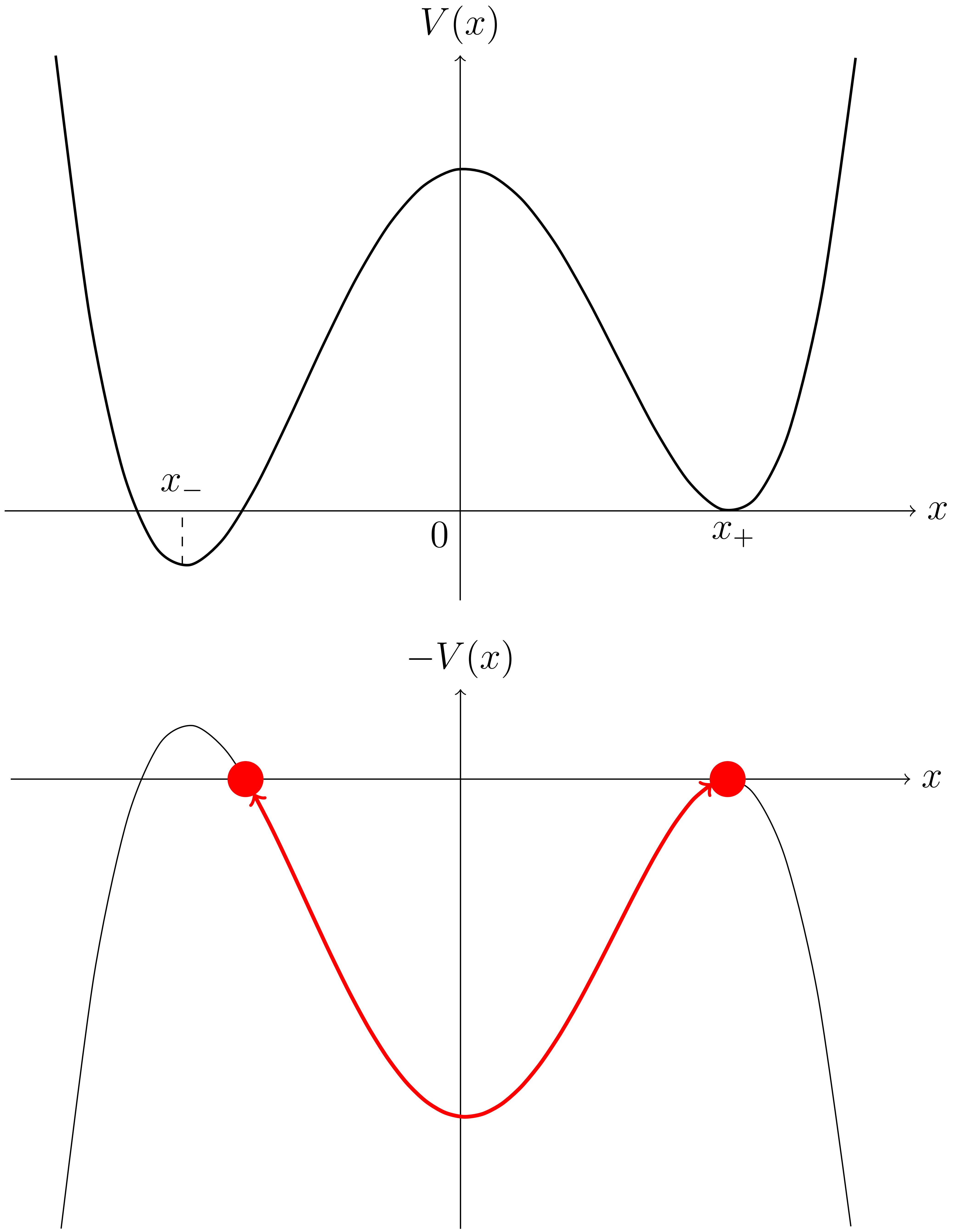
[Afzal et al., 2023]

String breaking rate

Tunneling and bounce

see e.g. [Coleman, 1985]

- Procedure:
  - Go to imaginary time
    - $\approx$ invert the potential
  - Find the bounce solution
    - Action: S_B
  - Decay rate: $\Gamma \sim \exp[-S_B]$



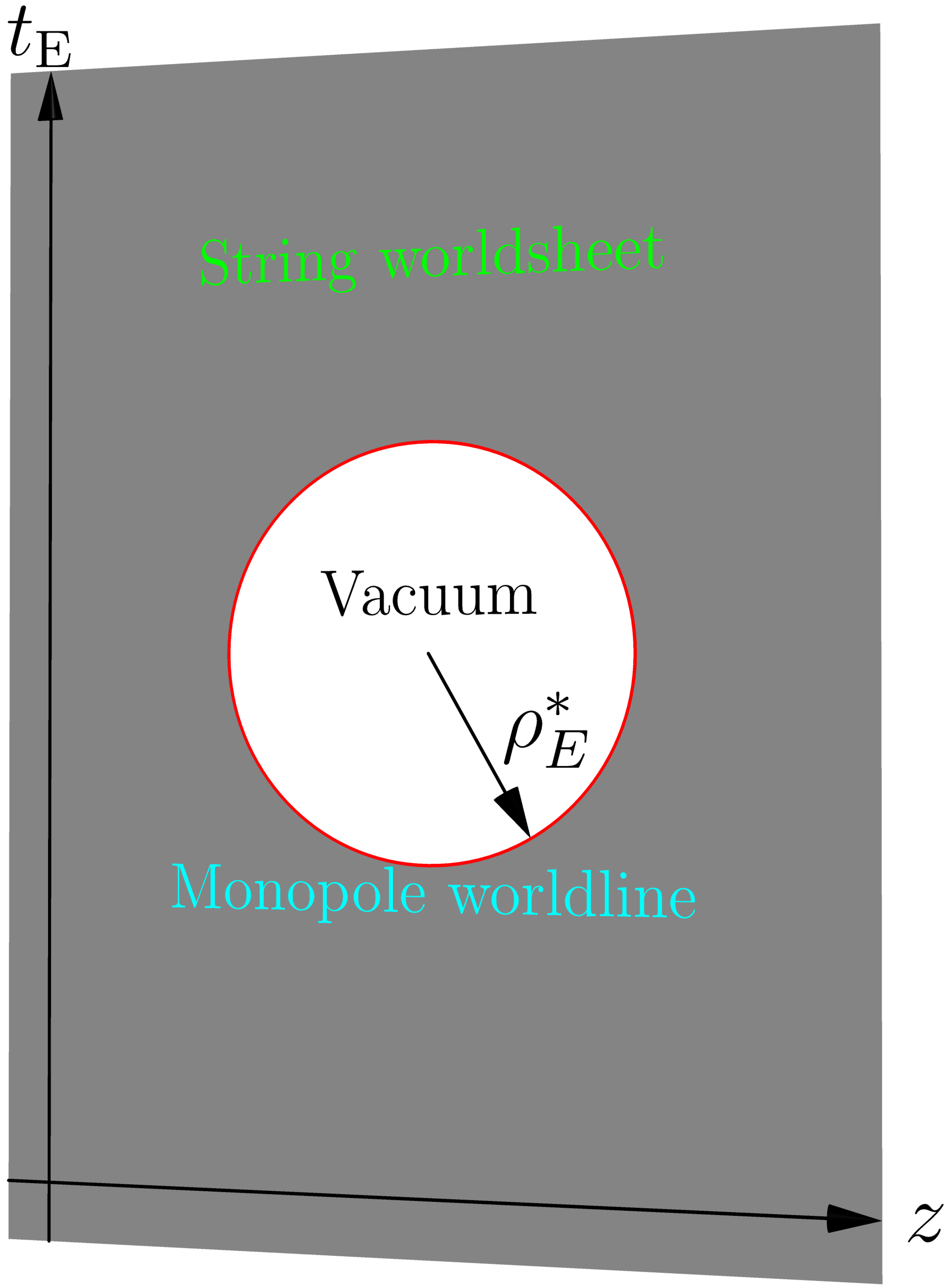
String breaking rate

Preskill-Vilenkin approximation [Preskill & Vilenkin, 1992]

- Neglect monopole size and string width
- $S_E = 2\pi\rho_E^* M_M - \pi\rho_E^{*2} T_{\rm str}$
  - $\rightarrow \rho_E^* = M_M/T_{\rm str}\ ,\ \ S_B = \pi M_M^2 \big/ T_{\rm str} = \pi\kappa$
    - M_M: monopole mass, $T_{\rm str}$: string tension
  - String width $T_{\rm str}^{-1/2} \ll \rho_E^*$ required

$\rightarrow \sqrt{\kappa} \gg 1 \cdots$ **Is this OK for PTA ($\sqrt{\kappa} \sim 8$)?**

$\rightarrow$ **Alternative evaluation desired**



Re-evaluation of bounce action

Setup

Symmetry breaking and topological defects

- SU(2) gauge theory with ϕ: triplet scalar and h: doublet scalar
- SSB step 1: SU(2) → U(1) by $\phi^a = V\delta_3^a$
  - $\pi_2(\mathrm{SU}(2)/\mathrm{U}(1)) = \mathbb{Z}$ → monopoles formed by ϕ
- SSB step 2: U(1) → 1 by $h_i = v\delta_i^1$
  - $\pi_1(\mathrm{U}(1)) = \mathbb{Z}$ → cosmic strings formed by h_1
  - Metastable because $\pi_1(\mathrm{SU}(2)) = 0$

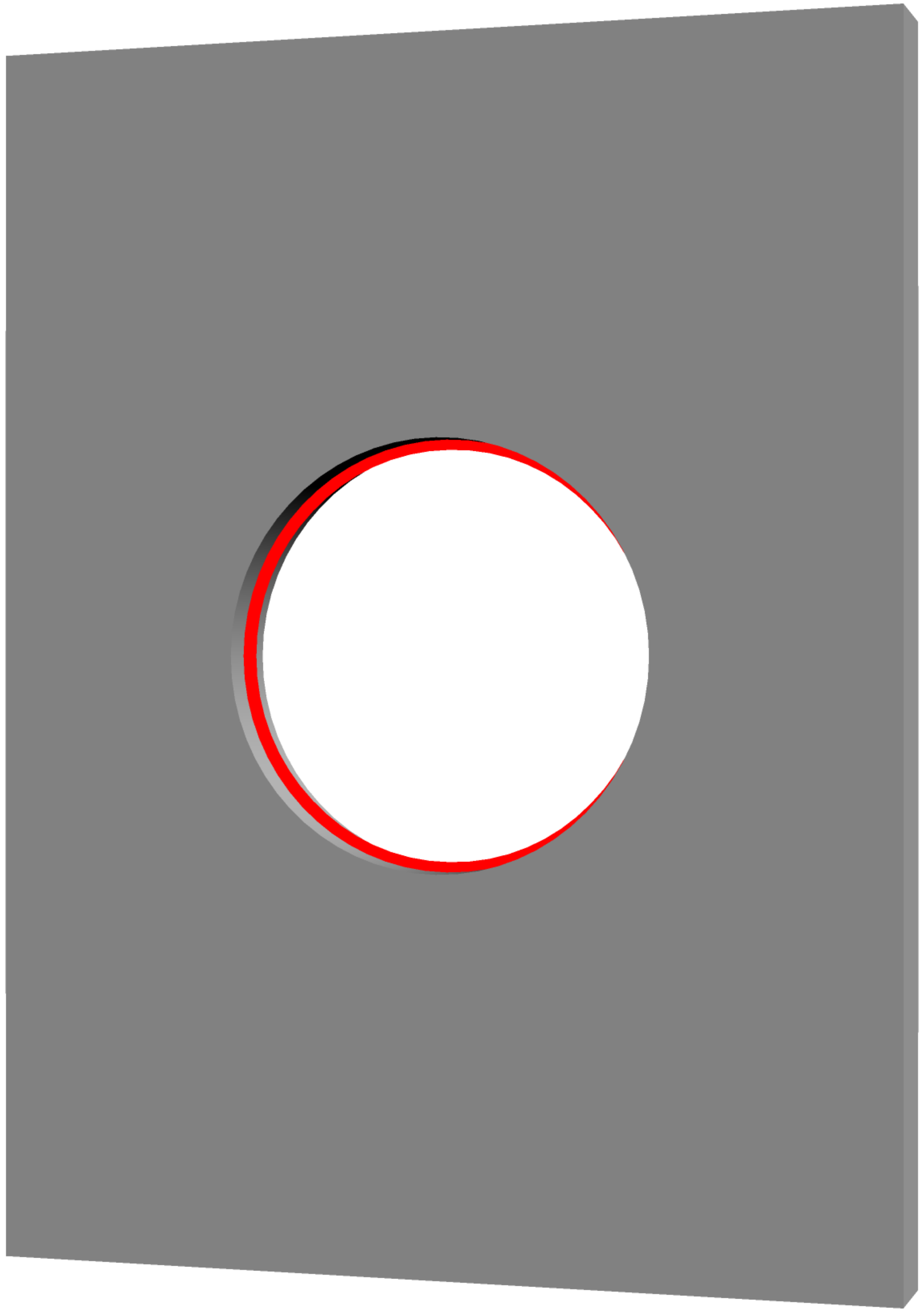
$\sqrt{\kappa_{PV}} \propto V/v$

→ interested in $V/v = \mathcal{O}(1)$

Strategy

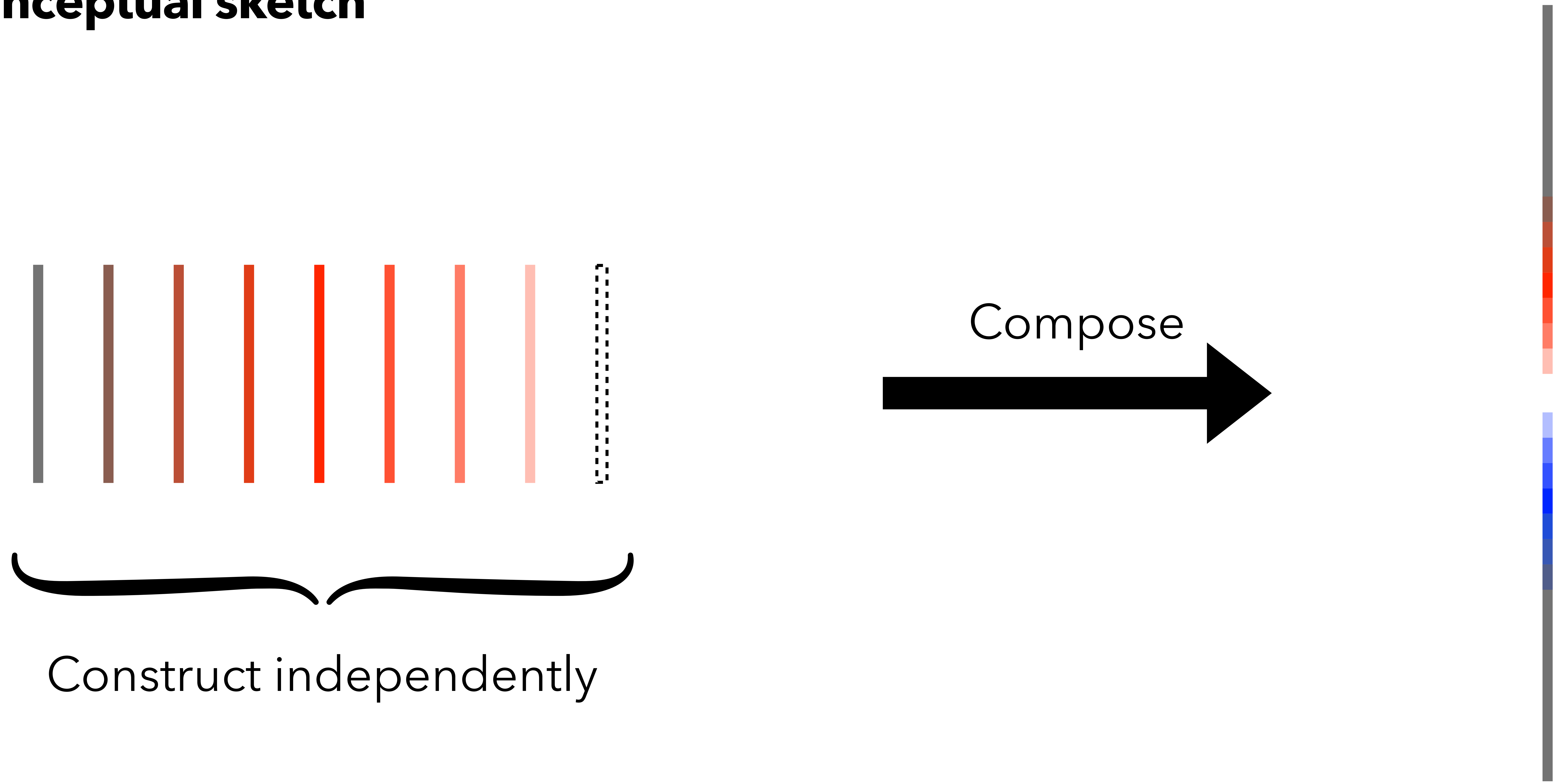
How to evaluate the bounce action?

- Solve 4D Euclidean field equation?
  - Easier said than done!
    - Bounce: saddle point of S_E
      → nontrivial algorithm needed
- → Alternative strategy



Strategy

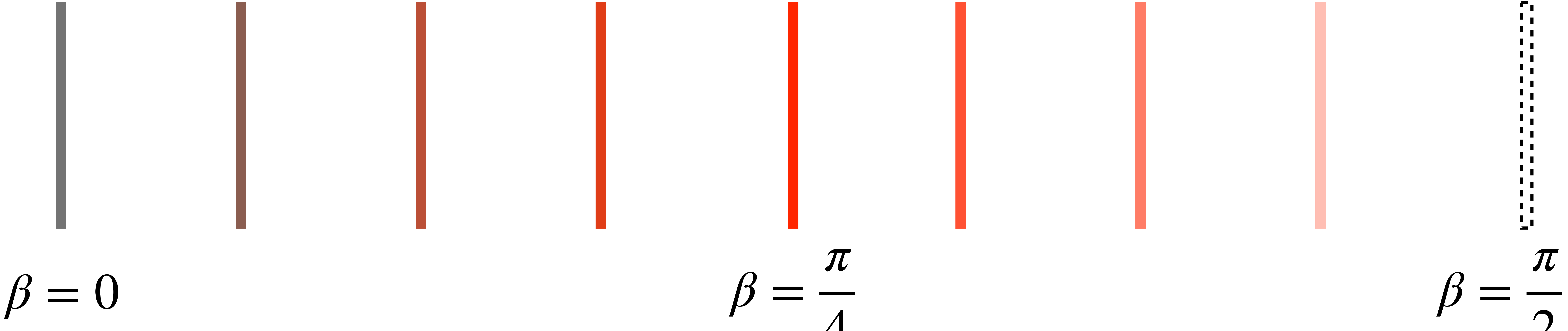
Conceptual sketch



Strategy

Step 1: Build "excited strings" with an Ansatz [Shifman & Yung, 2002]

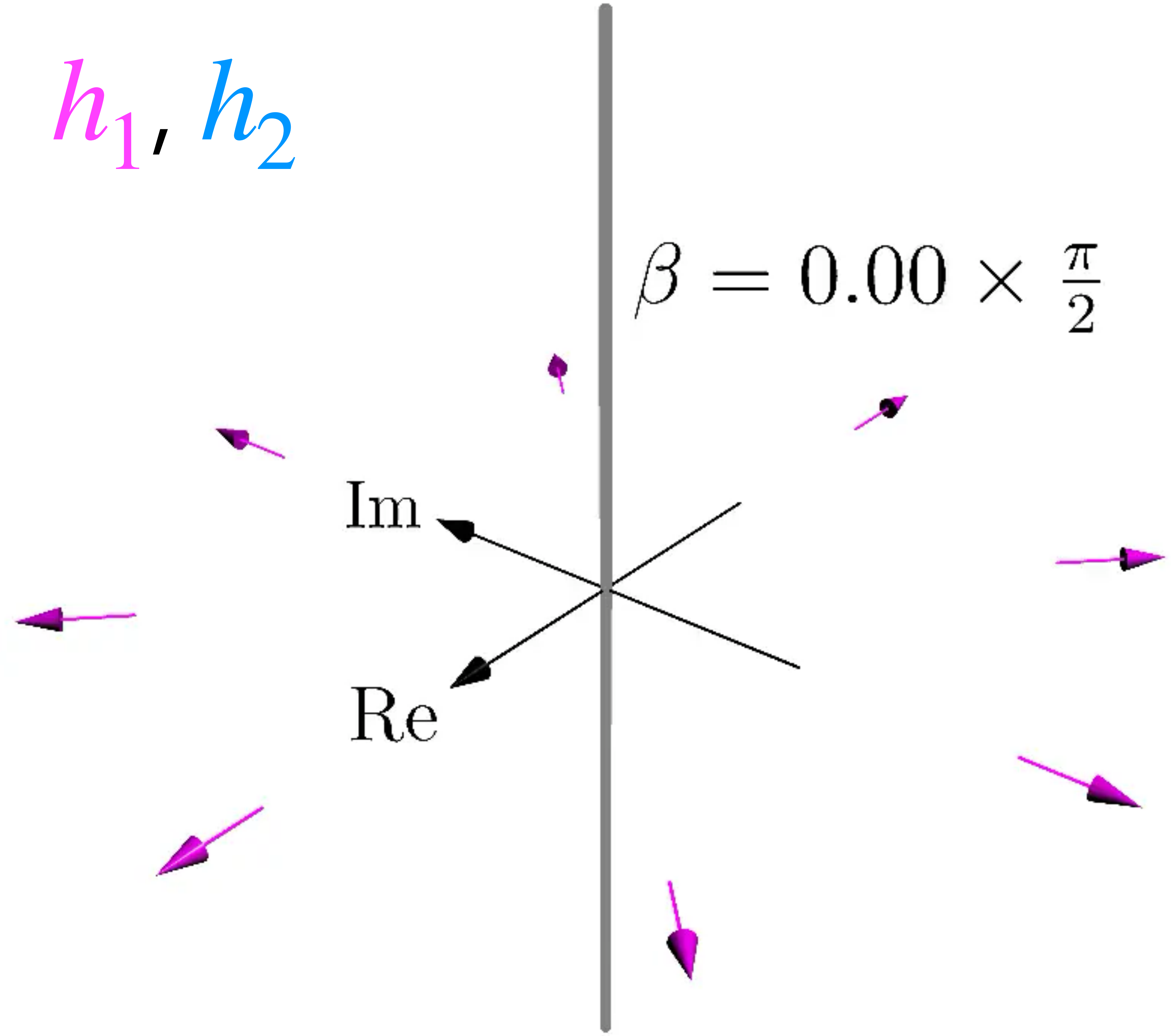
- Introduce β: unwinding parameter (ordinary string at 0, vacuum at $\pi/2$)
- Make β-dependent static string configuration
  - β-dependent Ansatz with a few profile functions
    - Minimize the string tension for each β



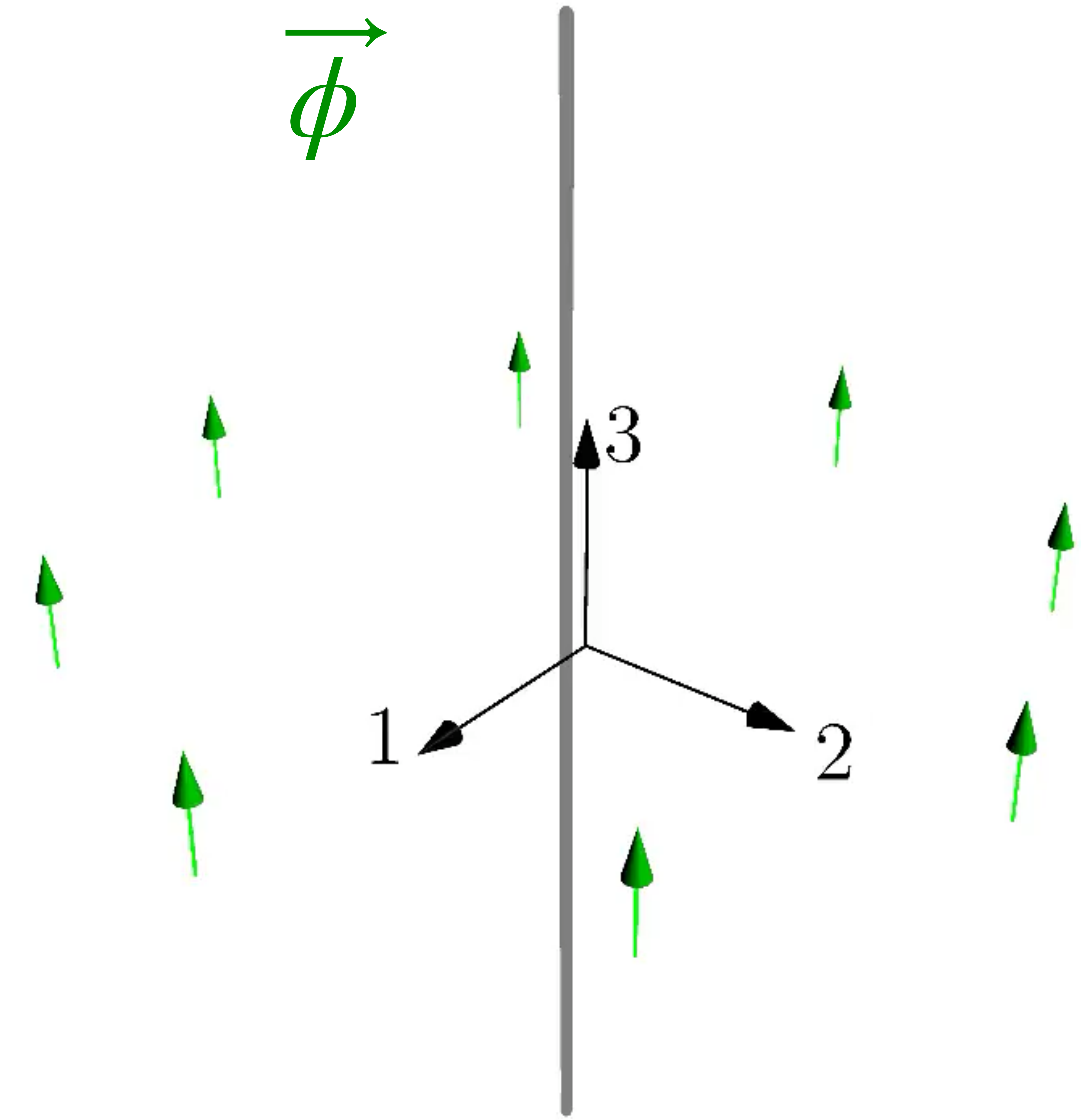
Strategy

Unwinding the string

h_1, h_2

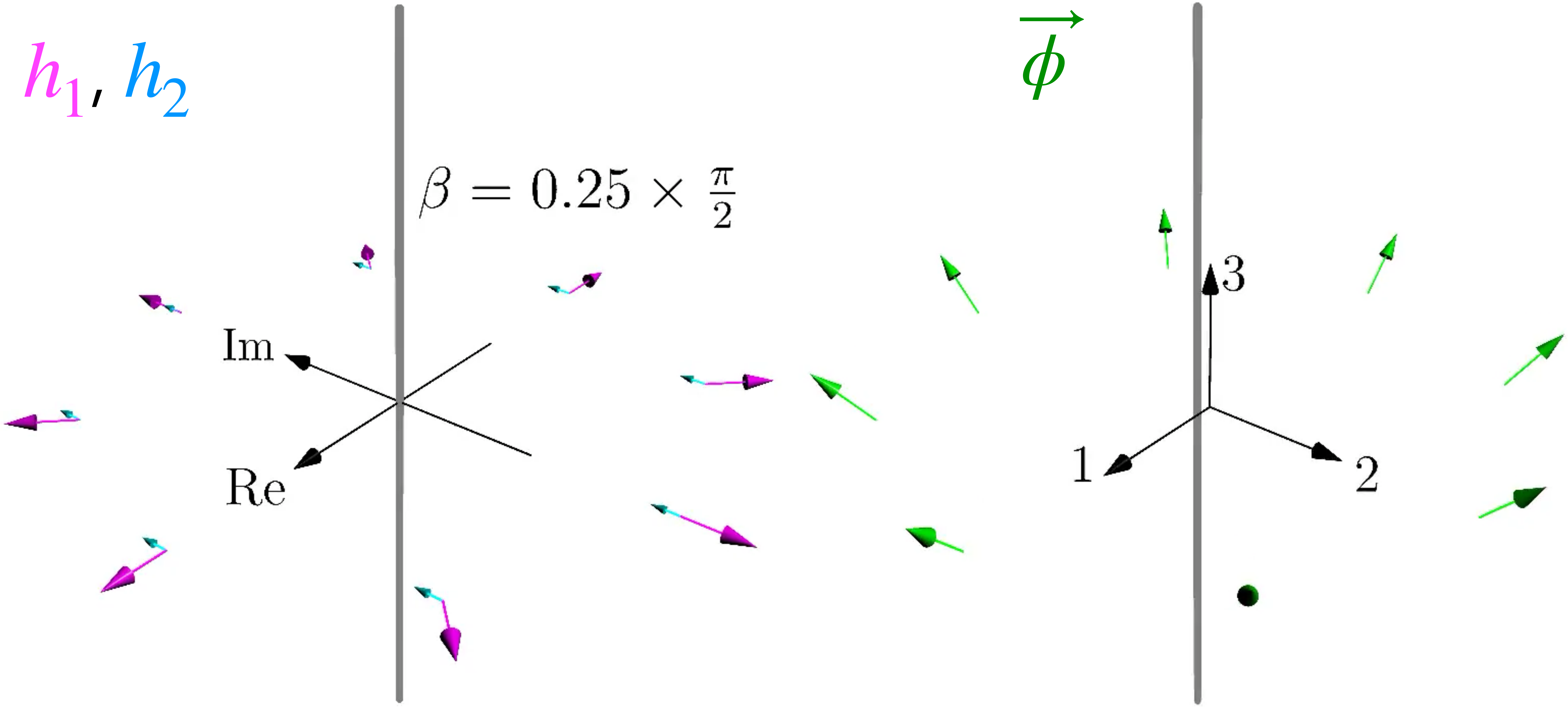


$\vec{\phi}$



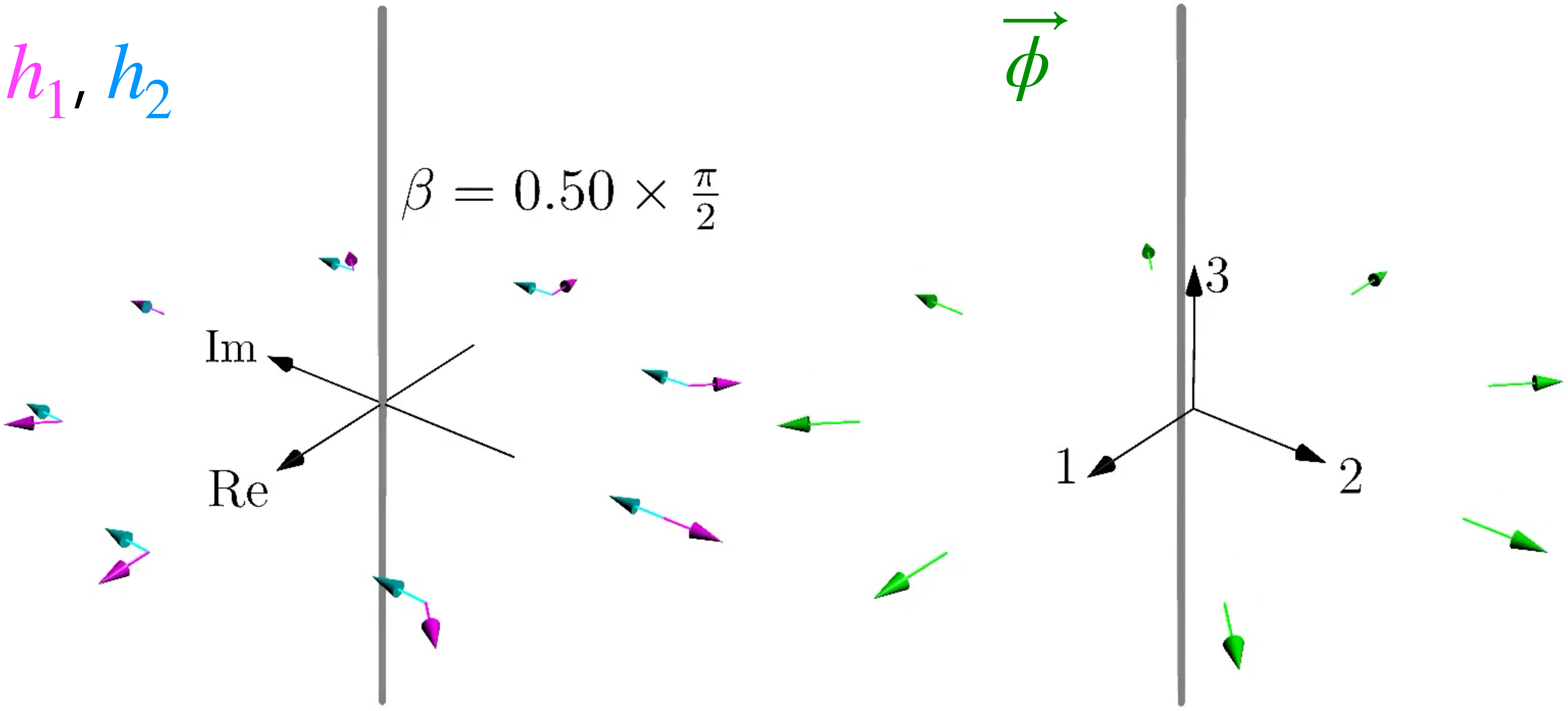
Strategy

Unwinding the string



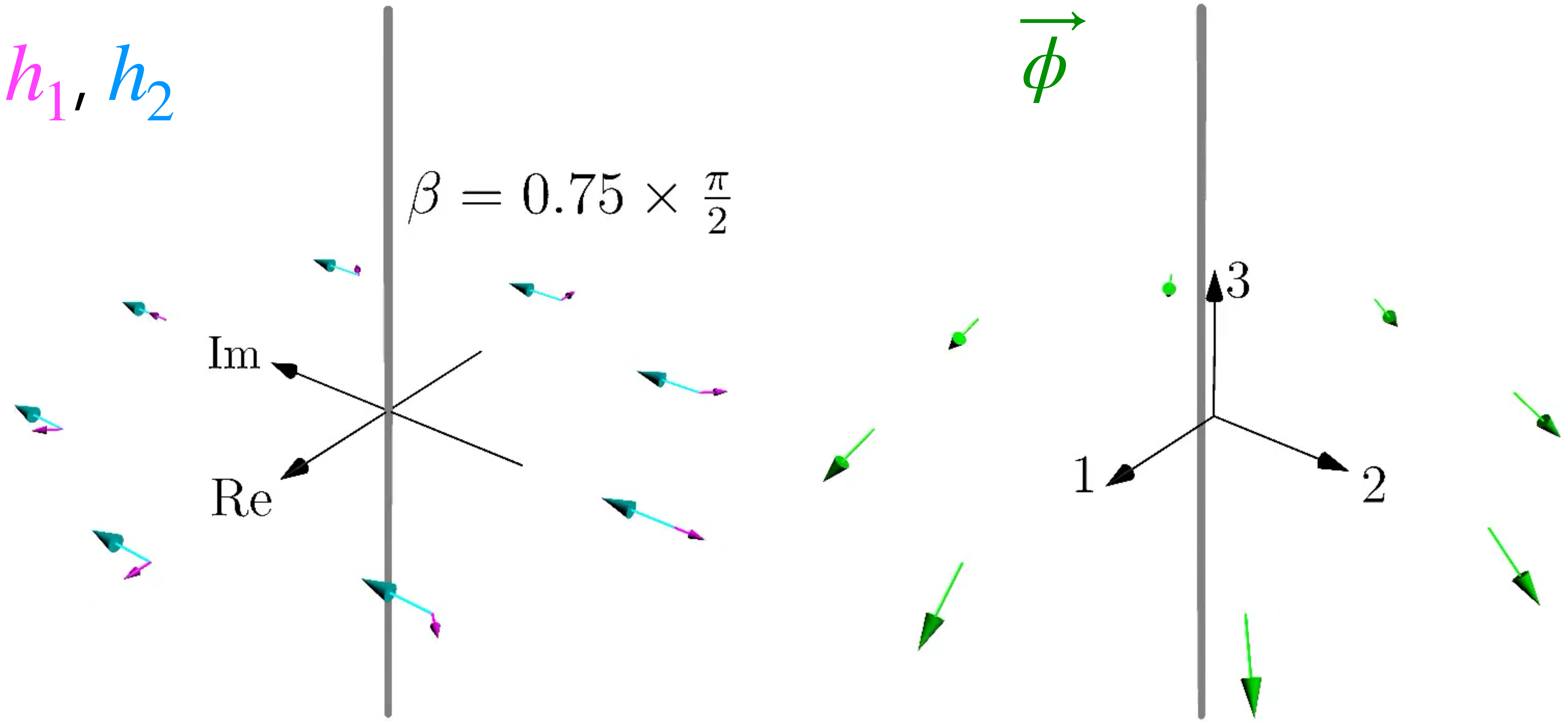
Strategy

Unwinding the string



Strategy

Unwinding the string



Strategy

Unwinding the string

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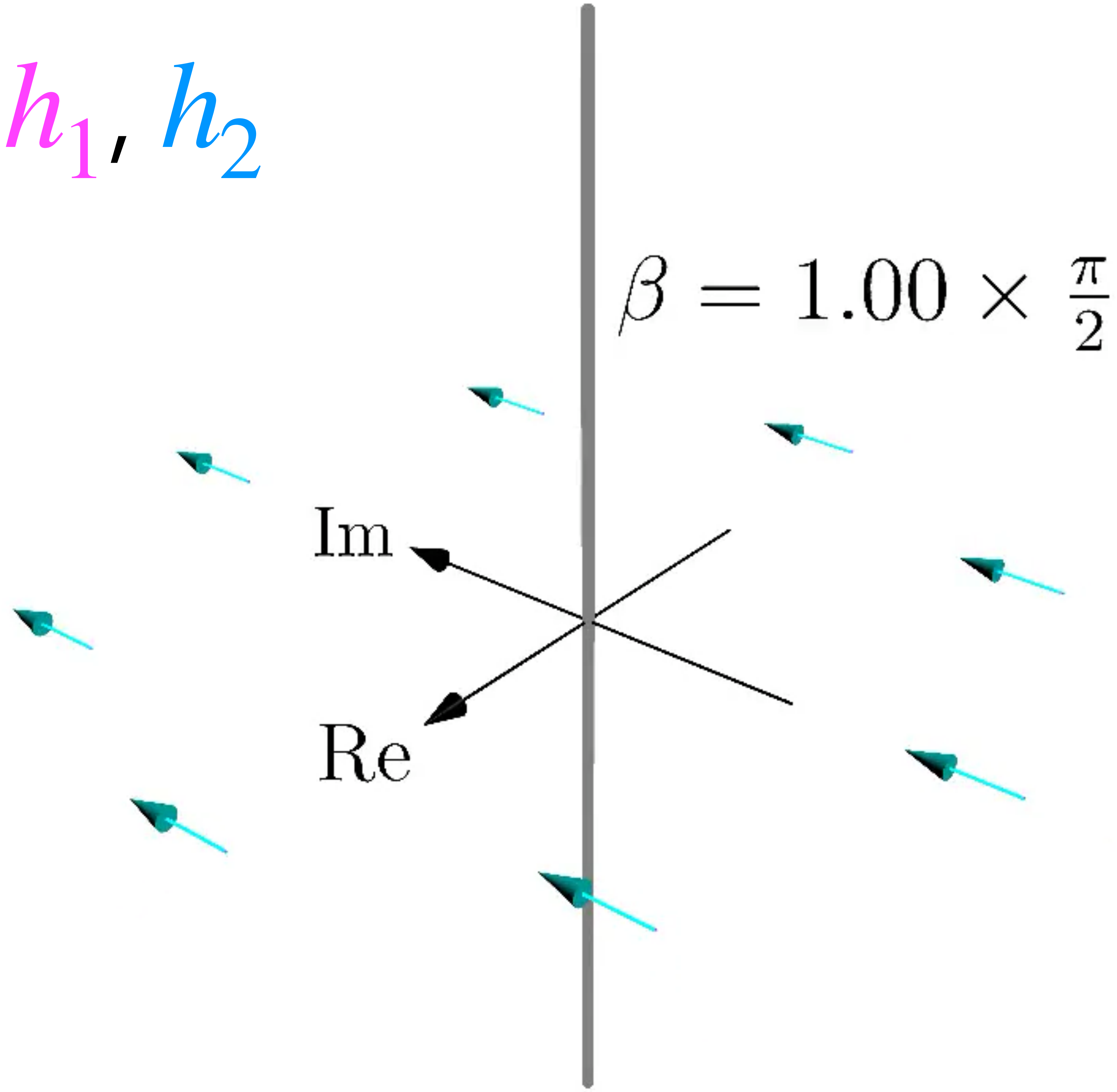
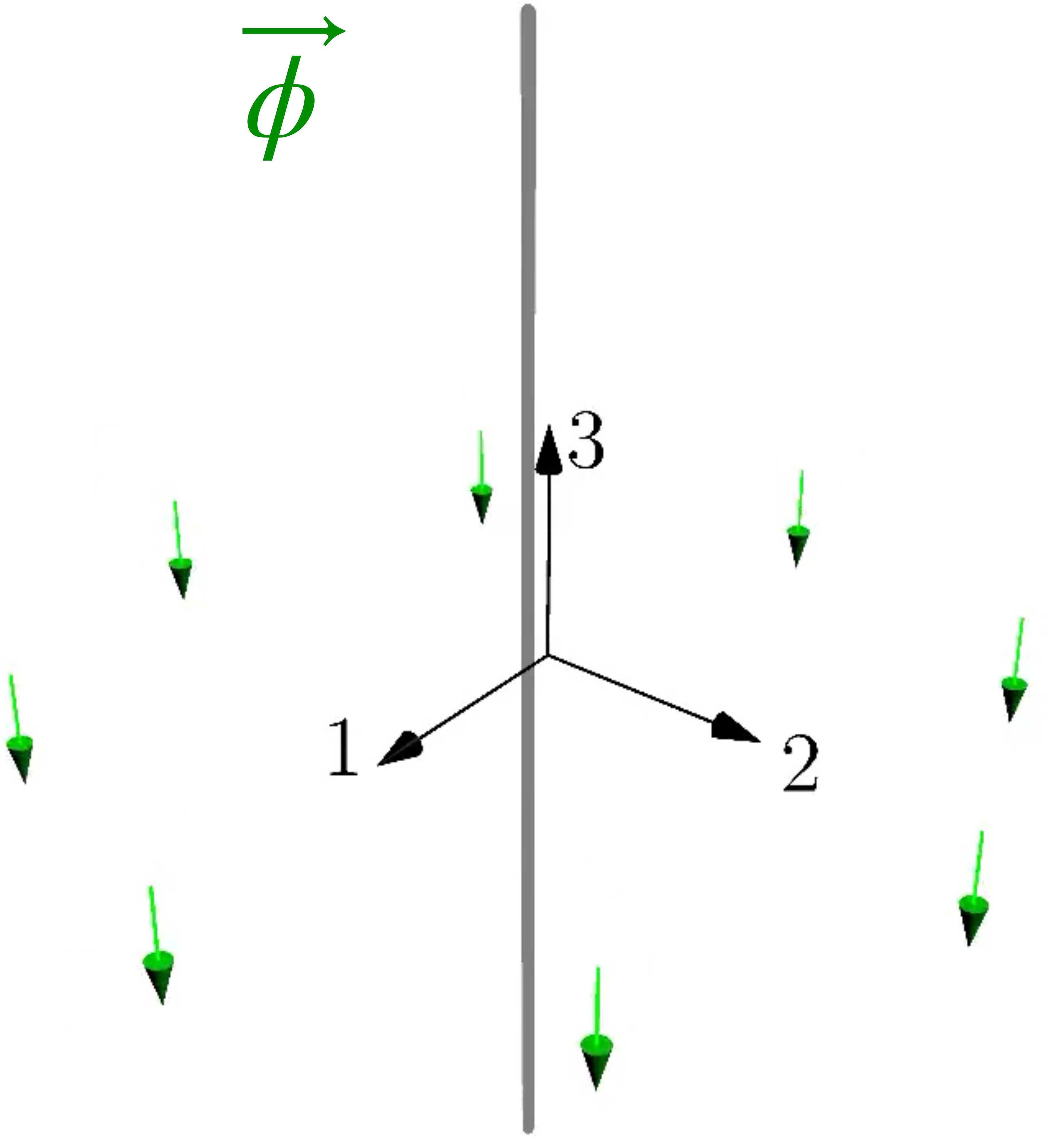


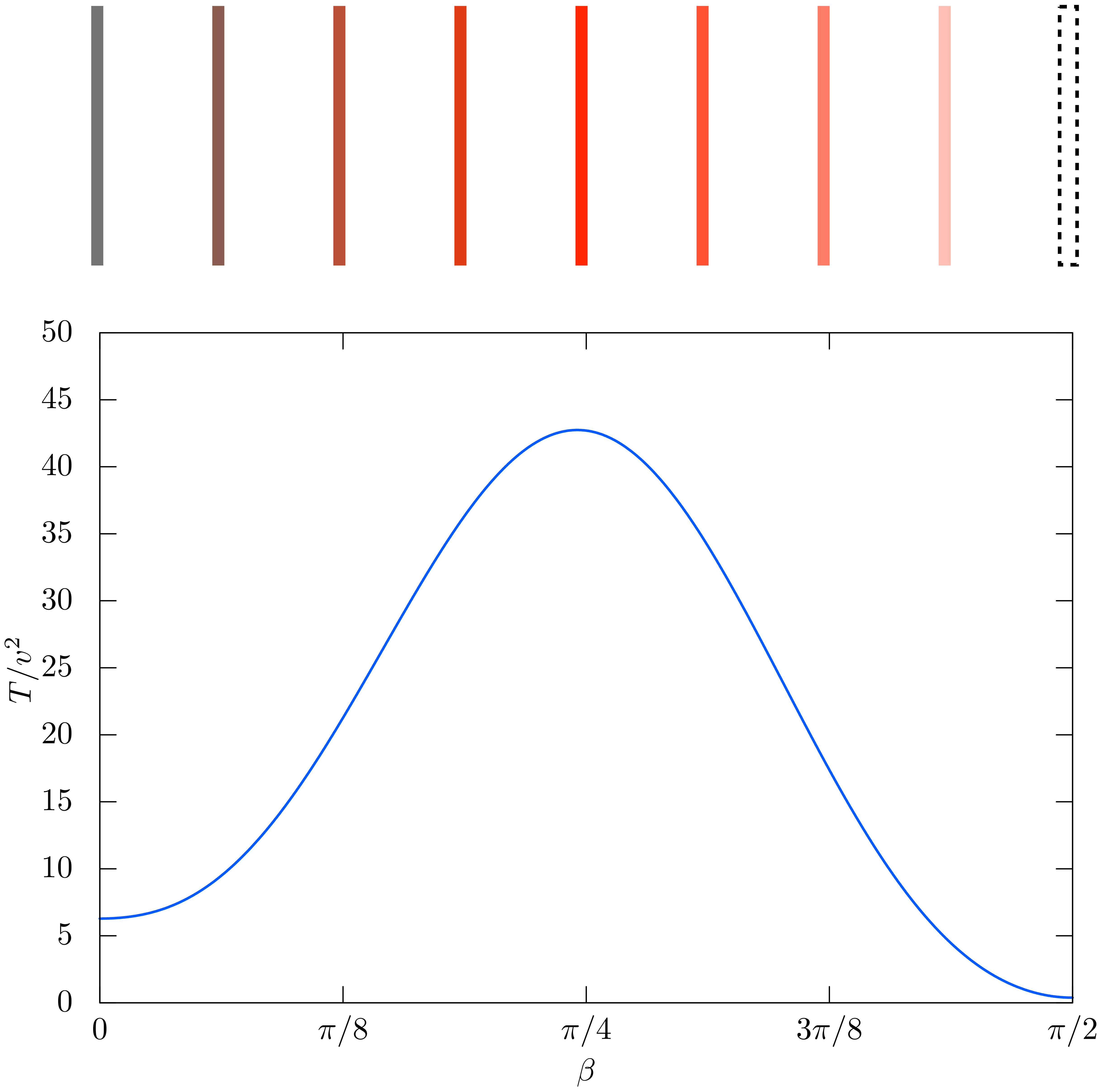
$\vec{\phi}$



Strategy

β-dependent tension

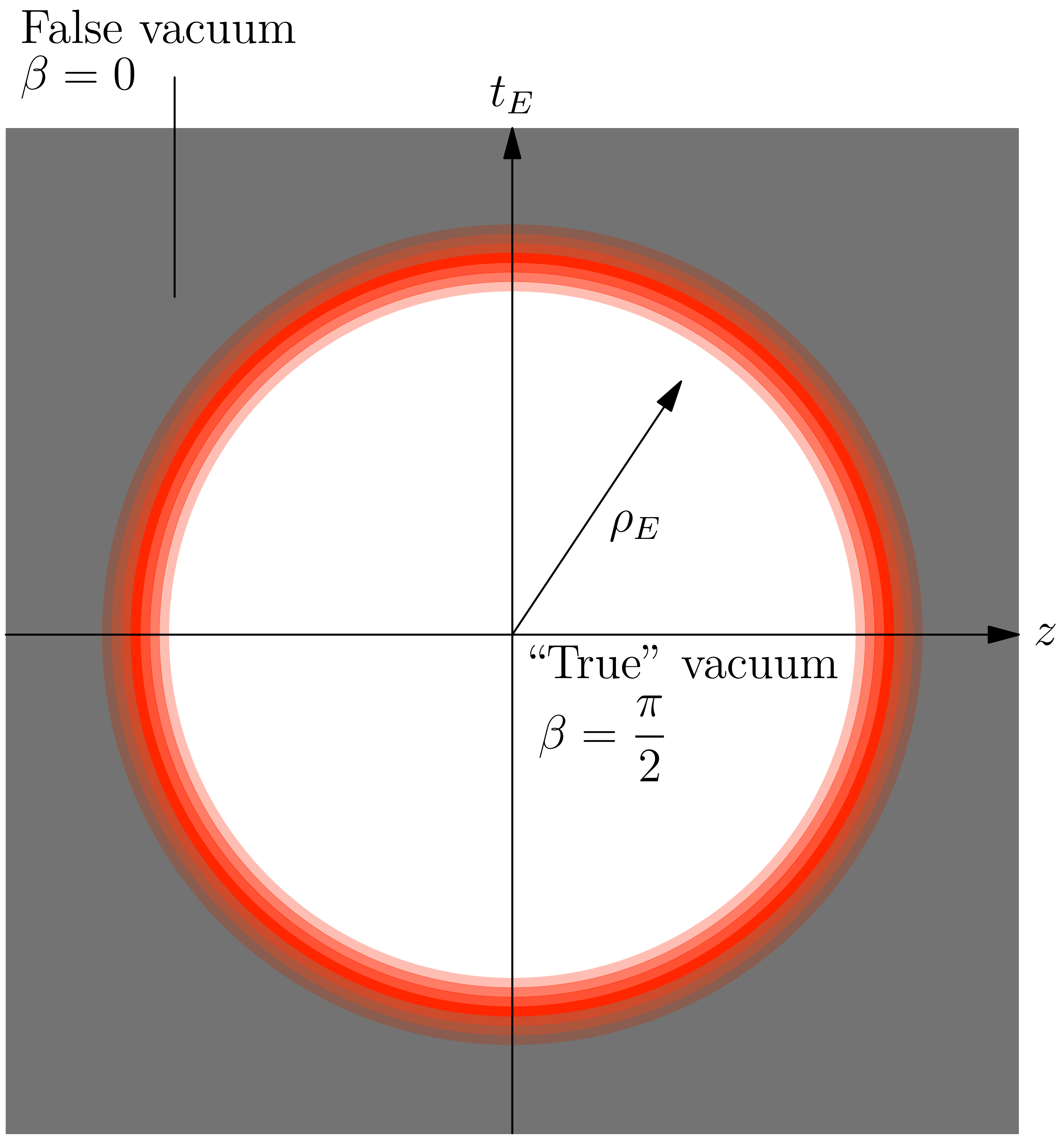
- $\beta = 0$: ordinary string tension
- $\beta \sim \pi/4$: monopole → potential wall
- $\beta = \pi/2$: vacuum i.e. tension=0



Strategy

Step 2: Promote β to a field on the string

- Construct effective 2D theory about $\beta(t_E, z)$
- The bubble is circular
  - Reduces to 1D theory:
    $$S_E = 2\pi \int_0^\infty \rho_E \, \mathrm{d}\rho_E \left[\frac{1}{2} \mathscr{K}_{\mathrm{eff}}(\beta) \beta'^2 + T(\beta) \right]$$
  - EoM solvable → bounce action



Strategy

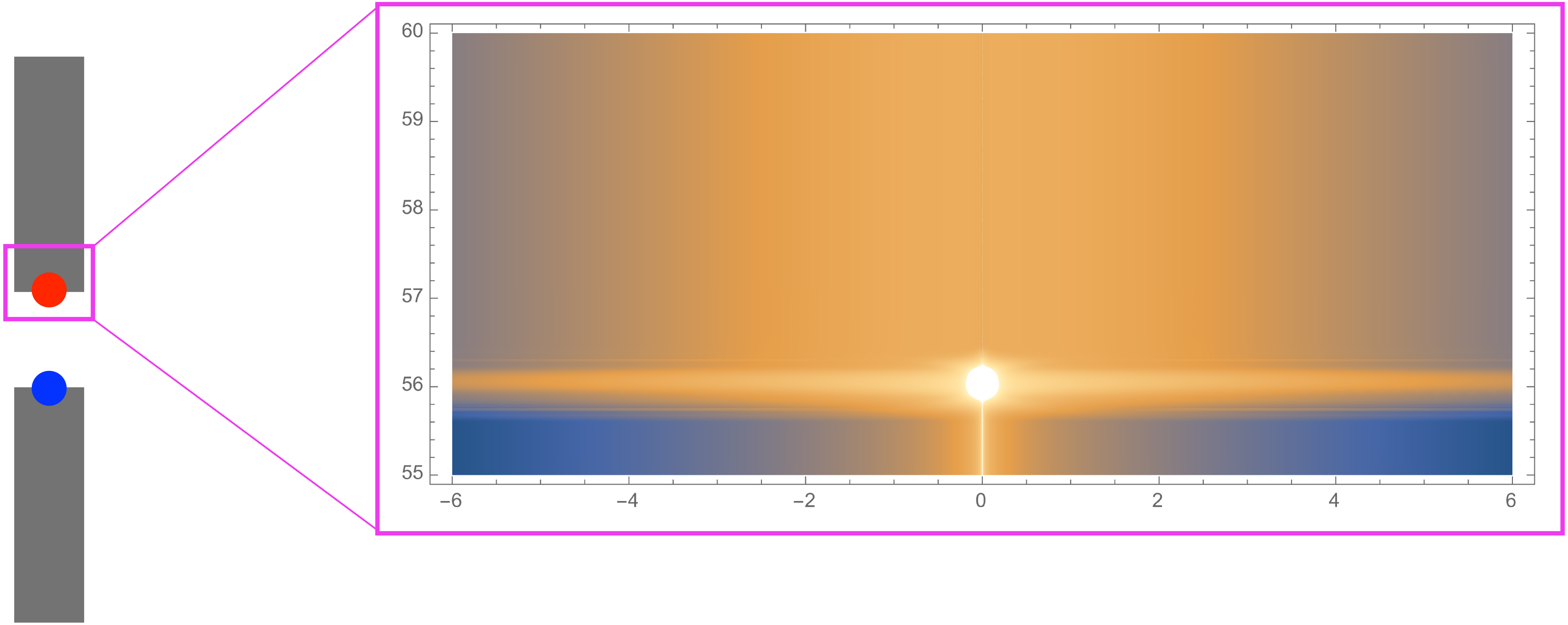
Summary

1. Build a "spectrum of excited strings"
    - $\beta = 0$: string, $\beta = \dfrac{\pi}{2}$: vacuum
    - Minimize the string tension within Ansatz → static profiles for each β
2. Promote β to a collective coordinate $\beta(t, z)$
    - Euclidean bubble: SO(2) symmetric → $\beta(\rho)$
    - Solve the Euclidean EoM for $\beta(\rho)$ and compute the bounce action

- → Upper bound on the optimal bounce action

Results

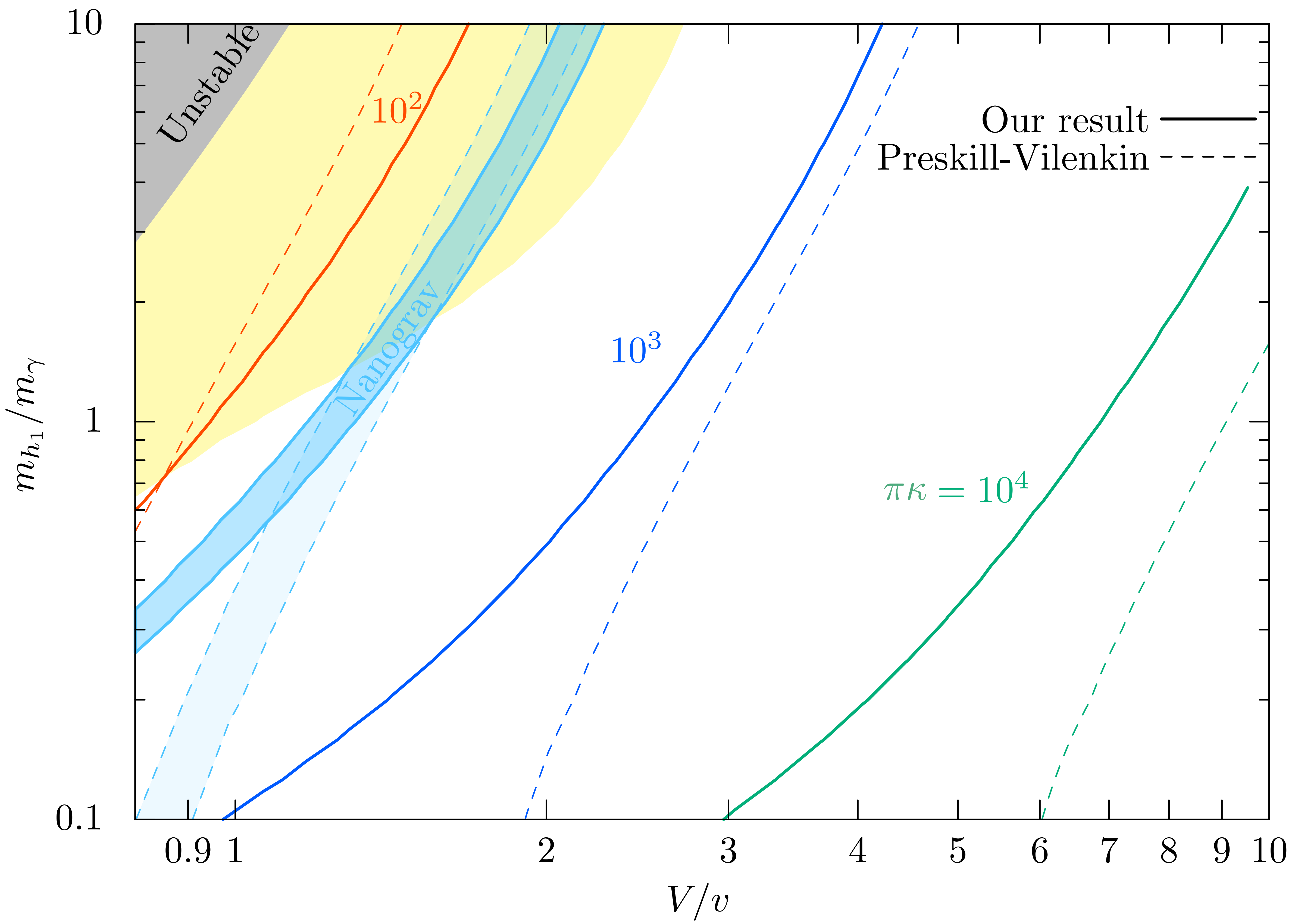
Cross Section of a Breaking String



Results

Interpretation of NANOGrav results

- Yellow: our $S_B <$ Preskill-Vilenkin
  - i.e. Preskill-Vilenkin is invalid
  - Overlaps with NANOGrav region
  - Modifies the interpretation



Conclusions & Outlooks

- A robust upper bound on the bounce action for string breaking was calculated
  - free of the conventional assumption (i.e. valid for finite string width)
- The Preskill-Vilenkin approximation may be unsuited to interpret the PTA data
- Next steps:
  - Optimal bounce action? (ongoing)
  - More realistic setup?
  - String formation process?

Thank you!

Backup

Setup

SU(2) gauge theory w/ adjoint Higgs & fundamental Higgs

- $\mathscr{L} = -\frac{1}{4g^2}F^2 - \quad Dh\ ^2 - \left(D\overrightarrow{\phi}\right)^2 - V_{\text{Higgs}}(h, \phi)$
  - ϕ: SU(2) adjoint, h: SU(2) fundamental
- $V_{\text{Higgs}}(h, \phi) = \lambda\left(\quad h\ ^2 - v^2\right)^2 + \tilde{\lambda}\left(\overrightarrow{\phi}^2 - V^2\right)^2 + \gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2$
  - Assumptions: λ, $\tilde{\lambda}$, $\gamma > 0$, $V > v$

Setup

Symmetry breaking pattern

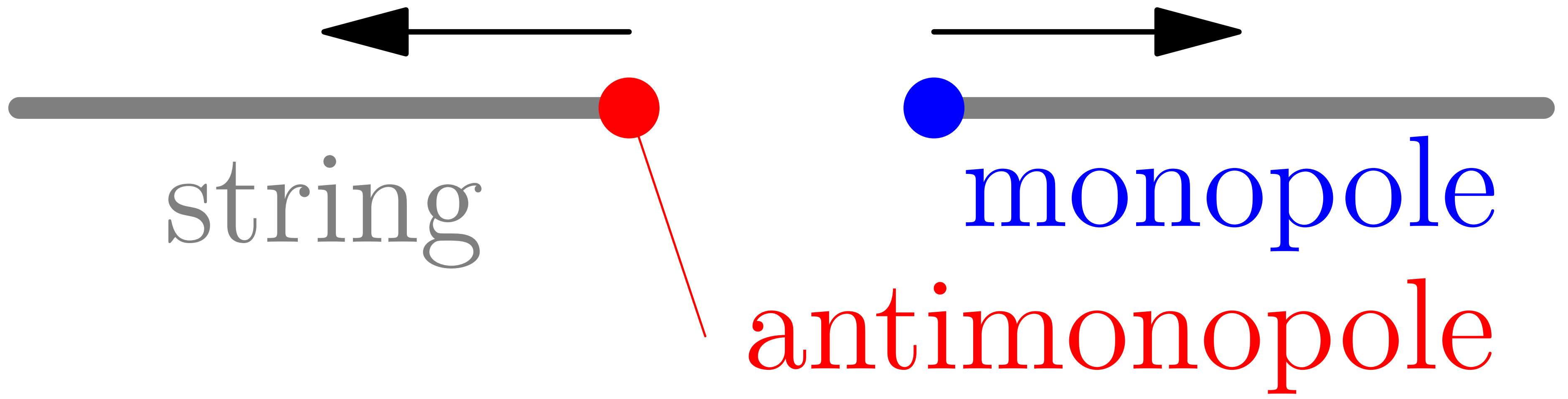
- $V_{\text{Higgs}}(h,\phi) = \lambda\left(h^{2} - v^2\right)^2 + \tilde{\lambda}\left(\vec{\phi}^{\,2} - V^2\right)^2 + \underbrace{\gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2}_{\Big\downarrow\ \phi^a = V\delta^a_3}$

$$\gamma V^2 \; h_2{}^{2}$$

- SU(2) → U(1) by $\phi^a = V\delta^a_3$
  - U(1) generator: $\tau^3/2$
- U(1) → 1 by $h_i = v\delta^1_i$

Setup

Cosmic Strings and Monopoles



- 1st SSB: SU(2) → U(1) by $\phi = V\delta_3^a$
  - $\pi_2(\mathrm{SU}(2)/\mathrm{U}(1)) = \mathbb{Z}$ → monopoles formed by ϕ
- 2nd SSB: U(1) → 1 by $h_1 = ve^{i\times 0}$
  - $\pi_1(\mathrm{U}(1)) = \mathbb{Z}$ → cosmic strings formed by h_1 (at least for $V \gg v$)
  - But also $\pi_1(\mathrm{SU}(2)) = 0$ → only metastable
    - Strings can break via monopole-antimonopole pair production

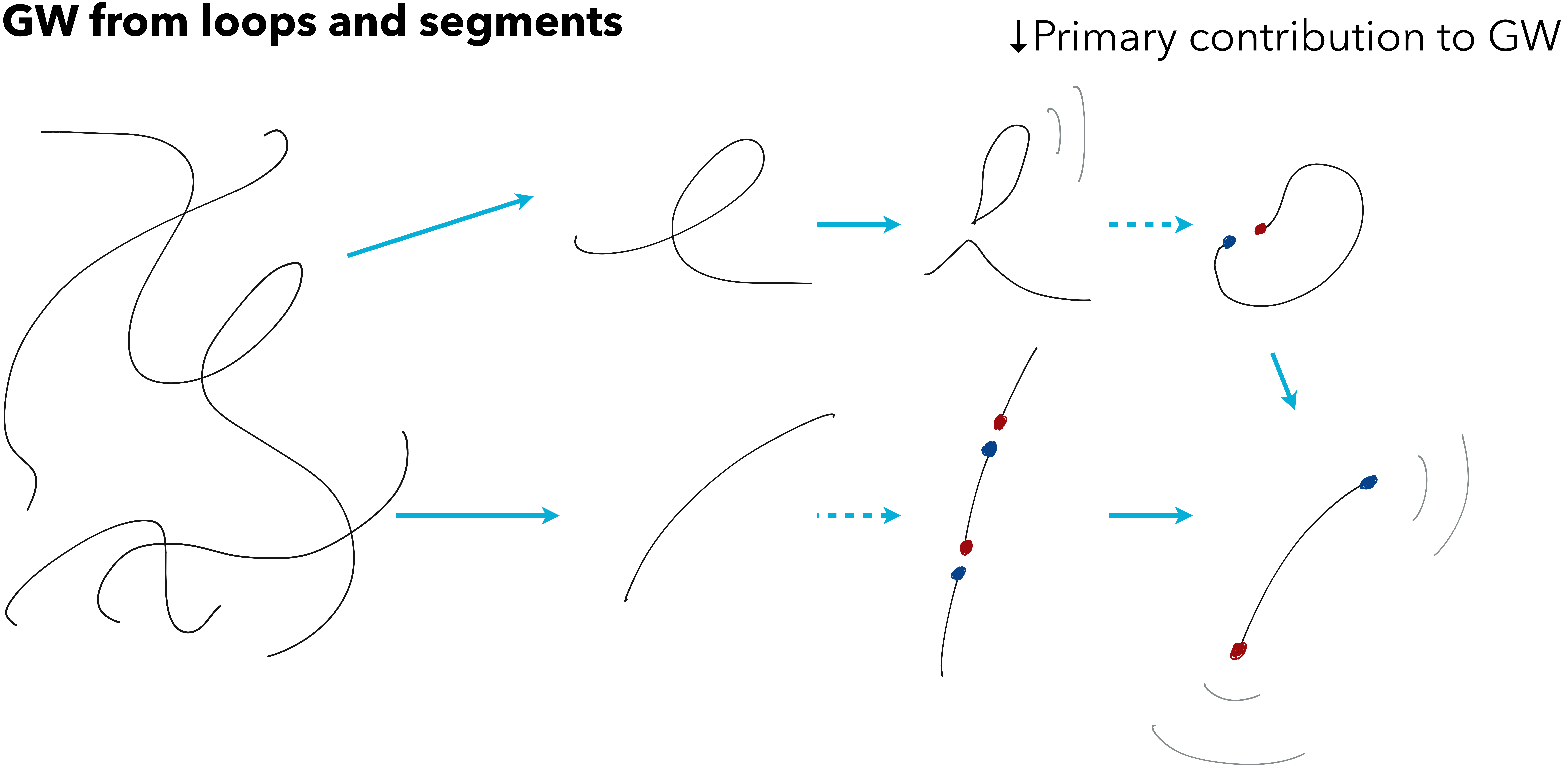
Strategy

Two Ansätze [Shifman & Yung, 2002]

- Primitive Ansatz: $A_\theta = \left[\bullet + \blacktriangle \right] f_\beta(\rho)$
  - $f_\beta(\rho)$: one of the profile functions
- Improved Ansatz: $A_\theta = \bullet \, f_\beta^\gamma(\rho) + \blacktriangle \, f_\beta^W(\rho)$
  - Contains the primitive Ansatz
- No numerical computations so far

Metastable Cosmic Strings

GW from loops and segments



Strategy

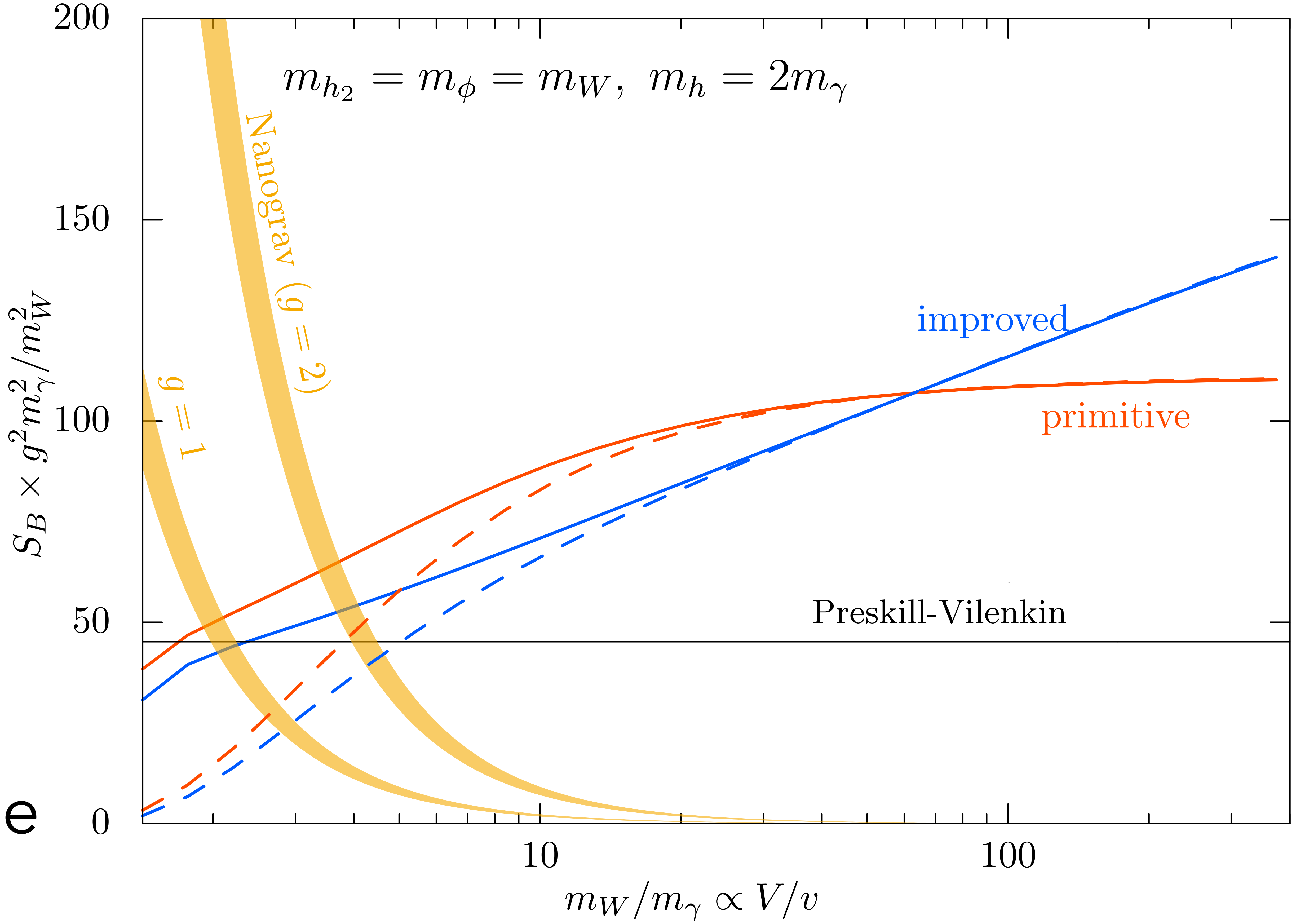
“β-thin-wall approximation” (vs. Preskill-Vilenkin)

- Thin-wall approximation to the 1D effective theory of $\beta(\rho_E)$
  - Valid only for $V \gg v$
- Preskill-Vilienkin approximation: similar but different
  - β-thin-wall: Ansatz → effective 1D theory → thin-wall
  - Preskill-Vilenkin: assume thin-wall in the 4D theory

More on Thin-Wall

Is Preskill-Vilenkin good enough?

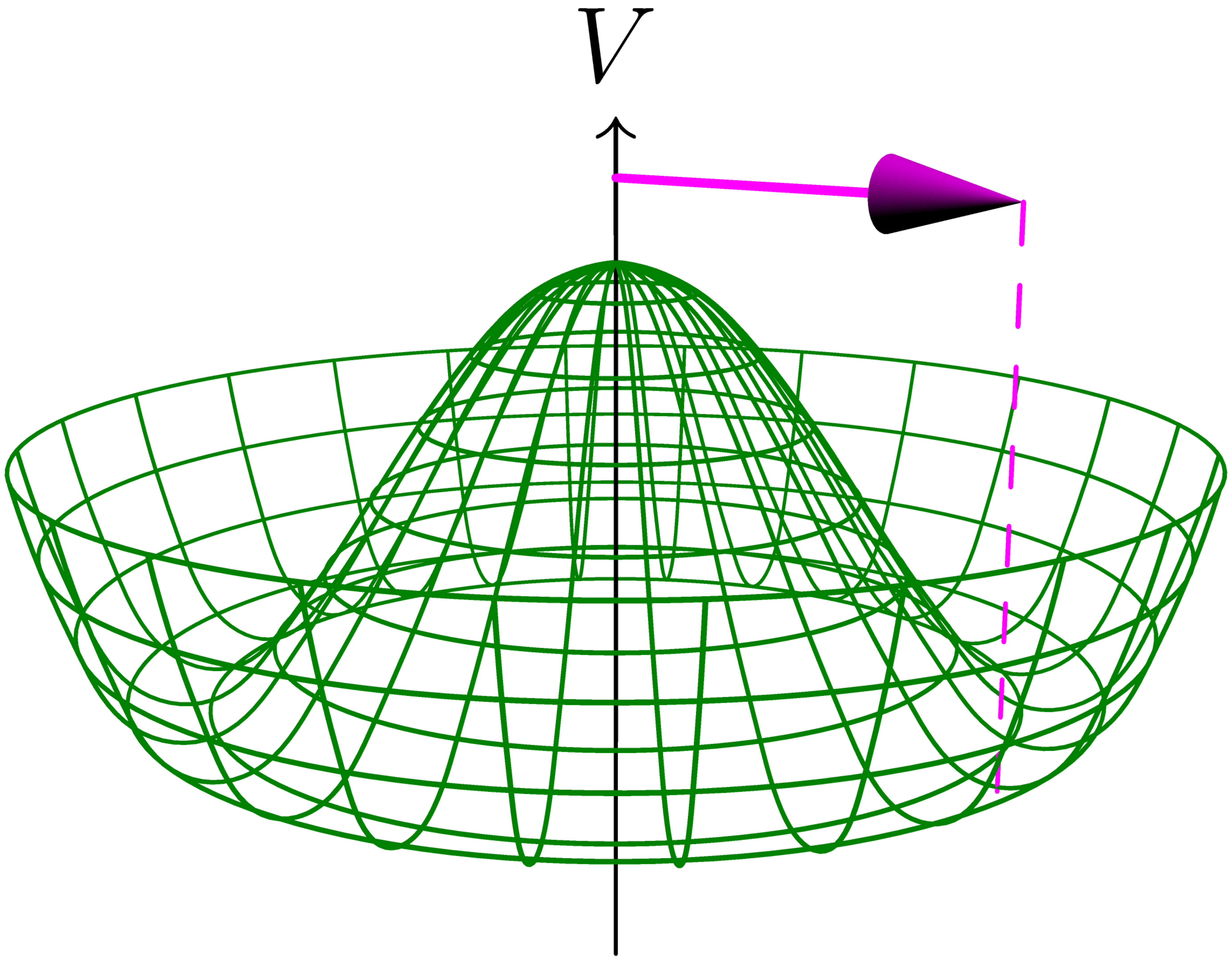
- solid: bounce, dashed: β-thin-wall
- For large hierarchy:
  - Primitive: Preskill-Vilenkin $\times \mathcal{O}(1)$
- For small hierarchy:
  - Deviation from β-thin-wall
    - Preskill-Vlienkin: also questionable



Cosmic Strings

from U(1) breaking

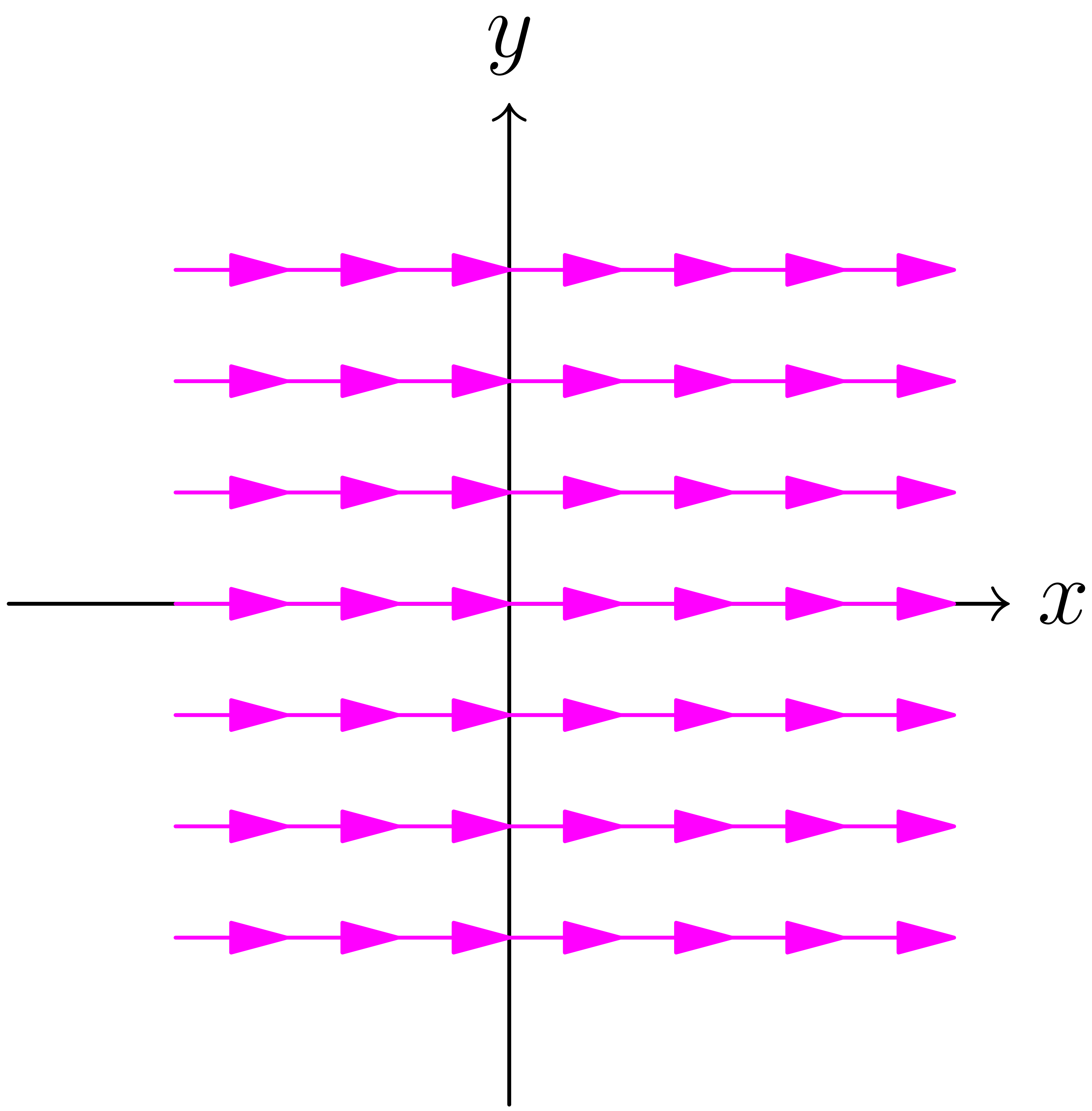
- Simplest setup: abelian Higgs
- $V(\phi) = \lambda \left(\phi^\dagger \phi - v^2\right)^2$
- U(1): $\phi \to e^{i\alpha}\phi$
  - broken by $\langle \phi \rangle = v$



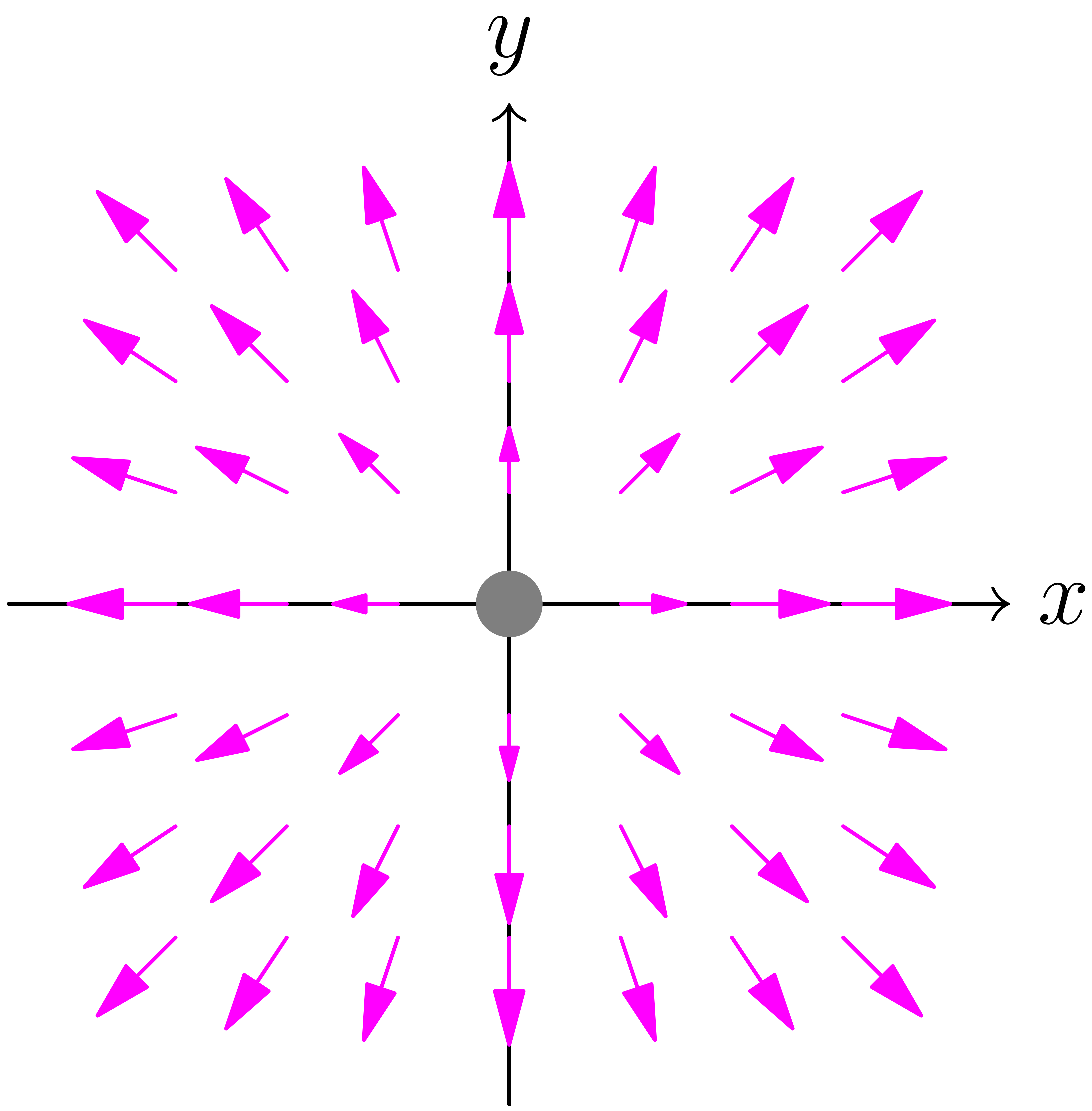
Cosmic Strings

from U(1) breaking (ctd.)

Vacuum

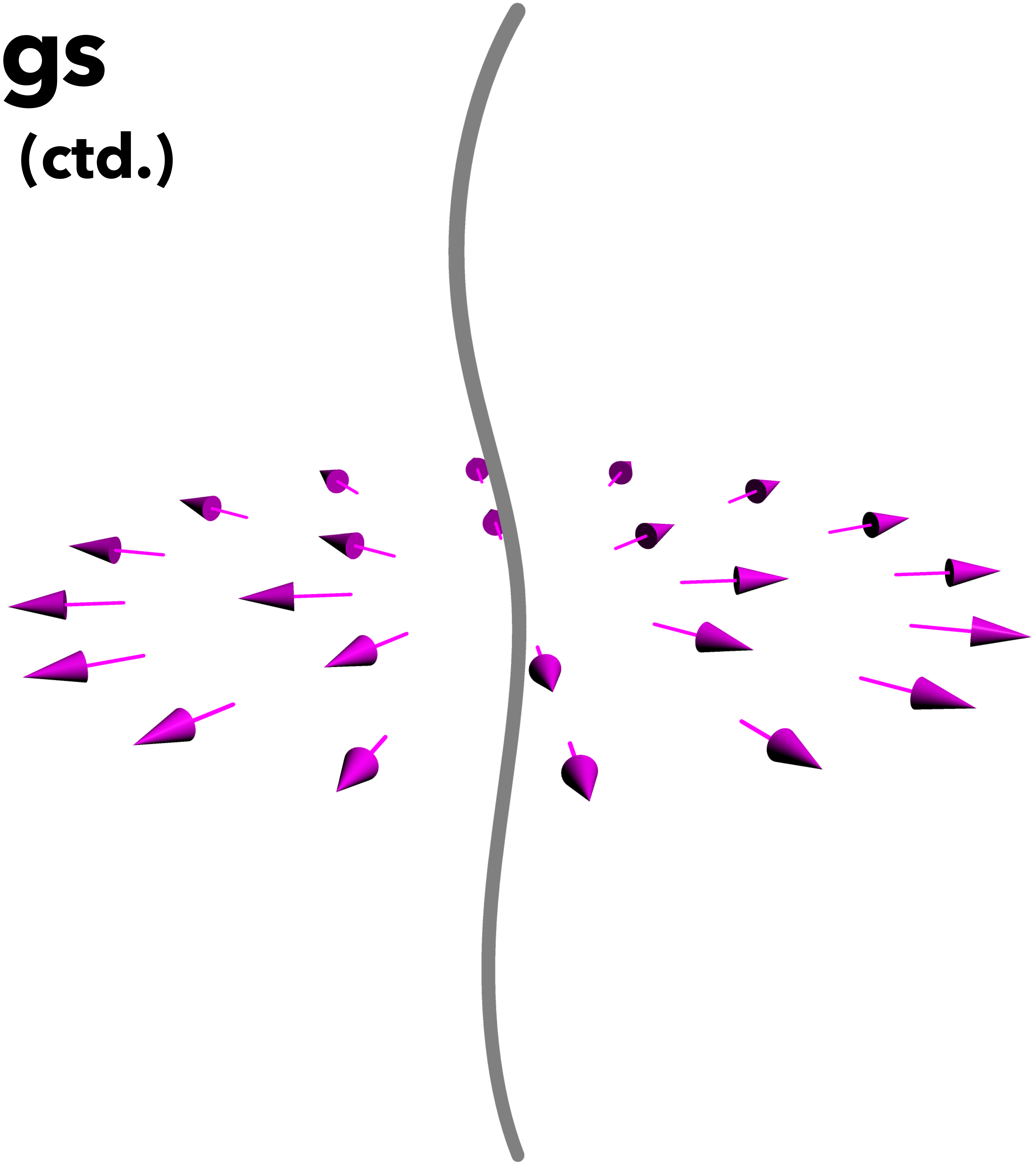


Wound about z axis: $\pi_1(\mathrm{U}(1)) = \mathbb{Z}$



Cosmic Strings

from U(1) breaking (ctd.)



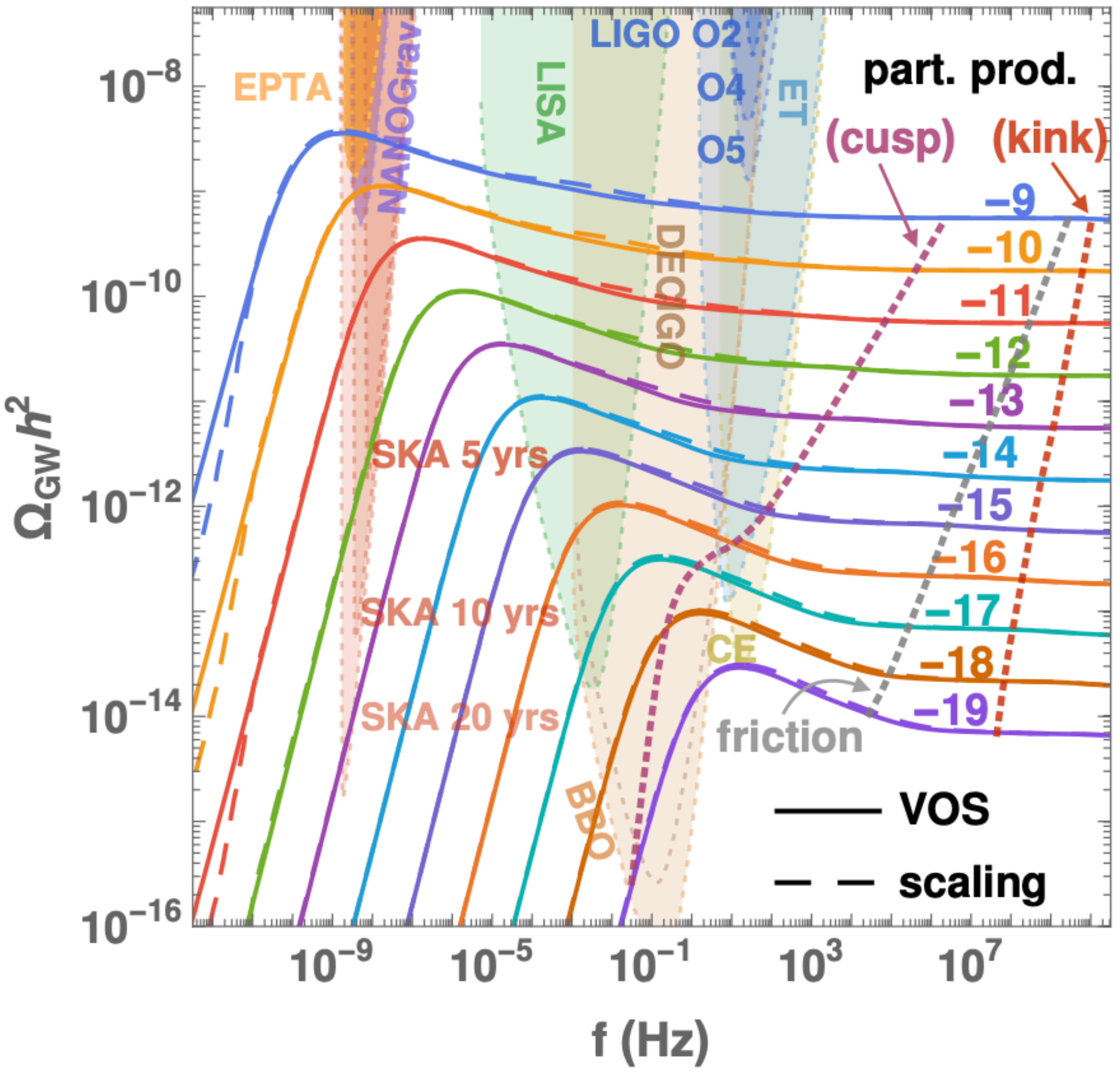
On Metastability

Stable strings vs. PTA

- Frequency ~ loop size ~ horizon
- Nanograv's spectrum: blue tilted
- The amplitude and
  the low-frequency cutoff correlate

→Mismatch with NANOGrav

late emission ← early emission



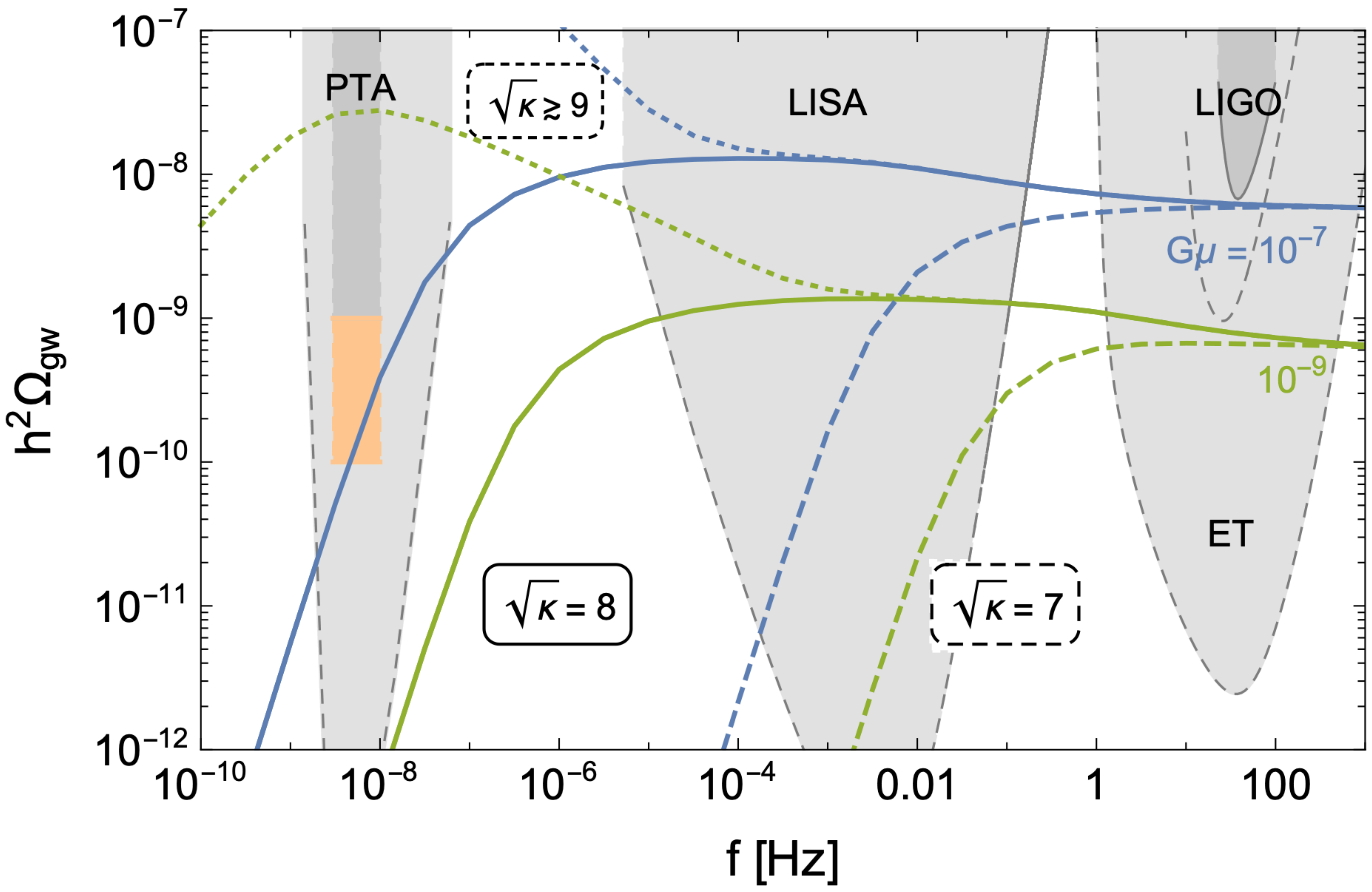
[Gouttenoire et al., 2019]

On Metastability

Metastable strings vs. PTA

- Less long loops
- IR cutoff moves to the right

  →better fit with the PTA data

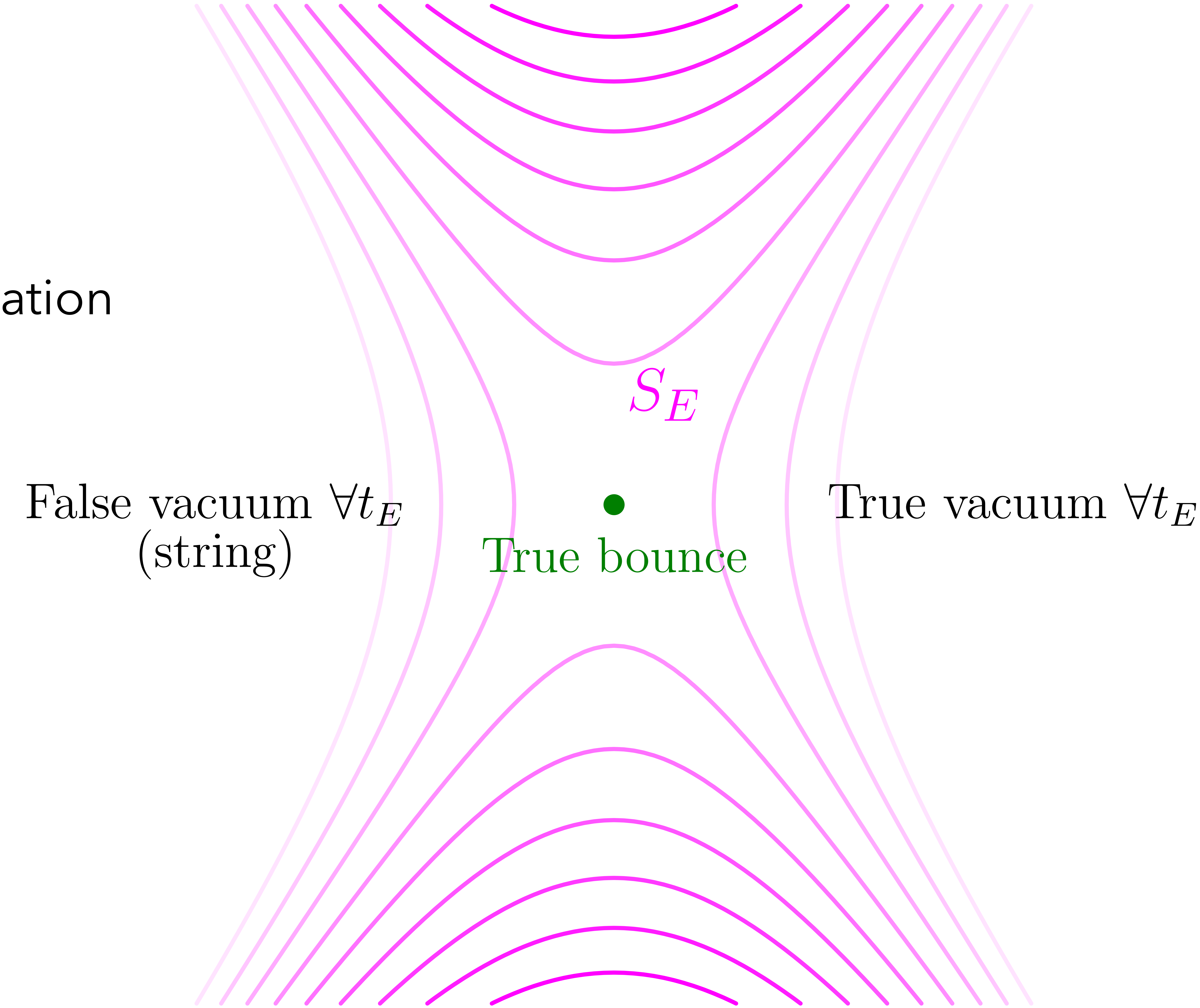


[Buchmüller et al., 2023]

Strategy

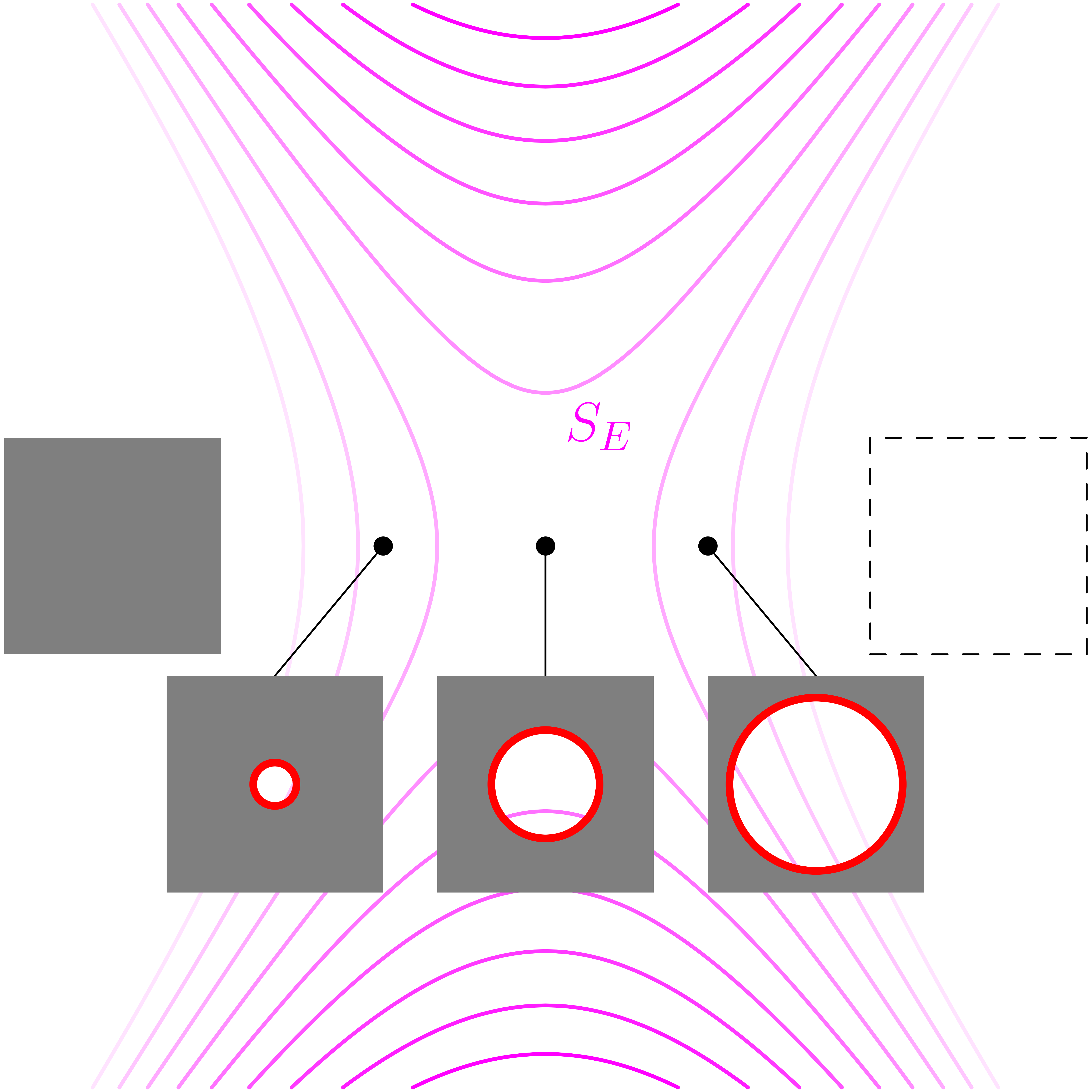
Why it is an upper bound

Each point: 4D field configuration



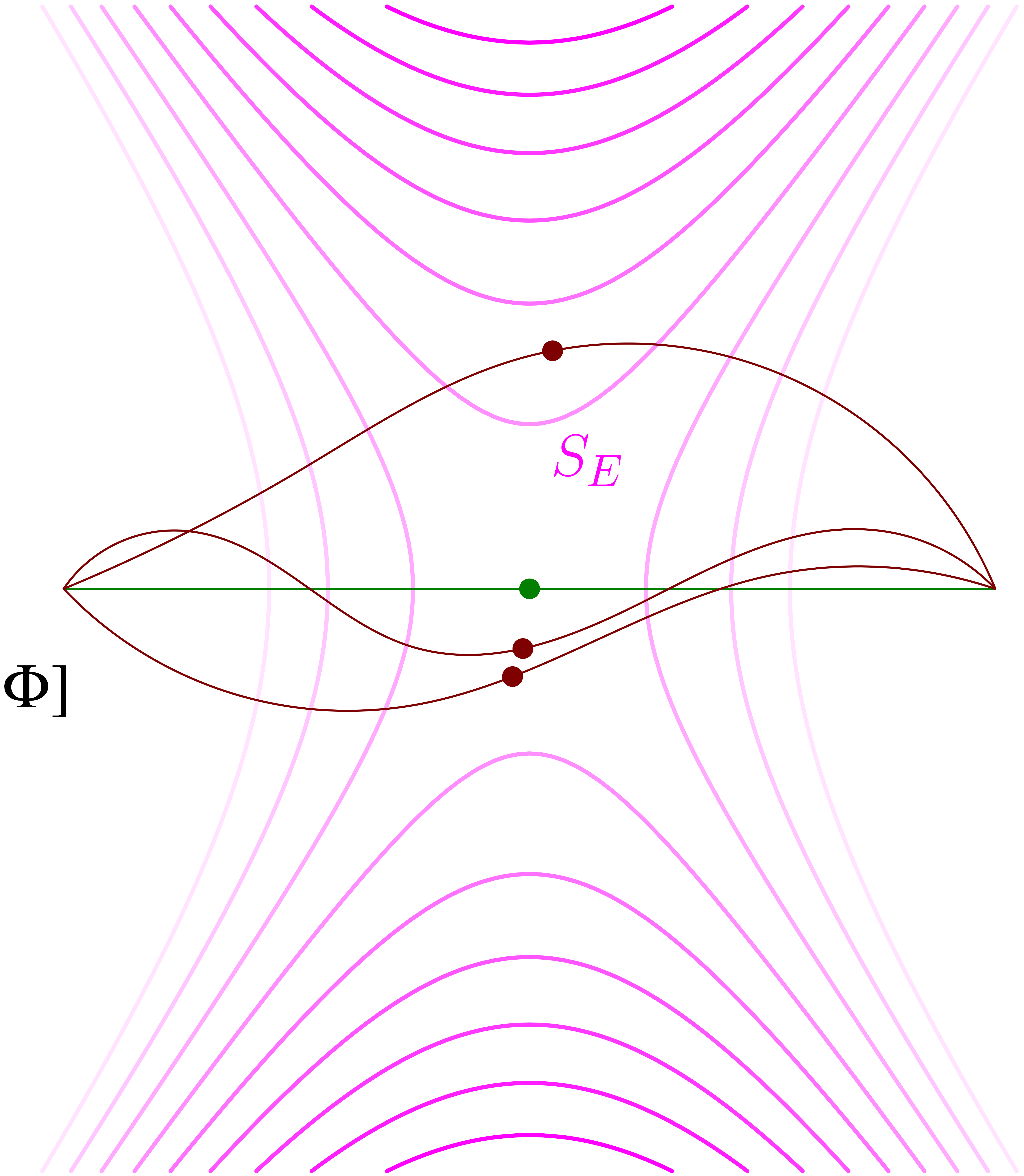
Strategy

Why it is an upper bound



Strategy

Why it is an upper bound



True (optimal) bounce action:

$$S_E[\bullet] = \min_{\text{path joining the two sides}} \; \max_{\Phi\in\text{path}} \; S_E[\Phi]$$

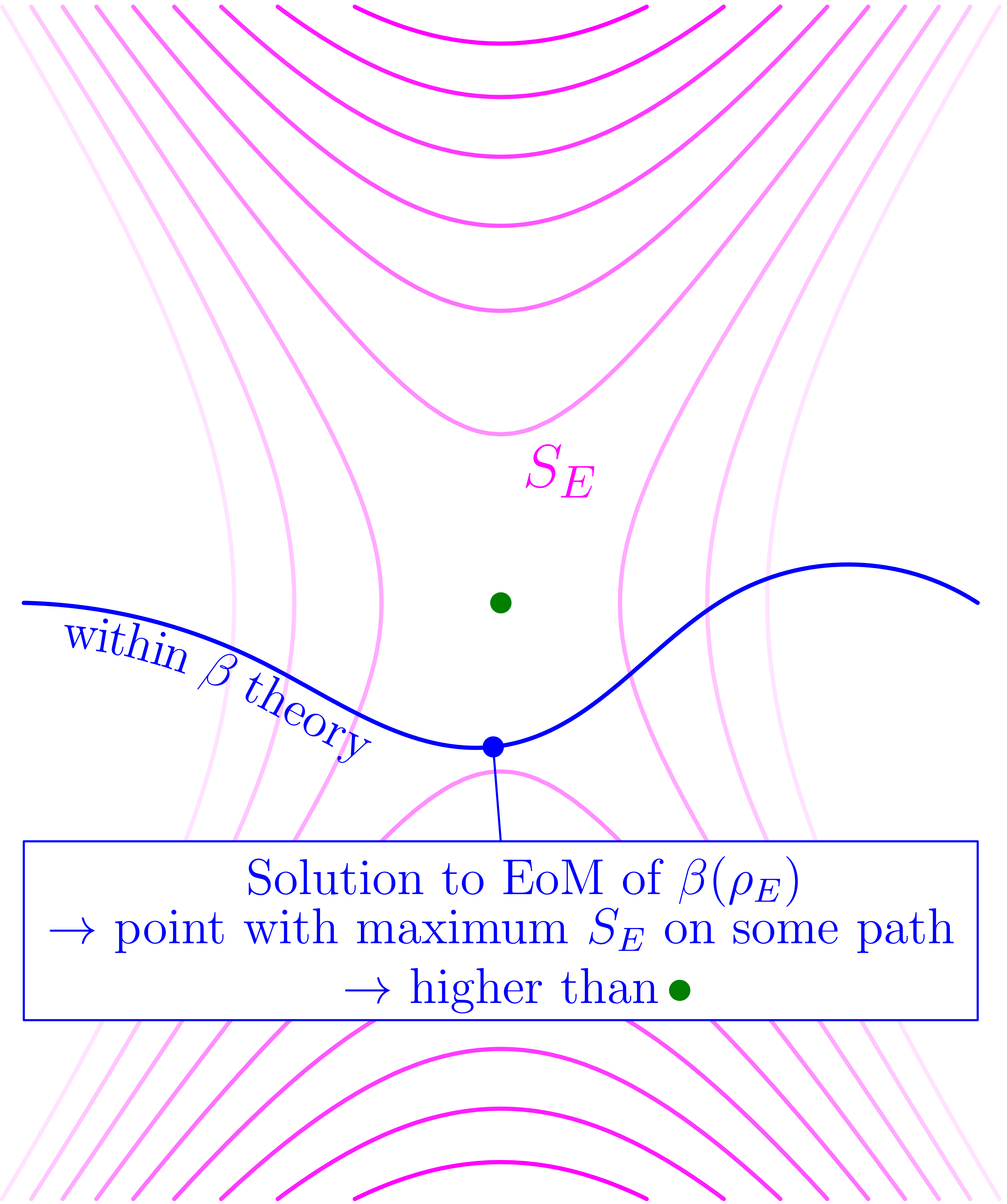
Strategy

Why it is an upper bound

$\exists$path that

- joins the two vacua
- stays within the effective β theory
- has maximum S_E at ●

→ $S_E[●] \geq S_E[●]$

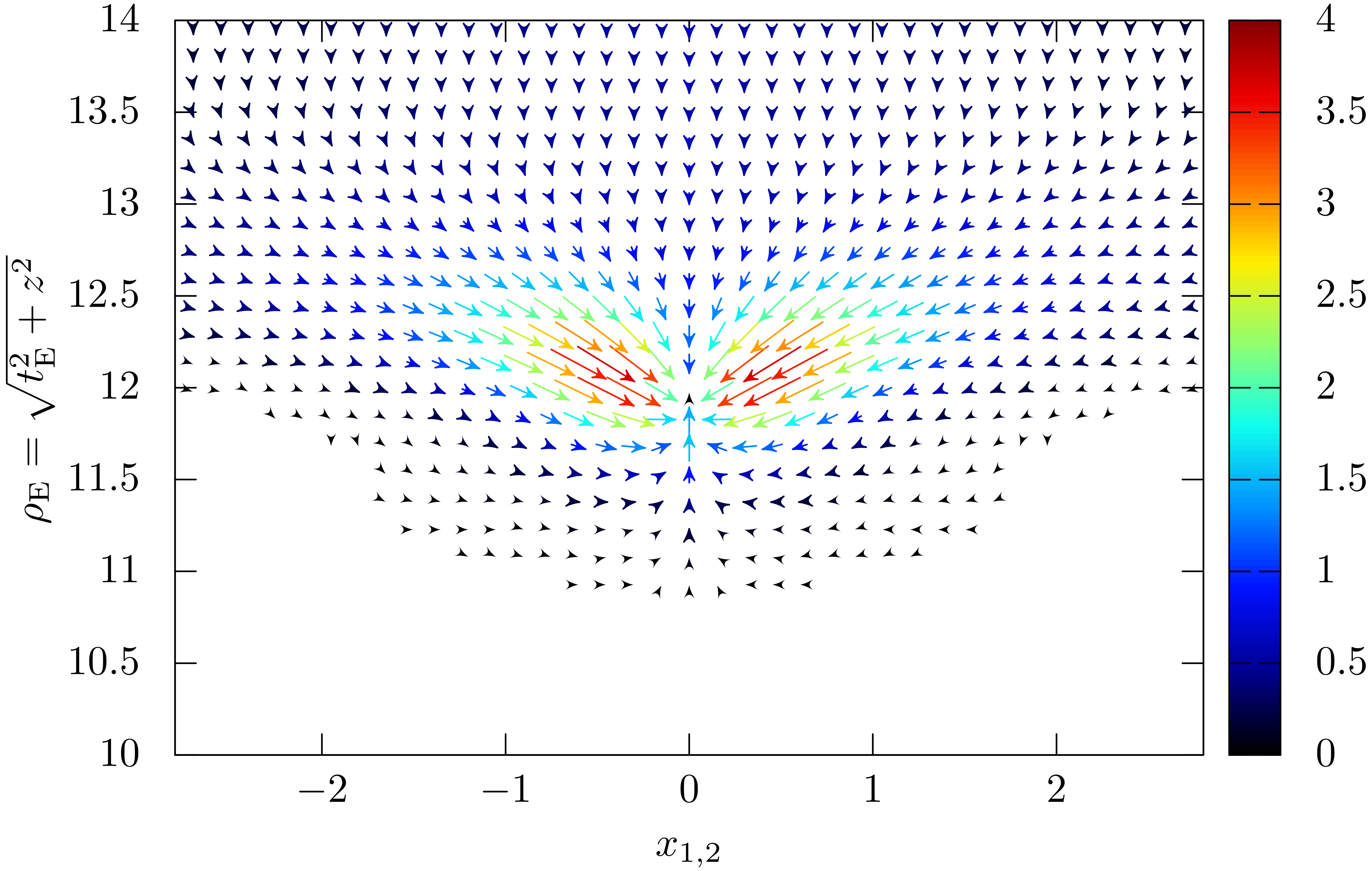


Solution to EoM of $\beta(\rho_E)$
$\rightarrow$ point with maximum S_E on some path
$\rightarrow$ higher than ●

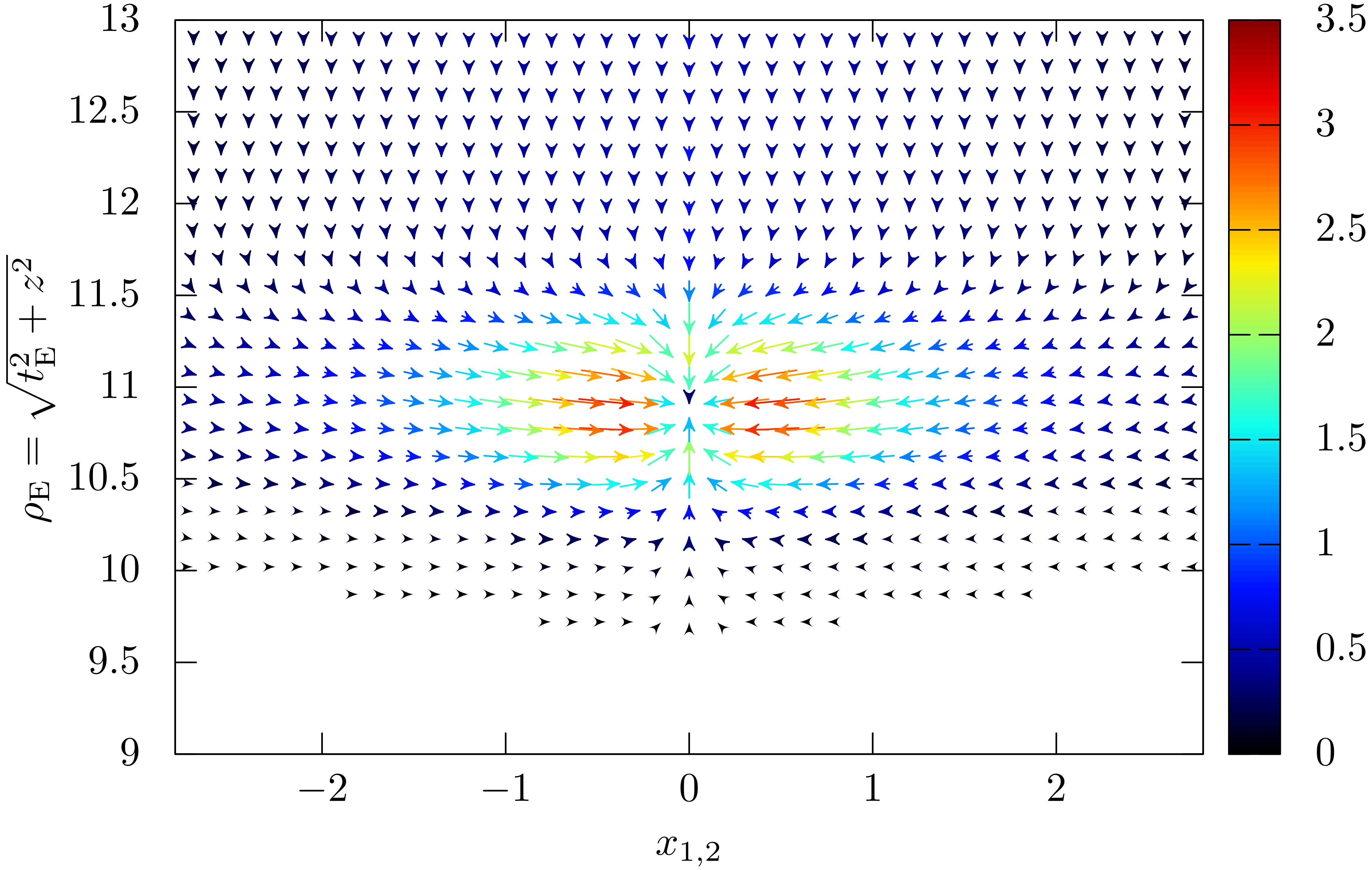
Magnetic fields

Cross section of the breaking string

$$B_i = \frac{1}{2}\epsilon^{ijk}\frac{\phi^a}{V}F^a_{jk}$$



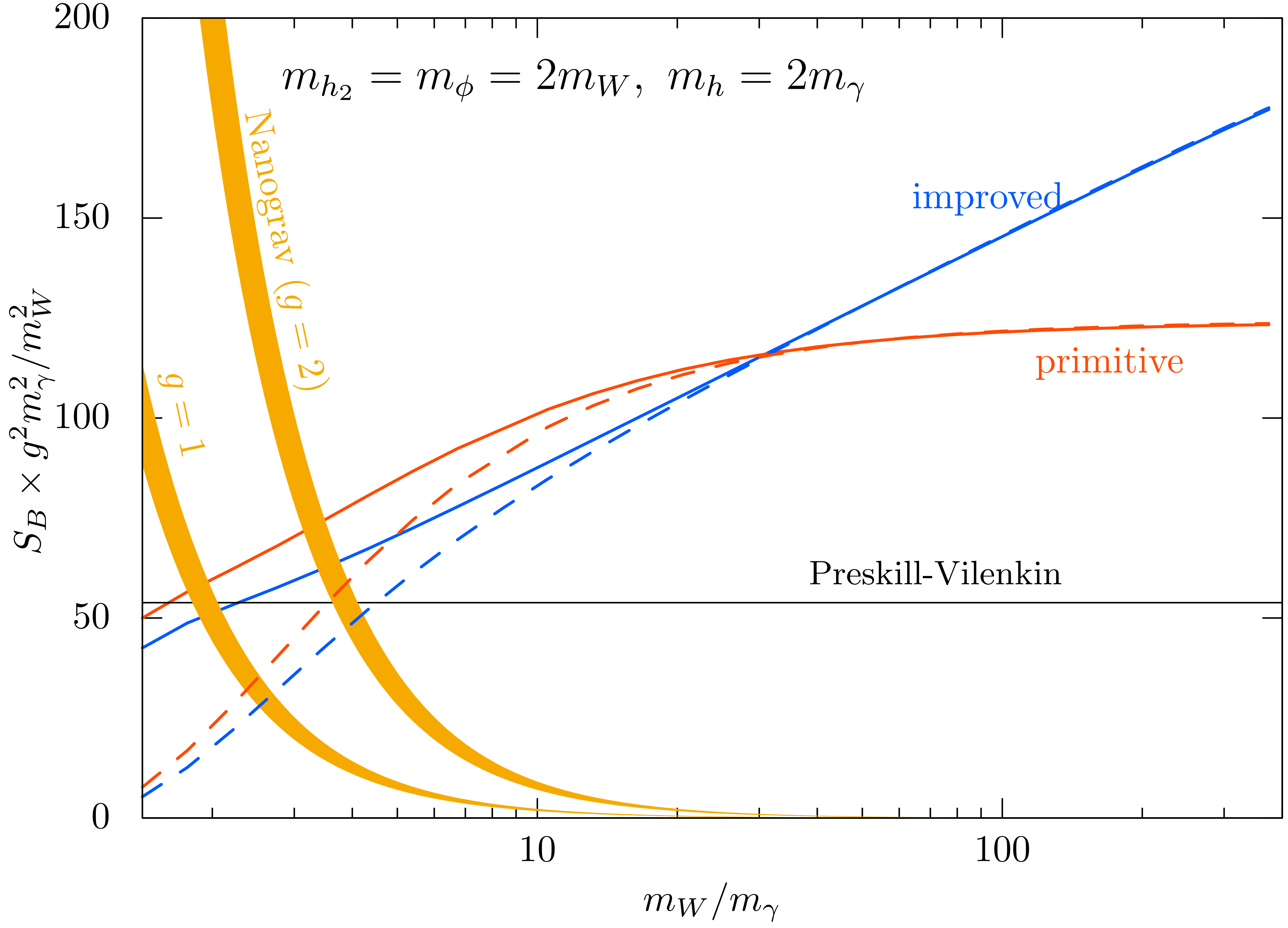
Primitive



Improved

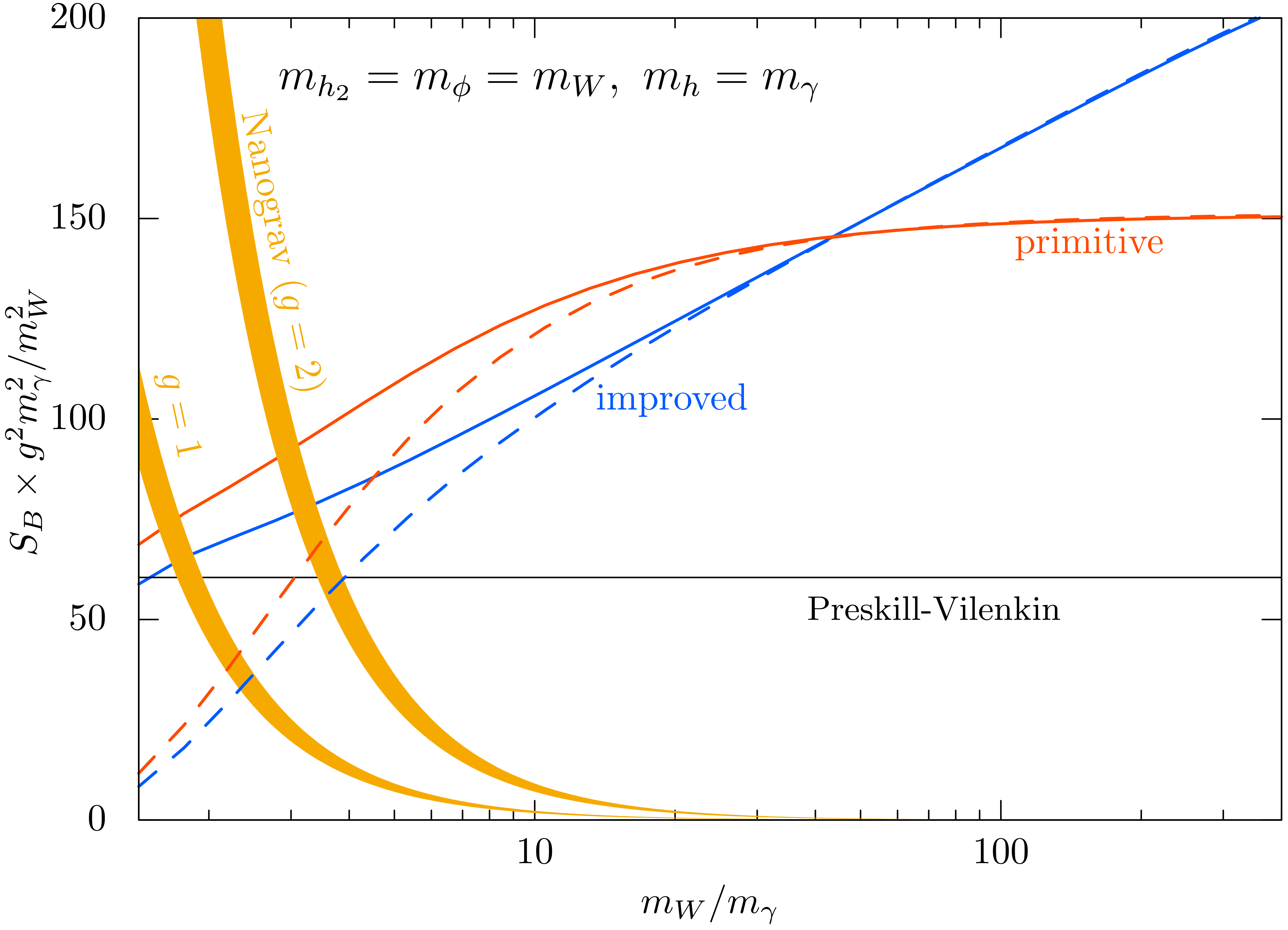
Other parameters

Light W



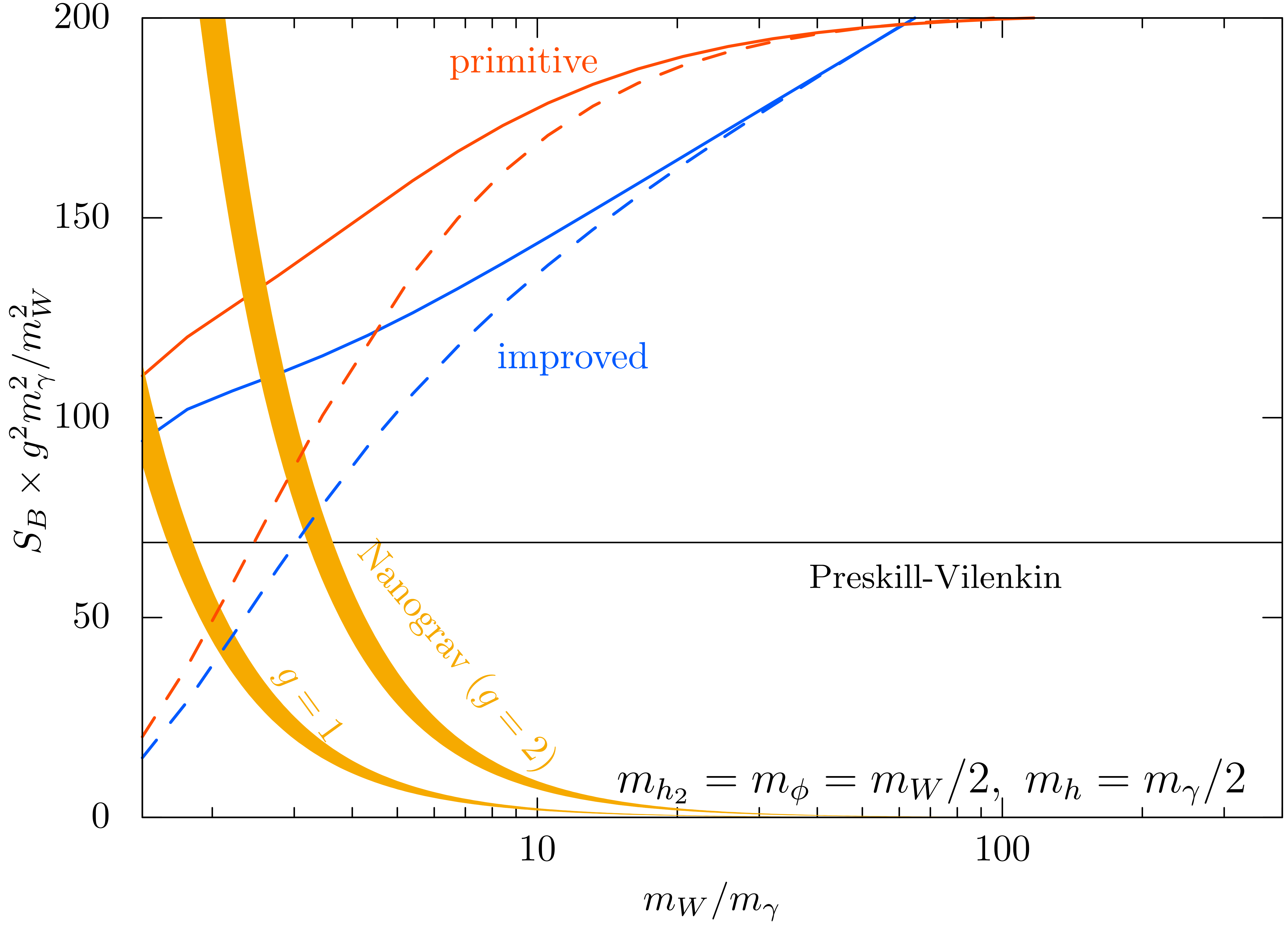
Other parameters

SUSY-like



Other parameters

Heavy W

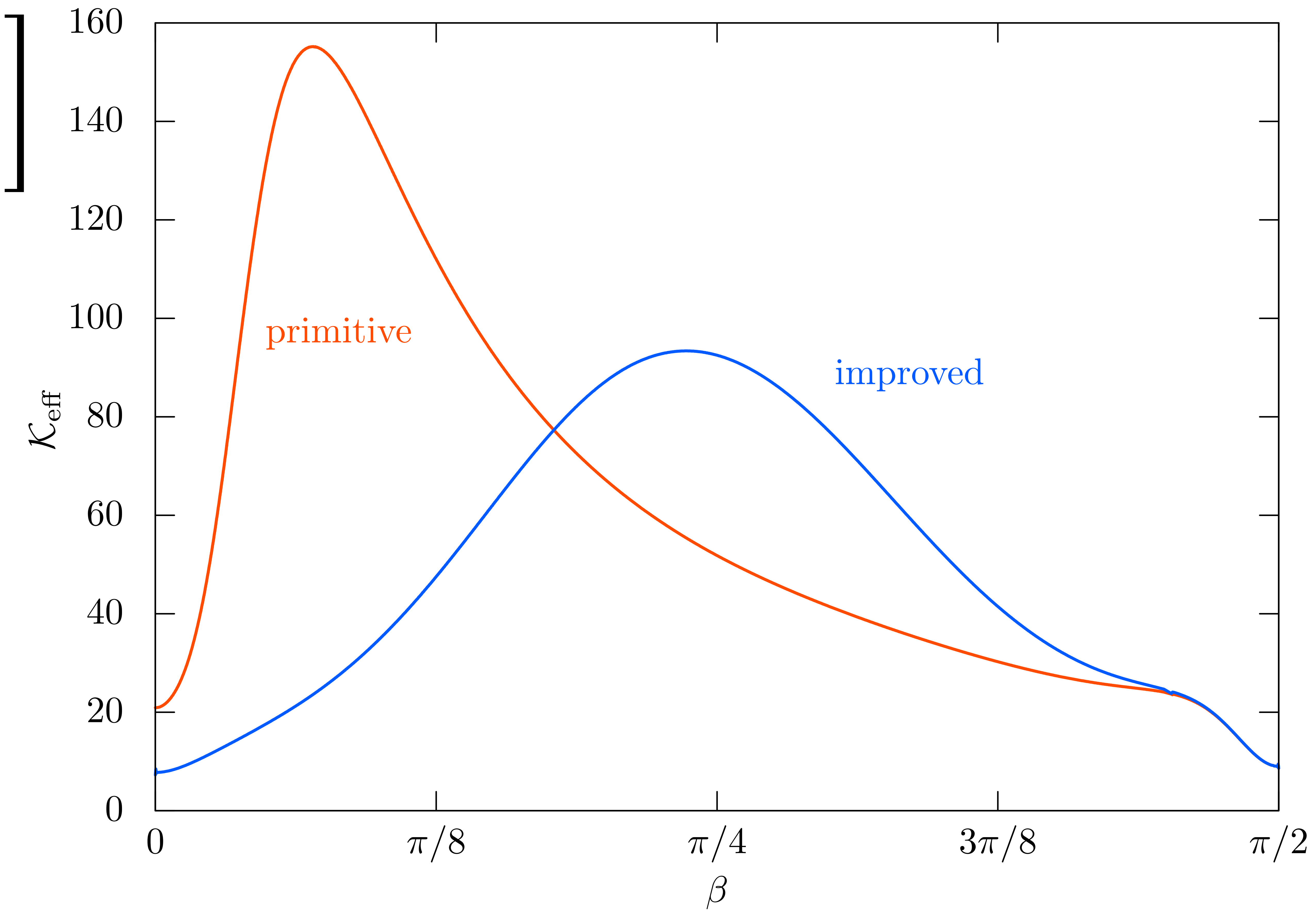


β-thin-wall approximation

- $$S_B = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
  $$\approx -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* \int_{\mathrm{wall}} \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
  $$= -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* m_{\mathrm{eff}}$$
  - $$m_{\mathrm{eff}} := \int_0^{\frac{\pi}{2}} \mathrm{d}\beta \sqrt{2\mathscr{K}_{\mathrm{eff}}(\beta)(T(\beta) - T(0))}$$
- Maximum: $S_B = \pi \dfrac{m_{\mathrm{eff}}^2}{T(0) - T(\pi/2)}$

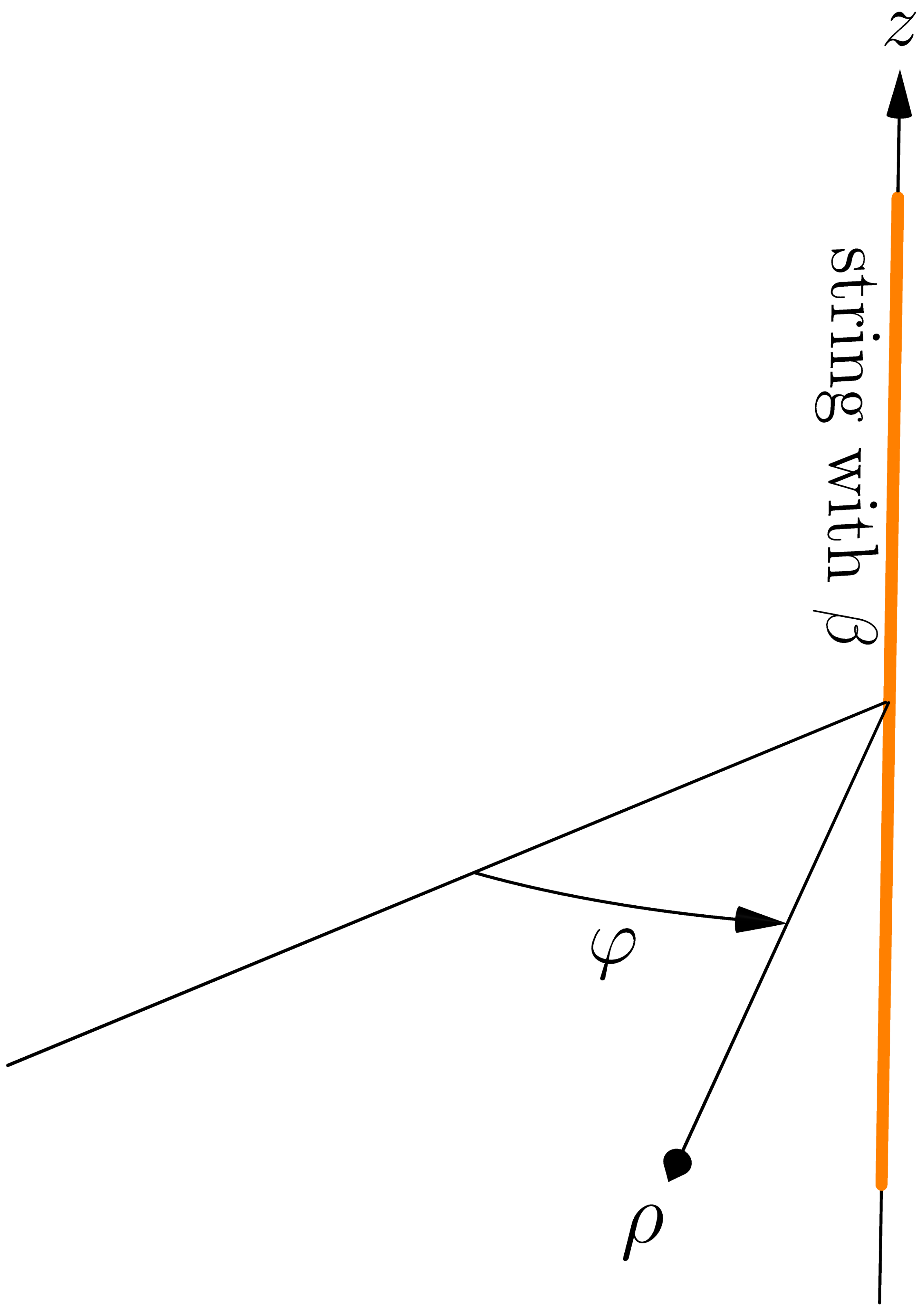
Kinetic term

- $S_E = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta)\right]$



Primitive Ansatz [Shifman & Yung, 2002]

- $h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$
- $A_\theta(x) = iU\partial_\varphi U^{-1}[1 - f_\beta(\rho)]$, other components: 0
- $\phi(x) = VU\frac{\tau_3}{2}U^{-1} + \varphi_\beta(\rho)\left[\frac{\tau_1}{2}\sin\beta - \frac{\tau_2}{2}\cos\beta\right]$
- $U = e^{-i\tau_3\varphi}\cos\beta + i\tau_1\sin\beta$
- $\xi_\beta(0) = 0$, $\xi_\beta(\infty) = v$, $f_\beta(0) = 1$, $f_\beta(\infty) = 0$, $\varphi_\beta(0) = V\sin 2\beta$, $\varphi_\beta(\infty) = 0$



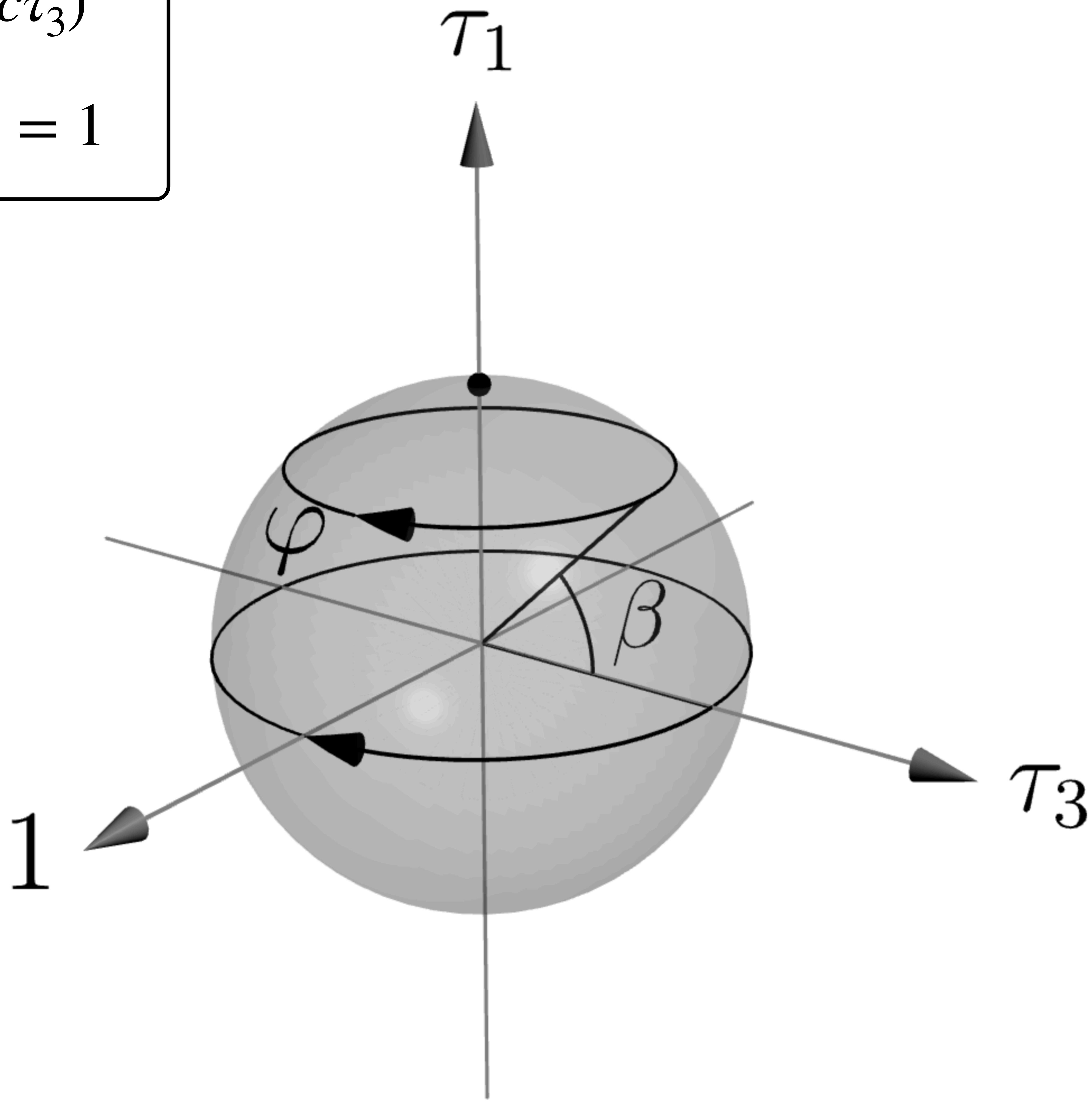
Strategy

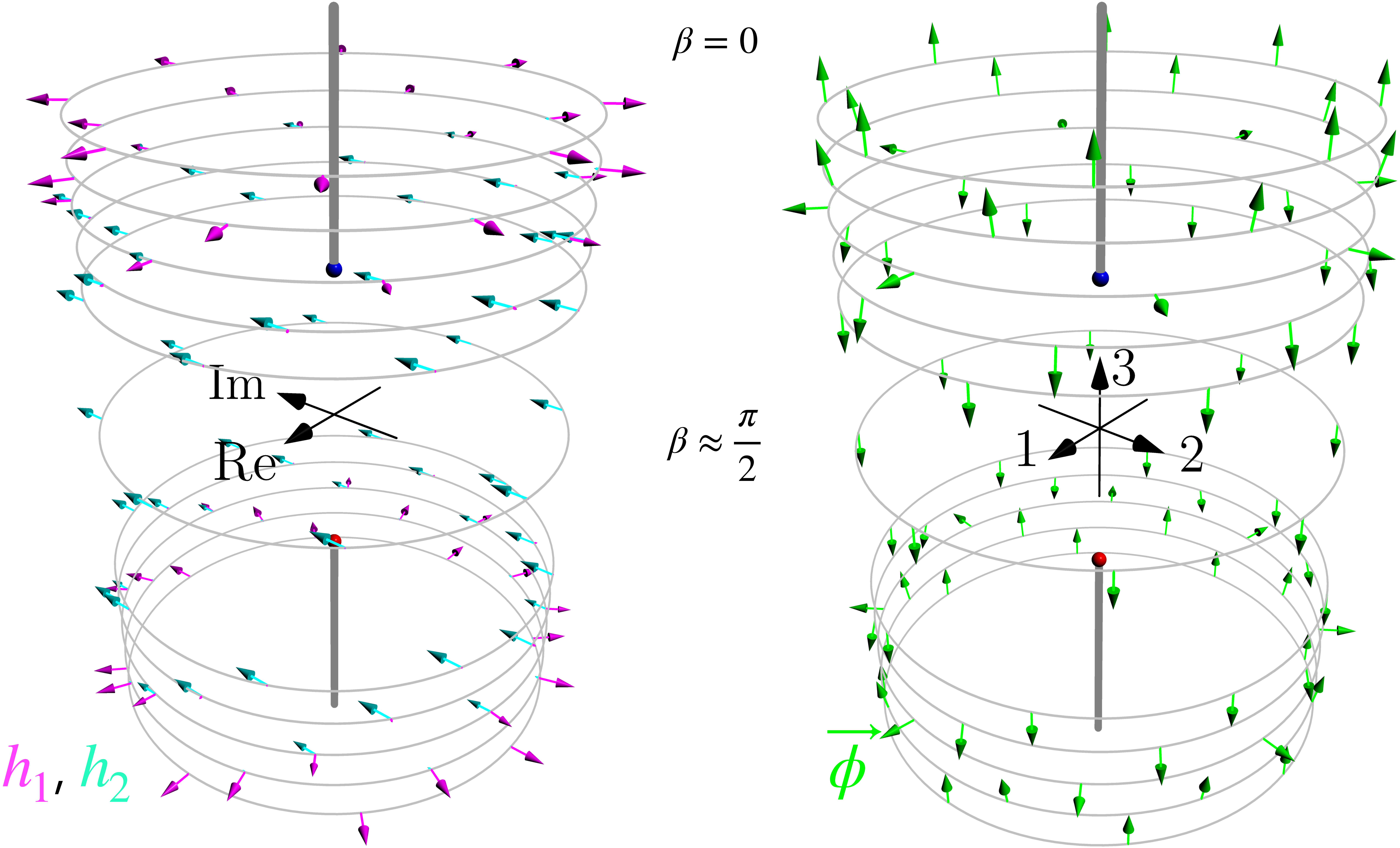
Unwinding the string

$$a + i(b\tau_1 + c\tau_3)$$

$$a^2 + b^2 + c^2 = 1$$

- $U = e^{-i\tau_3\varphi}\cos\beta + i\tau_1 \sin\beta \in S^2 \subset \mathrm{SU}(2)$
  - $h = U(v\ 0)^\top,\ \phi = U(\tau_3/2)U^\dagger$
  - controls the U(1) winding
    - $h_1 = e^{-i\varphi}v$ for $\beta = 0$
  - $U = i\tau_1 = \mathrm{const.}$ for $\beta = \pi/2$
    - completely unwound




$\beta = 0$
Im
Re
$\beta \approx \frac{\pi}{2}$
3
1
2
h_1, h_2
$\vec{\phi}$

Setup

Couplings vs. Masses

- Scale hierarchy: $\sqrt{\kappa_{PV}} = M_M/\sqrt{T_{\rm str}} \sim V/v \propto m_W/m_\gamma$
    - Gauge field : $m_W = gV$, $m_\gamma = \dfrac{1}{\sqrt{2}}gv$
    - (Scalars : $m_\phi = \sqrt{8\tilde{\lambda}}V$, $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$)
- Euclidean action in terms of the masses: $S_E = \dfrac{1}{g^2}[g \text{ independent}]$

Couplings vs. Masses (detailed)

- Gauge field : $m_W = gV$, $m_\gamma = \frac{1}{\sqrt{2}} g v$
  - Scale hierarchy: $V/v \propto m_W / m_\gamma$
- Scalar triplet : $m_\phi = \sqrt{8\tilde{\lambda}}V$
- Scalar doublet: $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$
- Euclidean action:

$$g^2 \mathscr{H} = \frac{1}{4}F^2 + \left| D\hat{h} \right|^2 + \frac{1}{2}\left(D\hat{\phi}\right)^2 + \frac{m_\phi^2}{8m_W^2}\left(\hat{\phi}^2 - m_W^2\right)^2 + \frac{m_{h_1}^2}{4m_\gamma^2}\left(\hat{h}^2 - 2m_\gamma^2\right)^2 + \frac{m_{h_2}^2}{m_W^2}\left| \left(\hat{\phi} - \frac{m_W}{2}\right)\hat{h} \right|^2$$