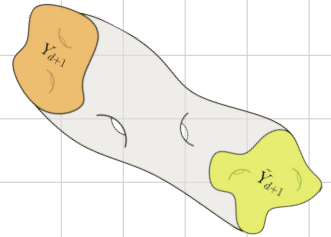


ANOMALIES & BORDISMS OF NON-SUPERSYMMETRIC STRINGS

Matilda Delgado

Based on:
[2310.06895] I. Basile, A. Debray, M.D., M. Montero



BIG PICTURE

Our world is non-supersymmetric

BIG PICTURE

Our world is non-supersymmetric

(at least at low energies)

BIG PICTURE

Our world is non-supersymmetric

(at least at low energies)

It is crucial for phenomenology to understand Quantum Gravity in setups without supersymmetry!

BIG PICTURE

String theory is a theory of Quantum Gravity

BIG PICTURE

String theory is a theory of Quantum Gravity

At low energies it looks *similar* to our world:

It has GR, chiral matter and gauge symmetries

BIG PICTURE

String theory is a theory of Quantum Gravity

At low energies it looks *similar* to our world:

It has GR, chiral matter and gauge symmetries

It is often said that String Theory needs supersymmetry

BIG PICTURE

String theory is a theory of Quantum Gravity

At low energies it looks *similar* to our world:

It has GR, chiral matter and gauge symmetries

It is often said that String Theory needs supersymmetry

FAKE NEWS!!

There are multiple string theories that have non-supersymmetric spacetimes

BIG PICTURE

String theory is a theory of Quantum Gravity

At low energies it looks *similar* to our world:

It has GR, chiral matter and gauge symmetries

It is often said that String Theory needs supersymmetry

FAKE NEWS!!

There are multiple string theories that have non-supersymmetric spacetimes

They are less understood/ less studied because we have less computational control w/o SUSY

BIG PICTURE

String theory is a theory of Quantum Gravity

At low energies it looks *similar* to our world:

It has GR, chiral matter and gauge symmetries

It is often said that String Theory needs supersymmetry

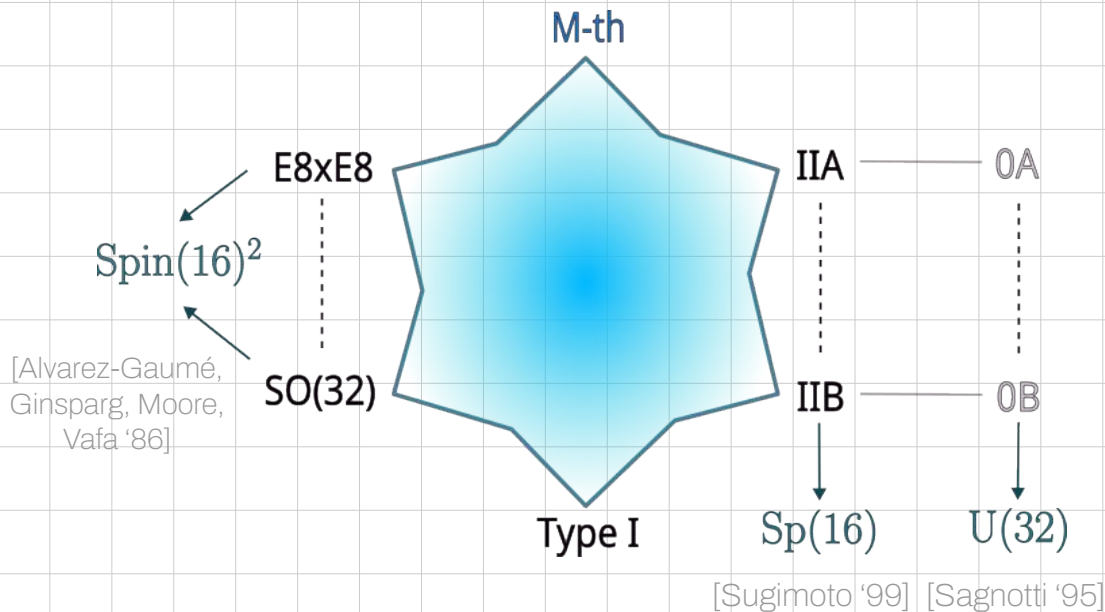
FAKE NEWS!!

There are multiple string theories that have non-supersymmetric spacetimes

They are less understood/ less studied because we have less computational control w/o SUSY

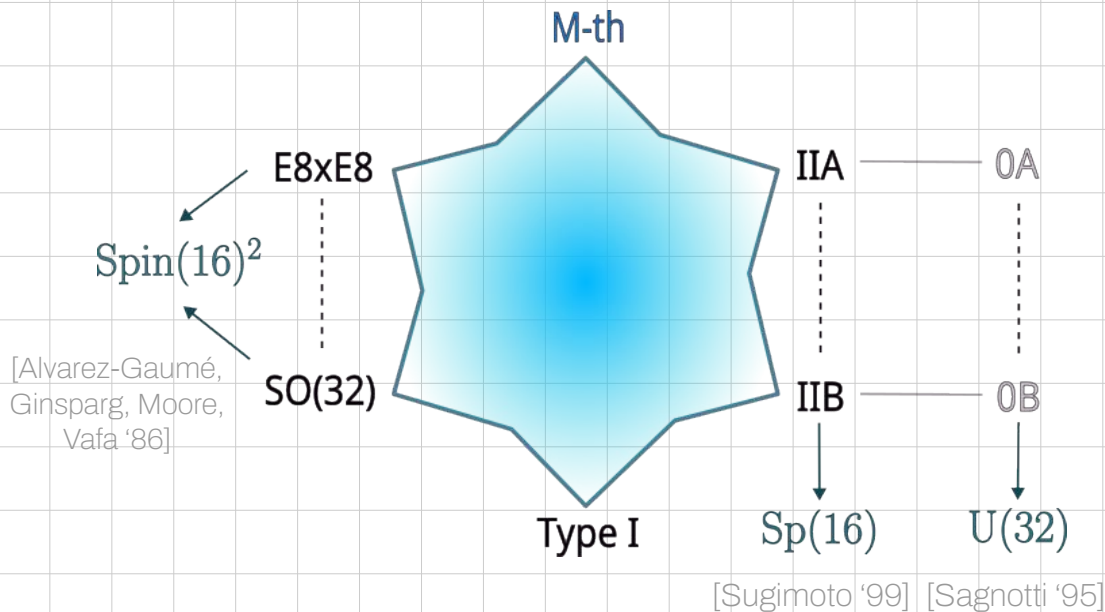
In ten dimensions, there are three of them

10D NON-SUPERSYMMETRIC STRING THEORIES



They can be related by **duality** to the supersymmetric (well-understood string theories).

10D NON-SUPERSYMMETRIC STRING THEORIES



KEY to understand
 Quantum Gravity
 away from SUSY !!

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

What do we know about them?

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

What do we know about them?

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86]

Sp(16)

Gauge symmetries

U(32)

[Sugimoto '99] [Sagnotti '95]

What do we know about them?

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86]

Sp(16)

[Sugimoto '99]

U(32)

[Sagnotti '95]

Gauge symmetries

Do we know if they are fully consistent?

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86]

Sp(16)

[Sugimoto '99]

U(32)

[Sagnotti '95]

Gauge symmetries

Do we know if they are fully consistent?

- worldsheet theory

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

[Alvarez-Gaumé,
Ginsparg, Moore,
Vafa '86]

Sp(16)

[Sugimoto '99]

U(32)

[Sagnotti '95]

Gauge symmetries

Do we know if they are fully consistent?

- worldsheet theory
- Do they have gauge/gravitational anomalies?

They all have chiral degrees of freedom that transform under the gauge syms

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

[Alvarez-Gaumé,
Ginsparg, Moore,
Vafa '86]

Sp(16)

[Sugimoto '99]

U(32)

[Sagnotti '95]

Gauge symmetries

Do we know if they are fully consistent?

- worldsheet theory ✓
- Do they have gauge/gravitational anomalies?
- local anomaly cancellation ✓

They all have chiral degrees of freedom that transform under the gauge syms

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

Sp(16)

U(32)

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86], [Sugimoto '99], [Sagnotti '95]

Gauge symmetries

They all have chiral degrees of freedom that transform under the gauge syms

Do we know if they are fully consistent?

- worldsheet theory ✓
- Do they have gauge/gravitational anomalies?
 - local anomaly cancellation ✓
 - global anomaly cancellation ?

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

Sp(16)

U(32)

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [Sugimoto '99] [Sagnotti '95]

Gauge symmetries

They all have chiral degrees of freedom that transform under the gauge syms

Do we know if they are fully consistent?

- worldsheet theory ✓
- Do they have gauge/gravitational anomalies?
- local anomaly cancellation ✓
- global anomaly cancellation ?

Our work answers this question

10D NON-SUPERSYMMETRIC STRING THEORIES

They all have low-energy effective actions that are schematically given by:

$$S \sim \frac{1}{2\kappa^2} \int dx^{10} \sqrt{-g} \left(R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} \text{tr}|F|^2 - T e^{a\phi} + \dots \right)$$

Dynamical Gravity

Spin(16)²

Sp(16)

U(32)

[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [Sugimoto '99] [Sagnotti '95]

Gauge symmetries

They all have chiral degrees of freedom that transform under the gauge syms

Do we know if they are fully consistent?

- worldsheet theory ✓
- Do they have gauge/gravitational anomalies?

- local anomaly cancellation ✓

- global anomaly cancellation ?

Our work answers this question

by computing the relevant bordism groups for these theories

ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike anomalies in global symmetries)

ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

$$Z[X_d] \implies \tilde{Z}[X_d] \neq Z[X_d]$$


ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

$$Z[X_d] \implies \tilde{Z}[X_d] \neq Z[X_d]$$

- Local anomalies = “*usual ones*”, associated to gauge transformations that can be made **arbitrarily small**
Think triangle (n-gon) diagrams \longrightarrow Cancelled by Green-Schwarz mechanism 


ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

$$Z[X_d] \implies \tilde{Z}[X_d] \neq Z[X_d]$$

- Local anomalies = “*usual ones*”, associated to gauge transformations that can be made **arbitrarily small**
Think triangle (n-gon) diagrams \longrightarrow Cancelled by Green-Schwarz mechanism 
- Global anomalies = associated to a transformation that cannot be *deformed to the identity*
Example: Witten's SU(2) anomaly [Witten '82]


ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

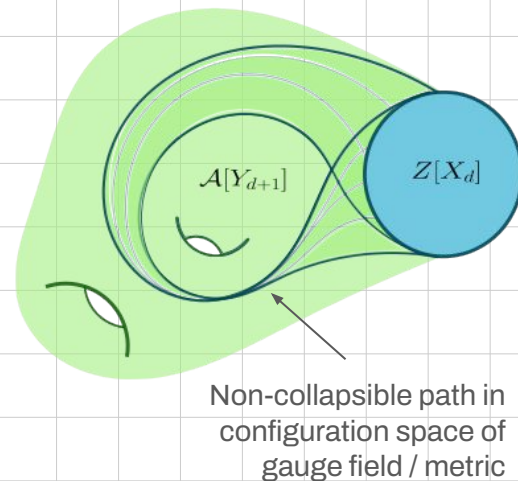
$$Z[X_d] \implies \tilde{Z}[X_d] \neq Z[X_d]$$

- Local anomalies = “*usual ones*”, associated to gauge transformations that can be made **arbitrarily small**
Think triangle (n-gon) diagrams \longrightarrow Cancelled by Green-Schwarz mechanism 
- Global anomalies = associated to a transformation that cannot be *deformed to the identity*
Example: Witten’s SU(2) anomaly [Witten ‘82]

GLOBAL ANOMALIES

The modern way of computing **global gauge and gravitational anomalies** of a theory on X_d is through a $(d+1)$ -dimensional *anomaly theory* on Y_{d+1} such that $\partial Y_{d+1} = X_d$

Review in: [García-Etxebarria, Montero '18]

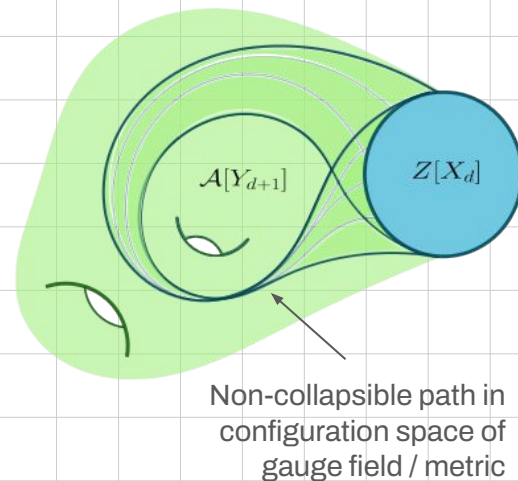


GLOBAL ANOMALIES

The modern way of computing **global gauge and gravitational anomalies** of a theory on X_d is through a $(d+1)$ -dimensional **anomaly theory** on Y_{d+1} such that $\partial Y_{d+1} = X_d$

The anomaly theory is **engineered** to give the **exact (opposite) anomaly** of the one you started with.

Review in: [García-Etxebarria, Montero '18]



GLOBAL ANOMALIES

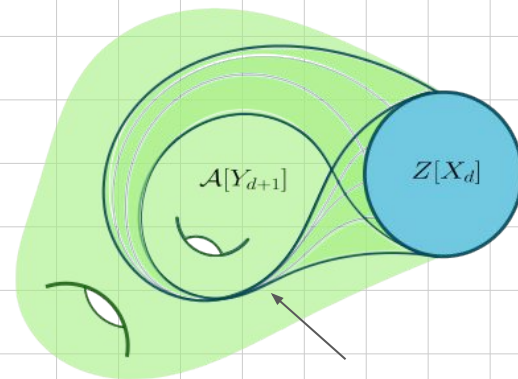
Review in: [García-Etxebarria, Montero '18]

The modern way of computing **global gauge and gravitational** anomalies of a theory on X_d is through a $(d+1)$ -dimensional **anomaly theory** on Y_{d+1} such that $\partial Y_{d+1} = X_d$

The anomaly theory is **engineered** to give the **exact (opposite) anomaly** of the one you started with.

To each anomalous dof in Z , you associate a contribution to the anomaly theory such that:

$$\mathcal{A}(Y_{d+1})Z[X_d] \text{ is anomaly-free}$$



Non-collapsible path in configuration space of gauge field / metric

GLOBAL ANOMALIES

Review in: [García-Etxebarria, Montero '18]

The modern way of computing **global gauge and gravitational** anomalies of a theory on X_d is through a $(d+1)$ -dimensional **anomaly theory** on Y_{d+1} such that $\partial Y_{d+1} = X_d$

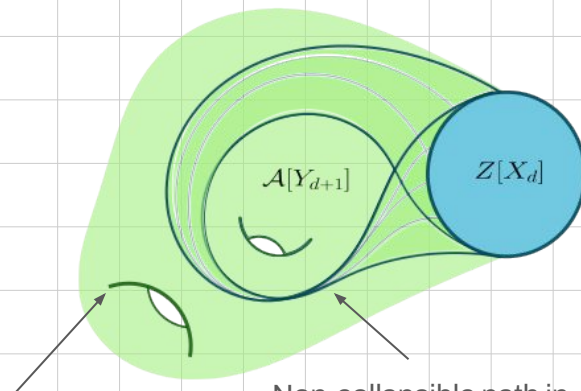
The anomaly theory is **engineered** to give the **exact (opposite) anomaly** of the one you started with.

To each anomalous dof in Z , you associate a contribution to the anomaly theory such that:

$$\mathcal{A}(Y_{d+1})Z[X_d] \text{ is anomaly-free}$$

In QG, allow for topology-change

⇒ “Dai-Freed anomalies”
Account for the possibility of a transformation that involves topology change



[García-Etxebarria, Montero '18]

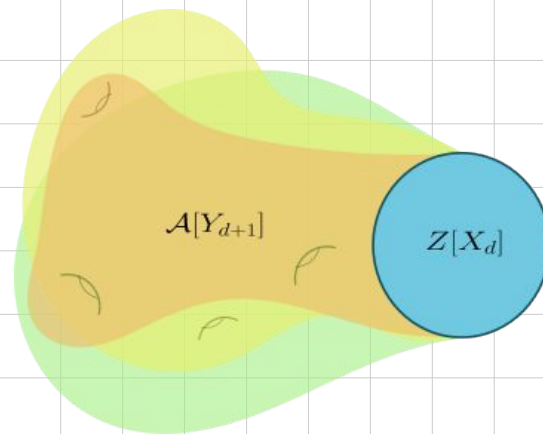
[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

The reason is the anomaly is much easier to detect in the anomaly theory.



[García-Etxebarria, Montero '18]

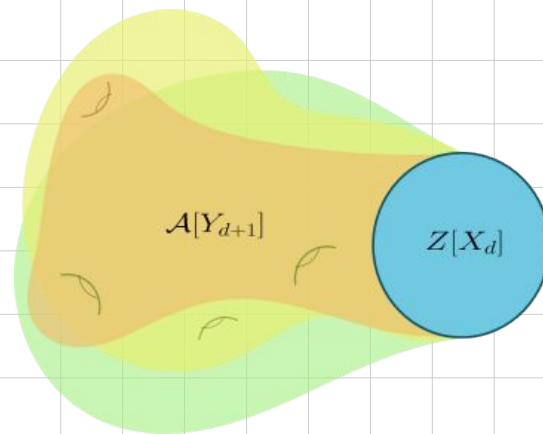
GLOBAL ANOMALIES

So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

The reason is the anomaly is much easier to detect in the anomaly theory.
Here's why:

How do we choose Y_{d+1} ?



[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

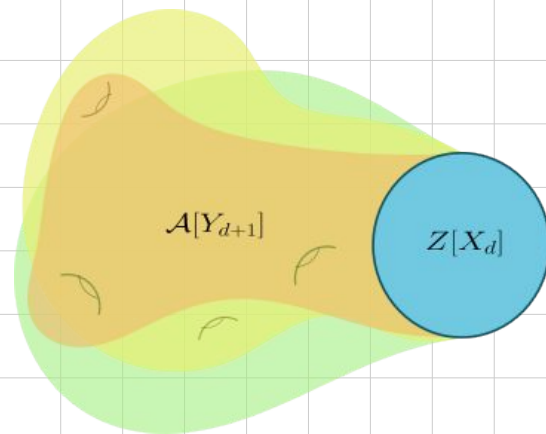
So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

The reason is the anomaly is much easier to detect in the anomaly theory. Here's why:

How do we choose Y_{d+1} ?

The anomaly $\mathcal{A}(Y_{d+1})$ should not depend on the choice of Y_{d+1} !



[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

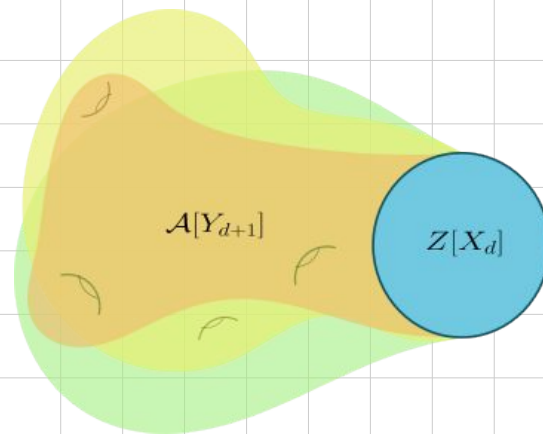
$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

The reason is the anomaly is much easier to detect in the anomaly theory.
Here's why:

How do we choose Y_{d+1} ?

The anomaly $\mathcal{A}(Y_{d+1})$ should not depend on the choice of Y_{d+1} !

You should be able to deform any two choices of Y_{d+1} into one another!



[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

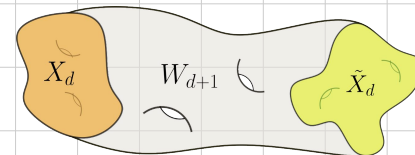
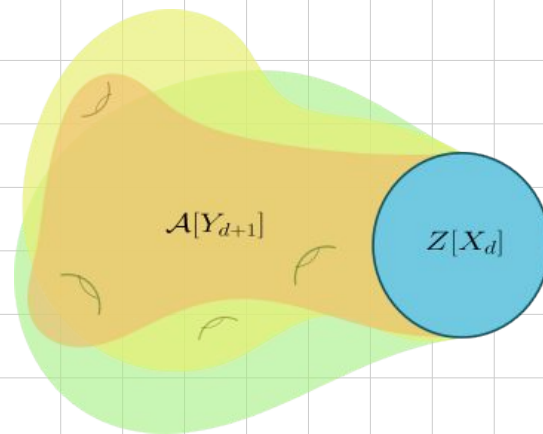
$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

The reason is the anomaly is much easier to detect in the anomaly theory.
Here's why:

How do we choose Y_{d+1} ?

The anomaly $\mathcal{A}(Y_{d+1})$ should not depend on the choice of Y_{d+1} !

You should be able to deform any two choices of Y_{d+1} into one another!



The two d -dimensional manifolds can be deformed into each other
-> They are in the same **bordism class!**

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

So we've constructed an anomaly theory in $(d+1)$ dimensions that gives us the **exact (opposite) anomaly** of the one in our theory:

$\mathcal{A}(Y_{d+1})Z[X_d]$ is anomaly-free

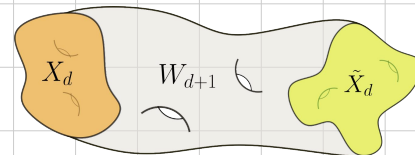
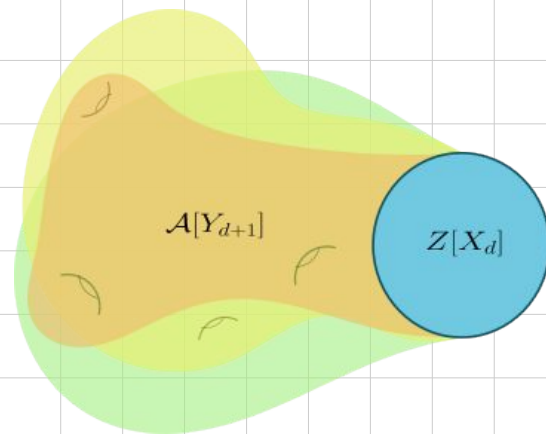
The reason is the anomaly is much easier to detect in the anomaly theory. Here's why:

How do we choose Y_{d+1} ?

The anomaly $\mathcal{A}(Y_{d+1})$ should not depend on the choice of Y_{d+1} !

You should be able to deform any two choices of Y_{d+1} into one another!

⇒ the anomaly is a **bordism invariant!**



The two d -dimensional manifolds can be deformed into each other
 -> They are in the same **bordism class**!

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a **bordism invariant** !

So now we *“just”* have to compute the relevant **11-dimensional bordism groups**!

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a **bordism invariant** !

So now we *“just”* have to compute the relevant **11-dimensional bordism groups**!

So what **11D cobordism groups** are the relevant ones for these three theories?

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a **bordism invariant** !

So now we “*just*” have to compute the relevant **11-dimensional bordism groups**!

So what **11D cobordism groups** are the relevant ones for these three theories?

All three theories only make sense on backgrounds that satisfy the non-trivial **Bianchi identity** associated to H :

$$dH \sim \text{tr}F^2 - \text{tr}R^2 = 0$$

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a **bordism invariant** !

So now we “*just*” have to compute the relevant **11-dimensional bordism groups**!

So what **11D cobordism groups** are the relevant ones for these three theories?

All three theories only make sense on backgrounds that satisfy the non-trivial **Bianchi identity** associated to H :

$$dH \sim \text{tr}F^2 - \text{tr}R^2 = 0$$

→ **twisted string bordism**

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a **bordism invariant** !

So now we *“just”* have to compute the relevant **11-dimensional bordism groups**!

So what **11D cobordism groups** are the relevant ones for these three theories?

All three theories only make sense on backgrounds that satisfy the non-trivial **Bianchi identity** associated to H :

$$dH \sim \text{tr}F^2 - \text{tr}R^2 = 0$$

→ **twisted string bordism**

Not many of them are known, we computed

$$\Omega_{11}^{string-Sp(16)}, \quad \Omega_{11}^{string-Spin(16)^2}, \quad \Omega_{11}^{string-U(32)}$$

using the Adams spectral sequence.

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

We find:

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

We find:

$$\underbrace{\Omega_{11}^{string-Sp(16)} = 0 \quad \Omega_{11}^{string-Spin(16)^2} = 0 \quad \Omega_{11}^{string-U(32)} = 0^*}_{}$$

All 3 bordism groups are completely trivial

*up to a subtlety for the Sagnotti string

i.e. all GLOBAL ANOMALIES VANISH !

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

We find:

$$\underbrace{\Omega_{11}^{string-Sp(16)} = 0 \quad \Omega_{11}^{string-Spin(16)^2} = 0 \quad \Omega_{11}^{string-U(32)} = 0^*}_{}$$

All 3 bordism groups are completely trivial

*up to a subtlety for the Sagnotti string

i.e. all GLOBAL ANOMALIES VANISH !

Huge consistency check!




NO GLOBAL ANOMALIES!




➤ NO GLOBAL ANOMALIES!

But that's not all ...

➔ We use anomaly inflow to gain insight into the worldvolume theory of 5-branes in these theories 

➤ NO GLOBAL ANOMALIES!

But that's not all ...

➔ We use anomaly inflow to gain insight into the worldvolume theory of 5-branes in these theories 


➔ **We also computed lower-dimensional cobordism groups for these theories!!**

$$\begin{array}{ll}
 \Omega_0^{\text{String-Sp}(16)} \cong \mathbb{Z} & \Omega_6^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 \\
 \Omega_1^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_7^{\text{String-Sp}(16)} \cong \mathbb{Z}_4 \\
 \Omega_2^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_8^{\text{String-Sp}(16)} \cong \mathbb{Z}^{\oplus 3} \oplus \mathbb{Z}_2 \\
 \Omega_3^{\text{String-Sp}(16)} \cong 0 & \Omega_9^{\text{String-Sp}(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
 \Omega_4^{\text{String-Sp}(16)} \cong \mathbb{Z} & \Omega_{10}^{\text{String-Sp}(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
 \Omega_5^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_{11}^{\text{String-Sp}(16)} \cong 0.
 \end{array}$$

Example: Sugimoto

➤ NO GLOBAL ANOMALIES!

But that's not all ...

➔ We use anomaly inflow to gain insight into the worldvolume theory of 5-branes in these theories 

➔ **We also computed lower-dimensional cobordism groups for these theories!!**

$$\begin{array}{ll}
 \Omega_0^{\text{String-Sp}(16)} \cong \mathbb{Z} & \Omega_6^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 \\
 \Omega_1^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_7^{\text{String-Sp}(16)} \cong \mathbb{Z}_4 \\
 \Omega_2^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_8^{\text{String-Sp}(16)} \cong \mathbb{Z}^{\oplus 3} \oplus \mathbb{Z}_2 \\
 \Omega_3^{\text{String-Sp}(16)} \cong 0 & \Omega_9^{\text{String-Sp}(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
 \Omega_4^{\text{String-Sp}(16)} \cong \mathbb{Z} & \Omega_{10}^{\text{String-Sp}(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
 \Omega_5^{\text{String-Sp}(16)} \cong \mathbb{Z}_2 & \Omega_{11}^{\text{String-Sp}(16)} \cong 0.
 \end{array}$$

Example: Sugimoto

By the **Cobordism Conjecture** we know they all have to **vanish in QG**.

[McNamara, Vafa '19]

This predicts the existence of **new extended objects** in these theories, that can trivialize these classes!

On the quest to characterizing these new extended objects:

[Andriot, Angius, Blumenhagen, Buratti, Carqueville, Cribiori, Calderon-Infante, DeBiasio, Debray, Delgado, Dierigl, Friedrich, Garcia-Etxebarria, Hebecker, Heckman, Huertas, Kneissl, Makridou, Montero, McNamara, Lust, Torres, Uranga, Vafa, Valenzuela, Velazquez, Walcher, Wang...'19-'24]

THANKS!