A bottom-up approach to nucleon decay RGEs, correlations and connection to UV

Arnau Bas i Beneito

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Based on work in collaboration with J. Gargalionis, J. Herrero-García, M. A. Schmidt. A. Santamaria [2312.13361] (Accepted for publication in JHEP)

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CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Standard story

$$\mathscr{L}_{SM}$$

$$SU(3)_C \times SU2)_L \times U(1)_Y$$

$$+$$

$$H,\ Q_L^i,\ u_R^i,\ d_R^i,\ L_L^i,\ e_R^i,\ i = 1,\ 2,\ 3$$

Individual Flavour Symmetries

↓ Yukawa couplings

$$U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times U(1)_B$$

↓ ν oscillations

[Super-K 1999, KamLAND 2003...]

$$U(1)_L \times U(1)_B$$

B and L accidentally conserved

(B + L violated in 3 units by sphaleron transitions)

↓

Proton stable

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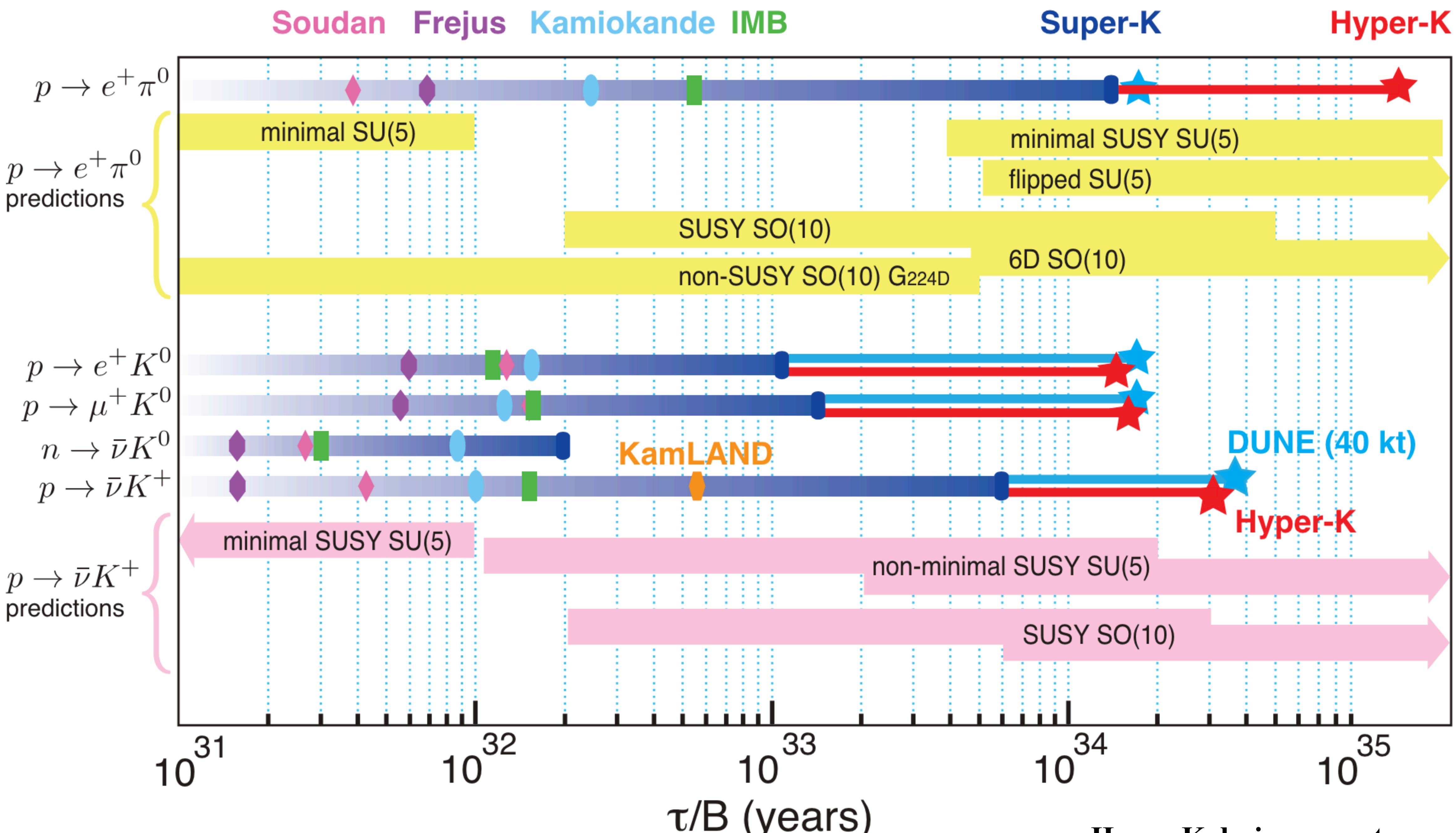
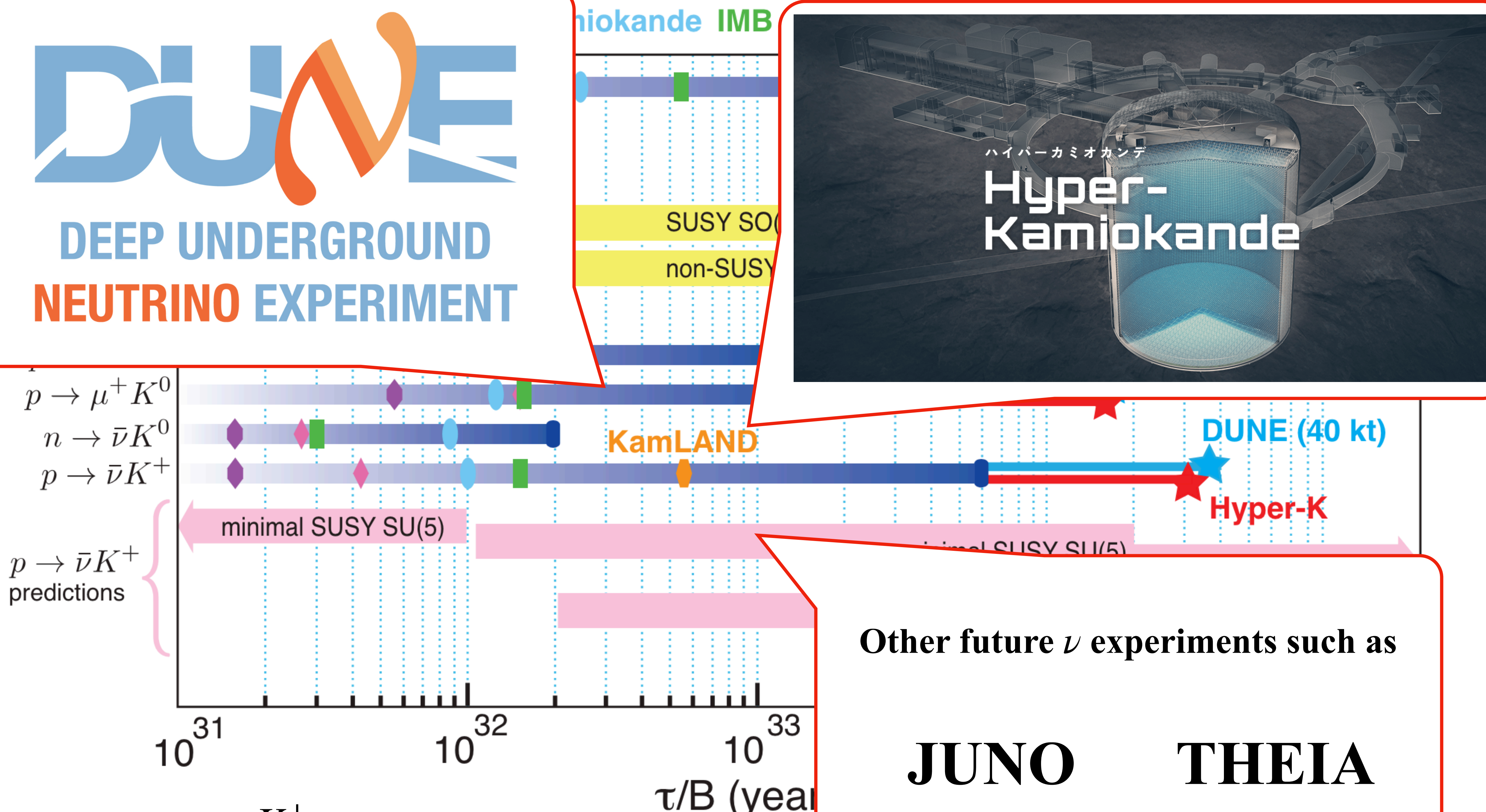
Experimental perspectives



Hyper-K design report

Experimental perspectives

DUNE
DEEP UNDERGROUND NEUTRINO EXPERIMENT



Other future ν experiments such as

JUNO **THEIA**

Sensitivity of $n \to K^+ e^-$ **increased by** 10^3 **in HK** !

BNV nucleon decay could be the next big discovery

Then… why nucleon decay?

· **There is** **no fundamental reason** **to have B and L conserved (Leptoquarks, Seesaw particles, SUSY, GUTs…)**

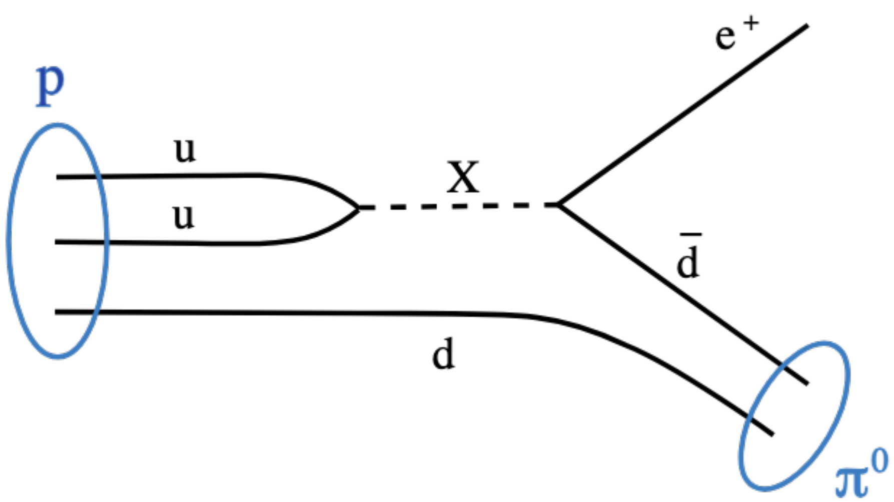
· **Experimental probes of BNV and LNV would constitute one of the** **strongest evidence** **for physics beyond SM (BSM) →PD will be looked for in future experiments (HK, DUNE…)**

Grand Unified Theories (GUTs)
[*Georgi et. al. 1973, H. Fritzsch et al. 1975*]

Baryogenesis
[Sakharov 1967]

→ **Baryon Number violation (BNV)**

E.g. proton decay (PD), neutron-antineutron oscillations



Large number of UV theories predicting PD

↓

Systematic study of PD in a model-independent way (bottom-up)

BNV within the SMEFT

Parametrization of new physics through Effective operators (d > 4)
SM Effective Field Theory (SMEFT)

Bounds on SMEFT WCs serve as a bridge to specific UV models

**[S. Weinberg 1979 ,
B. Grzadkowski et al. 2010,
W. Buchmuller et al. 1986,
Brivio et al. 2019,
B. Henning et al. 2016,
De Gouvea et al. 2014]**

$\Delta L = 2$ $\quad \Delta(B-L) = 0$ $\quad \Delta(B-L) = 2$

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{c^{d=5}}{\Lambda}\mathcal{O}_W + \frac{c^{d=6}}{\Lambda^2}\mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3}\mathcal{O}^{d=7} + \dots$$

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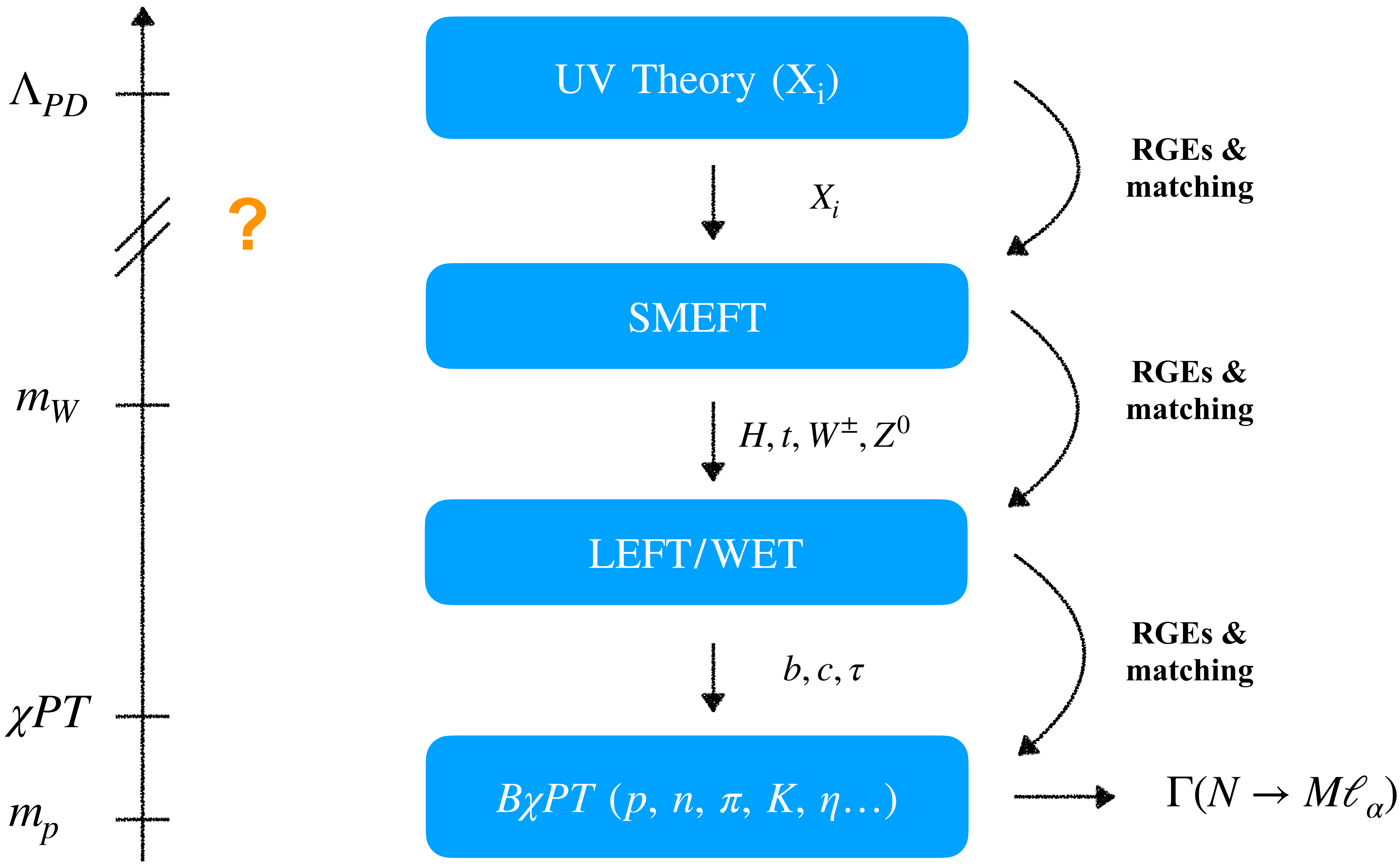
Different phenomenology $\longrightarrow$

$d=6$: $\Lambda \gtrsim 10^{15}$ GeV; $p \to \pi^0 e^+,\ p \to K^+\bar{\nu}$

$d=7$: $\Lambda \gtrsim 10^{10}$ GeV; $n \to \pi^+ e^-,\ p \to K^+\nu$

BNV within the SMEFT



· **Assumptions: Energy Desert and no SUSY in the TeV scale/RpV**

[S. Antusch et al. 2021, H. Dreiner et al. 2020]

SMEFT

$d\ =\ 6\ \to 4$ (273) operators **[L. F. Abbott et al. 1980, B. Grzadkowski et al. 2010]**

$$\mathcal{O}_{qqql,pqrs} = (Q_p^i Q_q^j)(Q_r^l L_s^k)\epsilon_{ik}\epsilon_{jl}\,, \quad \mathcal{O}_{qque,pqrs} = (Q_p^i Q_q^j)(\bar{u}_r^\dagger \bar{e}_s^\dagger)\epsilon_{ij}\,,$$

$$\mathcal{O}_{duue,pqrs} = (\bar{d}_p^\dagger \bar{u}_q^\dagger)(\bar{u}_r^\dagger \bar{e}_s^\dagger)\,, \quad \mathcal{O}_{duql,pqrs} = (\bar{d}_p^\dagger \bar{u}_q^\dagger)(Q_r^i L_s^j)\epsilon_{ij}\,,$$

$d\ =\ 7\ \to 6$ (297) operators **[L. Lehman 2014, Yi Liao et al. 2016]**

$$\mathcal{O}_{\bar{l}dddH,pqrs} = (L_p^\dagger \bar{d}_q^\dagger)(\bar{d}_r^\dagger \bar{d}_s^\dagger)H\,, \quad \mathcal{O}_{\bar{l}dqq\tilde{H},pqrs} = (L_p^\dagger \bar{d}_q^\dagger)(Q_r Q_s^i)\tilde{H}^j\epsilon_{ij}\,,$$

$$\mathcal{O}_{\bar{e}qdd\tilde{H},pqrs} = (\bar{e}_p Q_q^i)(\bar{d}_r^\dagger \bar{d}_s^\dagger)\tilde{H}^j\epsilon_{ij}\,, \quad \mathcal{O}_{\bar{l}dud\tilde{H},pqrs} = (L_p^\dagger \bar{d}_q^\dagger)(\bar{u}_r^\dagger \bar{d}_s^\dagger)\tilde{H}\,,$$

$$\mathcal{O}_{\bar{l}qdDd,pqrs} = (L_p^\dagger \bar{\sigma}^\mu Q_q)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger)\,, \quad \mathcal{O}_{\bar{e}dddD,pqrs} = (\bar{e}_p \sigma^\mu \bar{d}_q^\dagger)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger)\,,$$

SMEFT

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$$c^{d=6}(m_W) \sim (2-4)\ c^{d=6}(10^{15}\ \text{GeV})$$
$$c^{d=7}(m_W) \sim (1-2)\ c^{d=7}(10^{11}\ \text{GeV})$$

From gauge interactions and y_t
(Operator mixing subdominant)

· RGEs for d = 6 SMEFT [A. Manohar et al. 2014]
· RGEs for d = 7 SMEFT [Yi Liao et al. 2016]

LEFT

288 $\Delta(B-L)=0$ operators → 14 operators
228 $\Delta(B+L)=0$ operators → 9 operators

} LEFT operators involved in nucleon decay at tree-level

Name [52]	SMEFT matching
$[\mathcal{O}^{S,LL}_{udd}]_{pqrs}$	$V_{q'q}V_{r'r}(C_{qqql,r'q'ps} - C_{qqql,q'r'ps} + C_{qqql,q'pr's})$
$[\mathcal{O}^{S,LL}_{duu}]_{pqrs}$	$V_{p'p}(C_{qqql,rqp's} - C_{qqql,qrp's} + C_{qqql,qp'rs})$
$[\mathcal{O}^{S,LR}_{duu}]_{pqrs}$	$-V_{p'p}(C_{qque,p'qrs} + C_{qque,qp'rs})$
$[\mathcal{O}^{S,RL}_{duu}]_{pqrs}$	$C_{duql,pqrs}$
$[\mathcal{O}^{S,RL}_{dud}]_{pqrs}$	$-V_{r'r}C_{duql,pqr's}$
$[\mathcal{O}^{S,RR}_{duu}]_{pqrs}$	$C_{duue,pqrs}$
$[\mathcal{O}^{S,LR}_{udd}]_{pqrs}$	$-V_{q'q}C_{\bar{l}dqq\tilde{H},rspq'}\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,LR}_{ddd}]_{pqrs}$	$V_{p'p}V_{q'q}(C_{\bar{l}dqq\tilde{H},rsq'p'} - C_{\bar{l}dqq\tilde{H},rsp'q'})\frac{v}{2\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RL}_{ddd}]_{pqrs}$	$V_{s's}(C_{\bar{e}qdd\tilde{H},rs'qp} - C_{\bar{e}qdd\tilde{H},rs'pq})\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RR}_{udd}]_{pqrs}$	$C_{\bar{l}dud\tilde{H},rspq}\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RR}_{ddd}]_{pqrs}$	$C_{\bar{l}dddH,rspq}\frac{v}{\sqrt{2}\Lambda}$

Name [52] ([12])	Operator	Flavour
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$ (O^{ν}_{LL})	$(ud)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$ $(\tilde{O}^{\nu}_{LL1})$	$(us)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$ $(\tilde{O}^{\nu}_{LL2})$	$(ud)(s\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$ (O^{e}_{LL})	$(du)(ue_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$ $(\tilde{O}^{e}_{LL})$	$(su)(ue_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$ (O^{e}_{LR})	$(du)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$ $(\tilde{O}^{e}_{LR})$	$(su)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$ (O^{e}_{RL})	$(\bar{d}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$ $(\tilde{O}^{e}_{RL})$	$(\bar{s}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{111r}$ (O^{ν}_{RL})	$(\bar{d}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{211r}$ $(\tilde{O}^{\nu}_{RL1})$	$(\bar{s}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{112r}$ $(\tilde{O}^{\nu}_{RL2})$	$(\bar{d}^\dagger\bar{u}^\dagger)(s\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
~~$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$~~	~~$(\bar{d}^\dagger\bar{s}^\dagger)(u\nu_r)$~~	~~$(\mathbf{3},\bar{\mathbf{3}})$~~
$[\mathcal{O}^{S,RR}_{duu}]_{111r}$ (O^{e}_{RR})	$(\bar{d}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{duu}]_{211r}$ $(\tilde{O}^{e}_{RR})$	$(\bar{s}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\mathbf{1},\mathbf{8})$

Name	Operator	Flavour
~~$[\mathcal{O}^{S,LL}_{ddd}]_{[12]r1}$~~	~~$(ds)(\bar{e}_r d)$~~	~~$(\mathbf{8},\mathbf{1})$~~
$[\mathcal{O}^{S,LR}_{udd}]_{11r1}$	$(ud)(\nu^\dagger_r\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{udd}]_{12r1}$	$(us)(\nu^\dagger_r\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{udd}]_{11r2}$	$(ud)(\nu^\dagger_r\bar{s}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
~~$[\mathcal{O}^{S,LR}_{ddu}]_{[12]r1}$~~	~~$(ds)(\nu^\dagger_r\bar{u}^\dagger)$~~	~~$(\bar{\mathbf{3}},\mathbf{3})$~~
$[\mathcal{O}^{S,LR}_{ddd}]_{[12]r1}$	$(ds)(e^\dagger_r\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,RL}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger\bar{s}^\dagger)(\bar{e}_r d)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RR}_{udd}]_{11r1}$	$(\bar{u}^\dagger\bar{d}^\dagger)(\nu^\dagger_r\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{udd}]_{12r1}$	$(\bar{u}^\dagger\bar{s}^\dagger)(\nu^\dagger_r\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{udd}]_{11r2}$	$(\bar{u}^\dagger\bar{d}^\dagger)(\nu^\dagger_r\bar{s}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger\bar{s}^\dagger)(e^\dagger_r\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$

- RG effects universal in the LEFT

$c(2\ \mathrm{GeV}) \sim 1.26\ c(m_W)$

Not generated by D = 6, 7 SMEFT ops.

· **RGEs for d = 6 LEFT [A. Manohar et al. 2018]**

BχPT

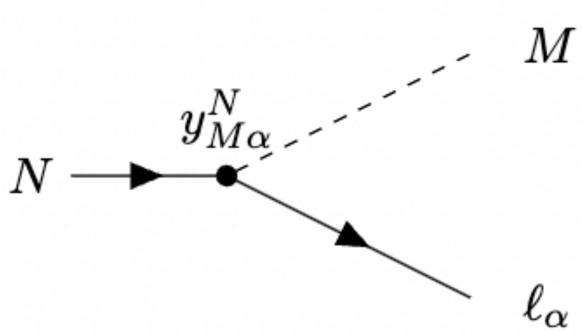
$$M = \sum_{a=1}^{8} M_a \frac{\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \qquad B = \sum_{a=1}^{8} B_a \frac{\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}$$

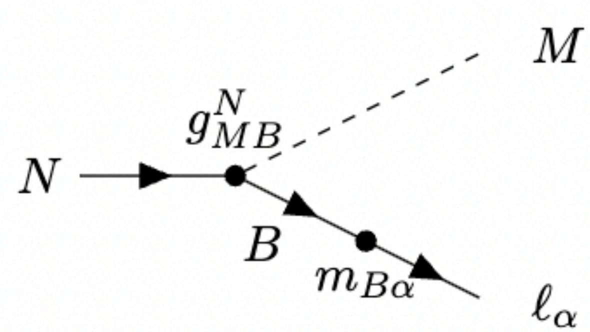
Flavour group $\mathrm{U(3)_L \times U(3)_R}$

BNC interactions **BNV interactions**

$$\mathcal{L} = g^N_{MB}\overline{B}\gamma^\mu\gamma_5 N\partial_\mu M + m_{B\alpha,X}\overline{\ell_\alpha}P_{\bar{X}}B + iy^N_{M\alpha,X}\overline{\ell_\alpha}P_{\bar{X}}NM$$

[M. Claudson et al. 1981,
P. Nath et al. 2007]





$\Gamma(N \to M\ell_\alpha)$

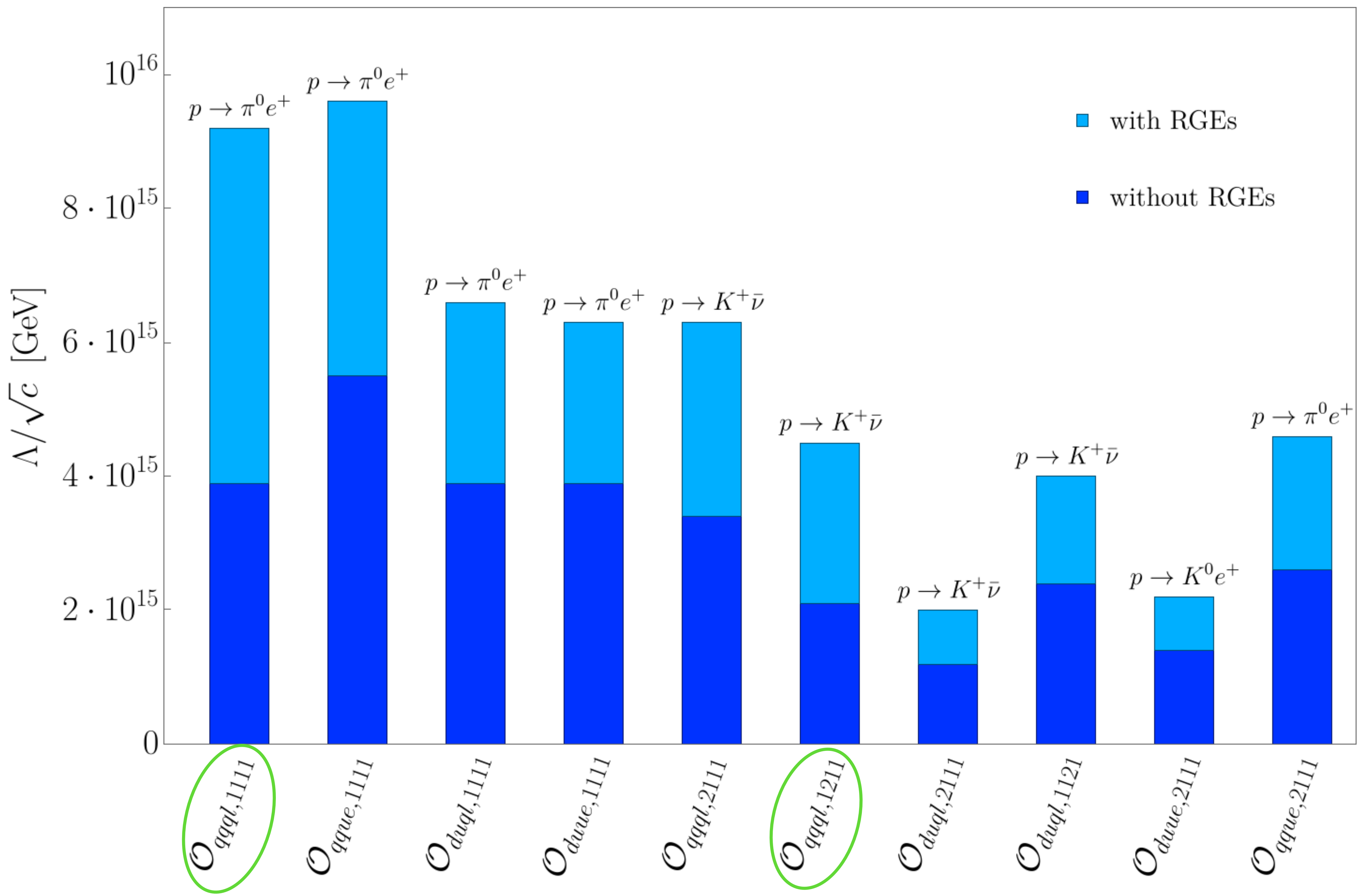
2 inputs from lattice α, β

[JLQCD 2000,
Y. Aoki et al. 2017,]

(First-time computation of $|\Delta(B-L)| = 2$ two-body decays in the BχPT formalism)

D = 6 limits

Bounds on other flavor components can be found in **[J. Gargalionis et al. 2024]**



$$\Lambda/\sqrt{c} \sim (1 \sim 10) \times 10^{15}\ \text{GeV}$$

D = 7 limits

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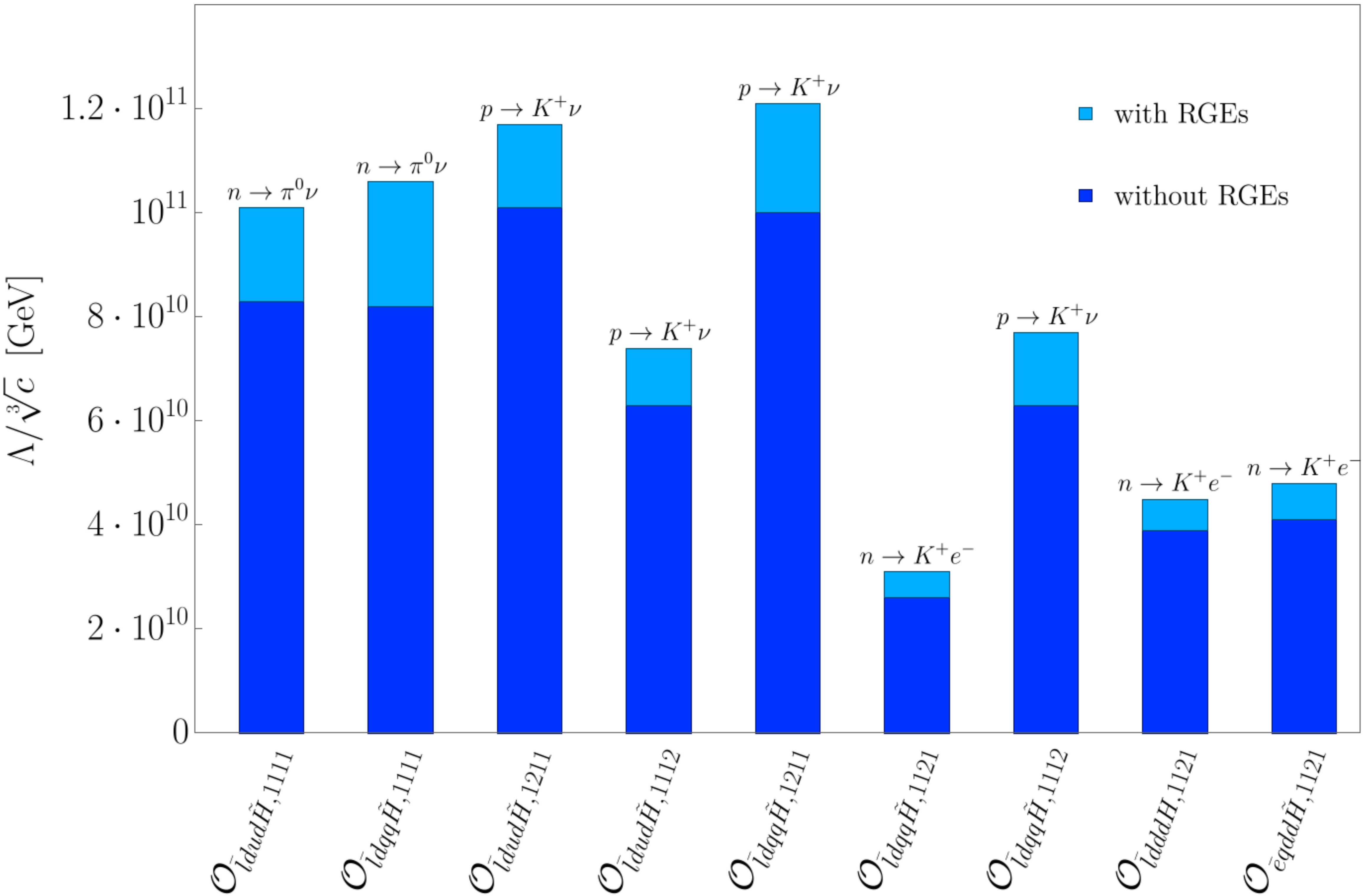
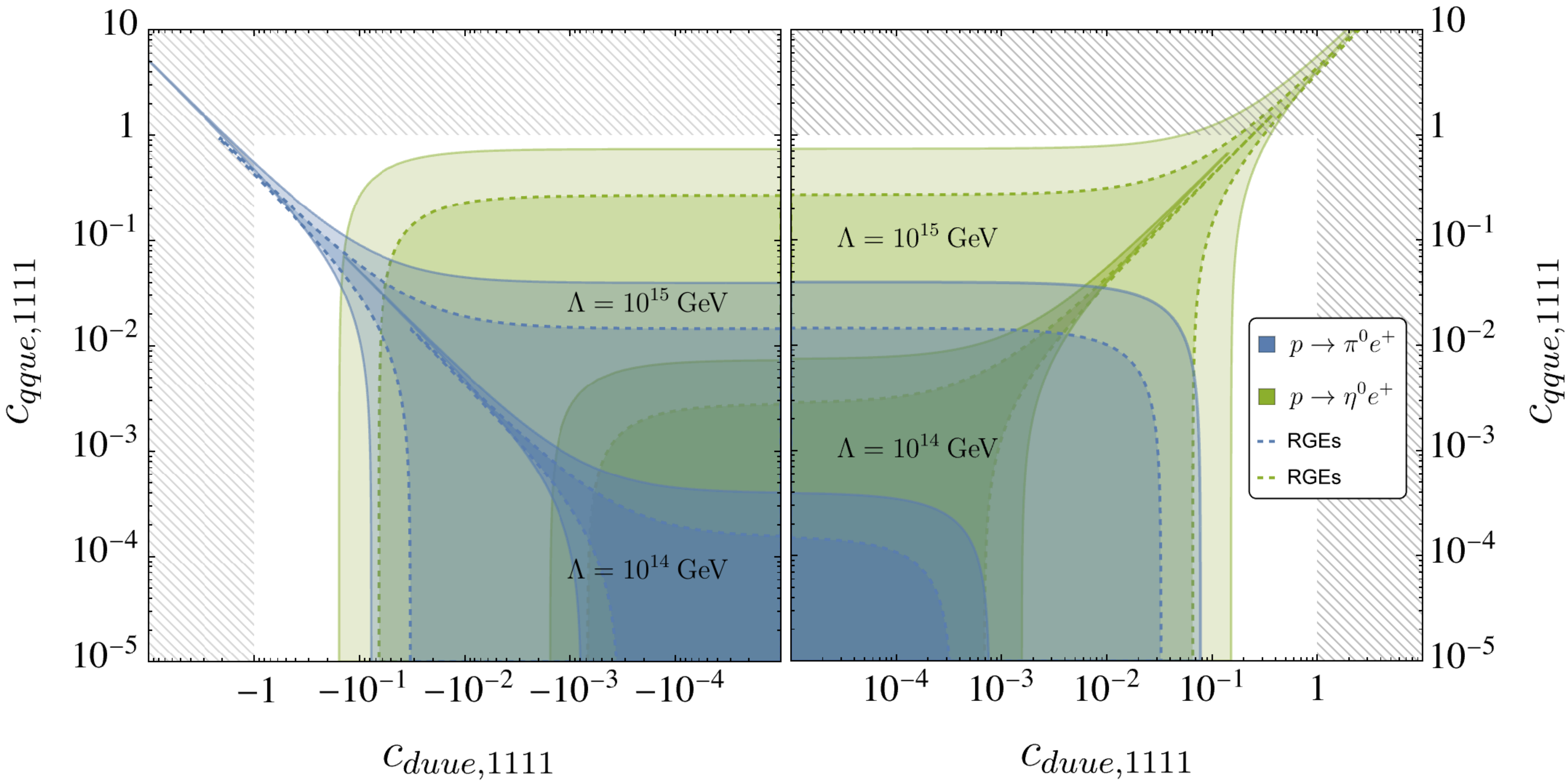


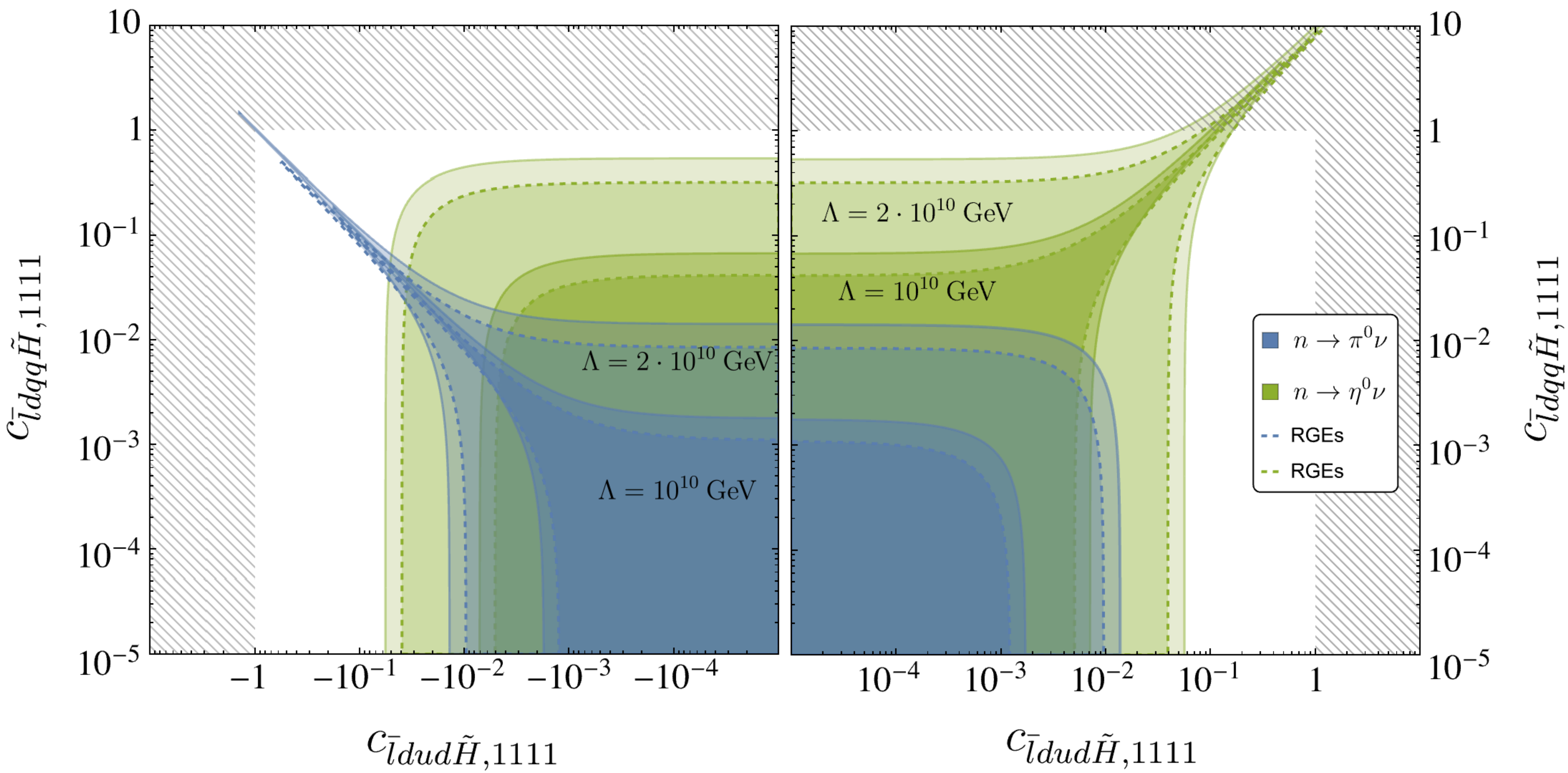
$$\Lambda/\sqrt[3]{c} \sim (2 \sim 12) \times 10^{10}\ \text{GeV}$$

D = 6 pairs of WCs



- Different search channels provide complementary constraints
- No flat directions

D = 7 pairs of WCs



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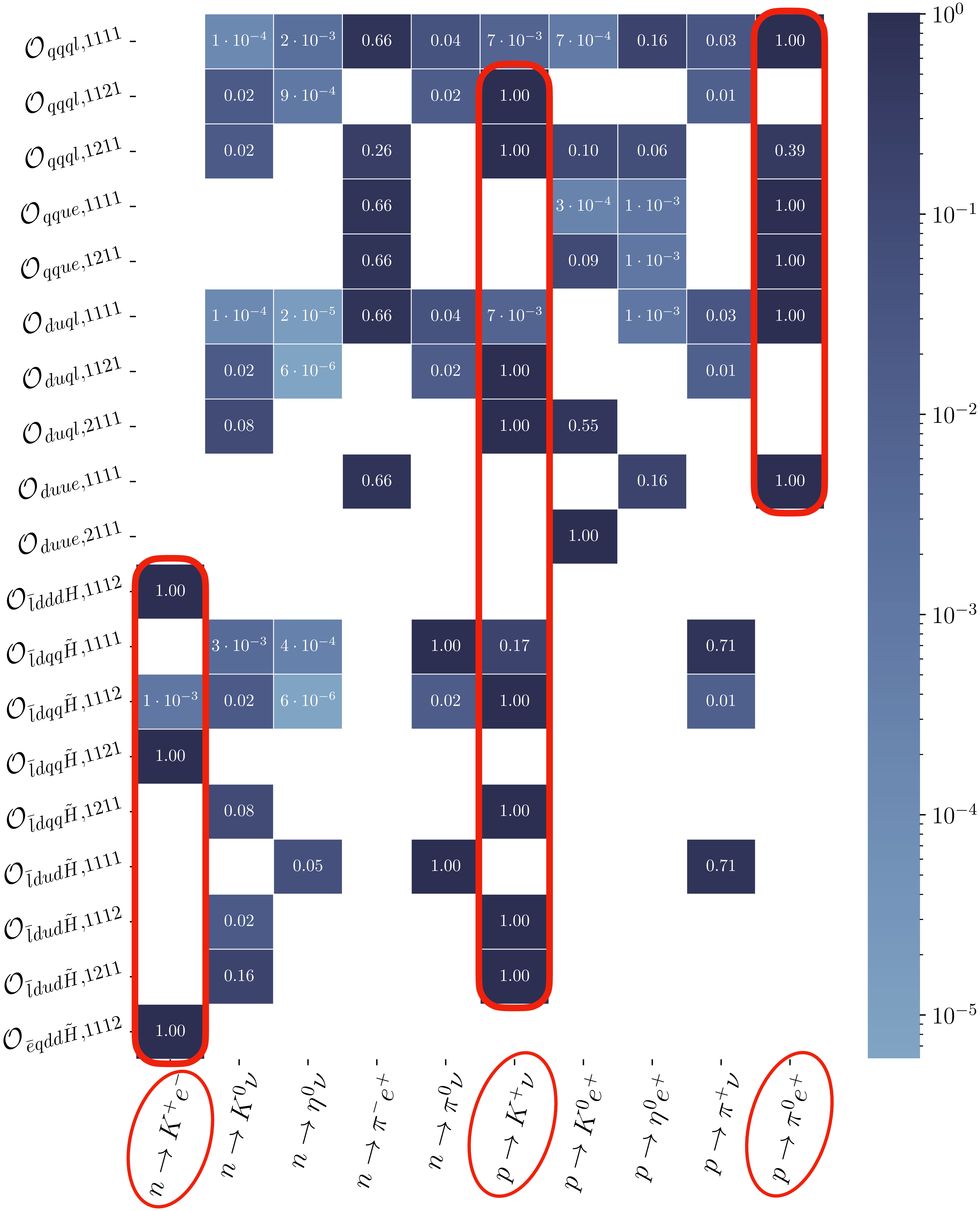
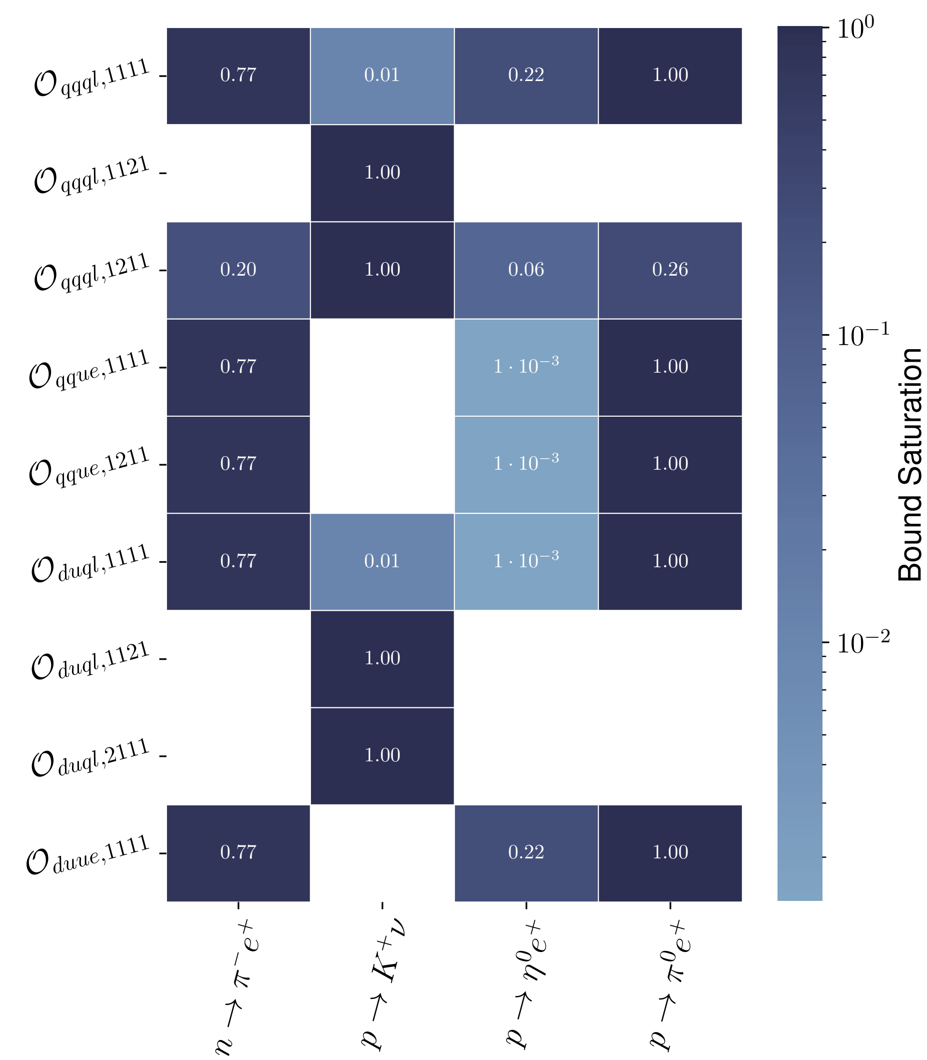


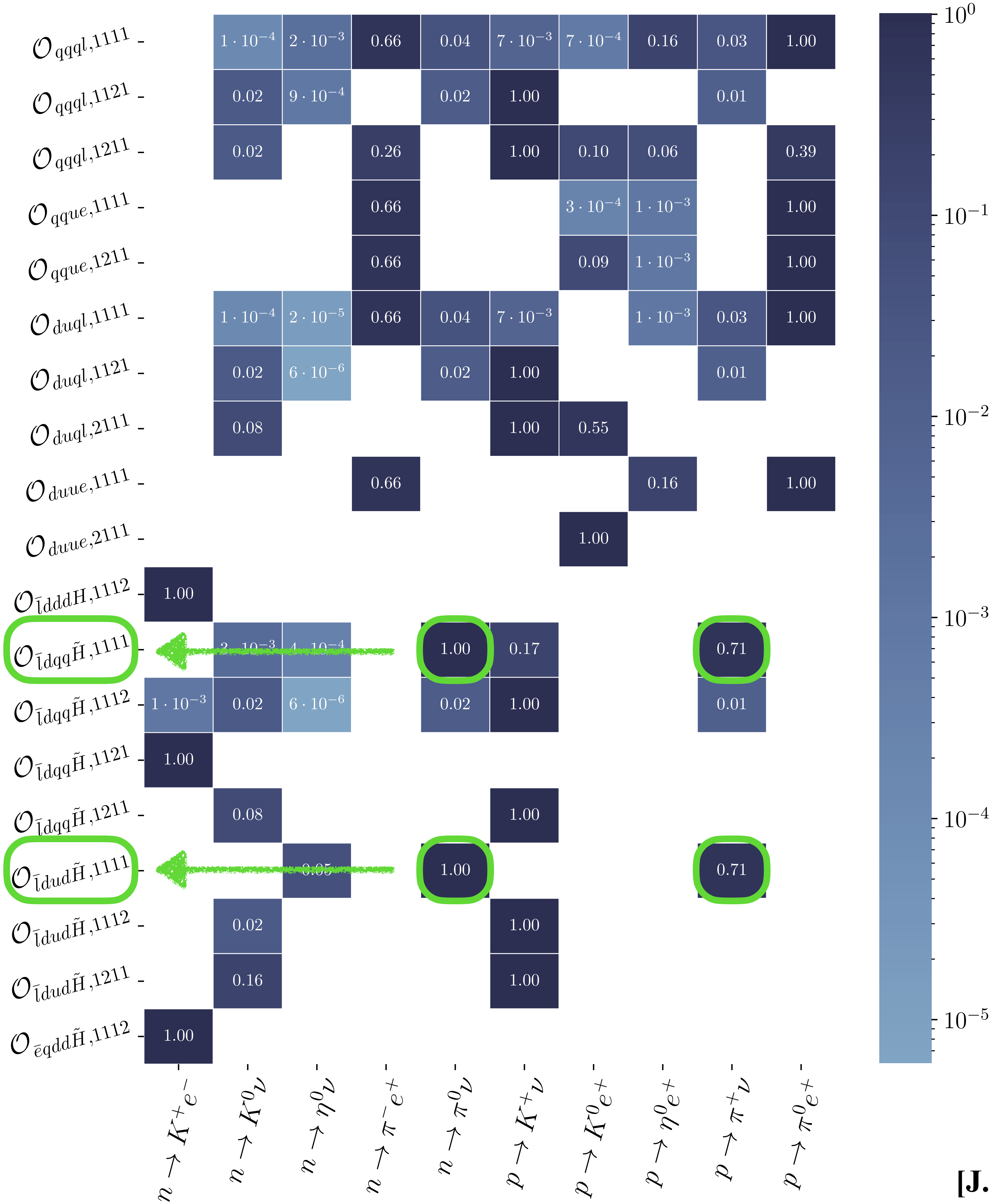
$$\text{Bound Saturation} \equiv \frac{\Gamma_{\text{i,th}}/\Gamma_{\text{max,th}}}{\Gamma_{\text{i,exp}}/\Gamma_{\text{max,exp}}}$$

$$\tau_{p\to\pi^0e^+} > 24 \cdot 10^{33} \text{ years}$$

$$\tau_{p\to K^+\nu} > 6.61 \cdot 10^{33} \text{ years}$$

$$\tau_{n\to K^+e^-} > 0.032 \cdot 10^{33} \text{ years}$$





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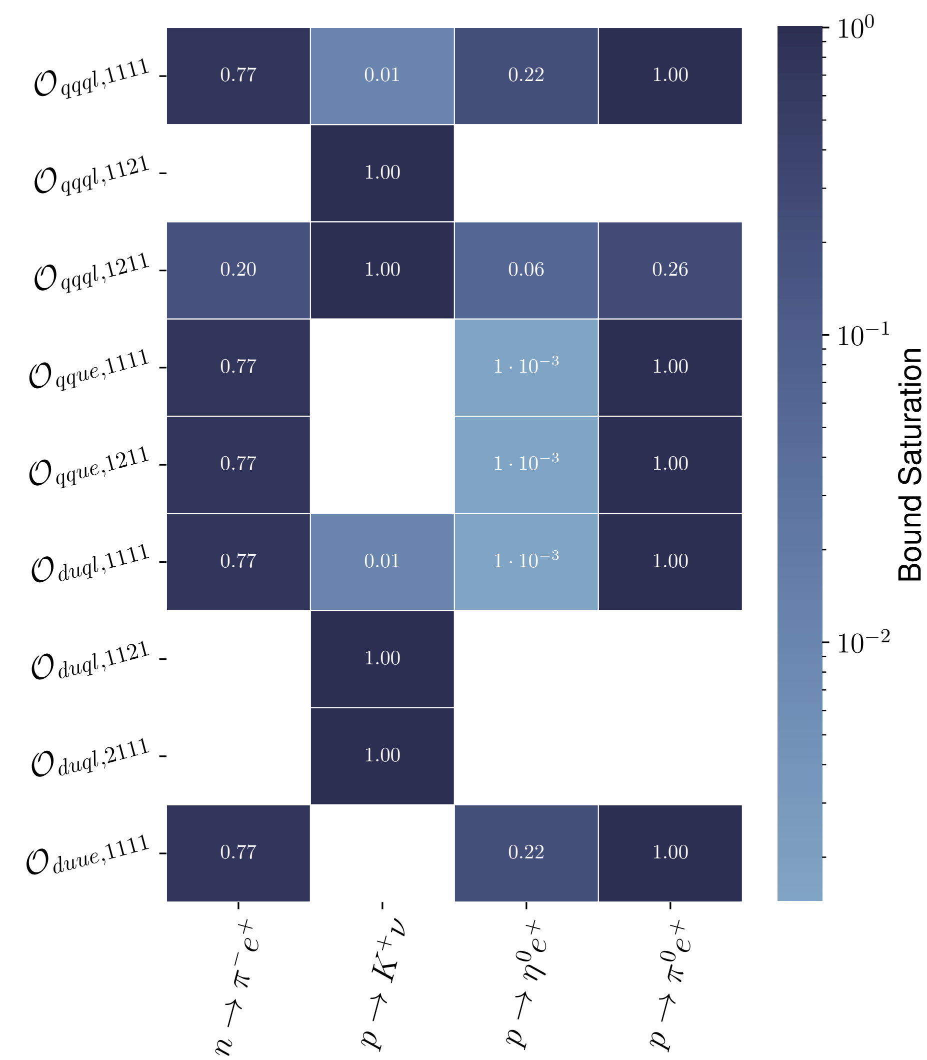


Tree-level UV completions in

[J. De Blas et al. 2018, Xu-Xiang Li et al. 2023]

Phenomenological matrices

Numerical κ-matrices available online

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^5}{\Lambda^4} \quad \text{for} \quad i = p \to \pi^0 e^+, p \to K^+ \nu \ldots \quad \text{(9 matrices)}$$

$$\Gamma_{(i)}^{|\Delta(B-L)|=2} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^7}{\Lambda^6} \quad \text{for} \quad i = p \to K^+ \nu, n \to K^+ e^- \ldots \quad \text{(6 matrices)}$$

$c_{qque,1111} \sim -0.44\, c_{duue,1111}$

$c_{qque,1111} \sim 4.1\, c_{duue,1111}$

$p \to \pi^0 e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	86.2		−44.4			−16.1				
$\mathcal{O}_{qque,1111}$		99.0		43.5						−22.9
$\mathcal{O}_{duql,1111}$	−44.4		22.9			8.31				
$\mathcal{O}_{duue,1111}$		43.5		19.1						−10.0
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−16.1		8.31			3.02				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−22.9		−10.0						5.28

$p \to \eta^0 e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	36.1		2.01			−6.76				
$\mathcal{O}_{qque,1111}$		0.48		−1.97						−0.11
$\mathcal{O}_{duql,1111}$	2.01		0.11			−0.38				
$\mathcal{O}_{duue,1111}$		−1.97		8.02						0.45
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−6.76		−0.38			1.27				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−0.11		0.45						0.03

Phenomenological matrices

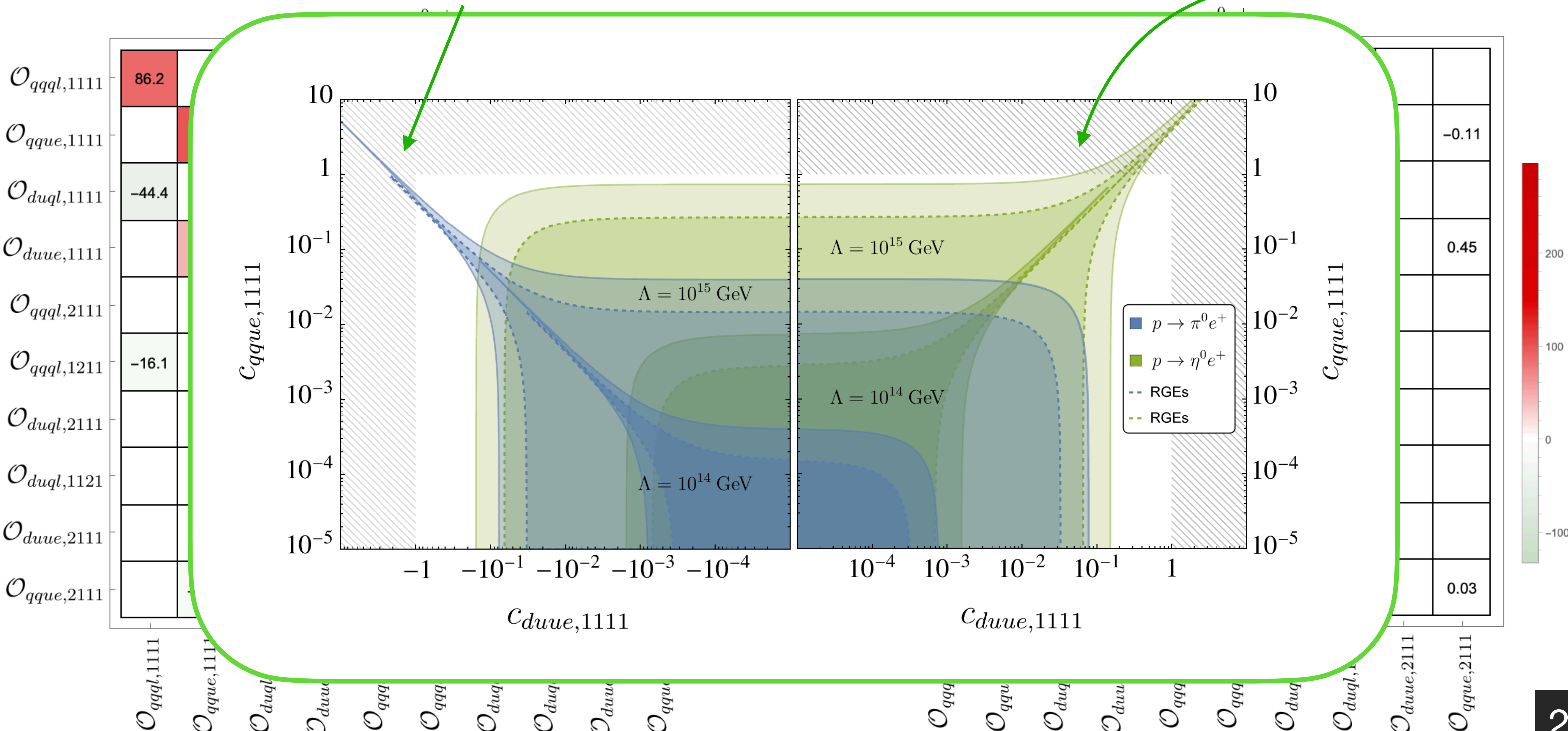
Numerical κ-matrices available online

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^5}{\Lambda^4} \quad \text{for} \quad i = p \to \pi^0 e^+, p \to K^+\nu \ldots \quad \text{(9 matrices)}$$

$$\Gamma_{(i)}^{|\Delta(B-L)|=2} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^7}{\Lambda^6} \quad \text{for} \quad i = p \to K^+\nu, n \to K^+ e^- \ldots \quad \text{(6 matrices)}$$

$c_{qque,1111} \sim -0.44\, c_{duue,1111}$

$c_{qque,1111} \sim 4.1\, c_{duue,1111}$

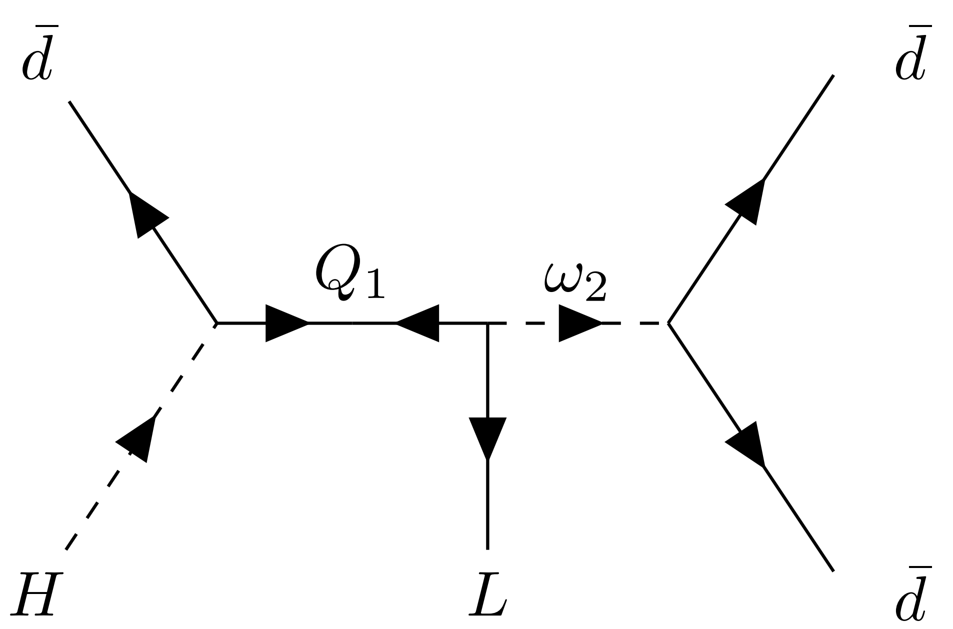


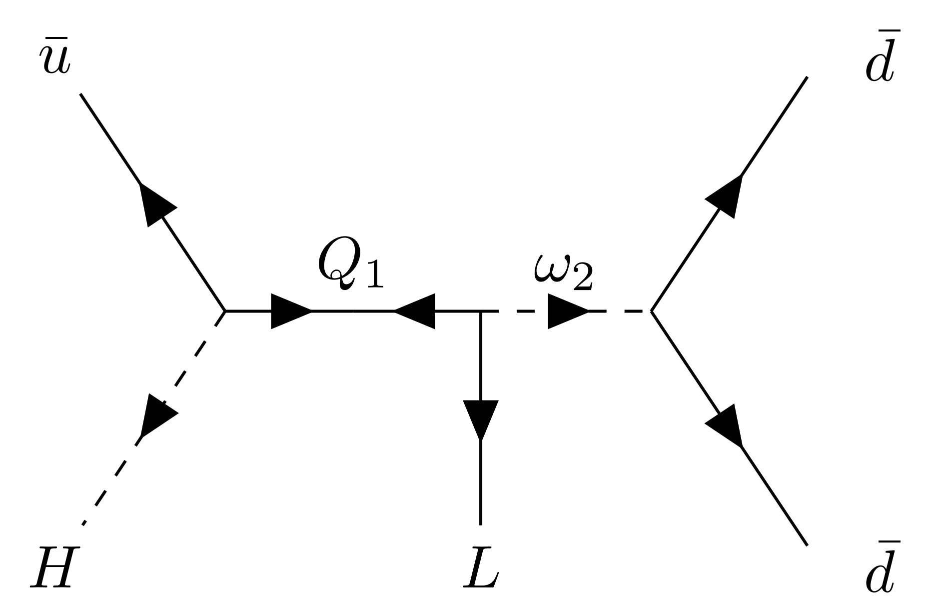
Example UV model

SM enhanced by a scalar LQ ω_2 and a VLF Q_1

$$\omega_2 \sim (3,\ 1,\ 2/3)\,,\ Q_1 + \bar{Q}_1^\dagger \sim (3,\ 2,\ 1/6)$$

$$\mathscr{L}_{\rm int} = y_{1,ij}\omega_2 \bar{d}^{\dagger i}\bar{d}^{\dagger j} + y_{2,k}H^\dagger Q_1 \bar{d}^k + y_{3,k}\ Q_1 \epsilon H \bar{u}^k + y_{4,l}\ \omega_2 \bar{Q}_1 L^l + \mathrm{h.c.}$$





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$$\downarrow$$

$$\mathscr{L}_{\rm eff} \supset C_{\bar{l}dddH,pqrs}\mathcal{O}_{\bar{l}dddH,pqrs} + C_{\bar{l}dud\tilde{H},pqrs}\mathcal{O}_{\bar{l}dud\tilde{H},pqrs} + \mathrm{h.c.}$$

y_1 antisymmetric $\Bigg\{$

$p \to K^+ \nu \quad n \to K^0 \nu \quad n \to K^+ e^-$

$$C_{\bar{l}dud\tilde{H},1211} = -\,C_{\bar{l}dud\tilde{H},1112}$$

ple UV

scalar LQ

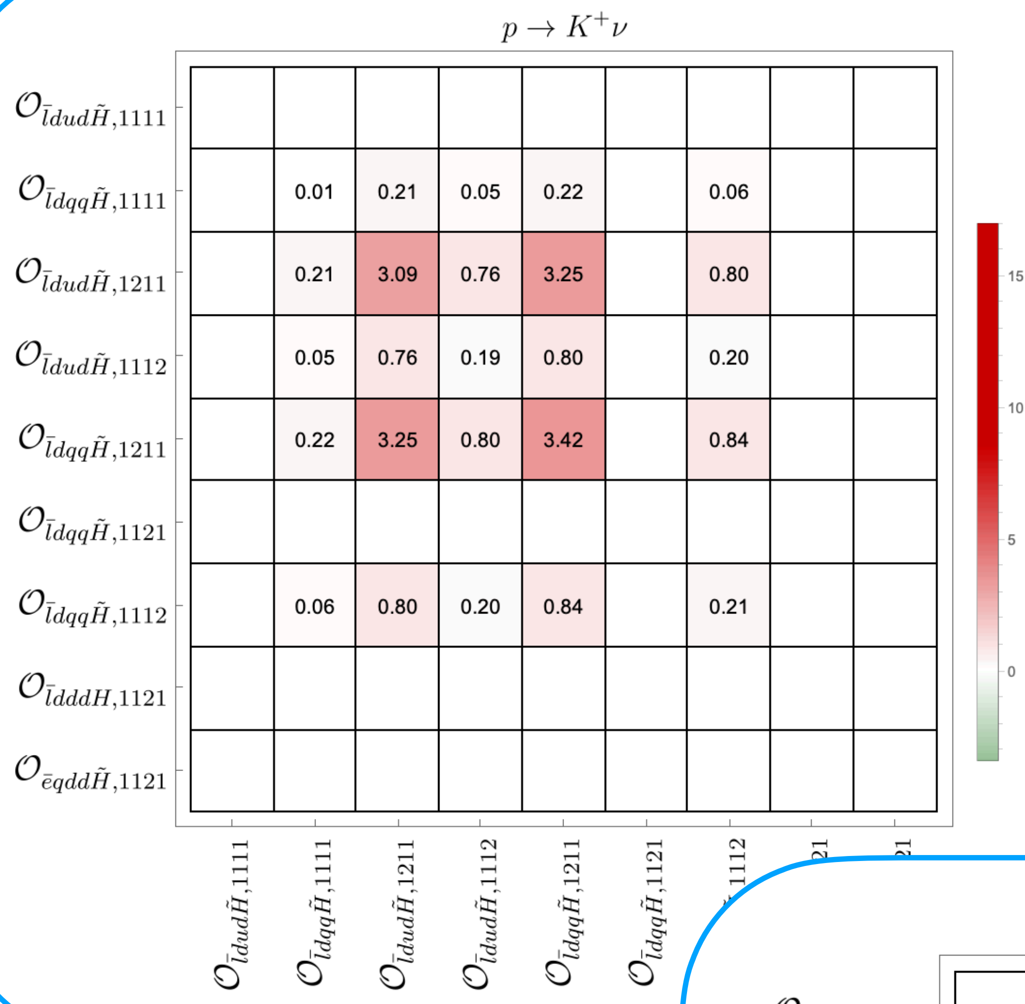
$), \; Q_1 + \bar{Q}$

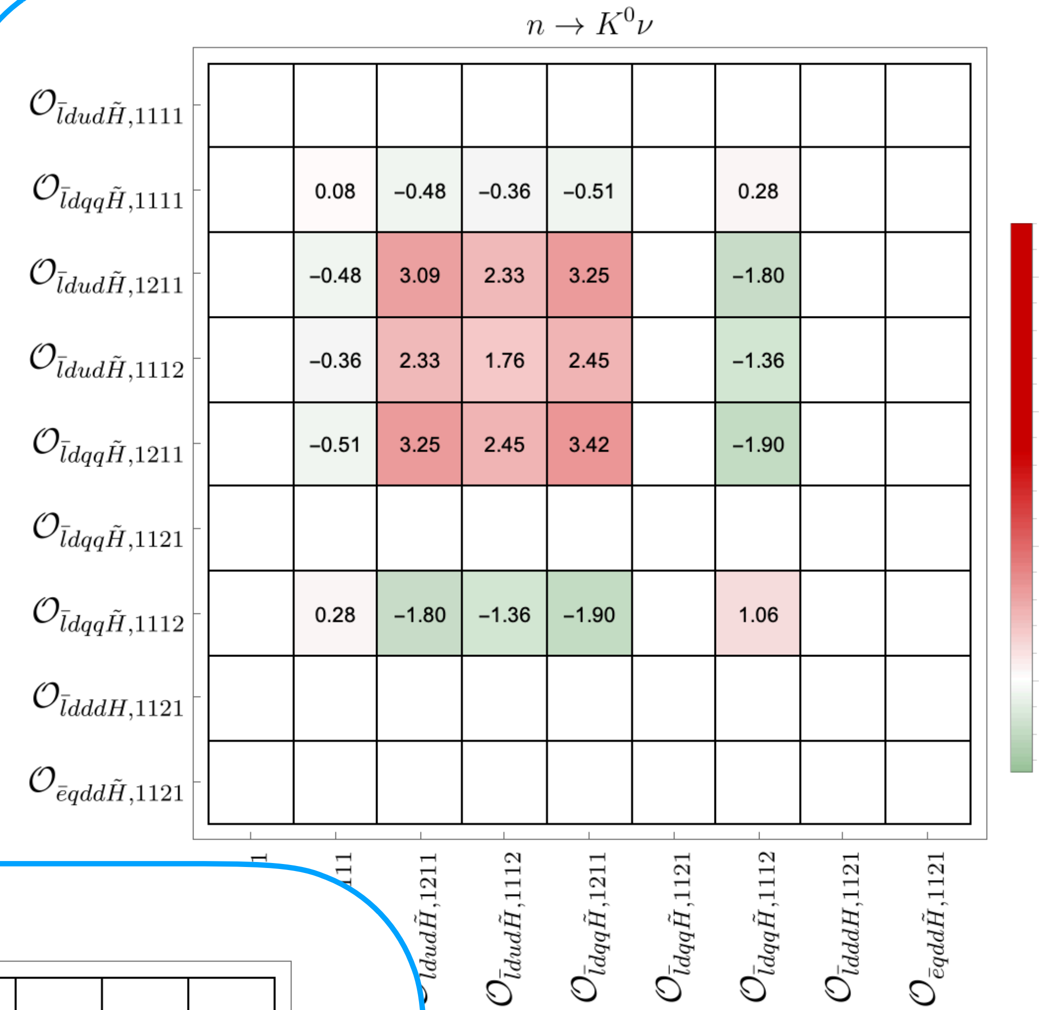
$Q_1 \bar{d}^k + y_{3,k}$

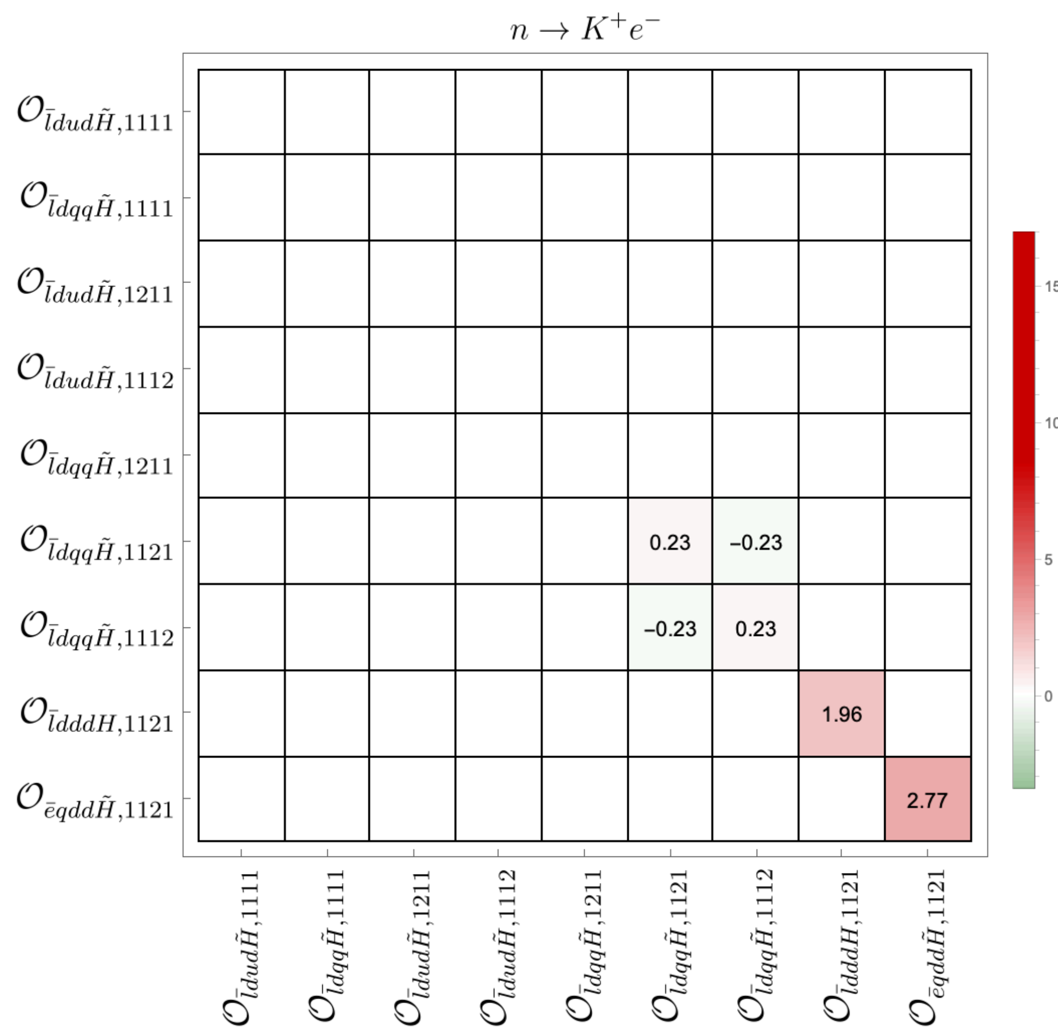
$\mathscr{L}_{\text{eff}}$ $\cdots$ h.c.

$K^+ e^-$

y_1 ant







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↓

$$\mathscr{L}_{\rm eff} \supset C_{\bar{l}dddH,pqrs}\mathcal{O}_{\bar{l}dddH,pqrs} + C_{\bar{l}dud\tilde{H},pqrs}\mathcal{O}_{\bar{l}dud\tilde{H},pqrs} + \mathrm{h.c.}$$

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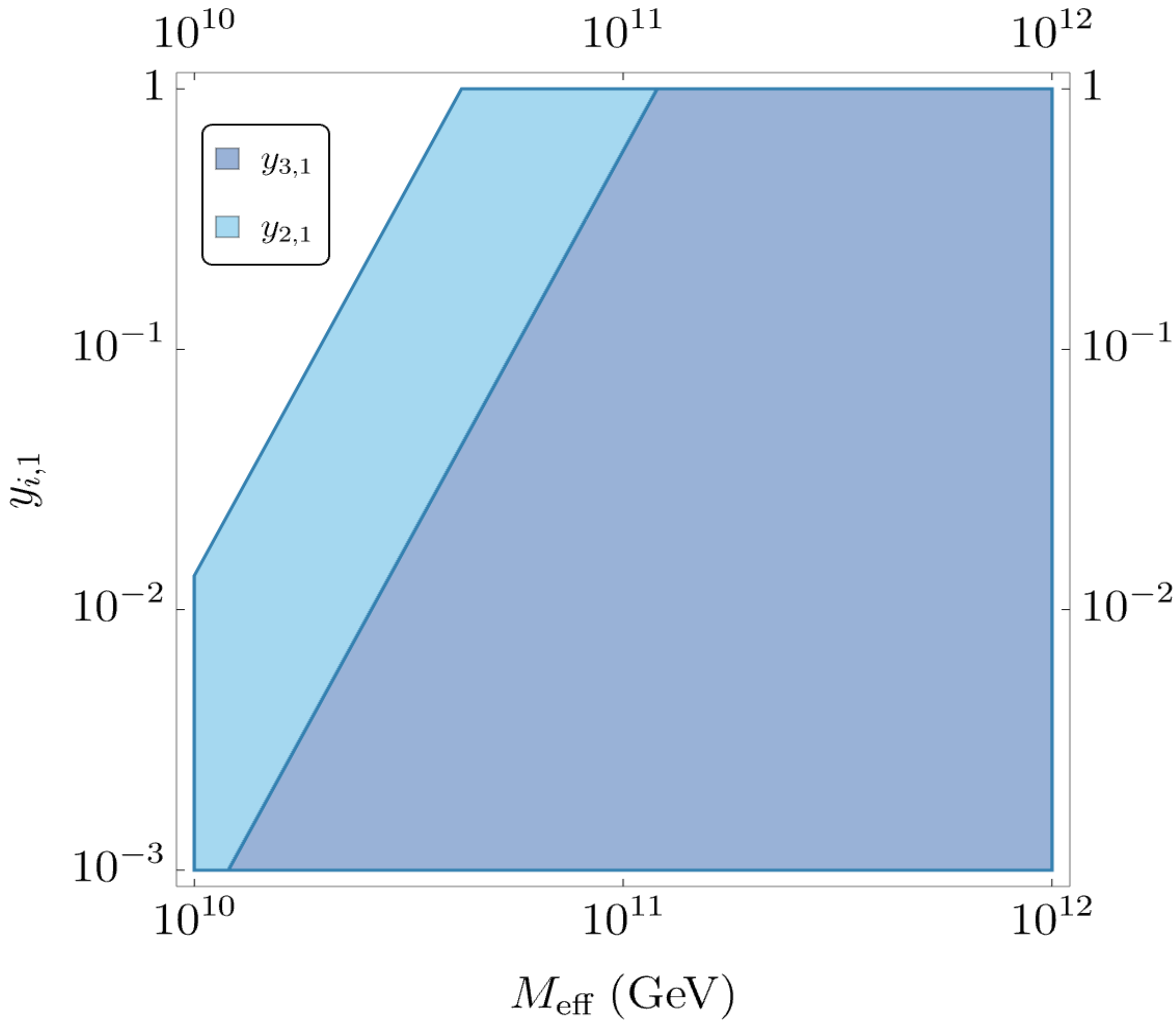
$$C_{\bar{l}dud\tilde{H},1211} = -\,C_{\bar{l}dud\tilde{H},1112}$$

κ-matrices for the 3 processes above, compute Γ and compare with $\Gamma^{\rm exp}$ ⟶ **$p \to K^+\nu$ the most constraining**

Example UV model



$$M_{\text{eff}}^3 \equiv \frac{M_\omega^2 M_Q}{y_{1,12}\, y_{4,1}^*}$$

Main results of this work

· Model-independent analysis on nucleon decay

· RG effects important: limits enhanced by 30% - 130% (d=6) , and 20 - 30% (d=7)

· Complementary analysis → Correlations and flat directions

· κ-matrices: SMEFT WC at $\Lambda \leftrightarrow$ observables at m_p

· Positive signals in 2-3 channels → SMEFT operators → GUT/Models

Main source of uncertainty: nuclear matrix elements α, β

Thank you!

Backup slides

RGEs

$$\left(\dot{C}_i \equiv 16\pi^2 \mu \frac{dC_i}{d\mu} = \sum_j \gamma_{ij} C_j\right)$$

$$\dot{C}_{duue,prst} = \left(-4g_3^2 - 2g_1^2\right) C_{duue,prst} - \frac{20}{3} g_1^2\, C_{duue,psrt}$$

$$\dot{C}_{duq\ell,prst} = \left(-4g_3^2 - \frac{9}{2} g_2^2 - \frac{11}{6} g_1^2\right) C_{duq\ell,prst}$$

$$\dot{C}_{qque,prst} = \left(-4g_3^2 - \frac{9}{2} g_2^2 - \frac{23}{6} g_1^2\right) C_{qque,prst}$$

$$\dot{C}_{qqq\ell,prst} = \left(-4g_3^2 - 3g_2^2 - \frac{1}{3} g_1^2\right) C_{qqq\ell,prst} \boxed{- 4g_2^2 \left(C_{qqq\ell,rpst} + C_{qqq\ell,srpt} + C_{qqq\ell,psrt}\right)}$$

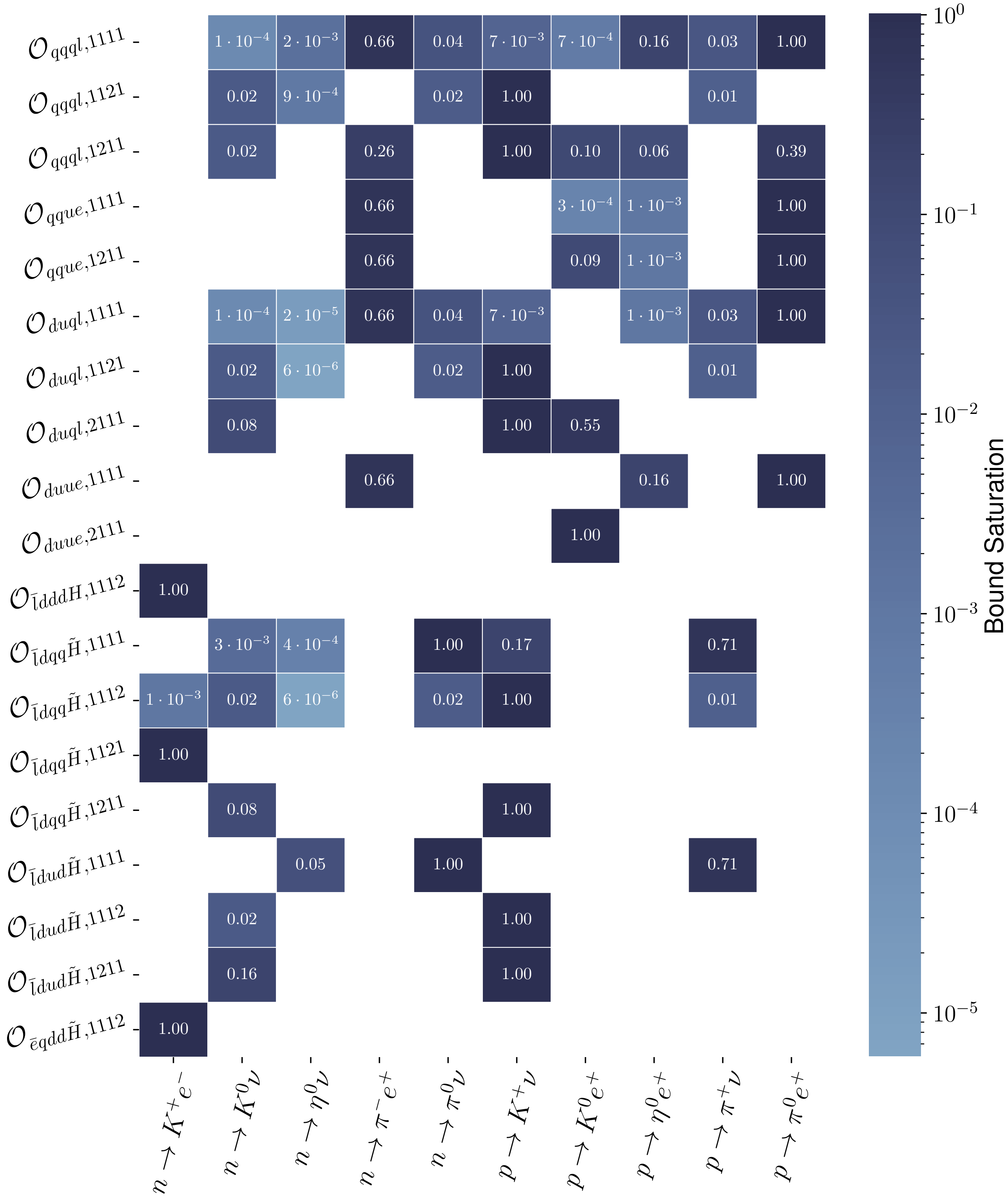
$$\dot{C}_{\bar{l}dud\tilde{H},prst} = \left(-4g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dud\tilde{H},prst} - \frac{10}{3} g_1^2\, C_{\bar{l}dud\tilde{H},ptsr}\,,$$

$$\dot{C}_{\bar{l}dddH,prst} = \left(-4g_3^2 - \frac{9}{4} g_2^2 - \frac{13}{12} g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dddH,prst}\,,$$

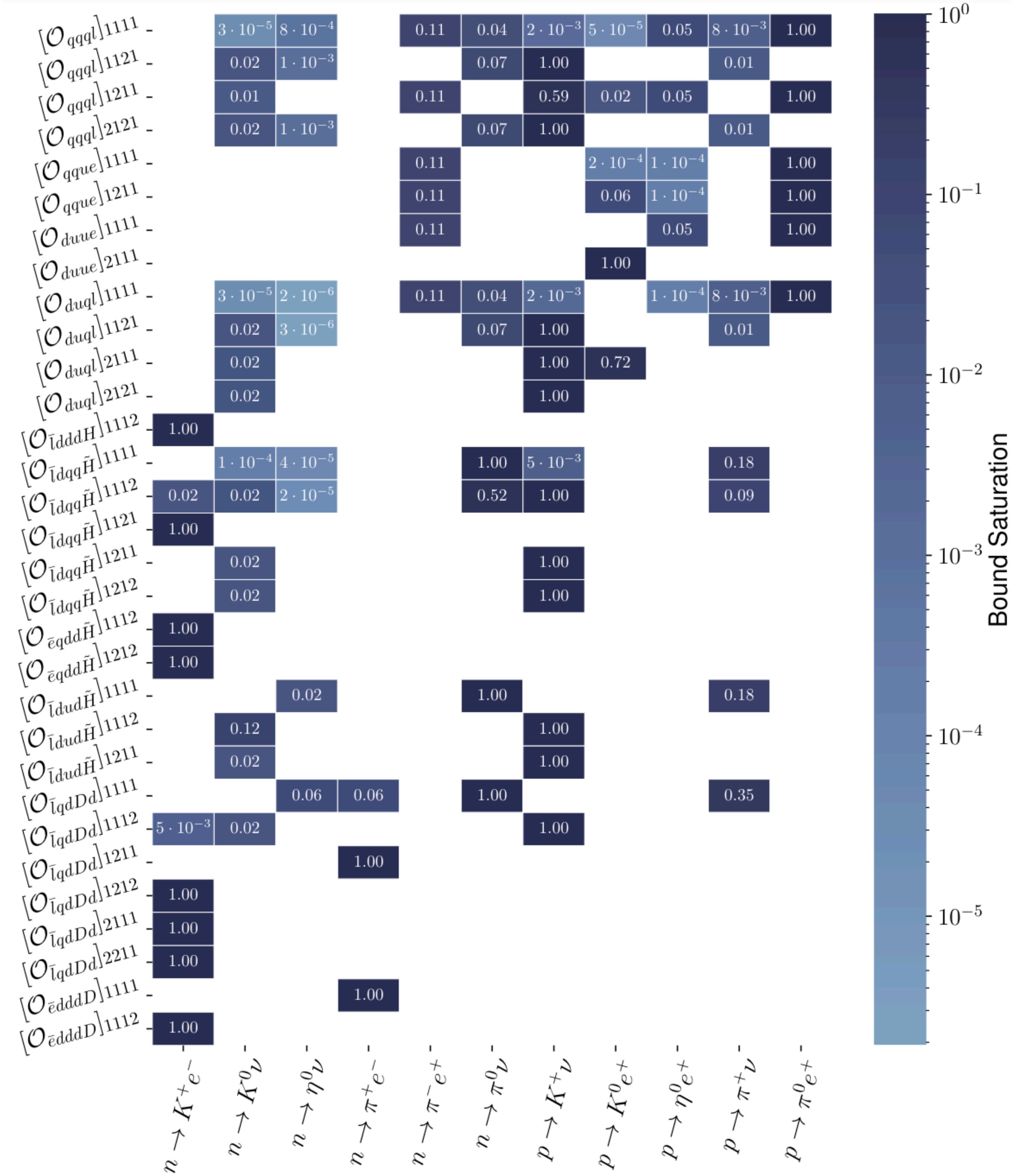
$$\dot{C}_{\bar{e}qdd\tilde{H},prst} = \left(-4g_3^2 - \frac{9}{4} g_2^2 + \frac{11}{12} g_1^2 \boxed{+ y_t^2}\right) C_{\bar{e}qdd\tilde{H},prst}\,,$$

$$\dot{C}_{\bar{l}dqq\tilde{H},prst} = \left(-4g_3^2 - \frac{15}{4} g_2^2 - \frac{19}{12} g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dqq\tilde{H},prst} - 3g_2^2\, C_{\bar{l}dqq\tilde{H},prts}\,.$$

Direct VS Indirect method



BχPT



Direct Method in [J. Gargalionis et al. 2024]

Direct VS Indirect method

$$\Gamma(N \to M + \ell) = \frac{m_N}{32\pi}\left(1 - \frac{m_M^2}{m_N^2}\right)^2 \left|\sum_I C_I W_0^I(N \to M)\right|^2$$

$W_0^I(N \to M)$ computed in the lattice
(Several parameters)

$$\Gamma\left(p \to \pi^+ \nu_r\right) = (32\pi f_\pi^2 m_p^3)^{-1}(m_p^2 - m_\pi^2)^2 \left|\alpha\left[L_{udd}^{S,LR}\right]_{11r1} + \beta\left[L_{udd}^{S,RR}\right]_{11r1}\right|^2 (1+D+F)^2$$

$$\Gamma\left(n \to K^+ e_r^-\right) = (32\pi f_\pi^2 m_n^3)^{-1}(m_n^2 - m_K^2)^2 \times$$
$$\left\{\left|\beta\left[L_{ddd}^{S,LL}\right]_{12r1} - \alpha\left[L_{ddd}^{S,RL}\right]_{12r1} + \frac{m_n}{m_\Sigma}\left(\alpha\left[L_{ddd}^{S,RL}\right]_{12r1} + \beta\left[L_{ddd}^{S,LL}\right]_{12r1}\right)(D-F)\right|^2\right.$$
$$\left. + \left|\beta\left[L_{ddd}^{S,RR}\right]_{12r1} - \alpha\left[L_{ddd}^{S,LR}\right]_{12r1} + \frac{m_n}{m_\Sigma}\left(\alpha\left[L_{ddd}^{S,LR}\right]_{12r1} + \beta\left[L_{ddd}^{S,RR}\right]_{12r1}\right)(D-F)\right|^2\right\}$$

D, F, f_π
low-energy BχPT constants

$$\Gamma\left(n \to K^0 \nu_r\right) = (32\pi f_\pi^2 m_n^3)^{-1}(m_n^2 - m_K^2)^2 \times$$
$$\left|-\alpha\left[L_{udd}^{S,LR}\right]_{12r1} + \beta\left[L_{udd}^{S,RR}\right]_{12r1} + \alpha\left[L_{udd}^{S,LR}\right]_{11r2} + \beta\left[L_{udd}^{S,RR}\right]_{11r2}\right.$$
$$- \frac{m_n}{2m_\Sigma}\left(\alpha\left[L_{udd}^{S,LR}\right]_{12r1} + \beta\left[L_{udd}^{S,RR}\right]_{12r1}\right)(D-F)$$
$$\left. + \frac{m_n}{6m_\Lambda}\left(\alpha\left[L_{udd}^{S,LR}\right]_{12r1} + \beta\left[L_{udd}^{S,RR}\right]_{12r1} + 2\alpha\left[L_{udd}^{S,LR}\right]_{11r2} + 2\beta\left[L_{udd}^{S,RR}\right]_{11r2}\right)(D+3F)\right|^2$$

α, β
computed in the lattice

κ-matrices

$\Gamma(p \to \pi^+\nu) = 2\ \Gamma(n \to \pi^0\nu)$

$n \to \pi^0\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	81.9		−42.2		−23.5			9.74		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	−42.2		21.7		12.1			−5.02		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−23.5		12.1		6.76			−2.80		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	9.74		−5.02		−2.80			1.16		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$p \to \pi^+\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	163.7		−84.4		−47.0			19.5		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	−84.4		43.5		24.2			−10.0		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−47.0		24.2		13.5			−5.60		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	19.5		−10.0		−5.60			2.32		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$n \to \eta^0\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	34.3		1.91		−9.86			−0.44		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	1.91		0.11		−0.55			−0.02		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−9.86		−0.55		2.83			0.13		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	−0.44		−0.02		0.13			0.006		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$p \to K^0e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	1.59					5.59	2.26			
$\mathcal{O}_{qque,1111}$		0.74							−2.21	3.20
$\mathcal{O}_{duql,1111}$										
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	5.59					19.6	7.93			
$\mathcal{O}_{duql,2111}$	2.26					7.93	3.20			
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$		−2.21							6.63	−9.58
$\mathcal{O}_{qque,2111}$		3.20							−9.58	13.9

$\Gamma(n \to \pi^-e^+) = 3\ \Gamma(p \to \pi^0e^+)$

$n \to \pi^-e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	258.5		−133.2			−48.4				
$\mathcal{O}_{qque,1111}$		297.1		130.5						−68.6
$\mathcal{O}_{duql,1111}$	−133.2		68.7			24.9				
$\mathcal{O}_{duue,1111}$		130.5		57.4						−30.1
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−48.4		24.9			9.06				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−68.6		−30.1						15.8

$p \to \pi^0e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	86.2		−44.4			−16.1				
$\mathcal{O}_{qque,1111}$		99.0		43.5						−22.9
$\mathcal{O}_{duql,1111}$	−44.4		22.9			8.31				
$\mathcal{O}_{duue,1111}$		43.5		19.1						−10.0
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−16.1		8.31			3.02				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−22.9		−10.0						5.28

BNC Lagrangian

$$\begin{aligned}\mathcal{L}_0 \supset & \left(\frac{D-F}{f_\pi}\,\overline{\Sigma^+}\gamma^\mu\gamma_5 p - \frac{D+3F}{\sqrt{6}f_\pi}\,\overline{\Lambda^0}\gamma^\mu\gamma_5 n - \frac{D-F}{\sqrt{2}f_\pi}\,\overline{\Sigma^0}\gamma^\mu\gamma_5 n\right)\partial_\mu \bar{K}^0 \\ & + \left(\frac{D-F}{\sqrt{2}f_\pi}\,\overline{\Sigma^0}\gamma^\mu\gamma_5 p - \frac{D+3F}{\sqrt{6}f_\pi}\,\overline{\Lambda^0}\gamma^\mu\gamma_5 p + \frac{D-F}{f_\pi}\,\overline{\Sigma^-}\gamma^\mu\gamma_5 n\right)\partial_\mu K^- \\ & + \frac{3F-D}{2\sqrt{6}f_\pi}\left(\overline{p}\gamma^\mu\gamma_5 p + \overline{n}\gamma^\mu\gamma_5 n\right)\partial_\mu \eta \\ & + \frac{D+F}{f_\pi}\,\overline{p}\gamma^\mu\gamma_5 n\,\partial_\mu \pi^+ \\ & + \frac{D+F}{2\sqrt{2}f_\pi}\left(\overline{p}\gamma^\mu\gamma_5 p - \overline{n}\gamma^\mu\gamma_5 n\right)\partial_\mu \pi^0 + \text{h.c.}\ .\end{aligned}$$

$$\xi B \xi \to L \xi B \xi R^\dagger \qquad \xi^\dagger B \xi^\dagger \to R \xi^\dagger B \xi^\dagger L^\dagger$$

$$\xi B \xi^\dagger \to L \xi B \xi^\dagger L^\dagger \qquad \xi^\dagger B \xi \to R \xi^\dagger B \xi R^\dagger$$

$$\xi B \xi \sim (\mathbf{3}, \bar{\mathbf{3}}), \quad \xi^\dagger B \xi^\dagger \sim (\bar{\mathbf{3}}, \mathbf{3}), \quad \xi B \xi^\dagger \sim (\mathbf{8}, \mathbf{1}), \quad \xi^\dagger B \xi \sim (\mathbf{1}, \mathbf{8})$$

$$\alpha \cdot \nu \ \text{tr}(\xi B \xi^\dagger P_{32}) = -(du)(d\nu) = [\mathcal{O}_{udd}]^{S,LL}_{1111}$$

$$\langle 0 | \epsilon^{abc} (\bar{u}^\dagger_a \bar{d}^\dagger_b) u_c | p^{(s)} \rangle = \alpha P_L u_p^{(s)}$$

$$\langle 0 | \epsilon^{abc} (u_a d_b) u_c | p^{(s)} \rangle = \beta P_L u_p^{(s)}$$

BNV Lagrangian

Name	LEFT	Flavour/BχPT
$[\mathcal{O}^{S,LL}_{udd}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi^\dagger P_{32}) \supset -\beta\overline{\nu^c_{Lr}}n - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(\sqrt{\frac{3}{2}}n\eta - \frac{1}{\sqrt{2}}n\pi^0 + p\pi^-\right)$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi^\dagger \tilde{P}_{22}) \supset -\beta\overline{\nu^c_{Lr}}\left(-\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}}\right) - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}n\bar{K}^0$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi^\dagger P_{33}) \supset \beta\sqrt{\frac{2}{3}}\overline{\nu^c_{Lr}}\Lambda^0 - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(n\bar{K}^0 + pK^-\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$-\beta\overline{e^c_{Lr}}\text{tr}(\xi B\xi^\dagger \tilde{P}_{31}) \supset \beta\overline{e^c_{Lr}}p + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}\left(\sqrt{\frac{3}{2}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$-\beta\overline{e^c_{Lr}}\text{tr}(\xi B\xi^\dagger P_{21}) \supset -\beta\overline{e^c_{Lr}}\Sigma^+ + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$
$[\mathcal{O}^{S,LR}_{uud}]_{[rs]tu}$	$(u_r u_s)(\bar{d}^\dagger_t \bar{e}^\dagger_u)$	—
$[\mathcal{O}^{S,LR}_{duu}]_{rstu}$	$(d_r u_s)(\bar{u}^\dagger_t \bar{e}^\dagger_u)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\text{tr}(\xi^\dagger B\xi^\dagger \tilde{P}_{31}) \supset -\alpha\overline{e^c_{Rr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}\left(-\frac{1}{\sqrt{6}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\text{tr}(\xi^\dagger B\xi^\dagger P_{21}) \supset \alpha\overline{e^c_{Rr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{uud}]_{[rs]tu}$	$(\bar{u}^\dagger_r \bar{u}^\dagger_s)(d_t e_u)$	—
$[\mathcal{O}^{S,RL}_{duu}]_{rstu}$	$(\bar{d}^\dagger_r \bar{u}^\dagger_s)(u_t e_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\text{tr}(\xi B\xi \tilde{P}_{31}) \supset \alpha\overline{e^c_{Lr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}\left(-\frac{1}{\sqrt{6}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\text{tr}(\xi B\xi P_{21}) \supset -\alpha\overline{e^c_{Lr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{dud}]_{rstu}$	$(\bar{d}^\dagger_r \bar{u}^\dagger_s)(d_t \nu_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi \tilde{P}_{32}) \supset -\alpha\overline{\nu^c_{Lr}}n + \frac{i\alpha}{f_\pi}\overline{\nu^c_{Lr}}\left(\frac{1}{\sqrt{6}}n\eta + \frac{1}{\sqrt{2}}n\pi^0 - p\pi^-\right)$
$[\mathcal{O}^{S,RL}_{dud}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi P_{22}) \supset \alpha\overline{\nu^c_{Lr}}\left(\frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}}\right) + \frac{i\alpha}{f_\pi}\overline{\nu^c_{Lr}}n\bar{K}^0$
$[\mathcal{O}^{S,RL}_{dud}]_{112r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi \tilde{P}_{33}) \supset \alpha\overline{\nu^c_{Lr}}\sqrt{\frac{2}{3}}\Lambda^0 - \frac{i\alpha}{f_\pi}\overline{\nu^c_{Lr}}\left(n\bar{K}^0 + pK^-\right)$
$[\mathcal{O}^{S,RL}_{dud}]_{212r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi P_{23}) \supset \alpha\overline{\nu^c_{Lr}}\Xi^0$
$[\mathcal{O}^{S,RL}_{ddu}]_{[rs]tu}$	$(\bar{d}^\dagger_r \bar{d}^\dagger_s)(u_t \nu_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$	$(\bar{d}^\dagger \bar{s}^\dagger)(u\nu_r)$	$-\alpha\overline{\nu^c_{Lr}}\text{tr}(\xi B\xi P_{11}) \supset \alpha\overline{\nu^c_{Lr}}\left(\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}}\right) - \frac{i\alpha}{f_\pi}\overline{\nu^c_{Lr}}pK^-$
$[\mathcal{O}^{S,RR}_{duu}]_{rstu}$	$(\bar{d}^\dagger_r \bar{u}^\dagger_s)(\bar{u}^\dagger_t \bar{e}^\dagger_u)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\beta\overline{e^c_{Rr}}\text{tr}(\xi^\dagger B\xi \tilde{P}_{31}) \supset -\beta\overline{e^c_{Rr}}p + \frac{i\beta}{f_\pi}\overline{e^c_{Rr}}\left(\sqrt{\frac{3}{2}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,RR}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\beta\overline{e^c_{Rr}}\text{tr}(\xi^\dagger B\xi P_{21}) \supset \beta\overline{e^c_{Rr}}\Sigma^+ + \frac{i\beta}{f_\pi}\overline{e^c_{Rr}}p\bar{K}^0$

Name	LEFT	Flavour/BχPT
$[\mathcal{O}^{S,LL}_{udd}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$\mathbf{(8,1)}$
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{32}) \supset -\beta\overline{\nu^c_{Lr}}n - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(\sqrt{\frac{3}{2}}n\eta - \frac{1}{\sqrt{2}}n\pi^0 + p\pi^-\right)$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger \tilde{P}_{22}) \supset -\beta\overline{\nu^c_{Lr}}\left(-\frac{\Lambda^0}{\sqrt{6}}+\frac{\Sigma^0}{\sqrt{2}}\right) - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}n\bar{K}^0$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{33}) \supset \beta\sqrt{\frac{2}{3}}\overline{\nu^c_{Lr}}\Lambda^0 - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(n\bar{K}^0 + pK^-\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$\mathbf{(8,1)}$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$-\beta\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger\tilde{P}_{31}) \supset \beta\overline{e^c_{Lr}}p + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}\left(\sqrt{\frac{3}{2}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$-\beta\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{21}) \supset -\beta\overline{e^c_{Lr}}\Sigma^+ + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$
$[\mathcal{O}^{S,LR}_{uud}]_{[rs]tu}$	$(u_r u_s)(\bar{d}^\dagger_t \bar{e}^\dagger_u)$	—
$[\mathcal{O}^{S,LR}_{duu}]_{rstu}$	$(d_r u_s)(\bar{u}^\dagger_t \bar{e}^\dagger_u)$	$\mathbf{(\bar{3},3)}$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\mathrm{tr}(\xi^\dagger B\xi^\dagger\tilde{P}_{31}) \supset -\alpha\overline{e^c_{Rr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}\left(-\frac{1}{\sqrt{6}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\mathrm{tr}(\xi^\dagger B\xi^\dagger P_{21}) \supset \alpha\overline{e^c_{Rr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{uud}]_{[rs]tu}$	$(\bar{u}^\dagger_r \bar{u}^\dagger_s)(d_t e_u)$	—
$[\mathcal{O}^{S,RL}_{duu}]_{rstu}$	$(\bar{d}^\dagger_r\bar{u}^\dagger_s)(u_t e_u)$	$\mathbf{(3,\bar{3})}$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger\bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi\tilde{P}_{31}) \supset \alpha\overline{e^c_{Lr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}\left(-\frac{1}{\sqrt{6}}p\eta+\frac{1}{\sqrt{2}}p\pi^0+n\pi^+\right)$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger\bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi P_{21}) \supset -\alpha\overline{e^c_{Lr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$

Nucleon decay channels

$$\Gamma(N \to M\ell_\alpha) \quad \Delta(B-L)=0 \quad \begin{cases} n \to \eta^0\nu \\ n \to \pi^0\nu \\ p \to \pi^+\nu \\ n \to \pi^- e^+ \\ p \to \eta^0 e^+ \\ p \to \pi^0 e^+ \\ p \to K^0 e^+ \\ n \to K^0\nu \\ p \to K^+\nu \end{cases}$$

$$\Gamma(N \to M\ell_\alpha) \quad |\Delta(B-L)|=2 \quad \begin{cases} n \to \eta^0\nu \\ n \to \pi^0\nu \\ p \to \pi^+\nu \\ n \to K^0\nu \\ p \to K^+\nu \\ n \to K^+ e^- \end{cases}$$

- All 2-body PS decays except for $p \to \bar{K}^0 e^+$ $n \to \bar{K}^0\nu$ $n \to K^- e^+$ $n \to \pi^+ e^-$

(No BχPT formalism developed for PD into vector mesons, e.g. $p \to \rho^0 e^+$)

SMEFT

- RGEs dominated by the SM gauge couplings → enhancement
- QCD contributions universal and dominant, BUT electroweak contributions are relevant for $\mathcal{O}_{qqql,1111}$
- top-loop contributions universal and suppressive for d = 7 WCs
- Operator Mixing subdominant for proton decay

$$c(m_W) \sim (2-4)\ c(10^{15}\ \text{GeV})$$

- RGEs for d = 6 SMEFT Manohar et. al. [2014]
- RGEs for d = 7 SMEFT Yi Liao et. al. [2016]

Phenomenological matrices

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4}\, c_j^* \, \boldsymbol{\kappa}_{(i)}^{jk} \, c_k \frac{m_p^5}{\Lambda^4} \quad \text{for} \quad i = p \to \pi^0 e^+, p \to K^+ \nu \ldots \quad \text{(9 matrices)}$$

$$\Gamma_{(i)}^{|\Delta(B-L)|=2} \equiv 10^{-4}\, c_j^* \, \boldsymbol{\kappa}_{(i)}^{jk} \, c_k \frac{m_p^7}{\Lambda^6} \quad \text{for} \quad i = p \to K^+ \nu, n \to K^+ e^- \ldots \quad \text{(6 matrices)}$$