Quantisation Across Bubble Walls and Friction

Symmetry–breaking & symmmetry–restoring PTs

SISSA & INFN Trieste

Giulio Barni

gbarni@sissa.it

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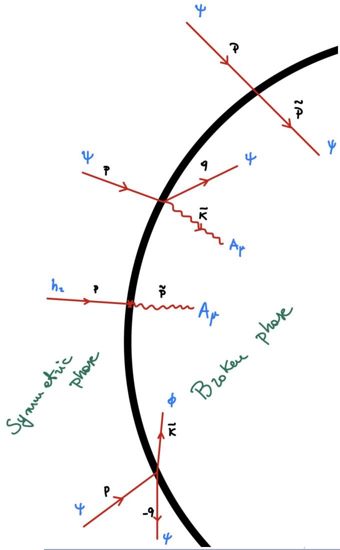
with *Aleksandr Azatov, Rudin Petrossian-Byrne and Miguel Vanvlasselaer*

Outline

1. FOPT: Why, What & bubbles dynamic
2. Toolkit for Quantisation across the wall (scalar example)
   1. Complete basis of solutions to the spatially dependent EOM
   2. Quantisation & construction of ‘In‘ and ‘Out‘ asymptotic states
   3. Amplitudes
   4. Approximations (Step wall and WKB)
3. **Gauge-fixing** and spin-interpolation
   1. EOM + gauge fixing
   2. Interpolation between Higgs and longitudinal polarisation
4. Results & Conclusions



FOPT: Why, What & bubbles dynamic

FOPT: Why?

Today, there is no known FOPT of the fundamental interactions in 4d at $\mu = 0$ for any T!

apart from Higgs instability, but not conclusive...

But...

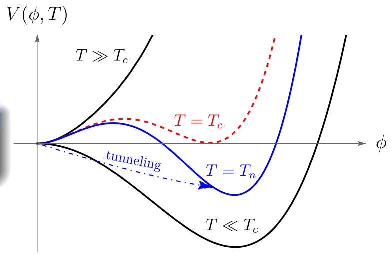
They frequently appear in **BSM** theories!

- **Many phenomenological consequences**: baryogenesis, dark matter, ...
- **Gravitational waves**: a stochastic background potentially observable at upcoming detectors (even completely decoupled sectors become interesting).

FOPT: What?

Vacuum decay rate

$$\Gamma \sim T^4 e^{-S_3/T}, \qquad \text{Euclidean action}$$



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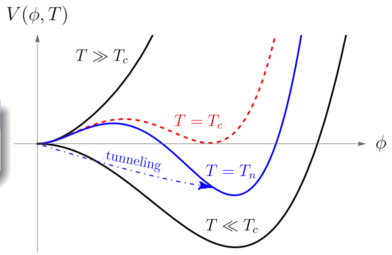
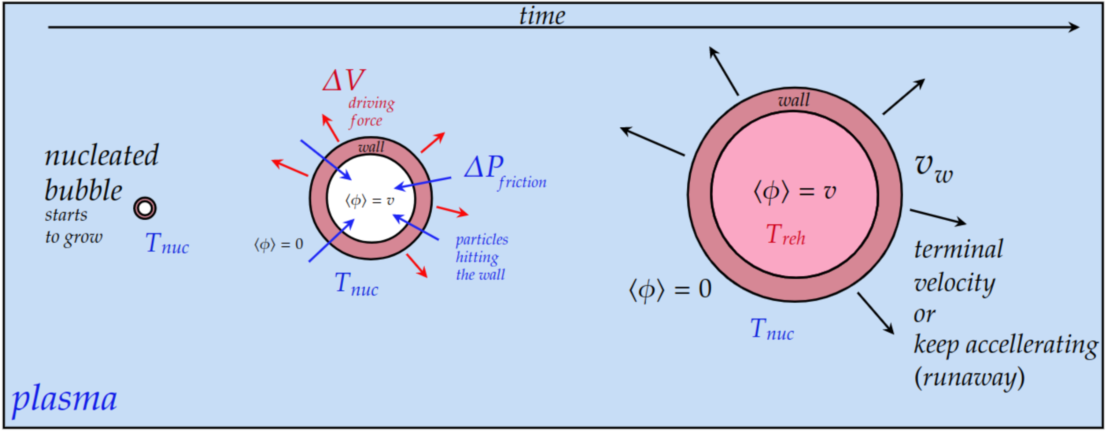


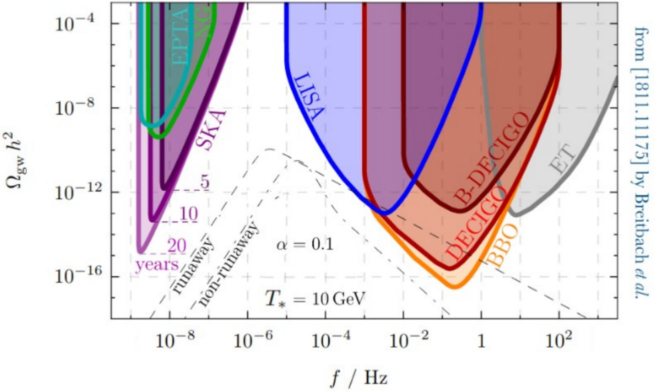
Solution w/ minimal action $\Rightarrow$ $O(d)$ spherical symm. $\Rightarrow$ **Bubbles!**

FOPT: Bubble dynamic



FOPT: Why distinguish between runaway or not?

Strong friction could change the GW signal



$$\alpha \equiv \frac{\Delta V}{\rho_{\mathrm{SM}}(T_*)}$$

Runaway or not is important to know the equilibrium velocity!

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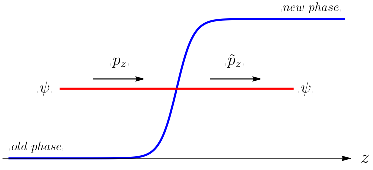

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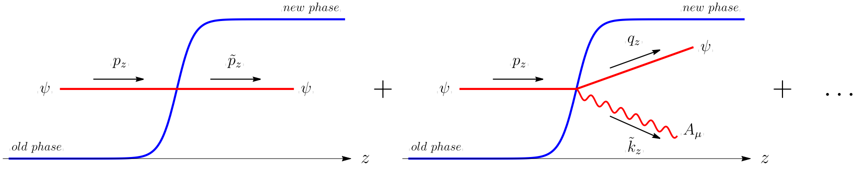

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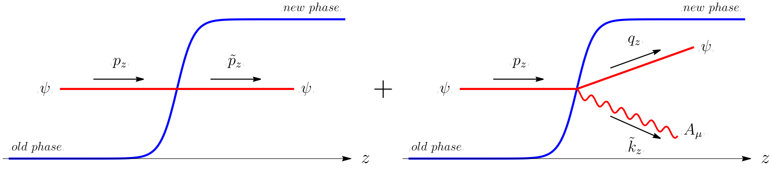

+ . . .

$$\mathcal{P} = \underbrace{\int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_i^{\text{eq}}}_{\text{incoming flux}} \times \underbrace{\sum_f \int d\mathbb{P}_{i\to f}\Delta p^z}_{\langle \Delta p^z\rangle} \sim (\gamma_w n v_w)\langle \Delta p\rangle$$

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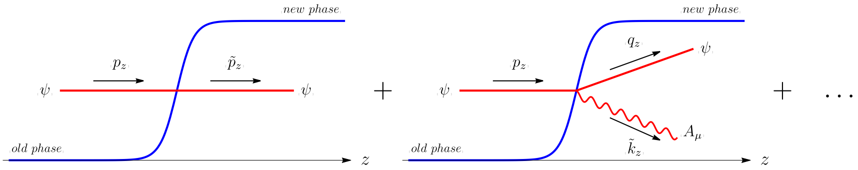

$$\mathcal{P} \sim (\gamma_w n v_w)\langle \Delta p \rangle$$

$\sim \Delta m^2 T_{\text{nuc}}^2$
constant as $\gamma_w \to \infty$
Bödeker, Moore ['09]

$\sim \dfrac{1}{16\pi^2} g^2 m_V \gamma_w T_{\text{nuc}}^3$
Bödeker, Moore ['17]
Azatov, Vanvlasselaer ['20]
Gouttenoire, Jinno, Sala ['21]

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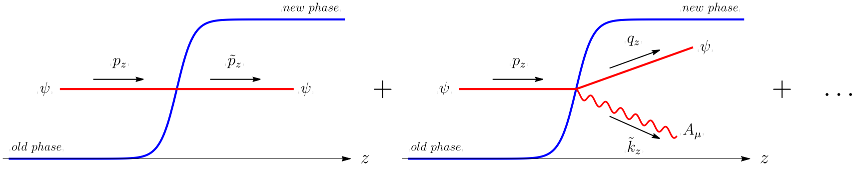
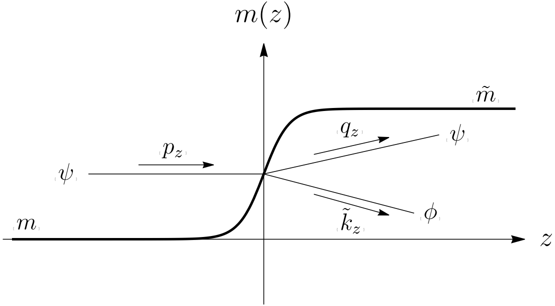


Weaknesses: { WKB approx. despite IR dominated emission; Longitudinal pol. not properly considered }



Toolkit for Quantisation across the wall

Scalar example

ϕ, ψ scalars: $-\mathcal{L} \supset \frac{1}{2}m_\phi^2(z)\phi^2 + \frac{1}{2}m_\psi^2\psi^2 + \frac{y}{2}\psi^2\phi$

$\rightarrow$ $m_\psi = const$ does not feel the wall

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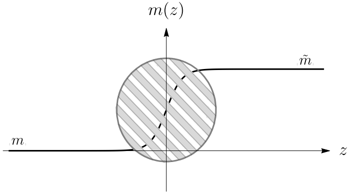

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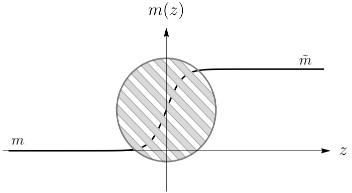
$$\chi_R(z) = \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z \rightarrow -\infty \\ t_k e^{i\tilde{k}^z z} & z \rightarrow +\infty \end{cases}$$



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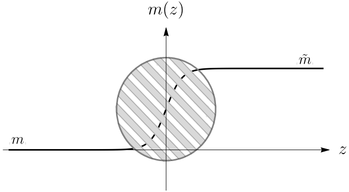

R with $k^z \lessgtr 0$ complete basis, but not orthogonal!

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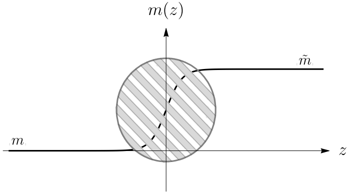

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NEW!

$$\chi_L(z) = \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z \to -\infty \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z \to +\infty \end{cases}$$

$$\int_{-\infty}^{\infty} dz\ \chi_{I,k^z}\chi^*_{J,q^z} = 2\pi\delta_{IJ}\delta(k^z - q^z), \quad I, J \in L, R$$

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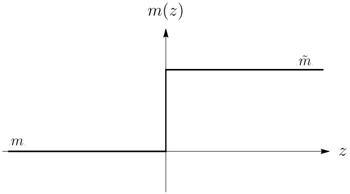
where $r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z}$ and $t_k = \frac{2k^z}{\tilde{k}^z + k^z}$



STEP WALL APPROX.
(valid in the IR)

Scalar emission: Quantisation

Having a complete set of states $\{\phi_{R,k^z}, \phi_{L,k^z}\}$ we can expand the field

$$\phi = \sum_{I=R,L} \int \frac{dk^3}{(2\pi)^3\sqrt{2k_0}} \left(a_{I,k^z}\phi_{I,k^z} + h.c.\right) \,, \qquad \begin{cases} [a_{I,k^z}, a^\dagger_{J,q^z}] = (2\pi)^3 \delta_{IJ}\delta^{(3)}(k-q) \\ [a_{I,k^z}, a_{J,q^z}] = [a^\dagger_{I,k^z}, a^\dagger_{J,q^z}] = 0 \end{cases}$$

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We can define two types of states

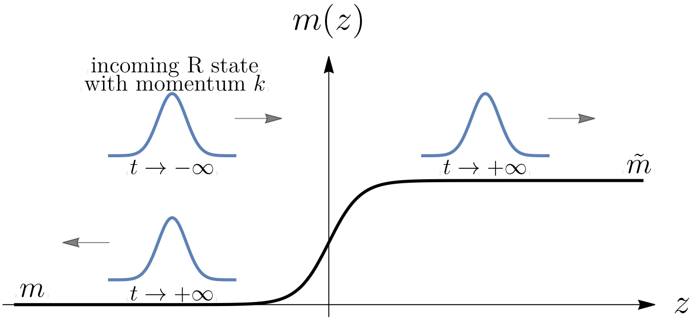
$$|k^z_R\rangle = \sqrt{2k_0}a^\dagger_{R,k^z}|0\rangle,$$
$$|k^z_L\rangle = \sqrt{2k_0}a^\dagger_{L,k^z}|0\rangle,$$

which should be thought as independent states in any process.

Scalar emission: complete basis for outgoing states

To compute $\langle \Delta p \rangle$ we need states with **definite final momentum**!

How we interpret the emission of a R movers?



Definite initial momentum, but not $\hat{P}$ eigenstate for $t \to +\infty$

Scalar emission: complete basis for outgoing states

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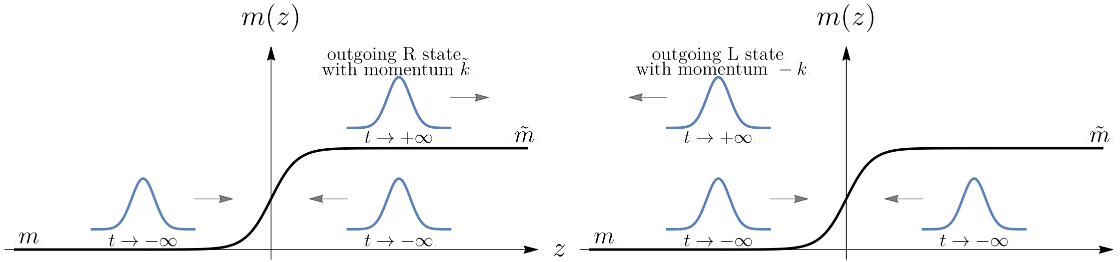
Then we define basis for outgoing states

$$|k_R^{\text{out}}\rangle = t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ |k_R^{\text{in}}\rangle - r_k^*\ |k_L^{\text{in}}\rangle,$$

$$|k_L^{\text{out}}\rangle = r_k^*\ |k_R^{\text{in}}\rangle + t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ |k_L^{\text{in}}\rangle\ \theta(\tilde{k})$$

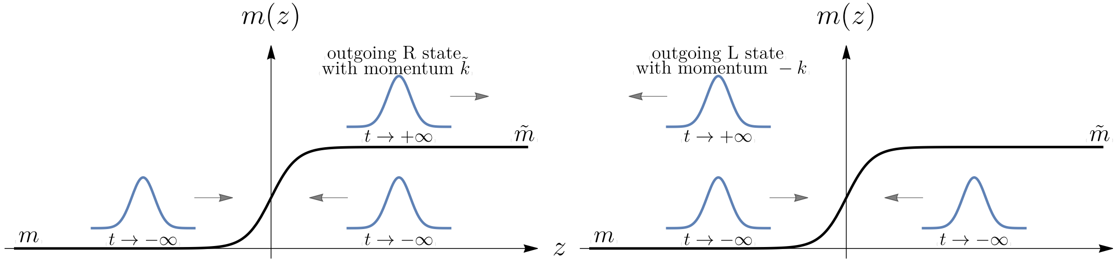


Scalar emission: complete basis for outgoing states

To compute $\langle \Delta p \rangle$ we need states with **definite final momentum**!
Then we define basis for outgoing states $\rightarrow \quad \{\zeta_{R,k^z}, \zeta_{L,k^z}\}$

$$\zeta_R = t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ \chi_R - r_k^*\ \chi_L \equiv \chi_L^*,$$

$$\zeta_L = r_k^*\ \chi_R + t_k^* \sqrt{\frac{\tilde{k}^z}{k^z}}\ \chi_L\ \theta(\tilde{k}) \equiv \chi_R^*$$



Scalar emission: Amplitudes & Phase Space

We are ready to compute the **amplitudes**

$$\mathcal{S} = \mathrm{T}\exp\left(-i\int d^4x \mathcal{H}_{\mathrm{Int}}\right) \qquad \mathcal{H}_{\mathrm{Int}} = -iy\psi^2(x)\phi(x)$$

$$\langle k_I^{\mathrm{out}}\, q|\, \mathcal{S}\, |p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n) i\mathcal{M}_I \overset{\mathrm{tree}}{=} -i\int d^4x \langle k_I^{\mathrm{out}}\, q|\, \mathcal{H}_{\mathrm{Int}}\, |p\rangle$$

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Then the **averaged exchanged momentum**

$$\begin{aligned}\langle\Delta p\rangle &= \langle\Delta p_R\rangle + \langle\Delta p_L\rangle \\ &= \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_R}}(p^z - q^z - \tilde{k}^z) + \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_L}}(p^z - q^z + k^z)\end{aligned}$$

$$\int d\mathbb{P}_{\psi\to\psi\phi_I}\Delta p_I^z = \int_{k^{z,I}_{\rm min}}^{k^z_{\rm max}} \frac{dk_z}{2\pi}\frac{1}{2k_0}\int_0^{k^2_{\perp,\rm max}} \frac{dk_\perp^2}{4\pi}\cdot\frac{1}{2p^z}\left[\frac{1}{2|q^z|}|\mathcal{M}_I|^2\Delta p_I^z\right]_{q^z=\pm q_k^z}$$

Scalar emission: Beyond step wall $\rightarrow$ WKB

When does the step wall approximation break?

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- If the z **momentum is large enough** $(k^z L_w \gtrsim 1)$ there will be **mostly transmission**! $\rightarrow$ WKB

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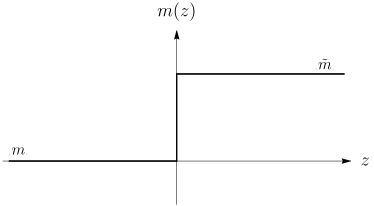
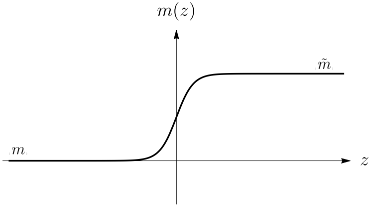
$k^z \lesssim L_w^{-1}$



Step wall: $\zeta_{R,L}$

$L_w^{-1} \lesssim k^z \leq k^z_{\max}$



WKB: $\chi_R(z) = \sqrt{\frac{k^z}{k^z(z)}} e^{-i\int_0^z dz' k^z(z')z'}$

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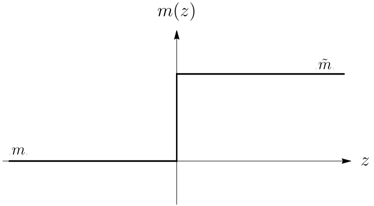
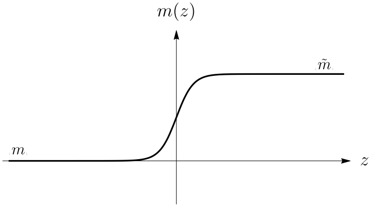




Step wall: $\zeta_{R,L}$ WKB: $\chi_R(z) = \sqrt{\frac{k^z}{k^z(z)}} e^{-i\int_0^z dz' k^z(z')z'}$

- When $\Delta p L_w \gg 1$ then $\mathcal{M} \to 0$ ($z-$momentum conservation is restored!)

Total contribution for $\langle \Delta p \rangle$

The integral over the phase space thus splits into two contributions and the averaged momentum exchange very schematically takes the form

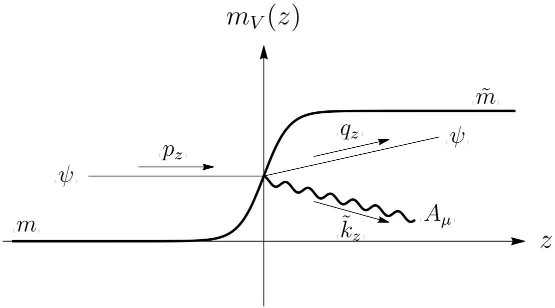
$$\langle \Delta p \rangle \sim \int^{k^z < L_w^{-1}} d^3k \, \Delta p \, |\mathcal{M}^{\text{step}}|^2 + \int_{k^z > L_w^{-1}} d^3k \, \Delta p \, |\mathcal{M}^{\text{wkb}}|^2 \, .$$

then we need to compute 3 contributions: $L-$**step,** $R-$**step and WKB**

$$\langle \Delta p_L^{\text{step}} \rangle = \int_0^{k^z_{\max}} \frac{dk_z}{2\pi} \frac{1}{2k_0} \int_0^{k^2_{\perp,\max}} \frac{dk_\perp^2}{4\pi} \cdot \frac{1}{2p^z} \left[\frac{1}{2|q^z|} |\mathcal{M}_L|^2 (p^z - q^z + k^z) \right] \Theta(L_w^{-1} - k^z) \, ,$$

$$\langle \Delta p_R^{\text{step}} \rangle = \int_{\Delta m}^{k^z_{\max}} \frac{dk_z}{2\pi} \frac{1}{2k_0} \int_0^{k^2_{\perp,\max}} \frac{dk_\perp^2}{4\pi} \cdot \frac{1}{2p^z} \left[\frac{1}{2|q^z|} |\mathcal{M}_R|^2 (p^z - q^z - \tilde{k}^z) \right] \Theta(L_w^{-1} - k^z) \, ,$$

$$\langle \Delta p^{\text{wkb}} \rangle = \int_{\Delta m}^{k^z_{\max}} \frac{dk_z}{2\pi} \frac{1}{2k_0} \int_0^{k^2_{\perp,\max}} \frac{dk_\perp^2}{4\pi} \cdot \frac{1}{2p^z} \left[\frac{1}{2|q^z|} |\mathcal{M}^{\text{wkb}}|^2 (p^z - q^z - \tilde{k}^z) \right] \times \Theta\left(k^z - L_w^{-1}\right) \Theta\left(L_w^{-1} - (p^z - q^z - \tilde{k}^z)\right) \, .$$



Gauge–fixing and spin–interpolation

(symmetry breaking PT)

Vector boson emission: Abelian Higgs model

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 - V(\sqrt{2}|H|) + |D_\mu \psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

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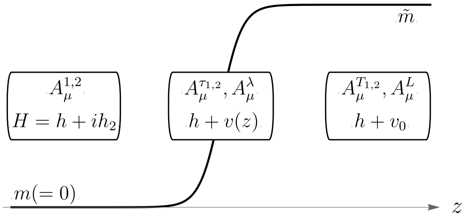
EOM: R_ξ gauge

$$\partial_z^2 v(z) = V'(v)$$
$$\Box h = -V''(v)h$$
$$\Box h_2 = -\xi g^2 v^2 h_2 - V'(v)\frac{h_2}{v} - 2g\partial_\mu v A^\mu$$
$$\partial_\nu F^{\mu\nu} = \frac{1}{\xi}\partial^\mu(\partial_\nu A^\nu) + g^2 v^2 A^\mu - 2g h_2 \partial^\mu v$$



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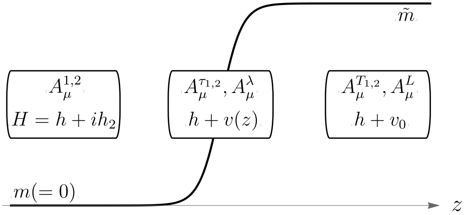
EOM: Unitary gauge $\xi \to \infty$

$$\partial_z^2 v(z) = V'(v)$$
$$\Box h = -V''(v)h$$
$$\partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \equiv m^2(z) A^\mu$$

(new!) Transversality condition $\begin{cases} \partial_\mu(m^2(z)A^\mu) = 0 \\ 3 \text{ propagating dofs} \end{cases}$



Vector boson emission: τ and λ polarisations

Generalized Lorentz condition: $\boxed{\partial_\mu(m^2(z)A^\mu) = 0}$

$k^\mu = (k_0, k_\perp, 0, k^z)$

Vector boson emission: τ and λ polarisations

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- $A_z = 0 \to \boxed{\partial_\mu A^\mu = 0} \Rightarrow \tau-$**polarisations**:

$$\epsilon_\mu^{\tau_1} = (0, 0, 1, 0), \qquad \epsilon_\mu^{\tau_2} = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2}$$

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- $A_z \neq 0 \Rightarrow \lambda-$**polarisation**: $A_\mu^\lambda = \partial_i a + A_z$ with $i = 0, 1, 2$ $\qquad E = \sqrt{k_0^2 - k_\perp^2}$

$$\epsilon_\mu^\lambda = \frac{k_\mu}{m(z)} \times \frac{k^z}{E} + \left(0, 0, 0, \frac{m(z)}{E}\right) \quad \to \quad A_\mu^\lambda = \partial_\mu a + \frac{m(z)^2}{E^2}(0,0,0,A_z)$$

(τ, λ) best suited for the problem, they differ from conventional (T, L) (agree only for $k_\perp = 0$).

Vector boson emission: λ interpolates between h_2 and $A^{(\lambda)}$

EOM for A_z: $$\left[-E^2 - \partial_z^2 + m(z)^2 - 2\left(\frac{m'}{m}\right)\partial_z + 2\left(\frac{m'}{m}\right)^2 - 2\frac{m''}{m}\right]A_z = 0$$

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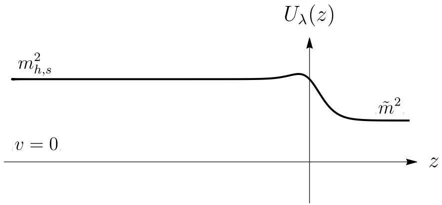
Now defining $A_z = \dfrac{E}{m(z)}\lambda$ we get

$$\left[-E^2 - \partial_z^2 + U_\lambda(z)\right]\lambda = 0$$

Symmetric $\rightarrow$ Broken



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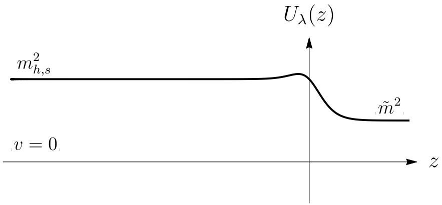

$$U_\lambda(-\infty) = m^2_{h,s} \quad \underset{-\infty \leftarrow z}{\overset{z \to +\infty}{\longleftrightarrow}} \quad U_\lambda(+\infty) = \tilde{m}^2$$

Can be proven $\forall\ V(v)$

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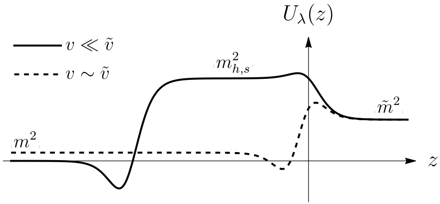

$$U_\lambda(-\infty) = m^2 \quad \xleftrightarrow[-\infty \leftarrow z]{z \rightarrow +\infty} \quad U_\lambda(+\infty) = \tilde{m}^2$$

Can be proven $\forall\ V(v)$

Vector boson emission: step wall solution

$\tau-$**polarisations**: $A^\tau_\mu = \sum a^\tau_{1,2}\epsilon^{\tau_1,\tau_2}_\mu$, only τ_2 gives a contribution, then

$$a^{\tau_2}_{R,k^z} = e^{-ik_0 t + ik_\perp x_\perp} \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z < 0 \\ t_k e^{i\tilde{k}^z z} & z > 0 \end{cases}$$

$$a^{\tau_2}_{L,k^z} = e^{-ik_0 t + ik_\perp x_\perp} \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z < 0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z > 0 \end{cases}$$

where $k_0 = \sqrt{k_z^2 - m^2 - k_\perp^2}$, $\tilde{k}^z = \sqrt{k_z^2 + m^2 - \tilde{m}^2}$ and $E = \sqrt{k_0^2 - k_\perp^2}$.

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Matching conditions: $\begin{cases} a^{\tau_2}|_{<0} = a^{\tau_2}|_{>0} \\ \partial_z a^{\tau_2}|_{<0} = \partial_z a^{\tau_2}|_{>0} \end{cases} \Rightarrow \begin{cases} r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z} \\ t_k = \frac{2k^z}{\tilde{k}^z + k^z} \end{cases}$

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Vector boson emission: step wall solution

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Matching conditions: $\begin{cases} \lambda v(z)|_{<0} = \lambda v(z)|_{>0} \\ \left.\frac{\partial_z \lambda}{v(z)}\right|_{<0} = \left.\frac{\partial_z \lambda}{v(z)}\right|_{>0} \end{cases} \Rightarrow \begin{cases} r_k = \frac{\tilde{v}^2 k^z - v^2 \tilde{k}^z}{\tilde{v}^2 k^z + v^2 \tilde{k}^z} \\ t_k = \frac{2k^z v \tilde{v}}{\tilde{v}^2 k^z + v^2 \tilde{k}^z} \end{cases}$

Vector boson emission: Quantisation

$$A^\mu = \sum_{I,\ell} \int \frac{d^3k}{(2\pi)^3 \sqrt{2k_0}} \left(a^{\text{out}}_{\ell,I,k} \; e^{-i(k_0 t - \vec{k}_\perp \vec{x})} \; \zeta^\mu_{\ell,I,k}(z) + h.c. \right) \; ,$$

where $I = R, L$ and $\ell = \tau_1, \tau_2, \lambda$.

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where $I = R, L$ and $\ell = \tau_1, \tau_2, \lambda$. The wave modes are constructed as follow

Outgoing states:

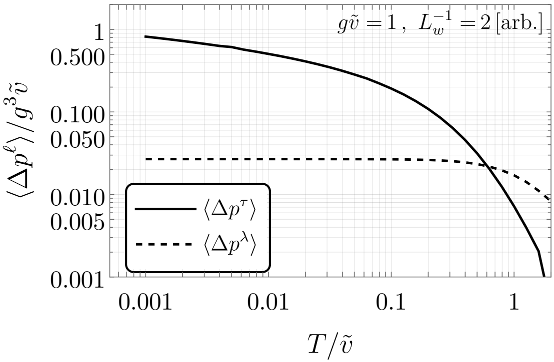
$$\zeta^\mu_{\tau_i,I,k} = \epsilon^\mu_{\tau_i}\ \chi^*_{\tau_i,I,k}(z)\ ,$$

$$\zeta^\mu_{\lambda,I,k} = \left(\frac{-ik^n \partial_z (v\lambda^*_{I,k})}{gEv^2}, \frac{E}{gv}\lambda^*_{I,k} \right) \overset{\text{on shell}}{=} \bar{\partial}^\mu \left(\frac{\partial_z (v\lambda^*_{I,k})}{Eg\ v^2} \right) + \frac{gv(z)}{E}\lambda^*_{I,k}\ \delta^\mu_z\ .$$

Results: $\langle \Delta p \rangle$ in the asymptotic limit $\gamma_w \to \infty$

Relative importance of τ and λ contributions

Symm. $\to$ Broken



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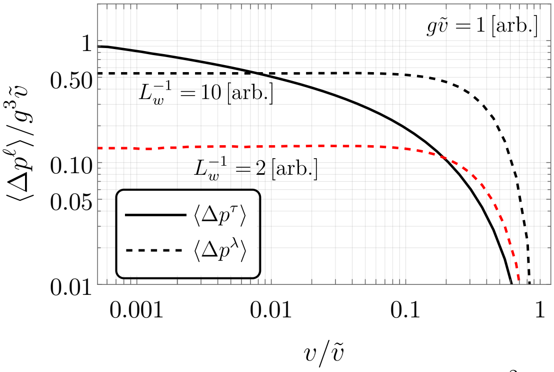
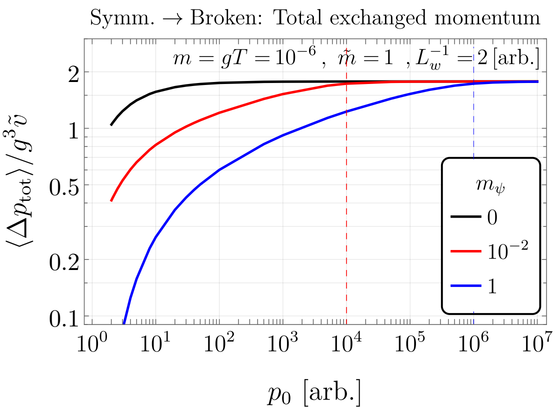


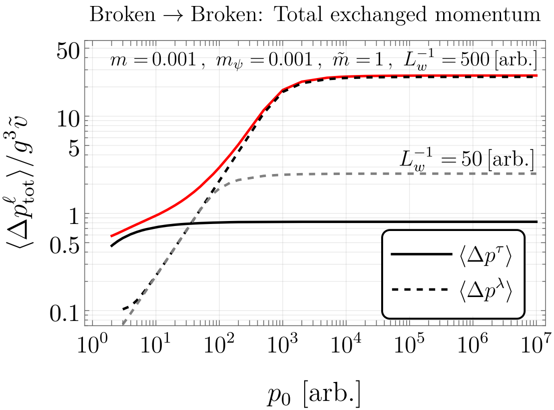
$$\langle \Delta p^\tau \rangle \simeq g^3 \tilde{v} \log \frac{\tilde{v}}{T} \qquad \langle \Delta p^\lambda \rangle \simeq g^3 \tilde{v}\, c_\lambda$$

$$\langle \Delta p^\tau \rangle \simeq g^3 \tilde{v}\, F_\tau \left(\frac{v}{\tilde{v}} \right) \qquad \langle \Delta p^\lambda \rangle \simeq g^2 \left(\frac{v^2 - \tilde{v}^2}{v^2 + \tilde{v}^2} \right)^2 L_w^{-1}$$

Results: Broken → Broken with transient regimes

We are also able to capture transient regimes → ultimately matters to determine equilibrium velocity



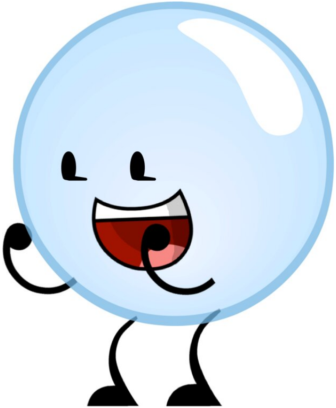


Main conclusions

1. We computed the **transition radiation emission on much more solid basis**, account for **longitudinal emission**, and analysed
   - **symmetric to broken**, aka symmetry–breaking PTs
   - **broken to broken** PTs
   - **broken to symmetric** aka symmetry–restoring PTs [done in 2405.19447]

2. We computed the **friction from transition radiation** in the most **minimal theory** (scalars) and in a **spontaneously broken Abelian gauge theory**.

3. We develop the tools for computing any particle process in such backgrounds.
   (A step towards systematically studying QFT (EFT?) for broken translations)



Thanks for your attention!

Backup slides

Why emission of vector bosons?

Vertex interactions

[JCAP05(2017)025]: Bodeker, Moore

In the relativistic regime the friction can be computed as

$$\Delta\mathcal{P} = \int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_A^{\rm eq} \times \sum_{b,c}\int d\mathbb{P}_{a\to b,c}\Delta p^z$$

where b is soft and

$$\int d\mathbb{P}_{a\to b,c} \sim \int d^2k_\perp \int dx \; |\mathcal{M}(a\to b,c)|^2, \qquad x \equiv \frac{E_c}{E_a}$$

The matrix element is related to the interaction via

$$\mathcal{M}(a\to b,c) = \int dz \; \chi_a(z)\chi_b^*(z)\chi_c^*(z)V(z), \qquad V(z) : \text{vertex}$$

Soft singularity in x matters! $\rightarrow$ **emission of vector bosons!**

$a(p)\to b(k)c(p-k)$	$\lvert V^2\rvert$
$S\to V_T S$ $F\to V_T F$ $V\to V_T V$	$4g^2C_2[R]\frac{1}{x^2}k_\perp^2$
$S\to V_L S$ $F\to V_L F$ $V\to V_L V$	$4g^2C_2[R]\frac{1}{x^2}m^2$
$F\to FV_T$	$2g^2C_2[R]\frac{1}{x}(k_\perp^2+m_b^2)$
$V\to FF$	$2g^2T[R]\frac{1}{x}(k_\perp^2+m_b^2)$
$S\to SV_T$	$4g^2C_2[R]k_\perp^2$
$F\to SF$	$y^2(k_\perp^2+4m_a^2)$
$S\to SS$	$\lambda^2\varphi^2$

FOPT: Bubble dynamic (General case)

Equilibrium velocity of the bubbles (or runaway), $\gamma_w^{\max}$, is setted by

$$\Delta V = \Delta\mathcal{P}(\gamma_w^{\max})$$

- ΔV is independent on the velocity of the wall
- $\Delta\mathcal{P}(\gamma_w^{\max})$ very difficult to compute in general and depends on the velocity

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GENERAL CASE: solve coupled differential system

$$\begin{cases} p^\mu \partial_\mu f_i + \frac{1}{2}\partial_z m_i[\phi]\partial_{p^z} f_i = \mathcal{C}[f_i, \phi] \\ \Box\phi + \frac{dV}{d\phi} + \sum_i \frac{dm_i^2[\phi]}{d\phi}\int \frac{d^3p}{(2\pi)^3}\frac{f_i}{2E_i} = 0 \end{cases}$$

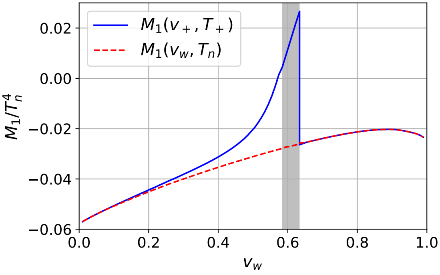

Figure: Friction acting on the bubble expansion. [2102.12490]

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Relativistic case ($\gamma_w \gg 1$)

$$\begin{cases} \mathcal{C} \to 0 \text{ (ballistic regime)}, & f_i \gg f_i^{\text{eq}} \\ \Box\phi + \dfrac{dV_T}{d\phi} = 0 \end{cases}$$

$$\boxed{\Delta\mathcal{P} = \int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_A^{\text{eq}} \times \sum_X \int d\mathbb{P}_{A\to X}\Delta p^z}$$

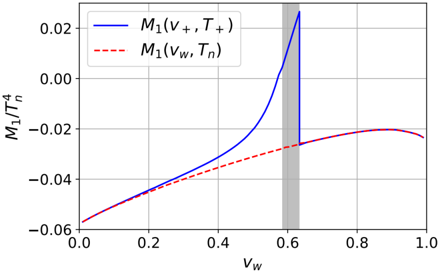

Figure: Friction acting on the bubble expansion. [2102.12490]

Relations between (τ, λ) and (T, L)

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Conventional pol. vectors:

$$\epsilon_{T_1} = (0,0,1,0), \quad \epsilon_{T_2} = \frac{1}{\sqrt{k_\perp^2 + k_z^2}}(0, k^z, 0, -k_\perp), \quad \epsilon_L = \frac{k_0}{m\sqrt{k_0^2 - m^2}}\left(\frac{k_0^2 - m^2}{k_0}, k_\perp, 0, k^z\right)$$

$$\begin{pmatrix} \epsilon_{T_1} \\ \epsilon_{T_2} \\ \epsilon_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{k_0 k^z}{E\sqrt{k_z^2 + k_\perp^2}} & -\frac{k_0 m}{E\sqrt{k_z^2 + k_\perp^2}} \\ 0 & \frac{k_0 m}{E\sqrt{k_z^2 + k_\perp^2}} & \frac{k_0 k^z}{E\sqrt{k_z^2 + k_\perp^2}} \end{pmatrix} \begin{pmatrix} \epsilon_{\tau_1} \\ \epsilon_{\tau_2} \\ \epsilon_\lambda \end{pmatrix}$$

- For $k^z, E \gg k_\perp, m$ mixing between τ, λ scales as m/E
- We are interested in the sum of all contributions $\rightarrow$ all computations in (τ, λ) basis

Amplitude from WKB (details)

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$$\mathcal{M}^{\text{wkb}} = \int_{-\infty}^{\infty} dz\; e^{i(p^z - q^z)z} \chi_R^*(k^z) V(z)$$

$$= \underbrace{\int_{-\infty}^{0} dz V(-\infty) e^{i\Delta p^z(-\infty) z} + e^{i\int_0^{L_w} dz' \Delta p(z')} \int_0^{\infty} dz V(+\infty) e^{i\Delta p^z(+\infty) z}}_{\mathcal{M}_{\text{outside}}} + \underbrace{\int_0^{L_w} dz V(z) e^{i\int_0^{z} dz' \Delta p(z')}}_{\mathcal{M}_{\text{inside}}}$$

$$= \underbrace{\int_{-\infty}^{0} dz V(-\infty) e^{i\Delta p^z(-\infty) z} + \cancel{e^{i\int_0^{L_w} dz' \Delta p(z')}} \int_0^{\infty} dz V(+\infty) e^{i\Delta p^z(+\infty) z}}_{\mathcal{M}_{\text{outside}}} + \cancel{\underbrace{\int_0^{L_w} dz V(z) e^{i\int_0^{z} dz' \Delta p(z')}}_{\mathcal{M}_{\text{inside}}}}$$

$\Delta p(z) L_w \ll 1$ On a case by case basis

$$\boxed{\mathcal{M}^{\text{wkb red.}} = \frac{V(-\infty)}{i\Delta p_z(-\infty)} - \frac{V(+\infty)}{i\Delta p_z(+\infty)}}$$

WKB approximation

WKB approximation

KG eom in Fourier space: $$\chi''(z) + \frac{p_z^2}{\hbar^2}\chi(z) = 0$$

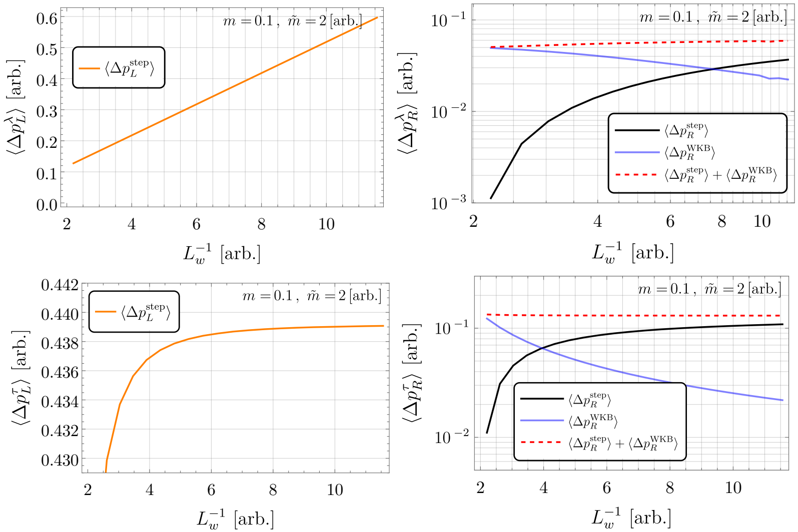
WKB ansatz: $\chi(z) = \exp\left[\frac{i}{\hbar}\left(S_0 + S_1\hbar + S_2\hbar^2 \dots\right)\right]$, put in the differential equation and match terms of the same order in $\hbar$.

$\implies$ Validity: $\hbar|S_0''(z)| \ll |S_0'(z)|$ & $2\hbar|S_0'S_1'| \ll |(\partial_z p^z(z))^2|$

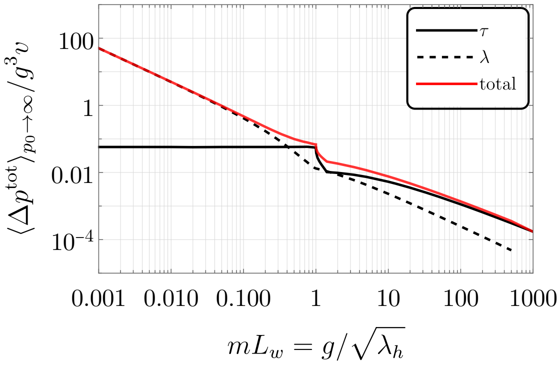
$$\boxed{\chi(z) = \sqrt{\frac{p_s^z}{p^z(z)}}\exp\left[i\int_0^z d\hat{z}\, p^z(\hat{z}) + i\int_0^z d\hat{z}\left(\frac{1}{2}\left(\frac{\partial_z p^z}{p^z}\right)^2 - \frac{\partial_z^2 p^z}{4p_z}\right)\dots\right]}$$

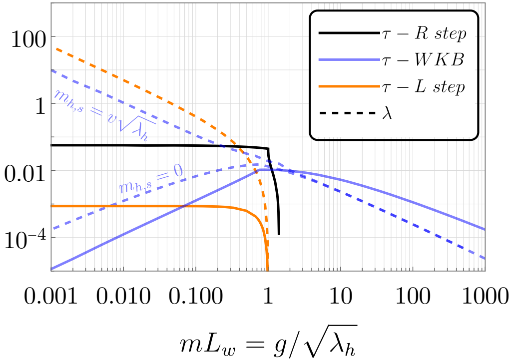
Sensitivity to the wall width

Sensitivity to the wall width: broken to broken



Sensitivity to the wall width: broken to symmetryc





Phase space for vector emission: thermal masses

Phase space for vector emission: thermal masses (vectors)

When thermal corrections become important? We cut the phase space at momentum $|\vec{k}|^2 \sim g^2T^2$, which is equivalent to use the following thermal masses for the vectors (symmetric $\to$ broken)

$$(\tau) : \begin{cases} \tilde{m} = m_\tau(z = +\infty) \approx g\sqrt{\tilde{v}^2 + T^2} \ , \\ m = m_\tau(z = -\infty) \approx gT \ , \end{cases} \quad (\lambda) : \begin{cases} \tilde{m} = m_\lambda(z = +\infty) \approx g\sqrt{\tilde{v}^2 + T^2} \ , \\ m = m_\lambda(z = -\infty) = m_{h,s}(T) \ . \end{cases}$$

gT is not the only scale possible. The self energy for transverse vectors receives ('magnetic mass') thermal corrections only at two loops of parametric order $\sim g^2T$ from charged matter.
For broken to broken transitions the vector masses are, for both λ and τ fields

$$m \approx g\sqrt{v^2 + T^2} \ , \quad \tilde{m} \approx g\sqrt{\tilde{v}^2 + T^2} \ , \qquad \text{(broken to broken)} \ .$$

Ward identity

Ward identity

If the gauge symmetry is preserved, vector bosons can couple only to conserved currents

$$\mathcal{M}^{(4,J)} \equiv \epsilon_k^\mu \mathcal{M}_\mu^{(4,J)} = (\epsilon_k^\mu + k^\mu)\mathcal{M}_\mu^{(4,J)} \quad , \qquad \text{(no wall)}.$$

where the $(4, J)$ label indicates full 4-momentum conservation, J^μ the conserved current and ϵ_k^μ the external particle's polarisation vector.

In the presence of a domain wall in the z direction, the generalised matrix element $\mathcal{M}^{(3)}$ includes an integral over z and the polarisation tensor is also a function thereof. The expression of conservation closest to the previous is

$$\mathcal{M}^{(3,J)} \equiv \int dz\, \chi^\mu_{\ell,I,k}(z)\mathcal{M}_\mu^{(3,J)}(z) = \int dz\, \left(\chi^\mu_{\ell,I,k}(z) + \bar{\partial}^\mu f(z)\right)\mathcal{M}_\mu^{(3,J)}(z) \; ,$$

$$\text{where} \quad \bar{\partial}^\mu \equiv (-ik^n, \partial^z) \; , \qquad \text{(with wall)}$$

and $f(z)$ is an arbitrary function. Example: $f(z) = a(z) = \dfrac{\partial_z \left(v^2\lambda\right)}{E^2\, v^2}$.

Interpolation between h_2 and $A^{(\lambda)}$

Interpolation between h_2 and $A^{(\lambda)}$

To understand this matching better let us look at the χ^μ_λ vector in the limit $v \to 0$

$$\chi^\mu_\lambda = (-ik^n a(z), \chi^z_\lambda) = \left(\frac{-ik^n \partial_z(v\lambda)}{gEv^2}, \frac{E}{gv}\lambda\right)$$
$$= \frac{\lambda}{gv}\left(-\frac{ik^n}{E}\left[\frac{v'}{v} + \frac{\lambda'}{\lambda}\right], E\right)_{v\to 0} = \frac{e^{-ikx}}{gv}\left(\frac{k^n}{E}\left[-im_{h,s} + k^z\right], E\right),$$

where we have used that λ becomes a plane wave far from the wall and $v'/v \to m_{h,s}$. Note that the factor $(-im_{h,s} + k^z)/E$ is a **pure phase** if the λ dof is on shell. Let us see whether we can build exactly the same **vector but from the Goldstone field** h_2. Indeed if we consider the vector

$$\partial^\mu\left(\frac{h_2}{gv}\right) = -\frac{e^{-ikx}}{gv}\left(k^n, k^z + i\frac{v'}{v}\right) = \frac{-iEe^{-ikx}}{gv(k^z - im_{h,s})}\left(\frac{k^n}{E}\left[-im_{h,s} + k^z\right], E\right).$$

So we can see that **the two vectors** χ^μ_λ **and** $\partial_\mu(h_2/gv)$ **are exactly the same apart from the constant phase factor**, so indeed λ field in the $z \to -\infty$ limit corresponds to the Goldstone boson.

Total pressure: fitting formulas

Total pressure: fitting formulas

Symmetric $\to$ Broken

$$\lim_{\gamma_w \to \infty} \mathcal{P}_{\text{th.}} \simeq \frac{\zeta(3)\gamma_w T^3}{\pi^2} \times g^3 \tilde{v} \left[0.135 \log\left(\frac{\tilde{v}}{T} + 2.26\right) - 0.085 - 0.2 \frac{\log\left(\frac{\tilde{v}}{T} + 2.26\right)}{\tilde{v}/T} + \frac{0.19}{\tilde{v}/T} \right]$$

Broken $\to$ Broken

$$\lim_{\gamma_w \to \infty} \mathcal{P}^{\lambda}_{v \neq 0} \sim 0.05 \frac{\zeta(3)\gamma_w T^3}{\pi^2} \times g^2 \frac{(v^2 - \tilde{v}^2)^2}{(v^2 + \tilde{v}^2)^2} \times L_w^{-1}.$$

Current conservation in the presence of the wall

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Considering a conserved current $\partial_\mu J^\mu = 0$, its conservation imposes that any interaction which can be written in the form

$$J^\mu \partial_\mu f \quad \Rightarrow \quad \langle final| \, \mathcal{S}_f \, |initial\rangle = \int d^4x \;\; \partial_\mu f(x) J^\mu(x) = 0 \ ,$$

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For example let us consider $f = \chi^\tau_{1,2}(z)$, where $\chi^\tau_{1,2}$ are the wave functions for the τ polarisations. Then the matrix element will be equal to

$$\begin{aligned} J^\mu \propto (p+q)^\mu \qquad \Rightarrow \qquad \mathcal{M} &= \frac{(p+q)_\mu k^\mu}{\Delta p} + r^\tau_k \frac{(p+q)_\mu k^\mu_r}{\Delta p_r} - t^\tau_k \frac{(p+q)_\mu \tilde{k}^\mu}{\Delta \tilde{p}} \\ &= (p+q)_z (1 + r^\tau_k - t^\tau_k) = 0 \; . \end{aligned}$$

where $k^\mu_r \equiv (k^m, -k^z), \quad \tilde{k}^\mu \equiv (k^m, \tilde{k}^z)$.

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Similarly we can choose $f = \alpha(z) = \frac{1}{gv^2 E}\partial_z(v\lambda)$, of the λ. Using the expression for reflection and transmission coefficients we can compute the amplitude for the processes $J^\mu \to \chi^\mu_\lambda$ corresponding to the interaction $J_\mu \chi^\mu_\lambda$. The computation goes as follows

$$J^\mu \propto (p+q)^\mu \quad \Rightarrow \quad \mathcal{M} = \frac{k^z}{gEv}\frac{(p+q)_\mu k^\mu}{\Delta p} - \frac{k^z}{Egv} r^\lambda_k \frac{(p+q)_\mu k^\mu_r}{\Delta p_r} - \frac{\tilde{k}^z}{gE\tilde{v}} t^\lambda_k \frac{(p+q)_\mu \tilde{k}^\mu}{\Delta \tilde{p}}$$
$$= \frac{(p+q)_z}{E} \underbrace{\left(\frac{k^z}{gv} - \frac{k^z}{gv} r^\lambda_k - \frac{\tilde{k}^z}{g\tilde{v}} t^\lambda_k\right)}_{=0} = 0 \; ,$$

The terms cancelling each other in the brackets are **growing in energy**, which makes crucially important the calculation of exact values of reflection and transmission coefficients.

Non−conserved currents

Non−conserved currents

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 - V(|H|) + |D_\mu\phi|^2 - m_\psi^2|\phi|^2 + (\kappa\phi^2 H + h.c.)$$

where $Q_{U(1)}(H) = 1, Q_{U(1)}(\phi) = -1/2$. The divergence of the current becomes equal to:

$$\partial_\mu J^\mu_\phi = \sqrt{2}v(z)\left(\kappa^*\phi^{*2} - \kappa\phi^2\right) \ , \quad J^\mu_\phi = i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) \ .$$

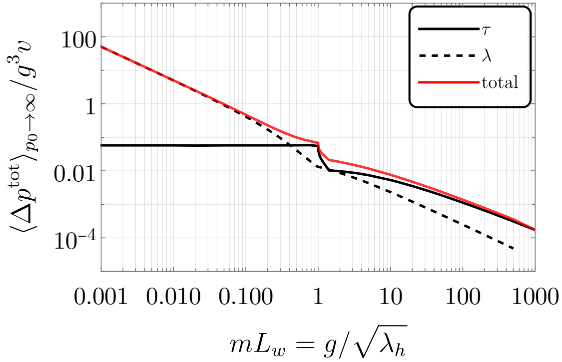
The interaction between the λ polarisation and J^μ_ϕ and can be written as follows:

$$gQ_\phi J^\mu_\phi A^{(\lambda)}_\mu \to Q_\phi\left[-\sqrt{2}\left(\kappa^*\phi^{*2} - \kappa\phi^2\right)\frac{1}{Ev(z)}\partial_z\left(v(z)\lambda(z)\right) - \frac{g^2 v(z)}{E}\lambda(z)J_z\right] \ .$$

We can see that on top of the term λJ_z present in the conserved current case, there is an additional interaction. However **this interaction is not growing with energy**, and in the limit $v(z) \to 0$, it is finite.

Symmetry–restoring PT

Results: Broken $\to$ Symmetric



$$\langle \Delta p^{\text{tot}} \rangle \simeq \frac{g^2}{2\pi^2} L_w^{-1} \simeq \frac{g^2}{2\pi^2} v \sqrt{\lambda_h}$$

Negative pressure?

Negative pressure?

$\rightarrow$ If some particle lose its mass, then $\rightarrow \mathcal{P}_{LO} \sim -\Delta m^2 T^2$, so what about $\mathcal{P}_{NLO}$?
$\rightarrow$ We have found that $\psi \rightarrow \psi A^\mu$ in symmetry restoring PTs produces $\mathcal{P}_{NLO} > 0$
And in general?

$$\Delta p \geq \Delta p_{\text{Min}} = p^z - q^0 - k^0 = p^z - p^0 = -\frac{m_a^2}{2p^z} + \mathcal{O}\left(\frac{1}{p_z^2}\right) \ , \tag{1}$$

which is the leading order minimum. Although this **absolute minimum** is negative, vanish for $p^z \rightarrow \infty$. A lower bound on the average momentum transfer is

$$\langle \Delta p \rangle = \int d\Pi_{\text{BTPH}} \ |\mathcal{M}|^2 \ \Delta p \geq \Delta p_{\text{Min}} \int d\Pi_{\text{BTPH}} \ |\mathcal{M}|^2 = \Delta p_{\text{Min}} \ \mathbb{P} \ , \tag{2}$$

$\mathbb{P}$ is an integrated probability for a physical process and it cannot grow arbitrarily to infinity with incoming particle energy $p_0 \approx p^z$ (less it break unitarity), and we conclude that

$$\langle \Delta p \rangle > -\frac{m_a^2}{2p^z} \ , \qquad \text{for} \quad p^z \rightarrow \infty \ . \tag{3}$$