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Which Standard Model?

— the SM gauge group, SMEFT, and generalized symmetries

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Renaissance of global symmetries

- Generalized symmetries: a mini revolution happened in the last 10 years in hep-th and condensed matter community.
- Global symmetries in QFT are defined as topological operators/defects. In this view, people found many generalizations.

Generalized Global Symmetries

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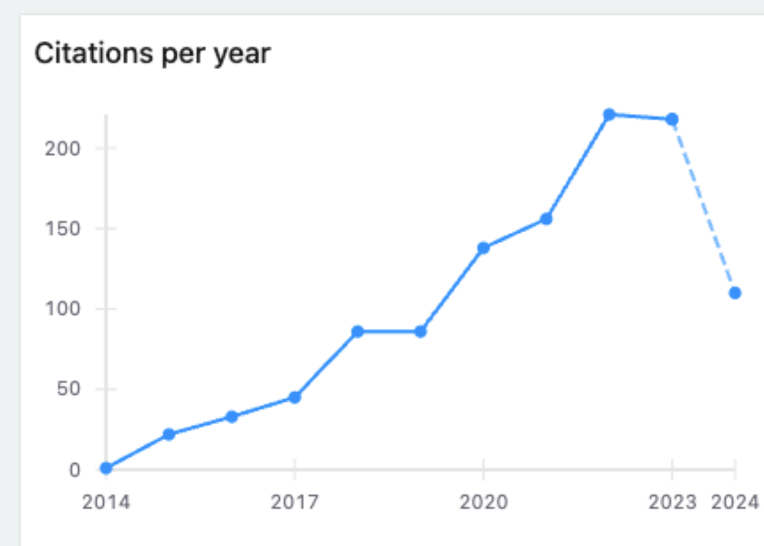
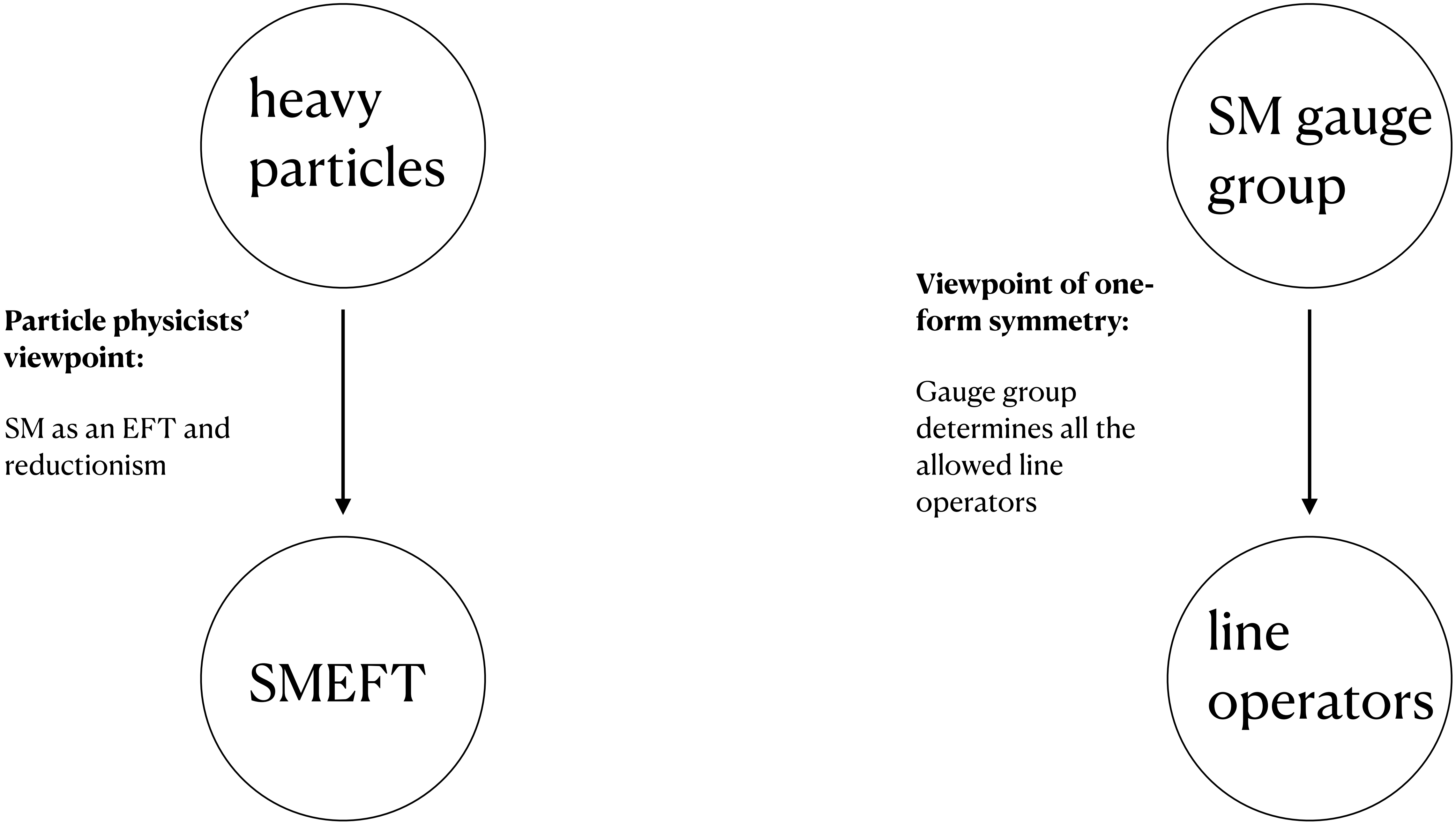


Renaissance of global symmetries

— a particle physicist's view

- Can generalized symmetries be used to solve particle physics problems?
- What are the simplest applications of generalized symmetries in particle physics? (Perhaps a more ambitious question is to find the most striking applications.)

Two perspectives



- As we will see in this talk, there is a natural connection between heavy particles, SMEFT, and line operators (with one-form global symmetry acting on them), hence to determine the SM gauge group.

Higher-form symmetries

- Free Maxwell theory with no matter:
  the Gauss law is understood as electric $U(1)$ 1-form symmetry

- Pure $SU(N)$ gauge theory with no matter:
  the center of the gauge group measures the N-ality of a Wilson line, which is understood as electric Z_N 1-form symmetry

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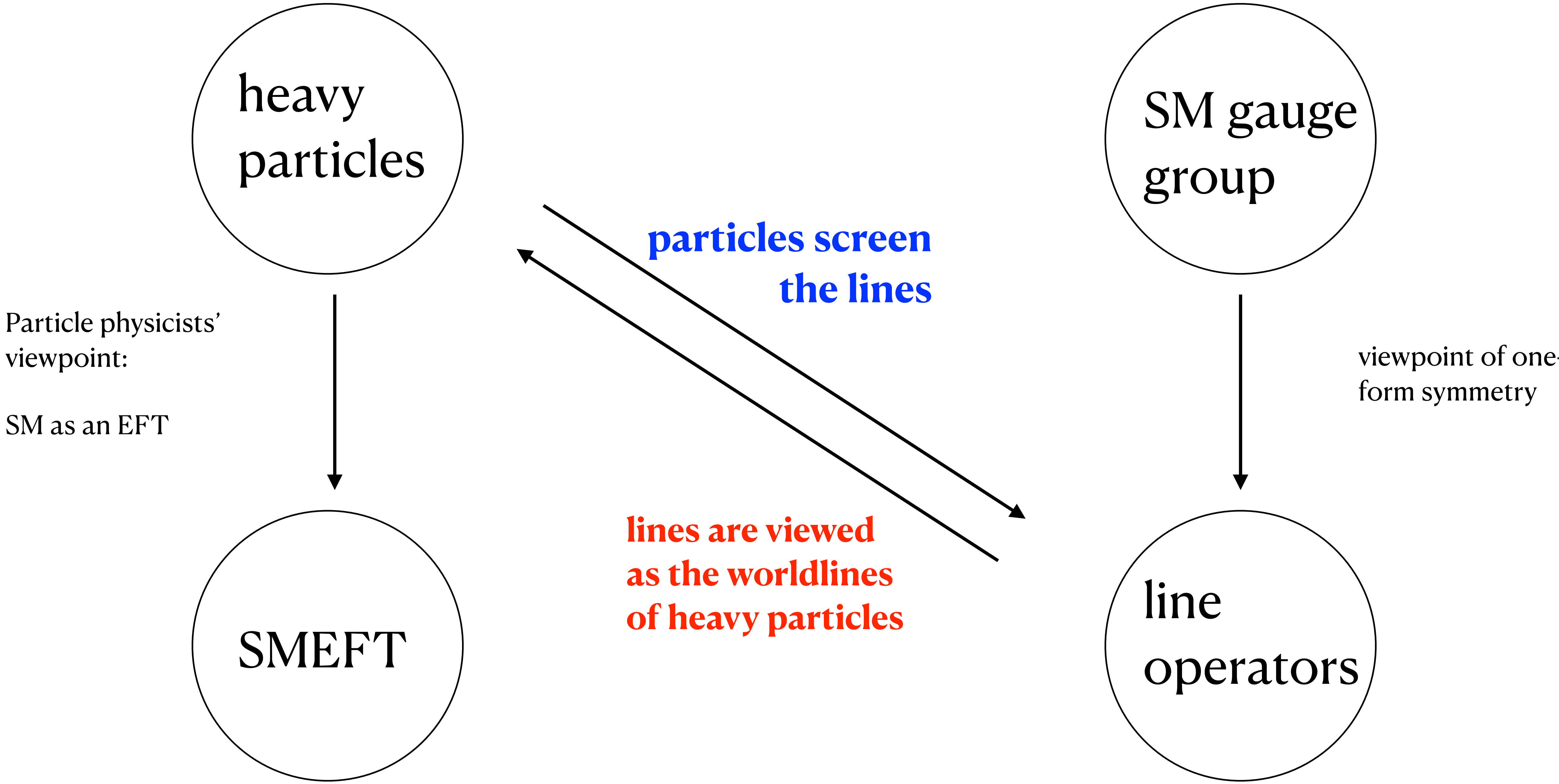
- Nevertheless, the notions of electric 1-form symmetry and Wilson lines are still valid at low energy, i.e. below the mass scale of the heavy particles that screen the Wilson lines. As such, the 1-form symmetry is viewed as accidental.



Unification of two perspectives: there is natural correspondence between heavy particles and line operators!

Toy Model

Example: SU(2) versus SO(3) groups

- They are sometimes use interchangeably

- But we have to keep in mind they are not exactly the same, namely $SO(3) \sim \dfrac{SU(2)}{\mathbb{Z}_2}$, where $\mathbb{Z}_2 = (e^{i\pi}, e^{2\pi i} = 1)$ is the center

- The consequence of the $\mathbb{Z}_2$ quotient:

  $SO(3)$ only has integer spin representations,

  $SU(2)$ can have both half-integer and integer spin representations

- In general, one can define $G \sim \dfrac{\tilde{G}}{H}$, where H is a subgroup of the center and all the allowed reps. are invariant under the H group

[Aharony, Seiberg, Tachikawa, 13]

Example: SU(2) versus SO(3) gauge theories

- Consider a low-energy theory with all the matter fields (including gauge bosons and Dirac fermions) in the adjoint representation of $SU(2)$. Suppose this is what has been discovered experimentally.

- The gauge group appears to be $SU(2)$. But this is not quite true.

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- Instead, the gauge group can be either $SU(2)$ or $SO(3)$

- In fancier language, the gauge group $G = \dfrac{SU(2)}{\Gamma}$, where $\Gamma = 1,\ \mathbb{Z}_2$
(The difference of the two theories can be rephrased in one-form symmetry.)

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- When it's $SO(3)$, since $\Gamma = \mathbb{Z}_2$ acts trivially in the full theory, this implies all the heavy particles have to be in the integer spin representations.

- Distinguishing $SU(2)$ vs. $SO(3)$ requires to discover at least one heavy particle in the half-integer spin representation.

- Coming back to low-energy EFT, heavy particle can be described by high dim. operators

The Standard Model

The Standard Model

- The matter content (+ gauge fields in the adjoints)

Table 29.1 Charges of Standard Model fields. □ indicates that the field transforms in the fundamental representation, and – indicates that a field is uncharged.

Field	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	ν_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
SU(3)	–	–	–	□	□	□	–
SU(2)	□	–	–	□	–	–	□
$U(1)_Y$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

[M. Schwartz QFT & SM textbook]

- The $SU(3)_c \times SU(2)_L \times U(1)_Y$ appears to be the gauge group, naively

- Nonetheless, much like the $SU(2)$ in the toy model, we are not sure this is the genuine gauge group. To find the genuine gauge group, we need to take a quotient to remove the trivial group elements.

Which Standard Model?

- The ambiguity comes from the following $\mathbb{Z}_6$ group acting trivially on all SM fields. (This is analogous to the $\mathbb{Z}_2$ center in the toy model.)

[… O'Raifeartaigh, 86; … Tong, 17; …]

$$\mathbb{Z}_6 = \{\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 = 1\} \qquad \alpha = \left(e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\pi i}\mathbb{1}_{2\times 2},\ e^{\frac{2\pi i}{6}}\right)$$

- The generator α act on a rep. (R_3, R_2, Q_Y) as

$$U_\alpha(R_3, R_2, Q_Y) = e^{\frac{2\pi i}{3}\mathcal{N}(R_3) + i\pi\mathcal{N}(R_2) + \frac{2\pi i}{6}(6Q_Y)} = e^{2\pi i\left(\frac{\mathcal{N}(R_3)}{3} + \frac{\mathcal{N}(R_2)}{2} + Q_Y\right)}$$

- Hence the condition for the $\mathbb{Z}_6$ group acting trivially is

$$\mathcal{N}(R_3) = 6Q_Y \bmod 3 \qquad \text{and} \qquad \mathcal{N}(R_2) = 6Q_Y \bmod 2$$

- All SM fields are invariant under the $\mathbb{Z}_6$ group (check it!)

Which Standard Model?

- There are ***<u>four</u>*** SM models from a low energy perspective, they differ by the global form of the gauge group (or one-form sym):

$$G = \frac{\tilde{G}}{\Gamma} = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\Gamma} \qquad\qquad \Gamma = \mathbb{Z}_6,\ \mathbb{Z}_3,\ \mathbb{Z}_2,\ 1$$

- Here $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are the two nontrivial subgroup of $\mathbb{Z}_6$, which are generated by α^3 and α^2, respectively. It's easy to see that they acts trivially when

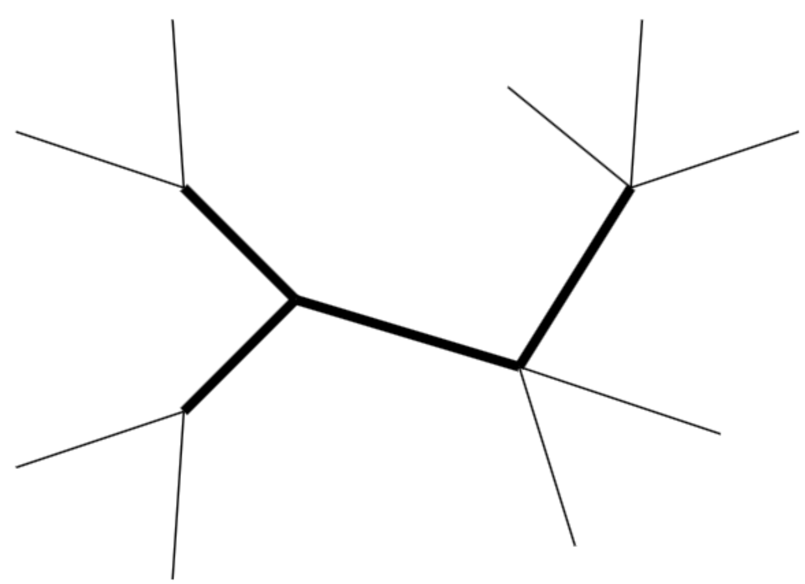
$$\mathbb{Z}_2: \qquad \mathcal{N}(R_2) = 6Q_Y \bmod 2 \qquad \text{and} \qquad R_3 \text{ unconstrained}$$

$$\mathbb{Z}_3: \qquad \mathcal{N}(R_3) = 6Q_Y \bmod 3 \qquad \text{and} \qquad R_2 \text{ unconstrained}$$

Heavy Particles & SMEFT

- Distinguishing them requires to discover new particles not invariant under $\mathbb{Z}_6$. (In the paper we call them "<u>$\mathbb{Z}_6$ ***exotics***</u>".) One can use SMEFT if they are heavy and have decoupling limit.

- No "<u>$\mathbb{Z}_6$ ***exotics***</u>" in tree-level UV completions, seen by cutting the following exemplifying graph. (The result is valid for operators of all mass dimensions.)



- Considering loop-level UV completion becomes **mandatory**!

Heavy Particles & SMEFT

- Example: adding one heavy complex scalar

$$\mathcal{L}_\phi \supset (D_\mu\phi^\dagger)(D^\mu\phi) - M^2\phi^\dagger\phi - \lambda_{\mathbf{3}}(H^\dagger\sigma^I H)(\phi^\dagger T^I\phi) - \lambda_{\mathbf{1}}(H^\dagger H)(\phi^\dagger\phi)$$

Representation	Solution
$\phi\ (\cdot,\cdot,Y_\phi)$	$Y_\phi^2 = \frac{4c_{HB}^2}{5(4c_{H\Box}-c_{HD})c_{HD}}$
$\phi(R_3,\cdot,0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15g_3c_{H\Box}c_{3G}}$
$\phi(\cdot,R_2,0)$	$\frac{\mu(R_2)}{d(R_2)} = \frac{80c_{HW}^2c_{ll}}{225c_{3W}^2\left[g_2^2(4c_{H\Box}+c_{HD})-3c_{ll}\right]}$
$\phi\ (R_3,\cdot,Y_\phi)$	$Y_\phi^2 = \frac{4g_3^2c_{HG}^2c_{HD}}{45g_1^4c_{3G}^2(4c_{H\Box}-c_{HD})}$ $\quad \frac{\mu(R_3)}{d(R_3)} = \frac{4c_{HG}^3c_{HD}}{45g_1^2c_{3G}^2c_{HB}(4c_{H\Box}-c_{HD})}$
$\phi\ (\cdot,R_2,Y_\phi)$	$Y_\phi^2 = \frac{g_2c_{HWB}^2}{15g_1^2c_{3W}(c_{ee}-c_{HD})}$ $\quad \frac{\mu(R_2)}{d(R_2)} = -\frac{g_1^2c_{HWB}^2}{5g_2^2c_{ee}(c_{ee}-c_{HD})}$
$\phi\ (R_3,R_2,0)$	$\frac{\mu(R_3)}{d(R_3)} = -\frac{c_{HG}^2}{15g_3c_{3G}(c_{H\Box}-3c_{ll}/(4g_2^2)+c_{HD}/4)}$ $\frac{\mu(R_2)}{d(R_2)} = -\frac{c_{HW}^2}{15g_2c_{3W}(c_{H\Box}-3c_{ll}/(4g_2^2)+c_{HD}/4)}$

group theoretical data ↙ ↘ measurable Wilson coeff.

Heavy particles & SM gauge group

- Q: What is the SM gauge group?
- A: It depends on what heavy particles we will discover. There are four scenarios as follows:
  - All particles are invariant under $\mathbb{Z}_6$, Γ remains undetermined as in the SM. However, if this is the case it might be better to write $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y/\mathbb{Z}_6$.
  - At least one heavy particle is not invariant under $\mathbb{Z}_3$ but invariant under $\mathbb{Z}_2$ (hence not invariant under $\mathbb{Z}_6$), Γ can be either $\mathbb{Z}_2$ or 1.
  - At least one heavy particle is not invariant under $\mathbb{Z}_2$ but invariant under $\mathbb{Z}_3$ (hence not invariant under $\mathbb{Z}_6$), Γ can be either $\mathbb{Z}_3$ or 1.
  - At least one heavy particle is not invariant under either $\mathbb{Z}_2$ or $\mathbb{Z}_3$ (hence not invariant under $\mathbb{Z}_6$), Γ is 1.

Conclusion & outlook

- The global form of the SM gauge group is unknown, but we can potentially determine it by discovering heavy particles, or using SMEFT at low energy.
- Heavy particles not invariant under $\mathbb{Z}_6$ can only appear in loop-level UV completions, hence studying models with one-loop matching becomes mandatory and more important than one might naively expect.
- Scalars that can trigger EWSB cannot be $\mathbb{Z}_6$ exotics. Easy to prove in general, see in the supplemental slides.
- Cosmological, astro-particle, and future collider studies are warranted. (We have a chance to bound the reheating temperature from above since $\mathbb{Z}_6$ exotic particle are stable.)
- Can we find striking applications of generalized symmetries in particle physics?

Supplemental slides

Examples of heavy particles & SM gauge group

- (R_3, R_2, Q_Y) = (fundamental, fundamental, 0) is allowed when $\Gamma = 1$ but forbidden when $\Gamma = \mathbb{Z}_{2,3,6}$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 2/3) is allowed when $\Gamma = 1$ or $\mathbb{Z}_3$, but forbidden when $\Gamma = \mathbb{Z}_2$ or $\mathbb{Z}_6$
- (R_3, R_2, Q_Y) = (fundamental, fundamental, 1/2) is allowed when $\Gamma = 1$ or $\mathbb{Z}_2$, but forbidden when $\Gamma = \mathbb{Z}_3$ or $\mathbb{Z}_6$
- Some well-known realistic examples include the original KSVZ fermions in axions models, fractionally-charged and milli-charged particles.

Electroweak symmetry breaking

- Q: What about the scalars that can trigger EWSB?
- A: They don't decouple and they are not $\mathbb{Z}_6$ exotics.
- Proof: 1) Since color is unbroken, the scalars must be neutral under $SU(3)_c$ (i.e. singlet rep. has N-ality zero). 2) In the notation of (j, Q_Y) the quantum numbers are subject to the following constraints to accommodate a electric neutral component:

$$-j \leq Q_Y \leq j \qquad \text{and} \qquad j + Q_Y \in \mathbb{Z}$$

- Q_Y is either integer or half-integer since j is, hence

$$0 = 6Q_Y \mod 3 \longrightarrow \text{invariant under } \mathbb{Z}_3$$

- Furthermore, let's compute $2j - 6Q_Y$

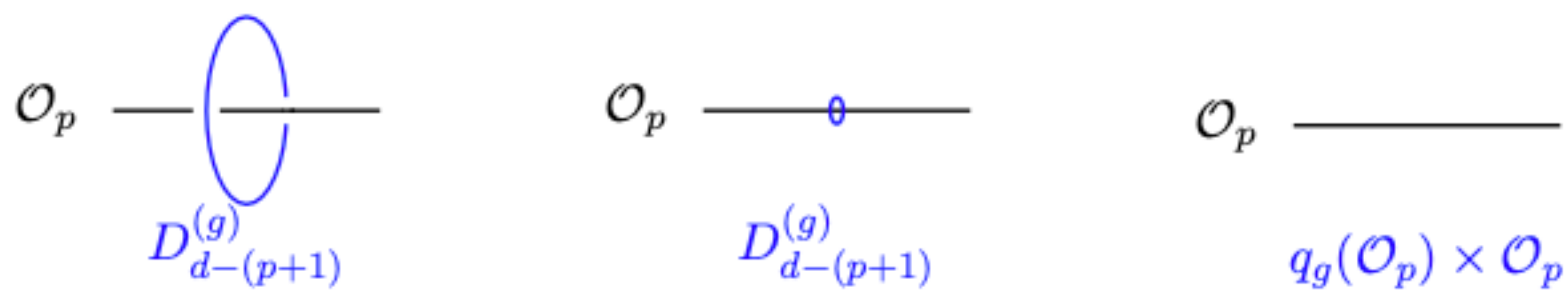
$$2(j - Q_Y) - 4Q_Y = 0 \mod 2 \longrightarrow \text{invariant under } \mathbb{Z}_2$$

- Invariance under both $\mathbb{Z}_{2,3}$ implies invariance under $\mathbb{Z}_6$

One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

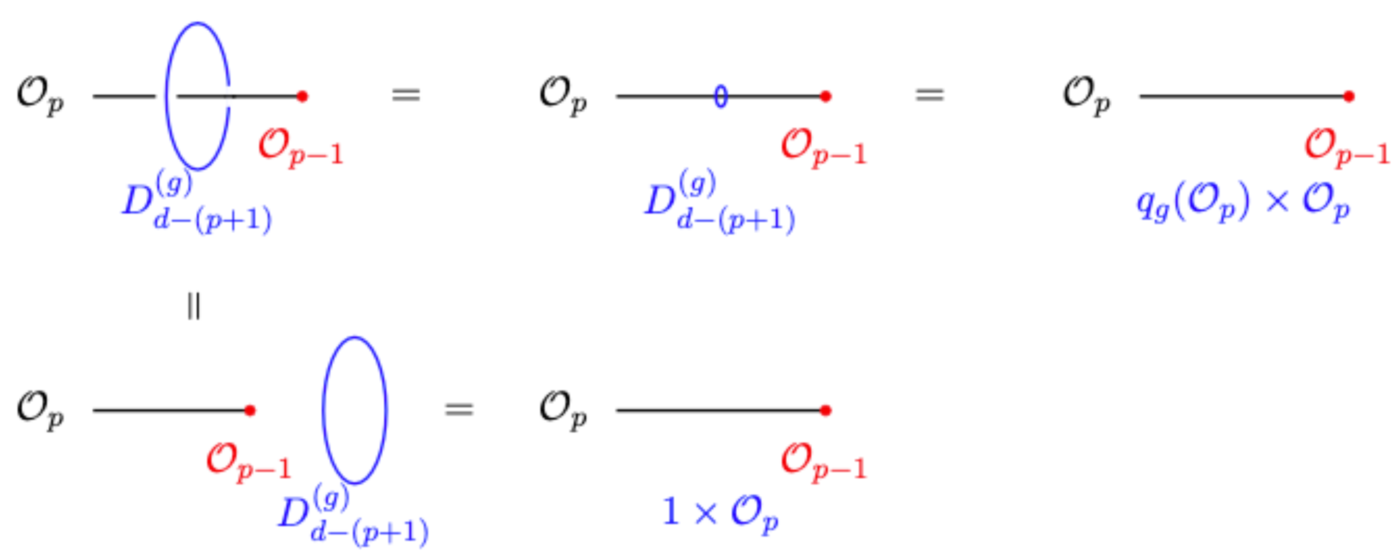
- A p-form global symmetry is generated by a dimension $(d-p-1)$ dimensional topological operator D_{d-p-1} acting on a p dimensional charged operator $\mathcal{O}_p$ as in the following:



(2.13)

- Higher form symmetries (i.e. $p > 1$) are abelian.

- Screening the charge: p-form symmetry can be screened (trivialized) by $p-1$ dimensional operators $\mathcal{O}_{p-1}$ which live at the end of $\mathcal{O}_p$



One-form symmetries and Line Operators

[ICTP lectures by Schafer-Nameki, 2023]

- One useful perspective is to think in terms of the equivalence relations between charged operators $\mathcal{O}_p$

$$\mathcal{O}_p^{(1)} \sim \mathcal{O}_p^{(2)} \quad \Leftrightarrow \quad \exists\, O_{p-1} \text{ at the junction between } \mathcal{O}_p^{(1)} \text{ and } \mathcal{O}_p^{(2)} \,. \qquad (2.28)$$

- Example: in a pure Yang-Mill theory with simply-connected gauge group G, Wilson lines of all possible charges under the center $\mathbb{Z}_G$ are allowed. Since the only local operators are in the adjoint which is not charged under center, all these $\mathbb{Z}_G$ charged Wilson lines are inequivalent and so the 1-form symmetry is the center. Also it's obvious that adding additional matter can trivialize some of the Wilson lines, hence breaking the 1-form symmetry to a subgroup.

- Taking the quotient Γ restricts the allowed Wilson lines, but it allows for more 't Hooft lines. There are different ways of adding the lines (called choices of "polarizations").

Centers for simply-connected groups

G	Z_G	$q(\boldsymbol{F})$
$SU(N)$	$\mathbb{Z}_N$	$1 \mod N$
$\mathrm{Spin}(4N)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,1) \mod (2,2)$
$\mathrm{Spin}(4N+2)$	$\mathbb{Z}_4$	$2 \mod 4$
$\mathrm{Spin}(2N+1)$	$\mathbb{Z}_2$	$1 \mod 2$
E_6	$\mathbb{Z}_3$	$1 \mod 3$
E_7	$\mathbb{Z}_2$	$1 \mod 2$
E_8	$\mathbb{Z}_1$	$1 \mod 1$

Table 1: Simply-connected Lie groups G and their centers Z_G, as well as the charge of the fundamental representation $\boldsymbol{F}$ under the generator(s) of the center.

$$Z_G = \mathrm{Center}(G) = \{g \in G : \; gh = hg \text{ for all } h \in G\}\,. \tag{2.18}$$

[ICTP lectures by Schafer-Nameki, 2023]