Small Instanton Effects on Composite Axion Mass

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arXiv:2404.19342 [hep-ph] with M. Ibe, S. Shirai and K. Watanabe.

Introduction

Strong CP problem

- CP-violating $\theta F\tilde{F}$ term in QCD: $|\theta| \lesssim 10^{-10}$

Peccei-Quinn (PQ) mechanism

1. **PQ symmetry, $\mathrm{U(1)_{PQ}}$**, is a global $\mathrm{U(1)}$ symmetry anomalous under QCD.
2. Spontaneous breaking of $\mathrm{U(1)_{PQ}}$ $\rightarrow$ pseudo-Goldstone boson is called **Axion**.
3. $\theta \sim 0$, dynamically.

Energy scale of $\mathrm{U(1)_{PQ}}$-breaking

- Observational constraint on decay constant: $f_a \gtrsim 10^9\,\mathrm{GeV}$

  (axion as dark matter $\leftarrow$ even larger f_a is prefererd)

A Challenge in Axion Models from Particle-physics Viewpoint:

“**Axion Quality Problem**”

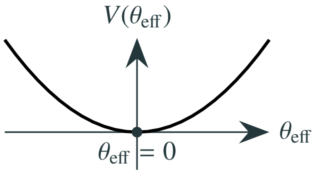
Introduction

"Axion Quality Problem"

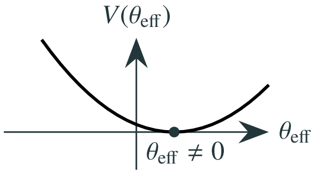
Known global symmetries are **accidentally** realized to preserve gauge symmetries (e.g. Baryon number etc).

With the **accidental** $\mathbf{U(1)_{PQ}}$,

- Higher-dimensional operators which **explicitly break** $\mathbf{U(1)_{PQ}}$
  $\rightarrow$ Non-zero effective θ-angle, easily exceeding the experimental upper bound.



Axion potential from anomaly.



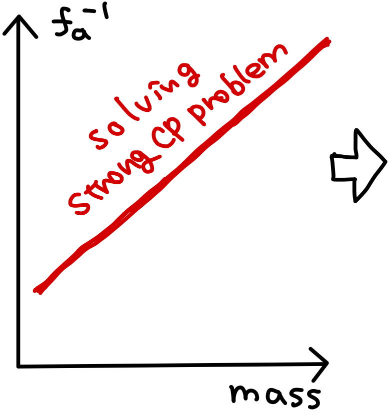
Additional PQ-breaking$\rightarrow$ $\theta \neq 0$.

Introduction

Axion quality problem $\rightarrow$ **Need for hidden dynamics** to solve the strong CP problem?

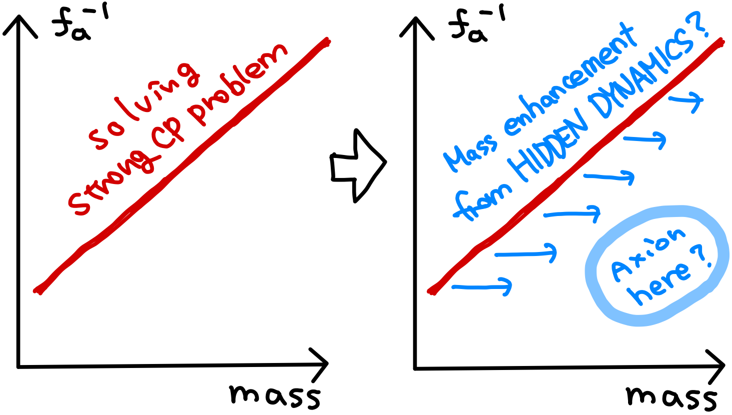
Introduction

Axion quality problem → **Need for hidden dynamics** to solve the strong CP problem?



Axion - like particles search

Axion quality problem → **Need for hidden dynamics** to solve the strong CP problem?



Axion - like particles search

Introduction

In axion models. . .

1. How is **spontaneous $\mathbf{U(1)_{PQ}}$-breaking** realized in a scale around $f_a \gtrsim 10^9$ GeV?

   (In this talk: **composite axion models**)

2. How is $\mathrm{U(1)_{PQ}}$ realized **accidentaly**, also avoiding **axion quality problem**?

3. **Is axion mass enhanced?**

   Mass enhancement in simplified models **not addressing the quality problem**:
   P. Agrawal and K. Howe (2018). C. Csáki, M. Ruhdorfer and Y. Shirman (2020)

Composite Axion Models

"Composite axion models"

1. A new, **strong-coupling gauge interactions** G_{ST} confine at a scale $\Lambda \sim f_a$.

2. $\mathrm{U}(1)_{\mathrm{PQ}}$: **axial rotation** of **new massless fermions** charged under G_{ST}

3. Confinement of $G_{\mathrm{ST}} \rightarrow$ **Spontaneous $\mathrm{U}(1)_{\mathrm{PQ}}$ breaking**, leaving the **axion**.

A **Simple** Composite Axion Model

before Addressing the Quality Problem

Composite Axion Models

A Simplest Example (with $G_{\rm ST} = \mathrm{SU}(N)$) [Choi & Kim (1985)] :

New fermions

	$\mathrm{SU}(N)$	$\mathrm{SU}(3)_{\rm QCD}$
ψ_1	**N**	$\bar{\mathbf{3}}$
ψ'_1	**N**	**1**
ψ_2	$\bar{\mathbf{N}}$	**3**
ψ'_2	$\bar{\mathbf{N}}$	**1**

ψ_1, ψ'_1 } $U(4)_N$

ψ_2, ψ'_2 } $U(4)_{\bar{N}}$

Maximal flavor symmetry for (3+1)-pairs of $(N, \bar{N})$:

(in vanishing coupling limit of $\mathrm{SU}(3)_{\rm QCD}$)

UNBROKEN: $\mathrm{SU}(4)_{\rm V} \times \mathrm{U}(1)_{\rm V}$

BROKEN: $\mathrm{SU}(4)_{\rm A} \times \mathrm{U}(1)_{\rm A}$

$$\mathbf{U(4)}_N \times \mathbf{U(4)}_{\bar{N}} = \mathrm{SU}(4)_{\rm V} \times \mathrm{U}(1)_{\rm V} \times \mathrm{SU}(4)_{\rm A} \times \mathrm{U}(1)_{\rm A}$$

$$\mathrm{SU}(4)_{\rm V} \supset \mathbf{SU(3)}_{\rm QCD}, \qquad \mathrm{SU}(4)_{\rm A} \supset \mathbf{U(1)}_{\rm PQ}$$

$\mathrm{U}(1)_{\rm A}$: anomalous w.r.t. $\mathrm{SU}(N)_{\rm ST}$

Note: $\mathbf{U(1)}_{\rm PQ} \subset \mathrm{U}(4)_N \times \mathrm{U}(4)_{\bar{N}}$ is realized only when mass terms are forbidden by hand.

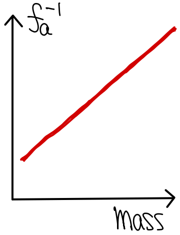
i.e. $\mathbf{U(1)}_{\rm PQ}$ **is NOT accidental** in this simple model.

Composite Axion Models

Axion Mass in This Simple Composite Axion Model

- $\mathrm{SU}(3)_{\mathrm{QCD}} \rightarrow$ mass from anomaly (coupling to $F\tilde{F}$) as usual.
- $G_{\mathrm{ST}} \rightarrow$ mass is **not affected**. ($\because$ $\mathrm{U}(1)_{\mathrm{PQ}}$ is **not anomalous with respect to** G_{ST})

> Axion mass is **unchanged** from the conventional prediction.



Note: The above situation is shared **in many composite axion models,** including models addressing the **quality problem**.

e.g. L. Randall(1992), B. Lillard and T. Tait(2018), R. Contino(2022), . . .

Axion Mass Can Be Enhanced?

→ Let us discuss a model in which $\mathrm{U}(1)_{\mathrm{PQ}}$ is **anomalous NOT ONLY by QCD**

Model

The Simple Model

(NOT addressing Quality Problem)

$$\begin{cases} G_{ST} = SU(N) \\ G_W = SU(3)_{QCD} \end{cases}$$

A Model Addressing the Quality Problem

[M. Redi & R. Sato (2016)]

$$\begin{cases} G_{ST} = SU(N)_1 \times SU(N)_2 \\ G_W = SU(3)_W \times SU(4)_W \end{cases}$$

$$\cup$$

$$SU(3)_{QCD}$$

The Simple Model

New fermions

	$SU(N)$	$SU(3)_{QCD}$
ψ_1	**N**	$\bar{\mathbf{3}}$
ψ'_1	**N**	**1**
ψ_2	$\bar{\mathbf{N}}$	**3**
ψ'_2	$\bar{\mathbf{N}}$	**1**

A Model Addressing the Quality Problem

[M. Redi & R. Sato (2016)]

New fermions

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$
ψ_1	**N**	$\bar{\mathbf{3}}$		
ψ'_1	**N**	**1**		
ψ_2		**3**	$\bar{\mathbf{N}}$	
ψ'_2		**1**	$\bar{\mathbf{N}}$	
ψ_3			**N**	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			**4**

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$	
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$			$\}\ U(4)^N_2$
ψ'_1	$\mathbf{N}$	$\mathbf{1}$			
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$		$\}\ U(4)^{\bar{N}}_1$
ψ'_2		$\mathbf{1}$	$\bar{\mathbf{N}}$		
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$	$\}\ U(4)^N_1$
ψ_4	$\bar{\mathbf{N}}$			$\mathbf{4}$	$\}\ U(4)^{\bar{N}}_2$

Maximal Flavor Symmetry (in vanishing coupling limit of $SU(3)_W$ and $SU(4)_W$)

$$\mathbf{U(4)^{N}_1 \times U(4)^{\bar{N}}_1 \times U(4)^{N}_2 \times U(4)^{\bar{N}}_2} = SU(3)_W \times SU(4)_W \times \mathbf{[U(1)]^4}$$

$$\times (SU(3)_W, SU(4)_W\text{−colored part})$$

$$\times (SU(N)_{ST}\text{−anomalous } [U(1)]^2)$$

Accidental $[\mathrm{U}(1)]^4$ Global Symmetry

$$\mathbf{U(4)_1^{N} \times U(4)_1^{\bar{N}} \times U(4)_2^{N} \times U(4)_2^{\bar{N}}} \supset \mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(4)_{\mathrm{W}} \times \mathbf{[U(1)]^4}$$

$$= \mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(4)_{\mathrm{W}} \times \mathbf{U(1)_{PQ} \times U(1)_1} \times \mathbf{U(1)_2 \times U(1)_3}$$

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$	$\mathrm{U}(1)_{\mathrm{PQ}}^{(\mathrm{SSB})}$	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_3$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$			1	1	1	1
ψ_1'	$\mathbf{N}$	$\mathbf{1}$			-3	1	-3	1
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$		1	1	-1	-1
ψ_2'		$\mathbf{1}$	$\bar{\mathbf{N}}$		-3	1	3	-1
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$	0	-1	0	1
ψ_4	$\bar{\mathbf{N}}$			$\mathbf{4}$	0	-1	0	-1

Two θ-angles of $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ $\leftarrow$ "nullified" by two anomalous $\mathrm{U}(1)$'s .

Axion Mass Enhancement?

Axion mass not only from QCD? (from $\mathbf{SU(3)_W}$, $\mathbf{SU(4)_W}$ **instantons**?)

→ Axion Mass Enhancement?

★ $\mathrm{SU(3)_W}$ and $\mathrm{SU(4)_W}$ instanton effects seems non-negligible because...

$$\frac{1}{g^2_{\mathrm{QCD}}(\Lambda)} = \frac{1}{g^2_{\mathrm{SU(3)_W}}(\Lambda)} + \frac{1}{g^2_{\mathrm{SU(4)_W}}(\Lambda)}$$

(Λ: "dynamical scale of $\mathrm{SU}(N)_{\mathrm{ST}}$" = "breaking scale")

→ $g_{\mathrm{SU(3)_W}}$ or $g_{\mathrm{SU(4)_W}}$ can be much larger than g_{QCD}.

However... conclusion: Mass enhancement is **absent**.

No Axion Mass Enhancement!

A rough explanation: **Ambiguous** $\mathrm{U}(1)_{\mathrm{PQ}}$ vs **Unambiguous** Goldstone Boson Mass

- Combination of anomalous symetries: $\mathrm{U}(1)_{\mathrm{PQ}}$ and $\mathrm{U}(1)_1$ → **redefined** $\mathbf{U(1)_{PQ}}$

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$	$\mathrm{U}(1)_{\mathrm{PQ}}^{(\mathrm{SSB})}$	$\mathrm{U}(1)_1$
ψ_1	**N**	$\bar{\mathbf{3}}$			$1+\alpha$	1
ψ_1'	**N**	**1**			$-3+\alpha$	1
ψ_2		**3**	$\bar{\mathbf{N}}$		$1+\alpha$	1
ψ_2'		**1**	$\bar{\mathbf{N}}$		$-3+\alpha$	1
ψ_3			**N**	$\bar{\mathbf{4}}$	$0-\alpha$	-1
ψ_4	$\bar{\mathbf{N}}$			**4**	$0-\alpha$	-1

Q. $g_{\mathrm{SU}(3)_{\mathrm{W}}} \gg g_{\mathrm{SU}(4)_{\mathrm{W}}} \sim g_{\mathrm{QCD}}$ → **Axion mass enhancement** by $\mathrm{SU}(3)_{\mathrm{W}}$ instantons?

A. **No**, since such enhancement is **absent with choosing** $\boldsymbol{\alpha = -1}$.

> $\mathbf{U(1)_1}$ **suppresses small instanton effects on the axion mass.** (while $\mathrm{U}(1)_1$ is necessary for solving the strong CP problem by **nullifying one of** $\boldsymbol{\theta}$ **parameters**).

Summary

1. **Quality problem** → A motivation for models with hidden dynamics.

2. Expectation: Is **axion mass enhanced** by hidden dynamics, in some models?

3. Such model ("at first sight") : Composite axion model by M. Redi and R. Sato (2016)
   Axion mass enhancement "seems" possible.

4. However, **anomalous (and unbroken) $U(1)$ symmetry nullifying one of θ parameters**, also **suppresses small instanton effects on the axion mass**.

   (More specifically, small instanton effects on the axion mass come only from configurations where total winding numbers of $SU(3)_W$ and $SU(4)_W$ coincide. See 2404.19342 [hep-ph].)

BACKUP

Quality in The Model

Comments on the **Quality of $\mathrm{U(1)_{PQ}}$**

1. $\mathrm{U(1)_{PQ}}$ is "**accidental**" = "guaranteed (for renormalizable terms) **by gauge symmetries**".

2. $\mathrm{U(1)_{PQ}}$ is "**high-quality**" = "guaranteed up to **high mass dimensions** of operators"
(In the present model, the quality is NOT sufficient)

3. Sufficiently high-quality $\mathrm{U(1)_{PQ}}$ needs **further extension of gauge symmetries**:

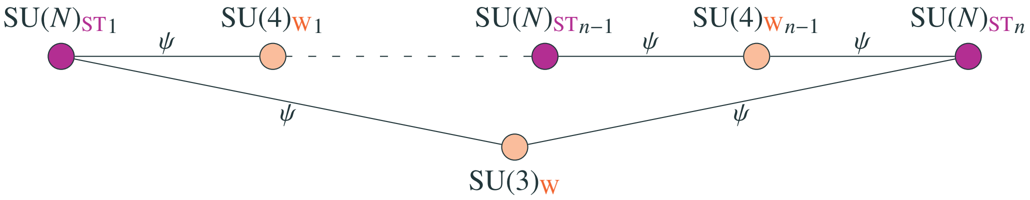
$$[\mathrm{SU}(N)_{\mathrm{ST}}]^2 \times \mathrm{SU}(4)_{\mathrm{W}} \times \mathrm{SU}(3)_{\mathrm{W}}$$
$$\rightarrow \quad [\mathrm{SU}(N)_{\mathrm{ST}}]^{\boldsymbol{n}} \times [\mathrm{SU}(4)_{\mathrm{W}}]^{\boldsymbol{n-1}} \times \mathrm{SU}(3)_{\mathrm{W}} \quad (n > 2)$$

In this talk: $n = 2$ for simplicity. The same arguments are applicable to $n > 2$.

Larger Model with Higher Quality

Composite accidental axion model [Redi & Sato (2016)].

$$G_{\text{CONFINE}} = [SU(N)_{\text{ST}}]^n$$
$$G = SU(3)_{\text{W}} \times [SU(4)_{\text{W}}]^{n-1}$$



$[SU(N)_{\text{ST}}]^n$ confine at a scale $\Lambda\,(\sim f_a) \gg \Lambda_{\text{QCD}}$.

(In this talk, $\underline{n = 2}$ as an example.)

Larger Model (n=3)

	$SU(N)_{ST3}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_{W1}$	$SU(N)_{ST2}$	$SU(4)_{W2}$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$				
ψ_1'	$\mathbf{N}$	$\mathbf{1}$				
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$			
ψ_2'		$\mathbf{1}$	$\bar{\mathbf{N}}$			
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$		
ψ_4				$\mathbf{4}$	$\bar{\mathbf{N}}$	
ψ_5					$\mathbf{N}$	$\bar{\mathbf{4}}$
ψ_6	$\bar{\mathbf{N}}$					$\mathbf{4}$

	$U(1)_{PQ}^{(SSB)}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
ψ_1	1	1	1	1	0
ψ_1'	−3	1	−3	1	0
ψ_2	1	1	−1	−1	0
ψ_2'	−3	1	3	−1	0
ψ_3	0	−1	0	1	0
ψ_4	0	0	0	−1	−1
ψ_5	0	0	0	1	1
ψ_6	0	−1	0	−1	0

- Only $U(1)_{PQ}$ is spontaneously broken, also for larger n.
- Additional $n - 2$ anomalous (and unbroken) $U(1)$s, cancelling the additional θ angles.

No Axion Mass Enhancement!

Another explanation of the same thing: **Influence of anomalous unbroken $\mathrm{U}(1)_1$**

- Axion potential ← obtained from vacuum amplitude:

$$W(a) = \int \prod DA\, D\psi^\dagger \mathcal{D}\psi \exp\left(-S_{\text{Euclidean}}(A, \psi, \psi^\dagger, a)\right)$$

- Contribution from "$\mathrm{SU}(3)_\mathrm{W}, \mathrm{SU}(4)_\mathrm{W}$ winding number $= (m, n)$" : $W(a)|_{(m,n)}$

By $\mathrm{U}(1)_1$ rotation of fermions in path integral: (See 2404.19342 [hep-ph])

$$W(a)|_{(m,n)} = \exp\left[2i\alpha(m-n)\right] \times W(a)|_{(m,n)}$$

→ **Vanishing unless $m = n$!**

Especially, single $\mathrm{SU}(3)_\mathrm{W}$ or $\mathrm{SU}(4)_\mathrm{W}$ instanton cannot enhance the axion mass.

$\mathrm{U}(1)_1$ suppresses small instanton effects on the axion mass. (while being necessary for solving the strong CP problem by **nullifying one of θ parameters**).