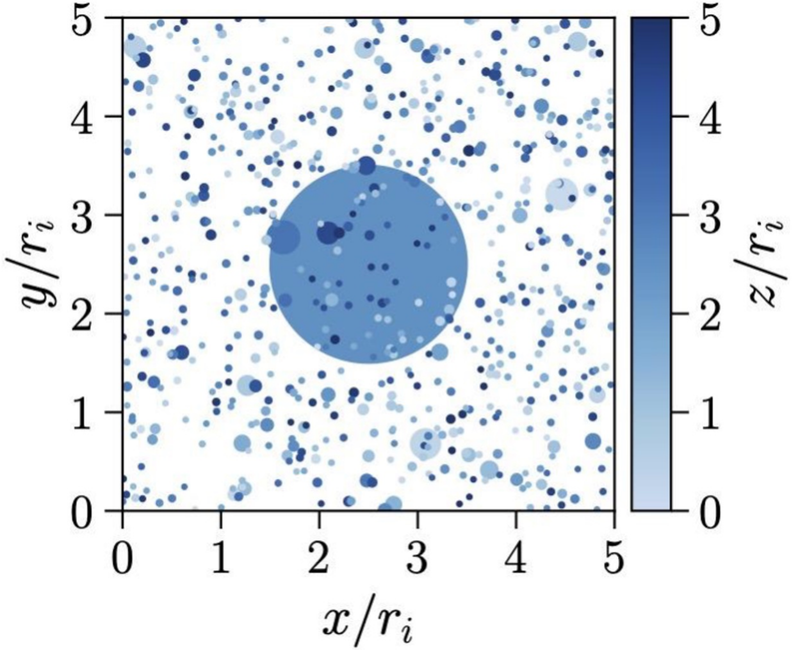
# Dark Radiation Isocurvature from Cosmological Phase Transitions

Mitchell J. Weikert



SUSY 2024
June 14th, 2024
IFT, Madrid

Based on 2402.13309:
Matthew R. Buckley, Peizhi Du, Nicolas Fernandez, & **MJW**

# Why are Cosmological Phase Transitions Interesting?

- Many models of Physics beyond the SM predict early universe first order phase transitions
    - Electroweak baryogenesis, early dark energy, etc…

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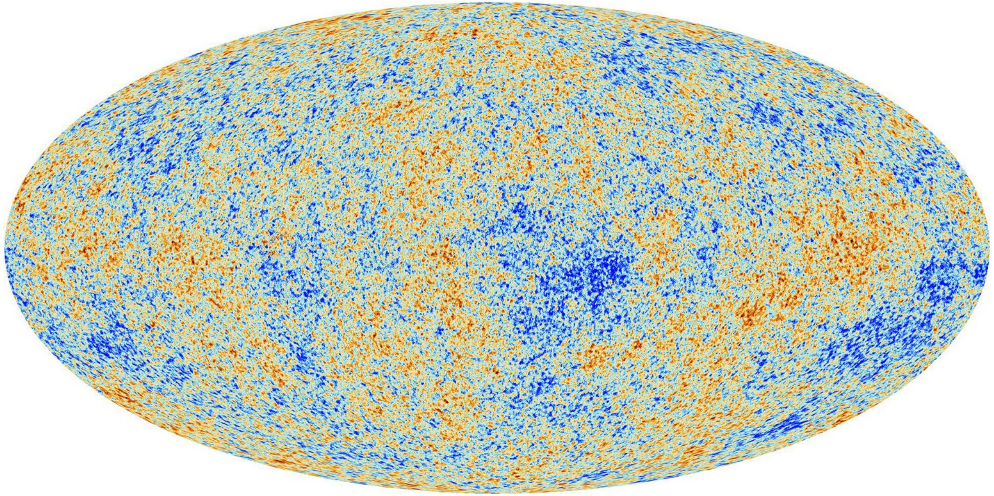
- Many models of Physics beyond the SM predict early universe first order phase transitions
    - Electroweak baryogenesis, early dark energy, etc…
- Such phase transitions are often accompanied by observable signatures which can be used to learn about the associated physics
    - We focus on **dark radiation**
        - Generic example: production of **gravitational waves** from bubble collisions

# Cosmic Microwave Background

- Dark radiation contributes to energy density, which influences CMB photons through the metric
  - Constraints on dark radiation: $\Delta N_{\text{eff}} < 0.3$ [1807.06209]

$$\Delta N_{\text{eff}} = \frac{\rho_{\text{dr}}}{\rho_\nu} N_{\text{eff}} \qquad N_{\text{eff}} = 3.044$$



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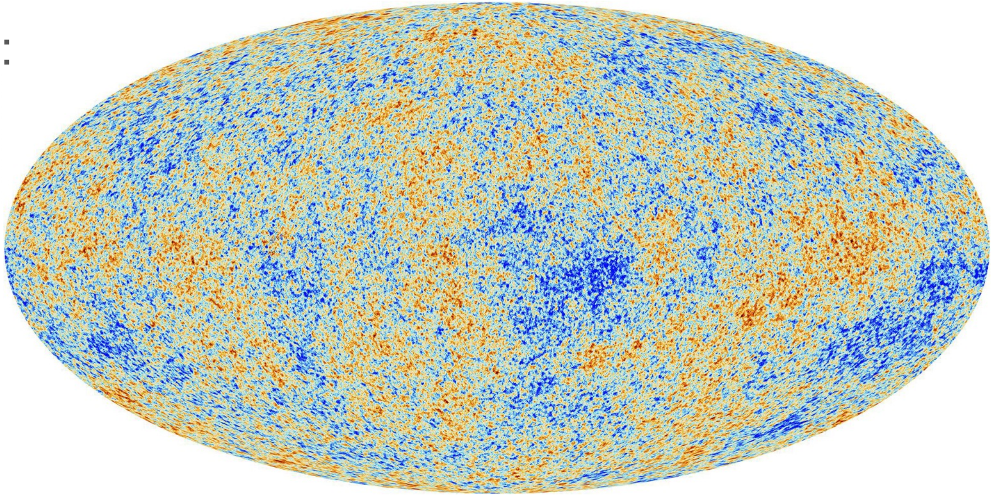
$$\mathcal{S}_{ab} = -3H \left( \frac{\delta\rho_a}{\dot{\rho}_a} - \frac{\delta\rho_b}{\dot{\rho}_b} \right) = 0$$



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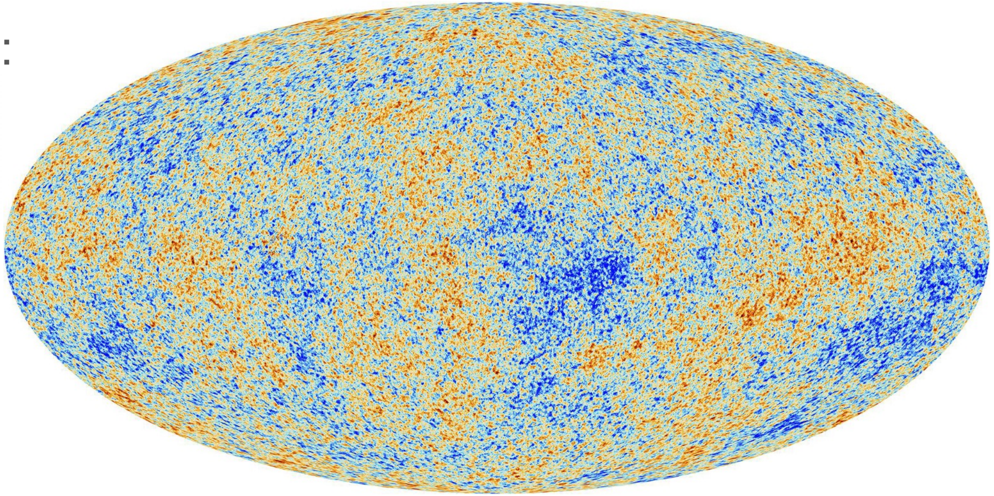
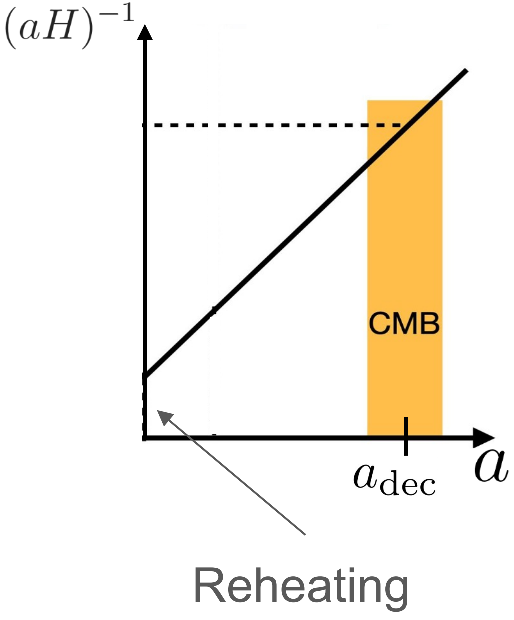


**Broad class of FOPT models that produce DR isocurvature can be constrained more stringently!**

# Inflationary Phase Transition

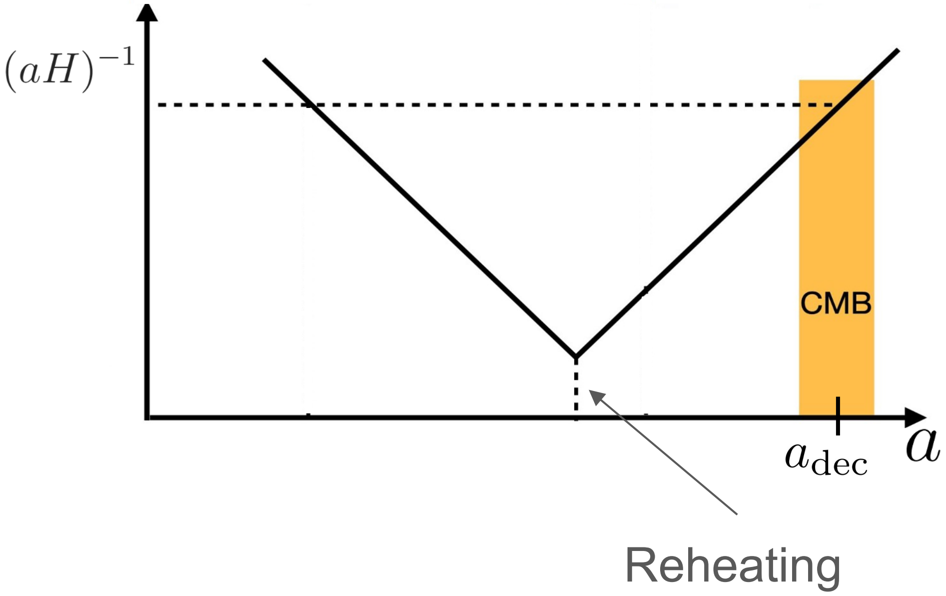
- Consider comoving horizon



# Inflationary Phase Transition

- Consider comoving horizon
- Single field inflation driven by $\phi$

$$H \approx H_{\mathrm{inf}} \to \mathrm{const}$$



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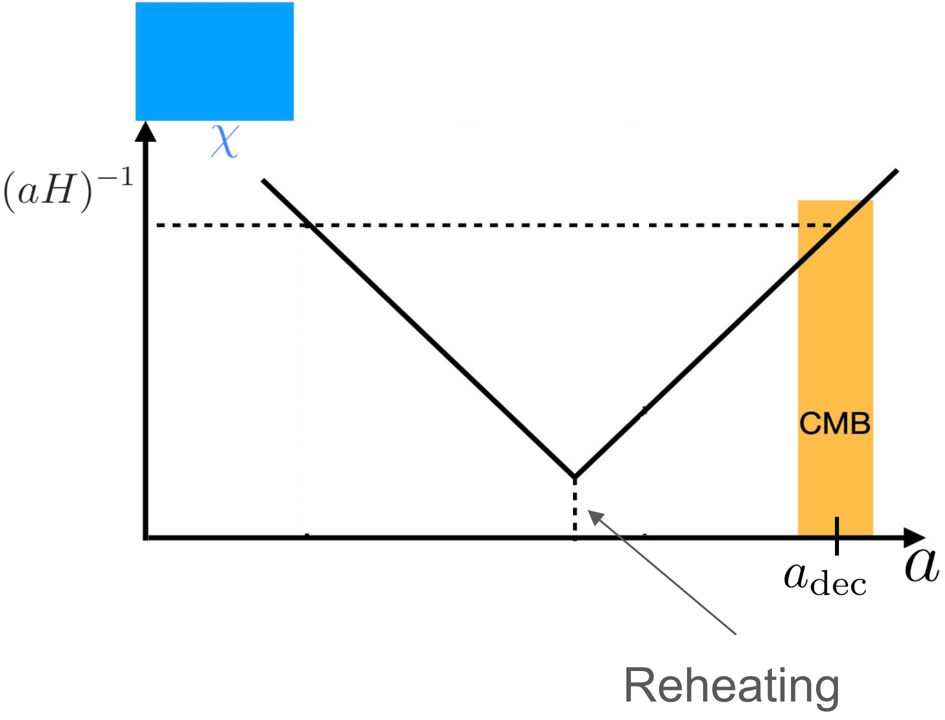

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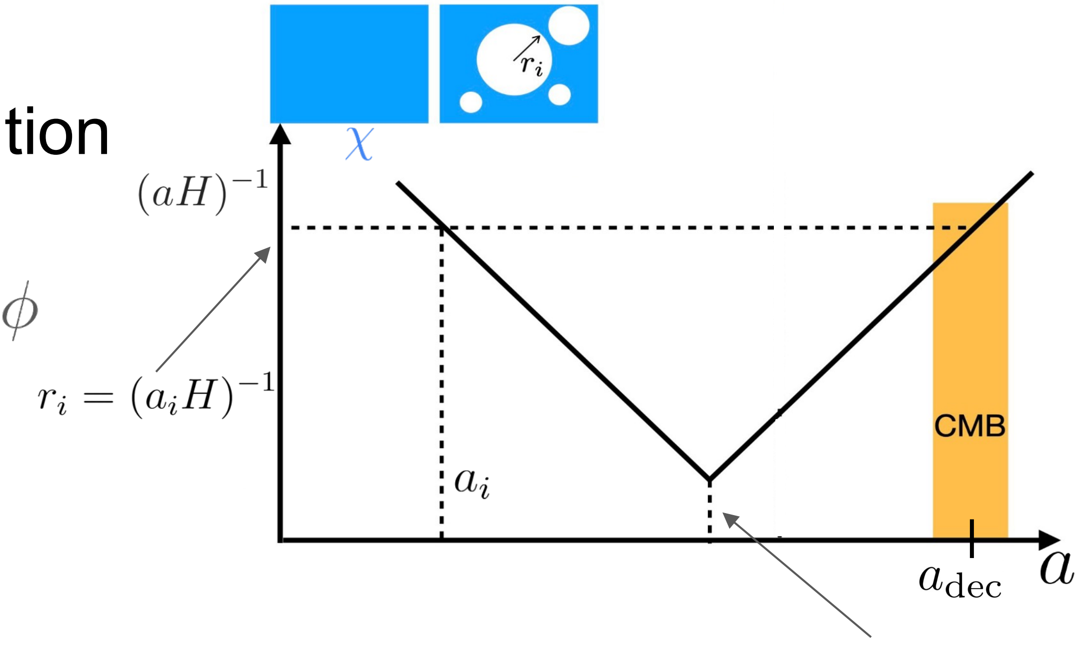

Reheating

# Inflationary Phase Transition

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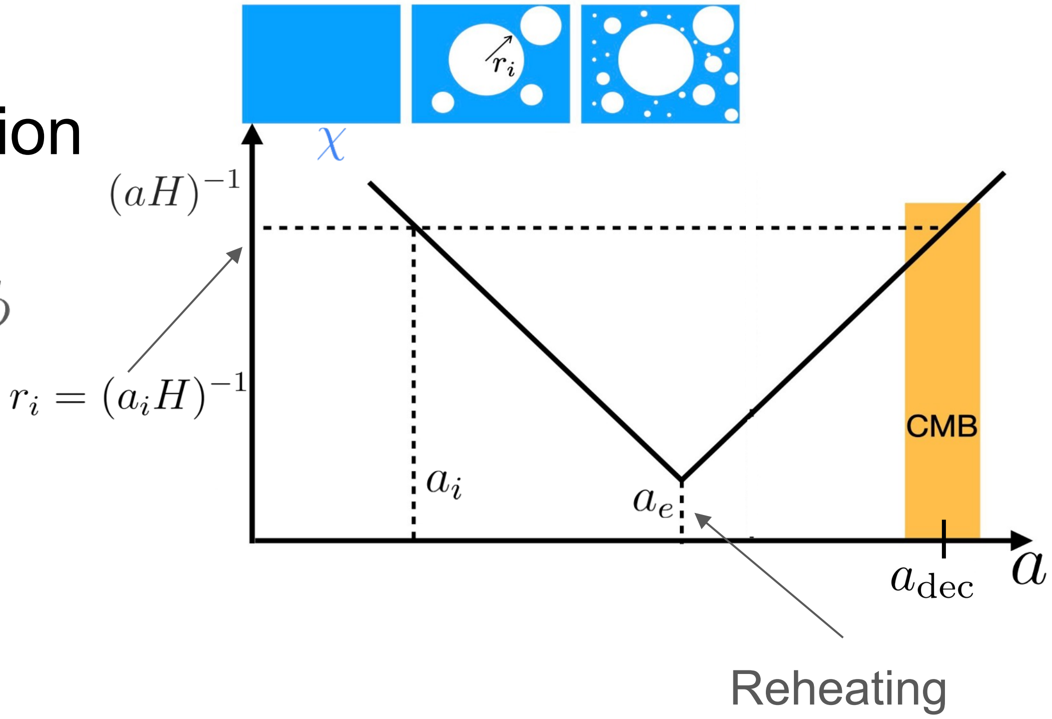
$$\gamma_{\text{PT}} \equiv \frac{\Gamma_{\text{PT}}}{H_{\text{inf}}^4} \ll 1 \Longrightarrow$$

PT remains incomplete



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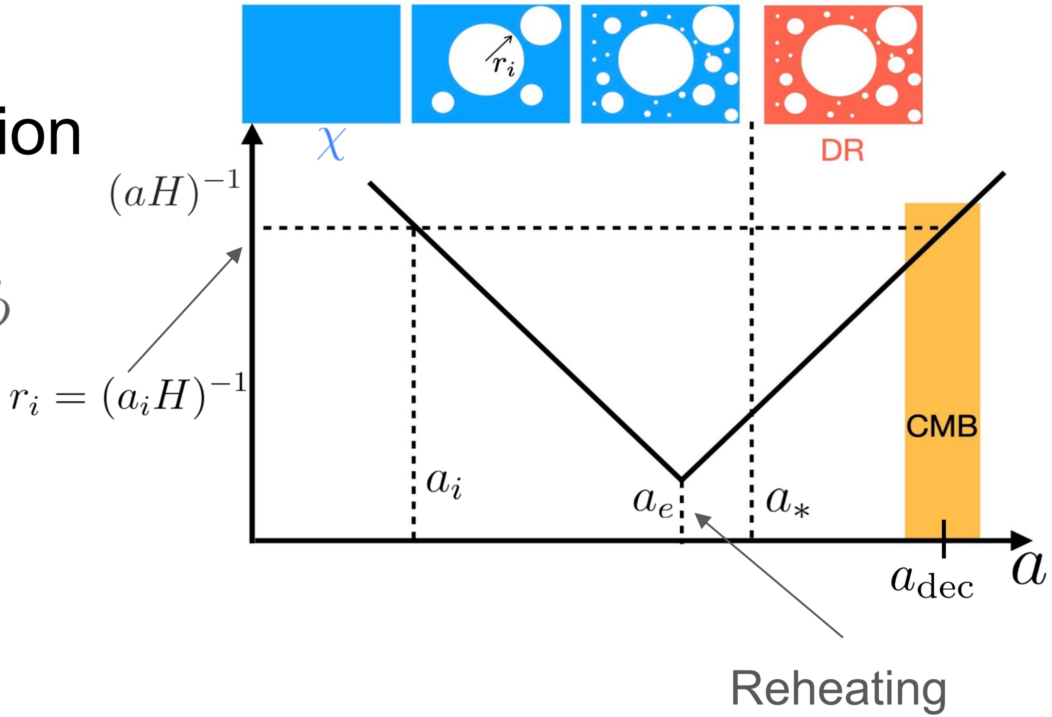
- PT completes: $\chi \longrightarrow$ DR
  - Large-scale features in CMB



# PT with Non-Thermal Trigger

$$V(\chi,\phi) = -\frac{1}{2}m^2\chi^2 + \frac{\mu(\phi)}{3}\chi^3 + \frac{\lambda}{4}\chi^4$$

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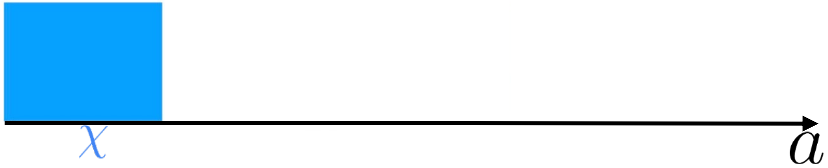

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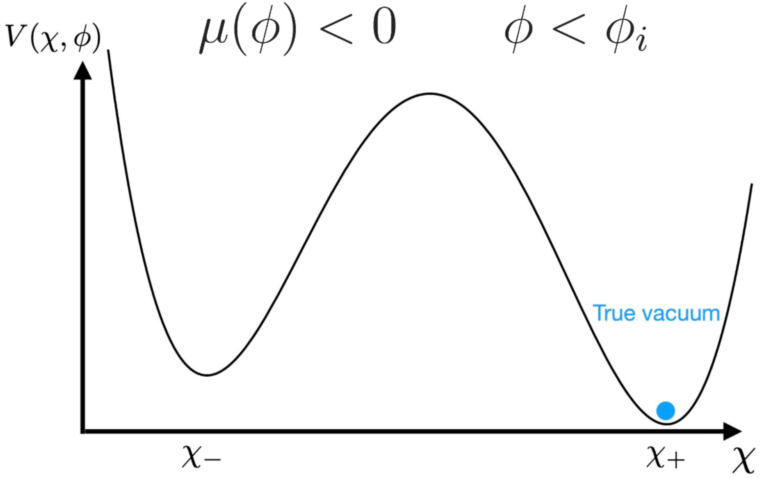
$\mu(\phi) < 0$ $\quad$ $\phi < \phi_i$



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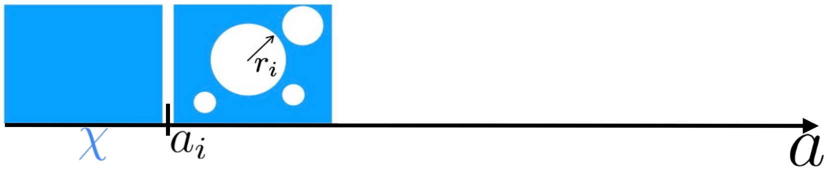

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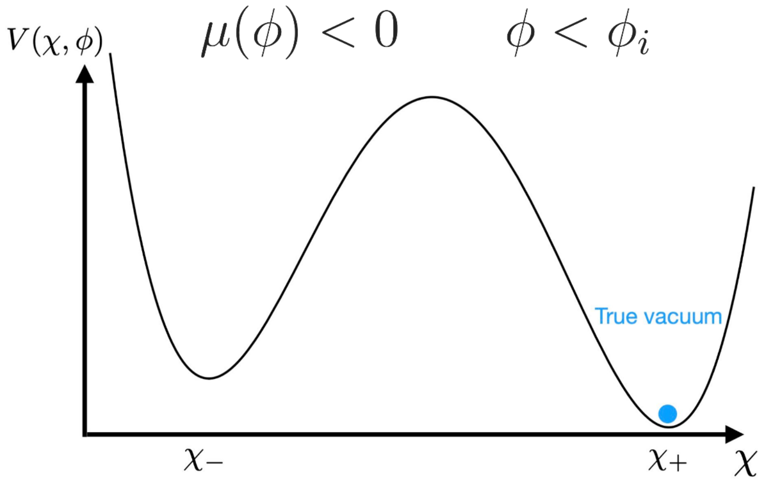

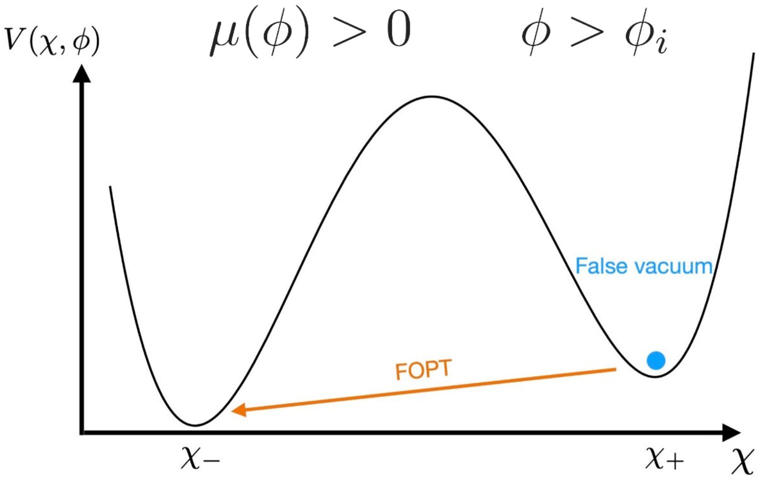

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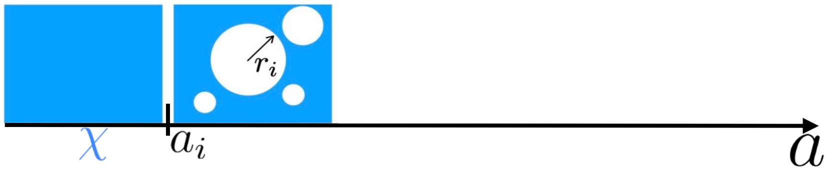

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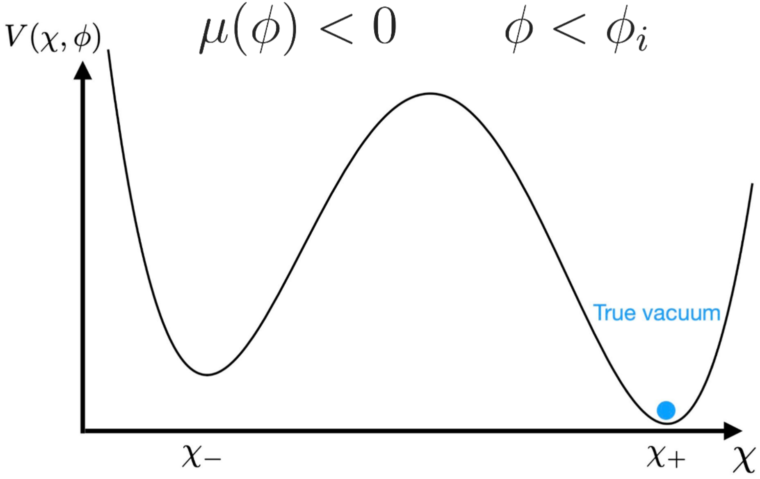
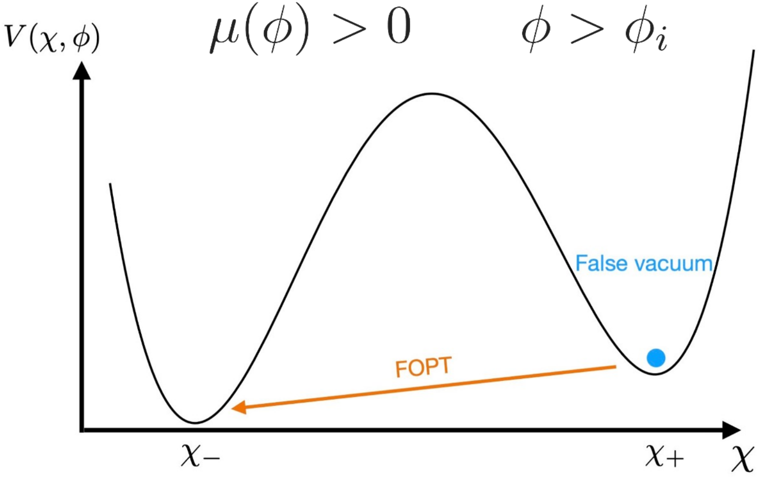
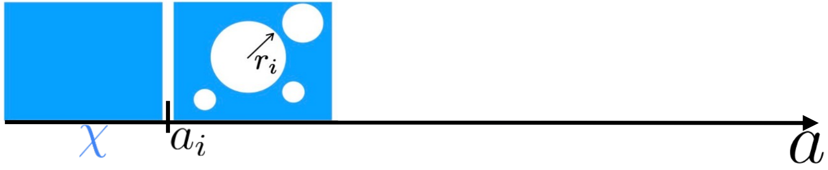




$$\mu(\phi) = \mu \tanh\left(\frac{\phi - \phi_i}{\Delta\phi}\right)$$

⇒ Behaves like a step function

# Cosmological PT Parameters



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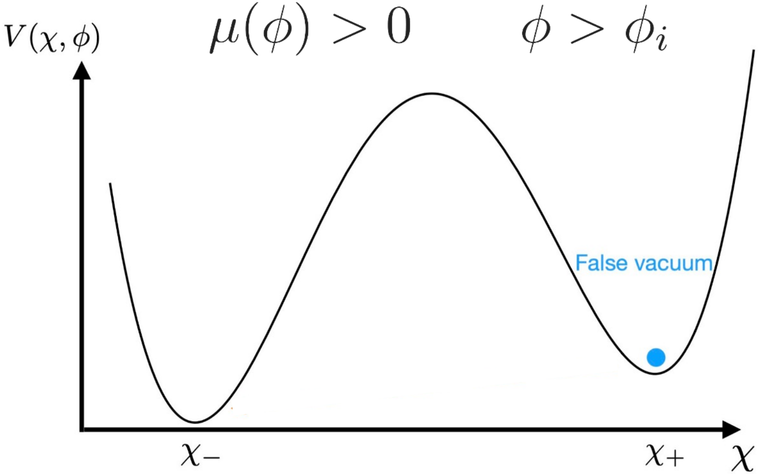

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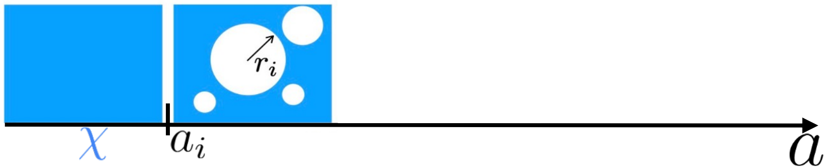

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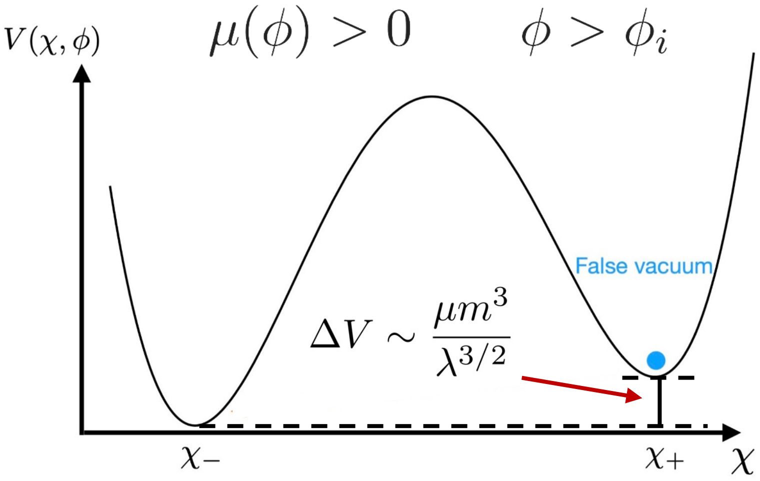

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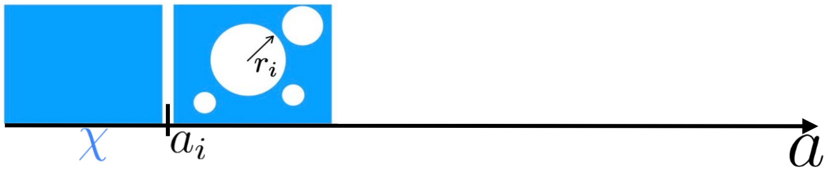

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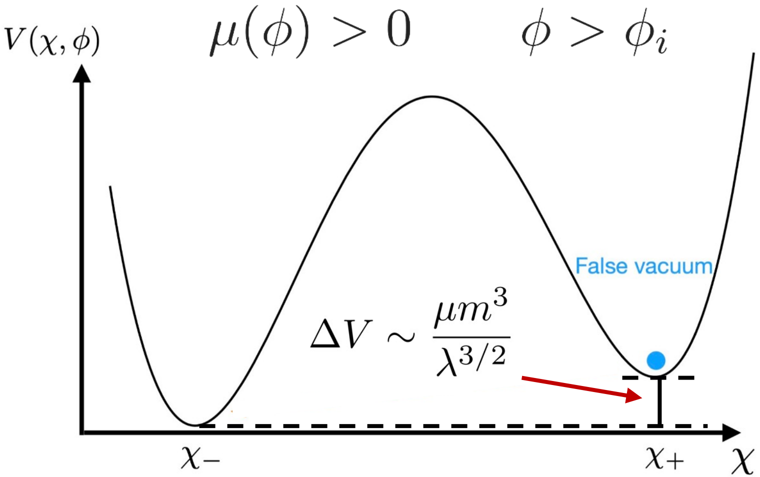
$$S_4 \sim \lambda^{1/2}\left(\frac{m}{\mu}\right)^3$$

$$r_c \sim \frac{3m^3}{\lambda\Delta V}$$



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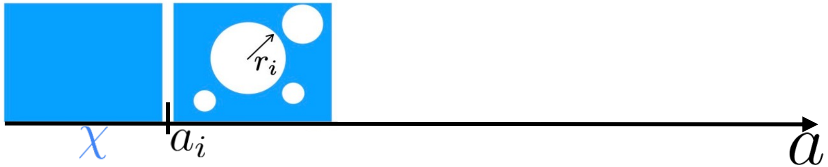

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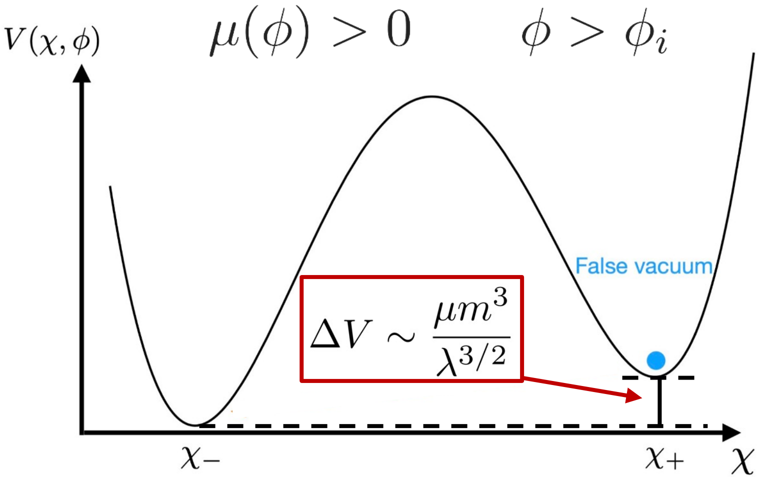

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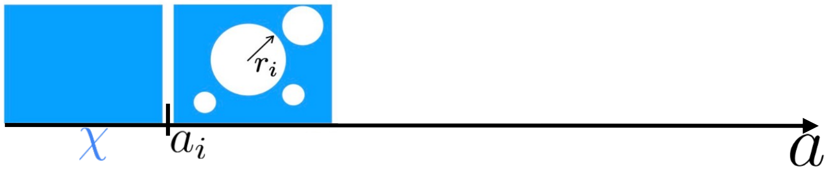
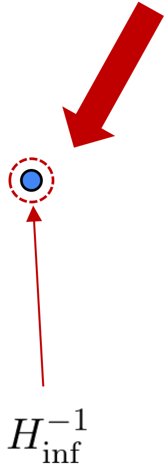


Shown in **physical** coordinates



Bubble wall eq. of motion

$$\partial_t(\gamma_w v_w) + \underbrace{\boxed{3H_{\text{inf}}(\gamma_w v_w)}}_{\text{Damping}} \approx \underbrace{\boxed{\frac{\Delta V}{\sigma}}}_{\text{Pressure}}$$

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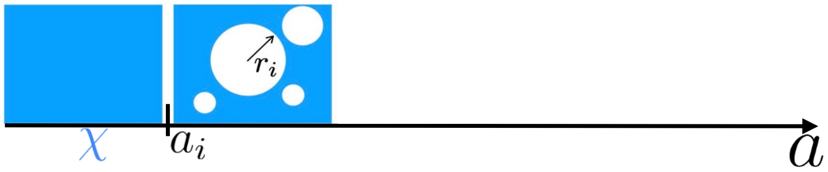

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Bubble walls reach terminal velocity

$$(\gamma_w v_w)|_\infty = \frac{\Delta V}{3\sigma H_{\rm inf}} = \frac{1}{r_c H_{\rm inf}} \gg 1$$

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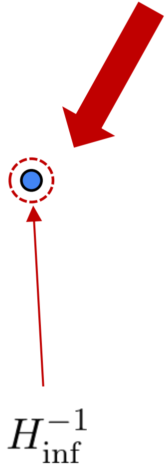

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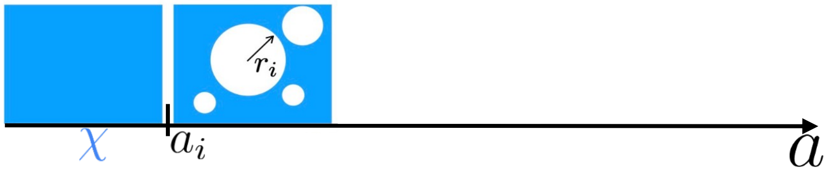

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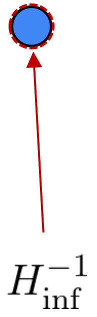

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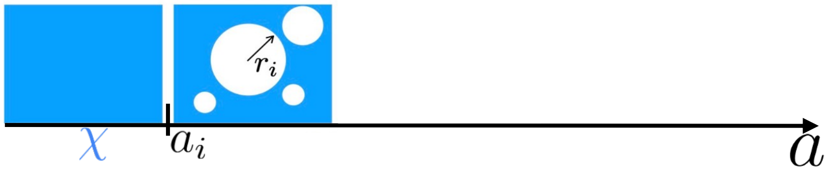

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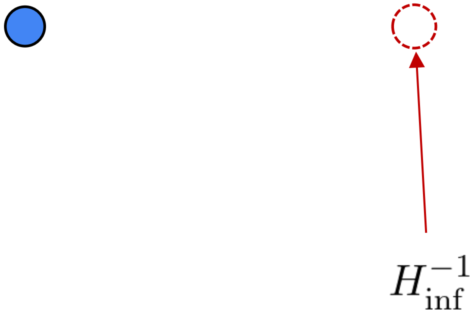

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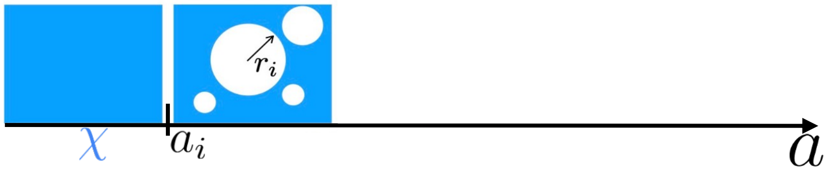

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Bubbles then expand with scale factor

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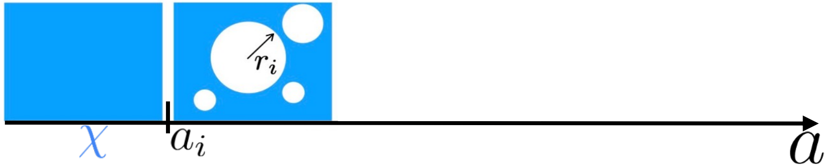

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Damping

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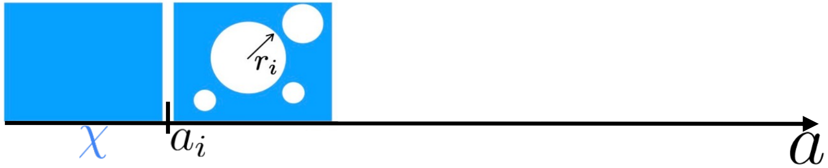

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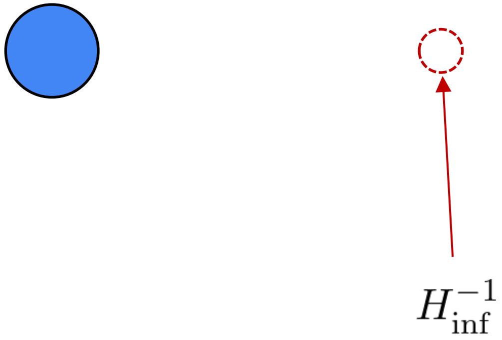

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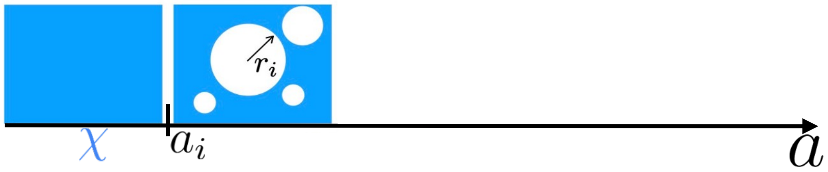

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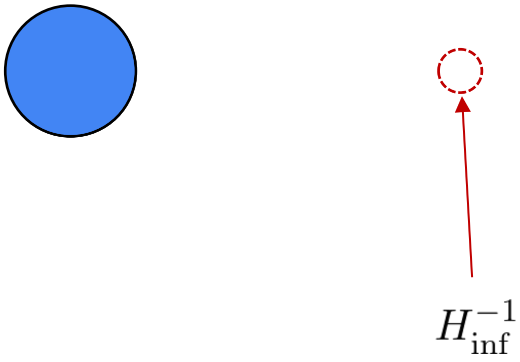

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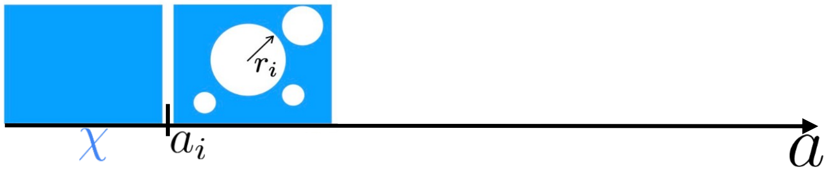

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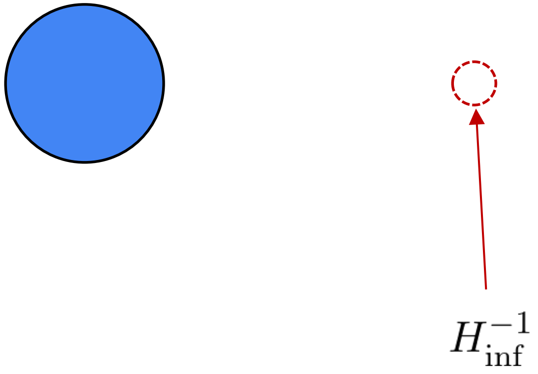

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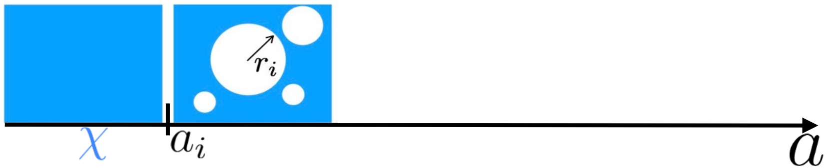

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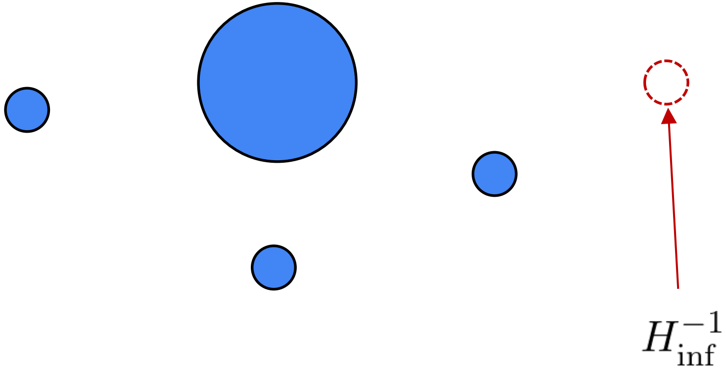
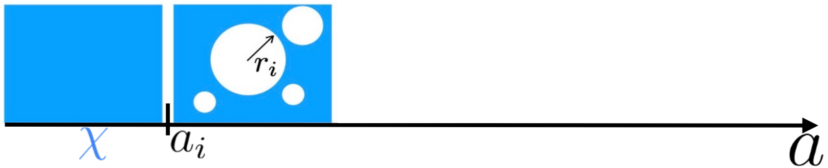
New bubbles nucleate





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Damping

Pressure

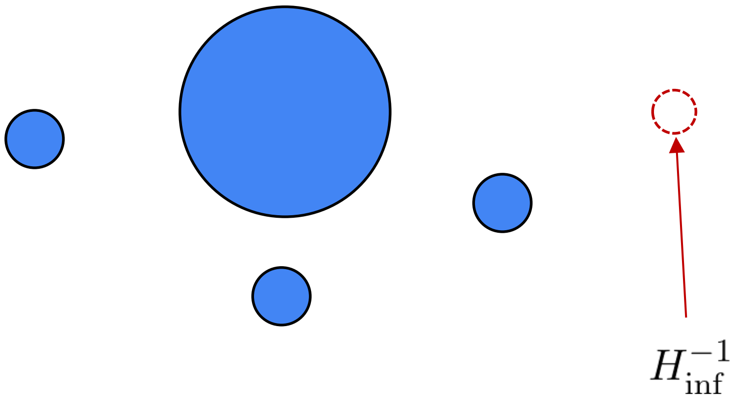
Bubble walls reach terminal velocity

$$(\gamma_w v_w)|_\infty = \frac{\Delta V}{3\sigma H_{\rm inf}} = \frac{1}{r_c H_{\rm inf}} \gg 1$$

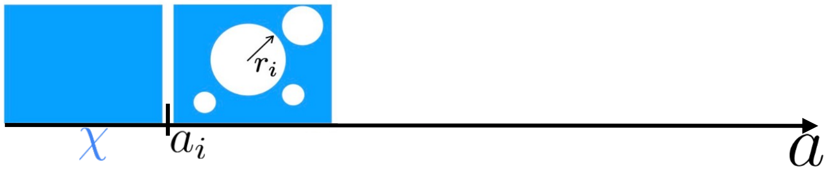
And quickly approaches the horizon

Bubbles then expand with scale factor

New bubbles nucleate



# Cosmological PT Parameters



Shown in **physical** coordinates

Bubble wall eq. of motion

$$\partial_t(\gamma_w v_w) + \underbrace{\boxed{3H_{\text{inf}}(\gamma_w v_w)}}_{\text{Damping}} \approx \underbrace{\boxed{\frac{\Delta V}{\sigma}}}_{\text{Pressure}}$$

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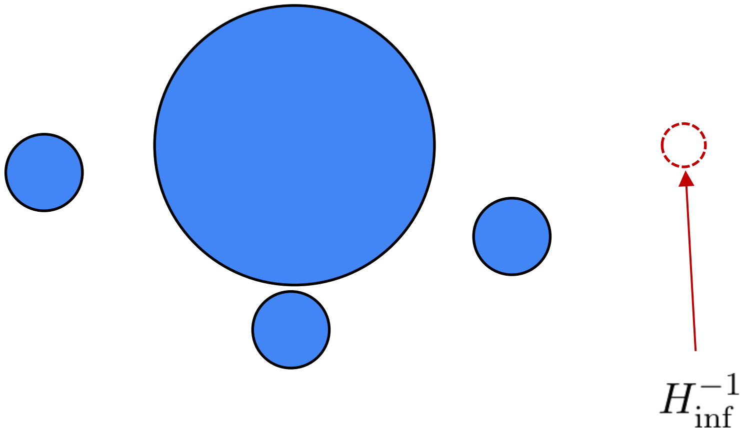

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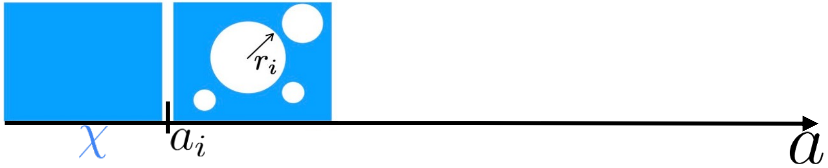

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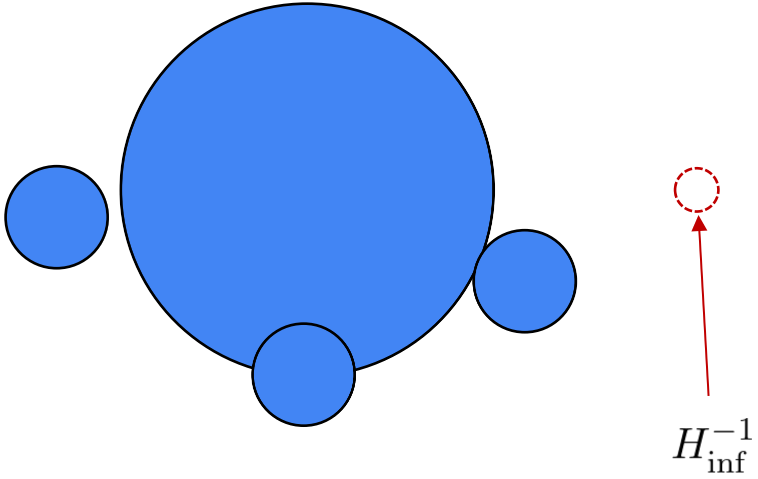

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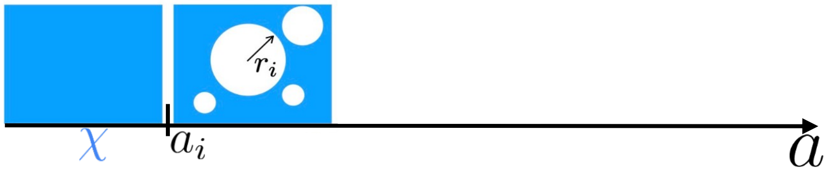

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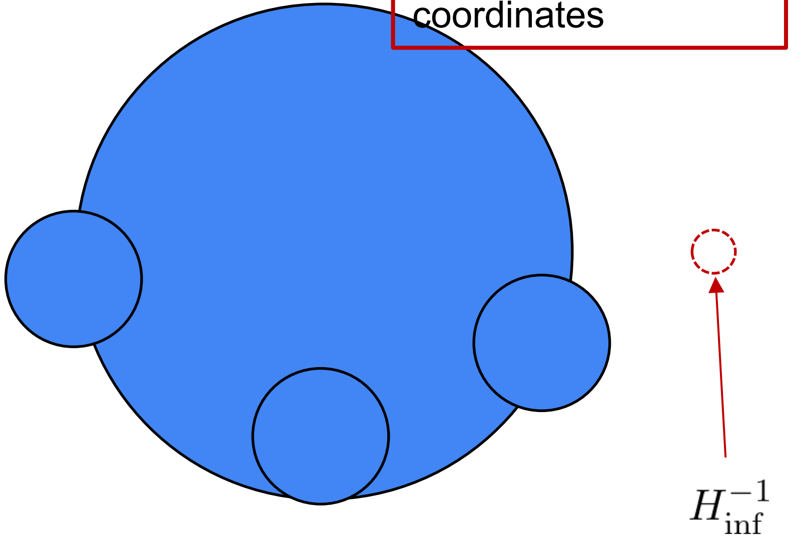

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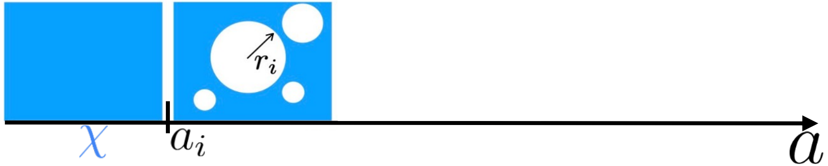

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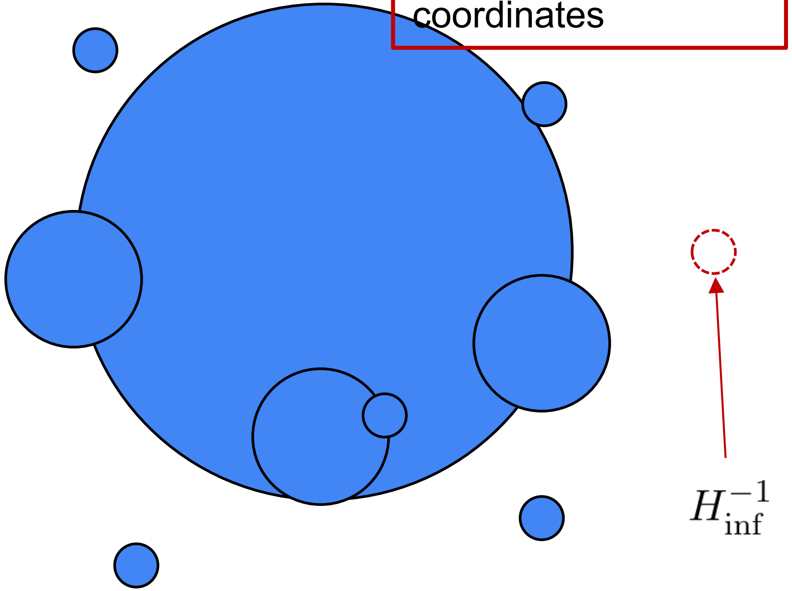
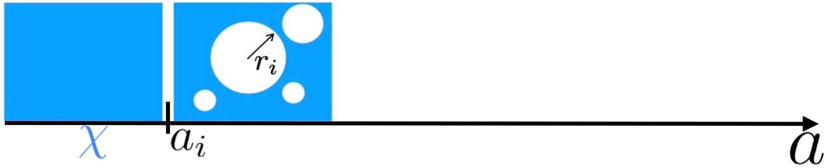
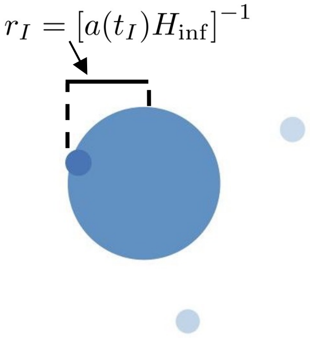


# Energy Density Distribution for $\chi$



Shown in **comoving** coordinates



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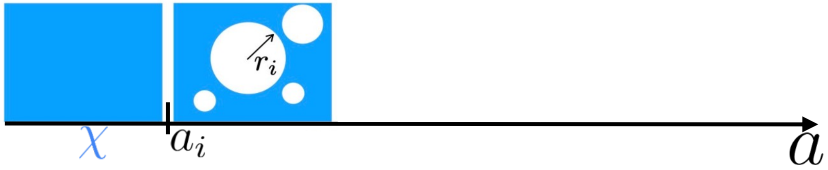
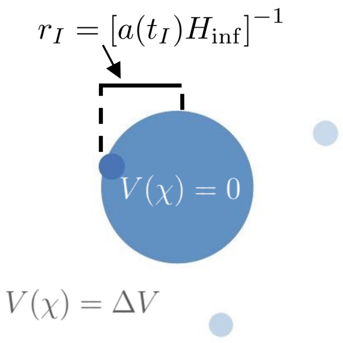


$$\rho_\chi(\mathbf{x},t) = \Delta V \left[1 - \sum_{I:t_I<t} \Theta(r_I - |\mathbf{x} - \mathbf{x_I}|)\right]$$

$t_I$ and $\mathbf{x_I}$ randomly sampled

Shown in **comoving** coordinates



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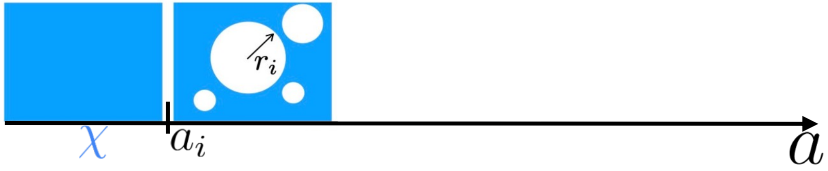

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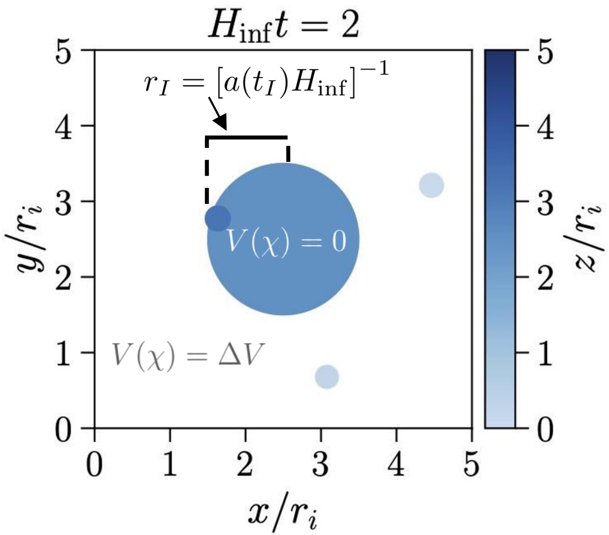

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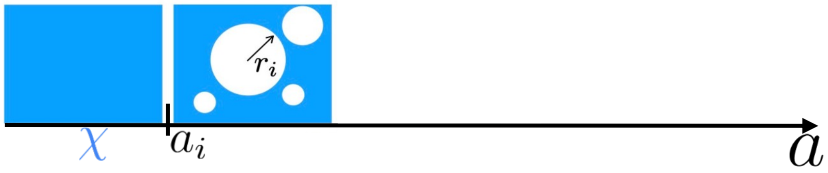

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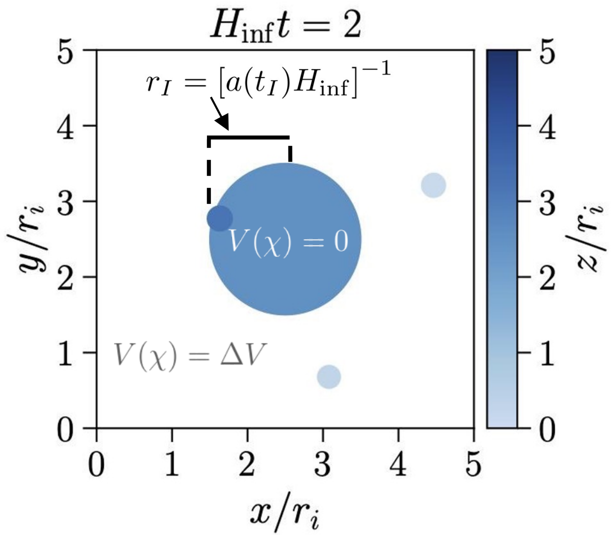

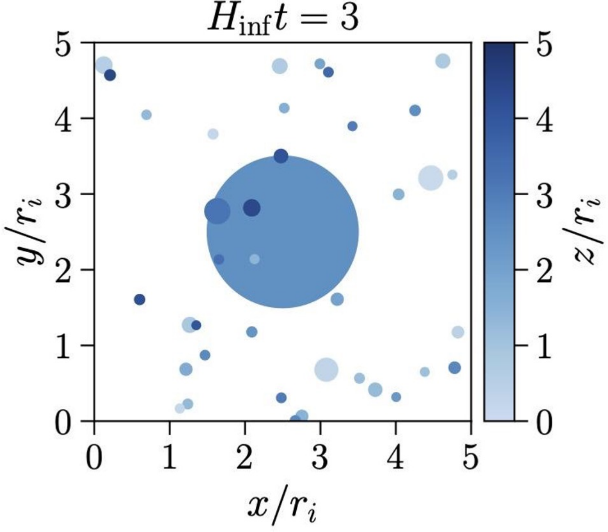

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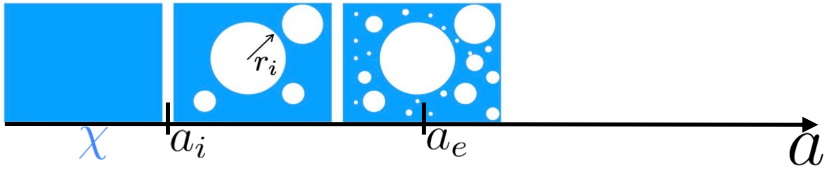

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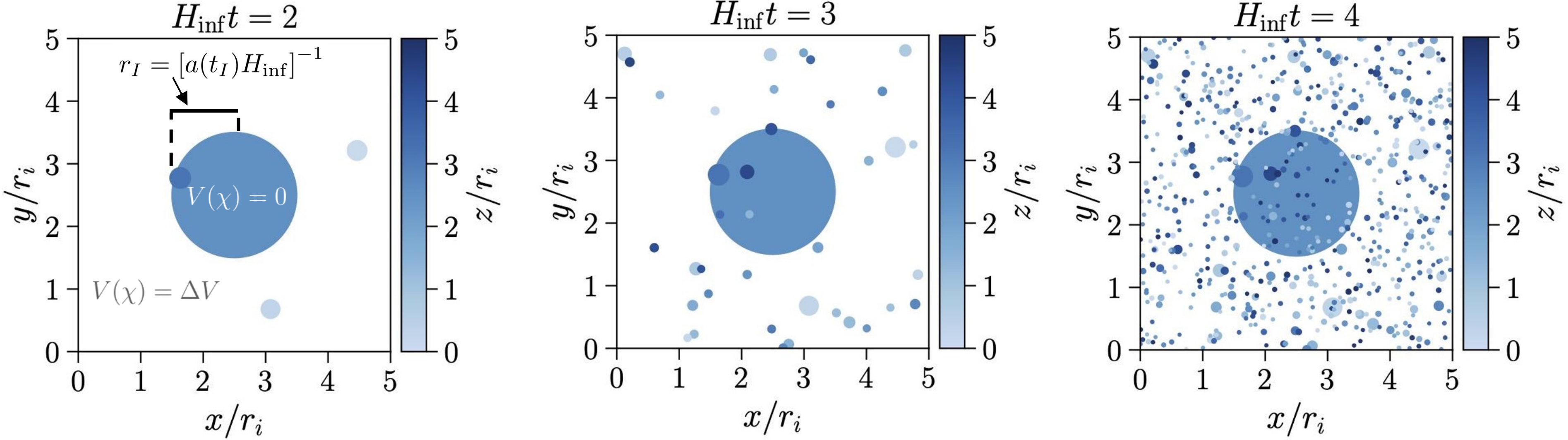
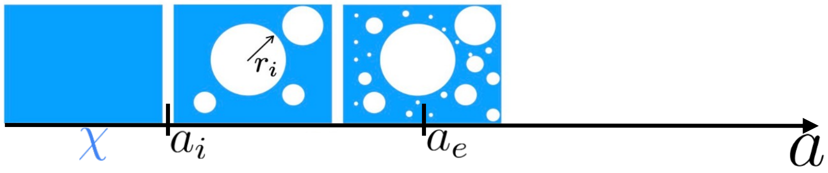
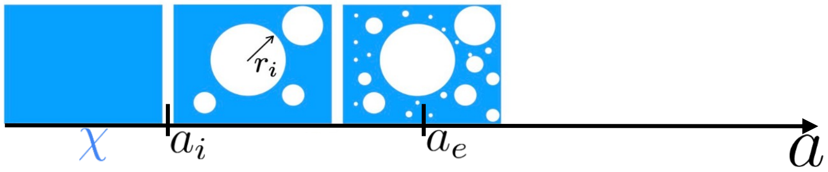




# Isocurvature Power Spectrum

$$\rho_\chi(\mathbf{x},t) = \Delta V\left[1 - \sum_{I:t_I<t}\Theta(r_I - |\mathbf{x} - \mathbf{x_I}|)\right]$$

# Isocurvature Power Spectrum

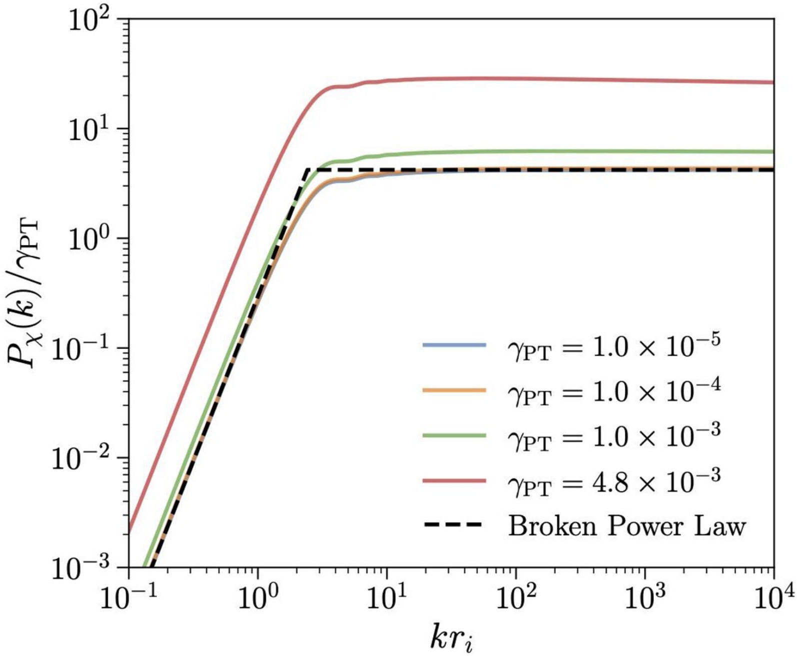


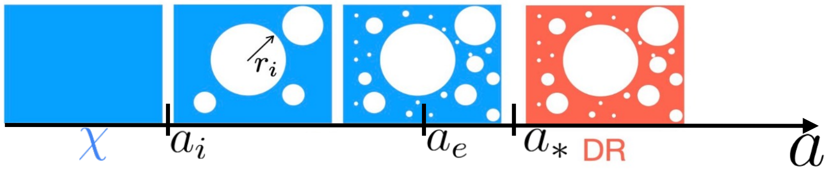
$$\rho_\chi(\mathbf{x},t) = \Delta V \left[1 - \sum_{I:t_I<t} \Theta(r_I - |\mathbf{x}-\mathbf{x_I}|)\right]$$

$$\Rightarrow$$

$$\langle \delta_\chi(\mathbf{k})\delta_\chi(\mathbf{k}')\rangle \equiv 2\pi^2 \frac{(2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')}{k^3} P_\chi(k)$$

$$P_\chi(k) \approx \gamma_{\mathrm{PT}} \begin{cases} \frac{8}{27}(kr_i)^3 & kr_i \ll 1 \\ 4.2 & kr_i \gg 1 \end{cases}$$





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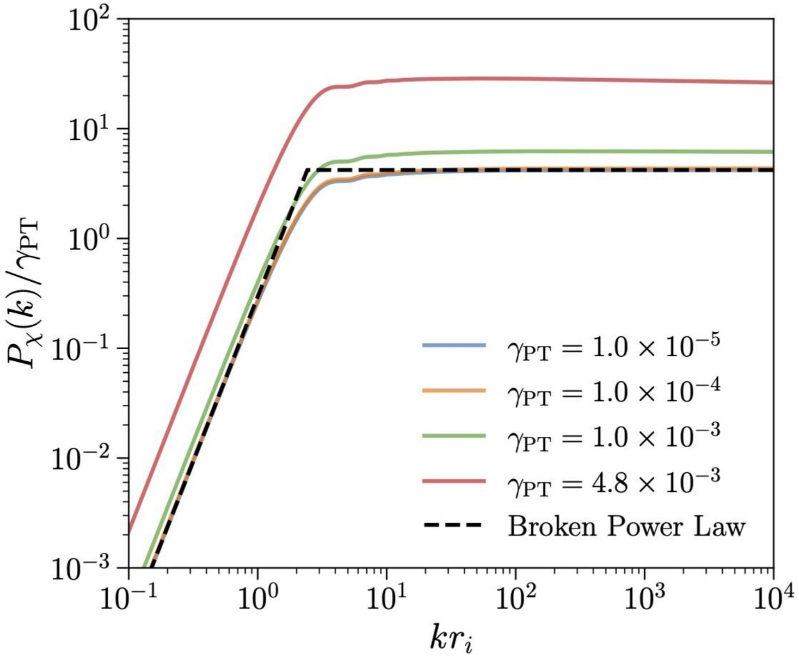
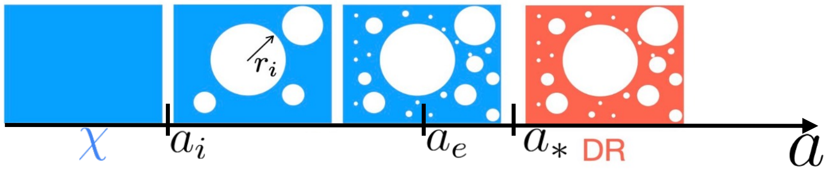
The PT completes when $\dfrac{\Gamma_{\rm PT}}{H^4} = \gamma_{\rm PT}\left(\dfrac{T_{\rm rh}}{T}\right)^8 \approx 1$





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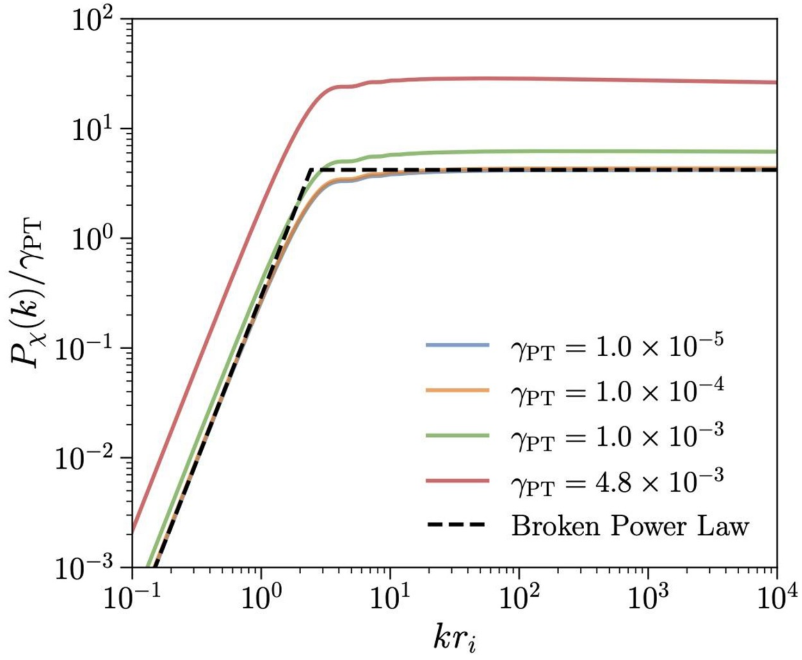

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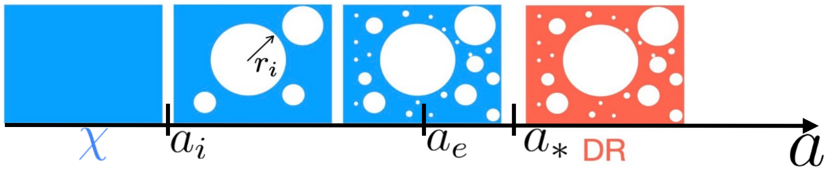
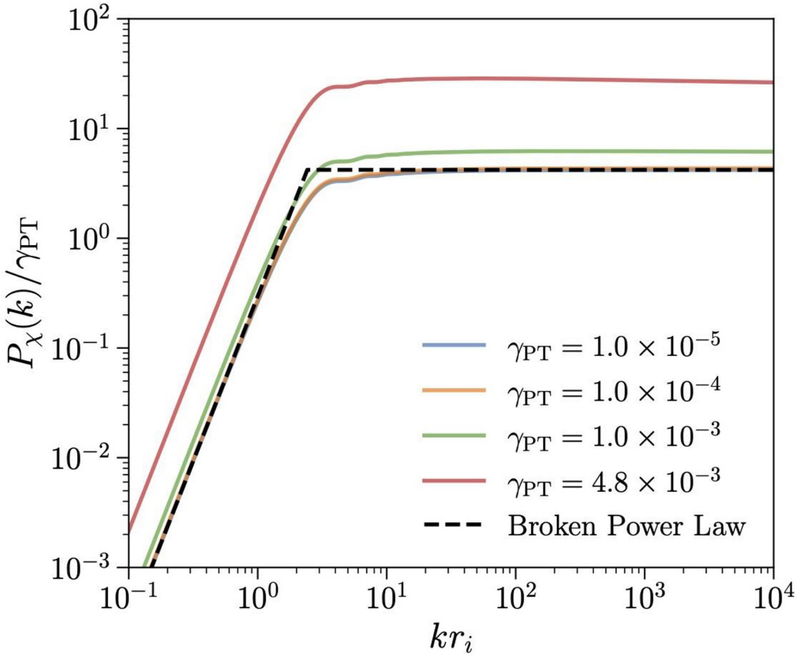
$$\mathcal{S}_{ab} = -3H\left(\frac{\delta\rho_a}{\dot\rho_a} - \frac{\delta\rho_b}{\dot\rho_b}\right) = 0$$

Adiabatic?



# Isocurvature Power Spectrum

$$\rho_\chi(\mathbf{x},t) = \Delta V \left[ 1 - \sum_{I:t_I<t} \Theta(r_I - |\mathbf{x} - \mathbf{x_I}|) \right] \Longrightarrow \langle \delta_\chi(\mathbf{k}) \delta_\chi(\mathbf{k'}) \rangle \equiv 2\pi^2 \frac{(2\pi)^3 \delta^3(\mathbf{k}+\mathbf{k'})}{k^3} P_\chi(k)$$



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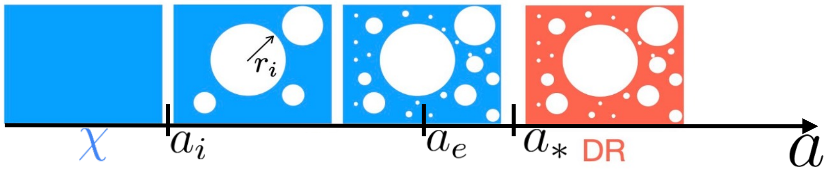
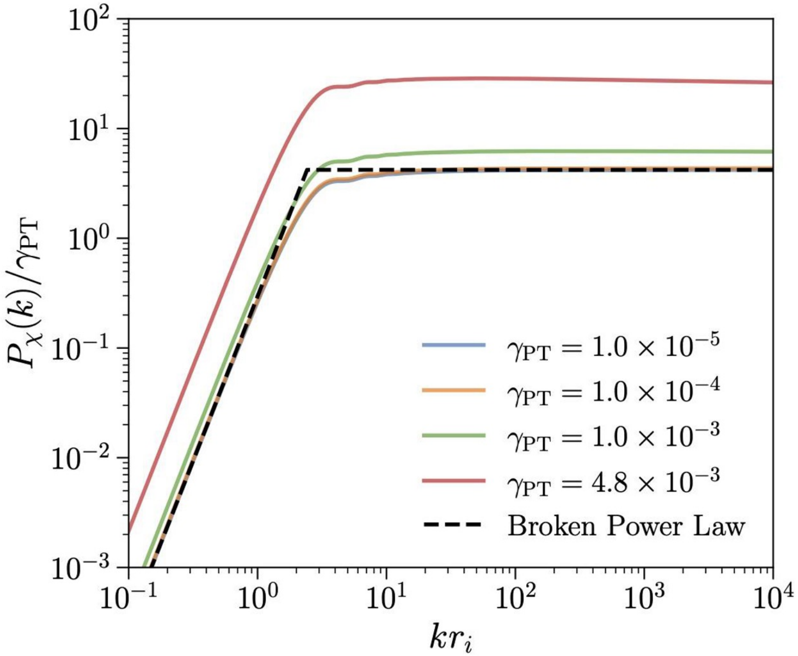


$$\rho_\chi(\mathbf{x},t) = \Delta V\left[1 - \sum_{I:t_I<t}\Theta(r_I - |\mathbf{x}-\mathbf{x_I}|)\right]$$

⇒

$$\langle\delta_\chi(\mathbf{k})\delta_\chi(\mathbf{k}')\rangle \equiv 2\pi^2\frac{(2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')}{k^3}P_\chi(k)$$



$$P_{\rm iso}(k) \approx P_\chi(k) \approx \gamma_{\rm PT}\begin{cases}\frac{8}{27}(kr_i)^3 & kr_i \ll 1\\ 4.2 & kr_i \gg 1\end{cases}$$

The PT completes when $\frac{\Gamma_{\rm PT}}{H^4} = \gamma_{\rm PT}\left(\frac{T_{\rm rh}}{T}\right)^8 \approx 1$

$$\mathcal{S}_{{\rm dr},\gamma} = \frac{3}{4}\delta_{\rm dr} - \frac{3}{4}\delta_\gamma \neq 0$$

Isocurvature!

$$P_{\rm iso}(k) = f_{\rm iso}^2 A_s\begin{cases}(k/k_i)^3 & k \le k_i\\ 1 & k > k_i\end{cases}$$

# Temperature Anisotropy Angular Power Spectrum

$$\Delta(\mathbf{x}, \hat{\mathbf{n}}, \tau) \equiv \frac{\Delta T}{\bar{T}} \xrightarrow[\text{Transform}]{\text{Fourier and Legendre}} \Delta_\ell(\mathbf{k}, \tau) = c^{\rm ad}(\mathbf{k})\Delta_\ell^{\rm ad}(k, \tau) + c^{\rm iso}(\mathbf{k})\Delta_\ell^{\rm iso}(k, \tau)$$

Only curvature initially

Only isocurvature initially

# Temperature Anisotropy Angular Power Spectrum

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Only curvature initially

Only isocurvature initially

$$C_\ell^{TT} = 4\pi \int d(\ln k) \left[ P_{\rm ad}(k) \left| \Delta_\ell^{\rm ad}(k, \tau_0) \right|^2 + P_{\rm iso}(k) \left| \Delta_\ell^{\rm iso}(k, \tau_0) \right|^2 \right]$$

# Temperature Anisotropy Angular Power Spectrum

$$\Delta(\mathbf{x},\hat{\mathbf{n}},\tau) \equiv \frac{\Delta T}{\bar{T}} \xrightarrow{\phantom{xxxxxxxx}} \Delta_\ell(\mathbf{k},\tau) = c^{\mathrm{ad}}(\mathbf{k})\Delta_\ell^{\mathrm{ad}}(k,\tau) + c^{\mathrm{iso}}(\mathbf{k})\Delta_\ell^{\mathrm{iso}}(k,\tau)$$

Fourier and Legendre Transform

Only curvature initially

Only isocurvature initially

$$C_\ell^{TT} = 4\pi \int d(\ln k) \left[ P_{\mathrm{ad}}(k) \left|\Delta_\ell^{\mathrm{ad}}(k,\tau_0)\right|^2 + P_{\mathrm{iso}}(k) \left|\Delta_\ell^{\mathrm{iso}}(k,\tau_0)\right|^2 \right]$$

$$P_{\mathrm{iso}}(k) = \boxed{f_{\mathrm{iso}}^2} A_s \begin{cases} (k/k_i)^3 & k \le k_i \\ 1 & k > k_i \end{cases}$$

$$\propto \boxed{\Delta N_{\mathrm{eff}}^2}$$

$$\boxed{k_i} \sim r_i^{-1}$$

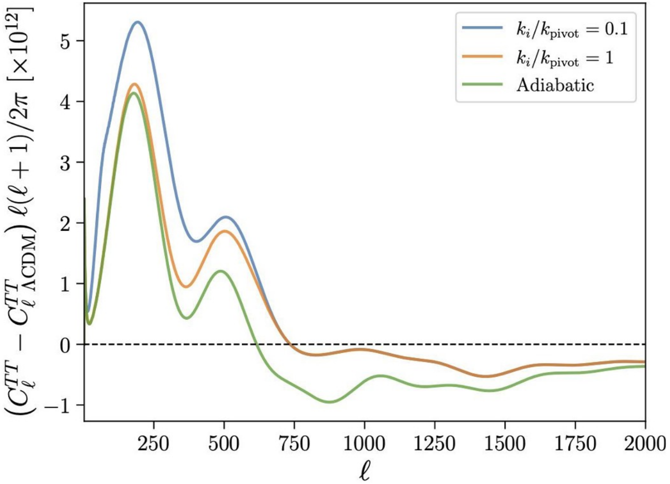
# Calculation of Angular Power Spectrum with CLASS

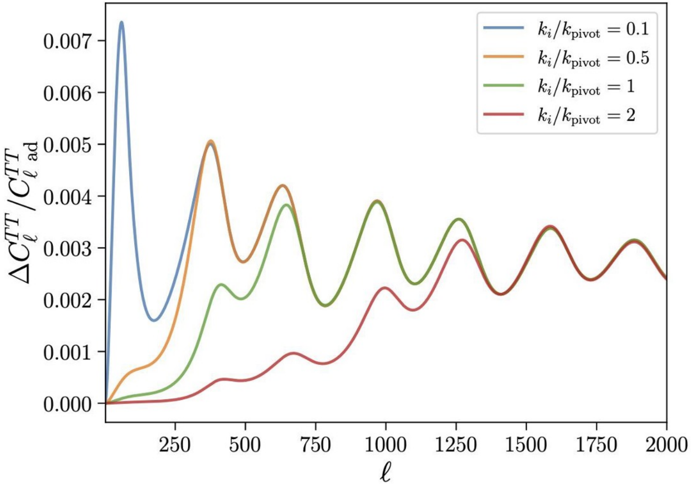
$$f_{\rm iso} = 10$$

$$\Delta N_{\rm eff} = 0.1$$

$$k_{\rm pivot} \equiv 0.05\,{\rm Mpc}^{-1}$$

$$\ell \sim 500 \times (k/k_{\rm pivot})$$





$$C_\ell^{TT} = 4\pi \int d(\ln k) \left[ P_{\rm ad}(k) \left|\Delta_\ell^{\rm ad}(k,\tau_0)\right|^2 + P_{\rm iso}(k) \left|\Delta_\ell^{\rm iso}(k,\tau_0)\right|^2 \right]$$

$$P_{\rm iso}(k) = \boxed{f_{\rm iso}^2} A_s \begin{cases} (k/k_i)^3 & k \le k_i \\ 1 & k > k_i \end{cases}$$

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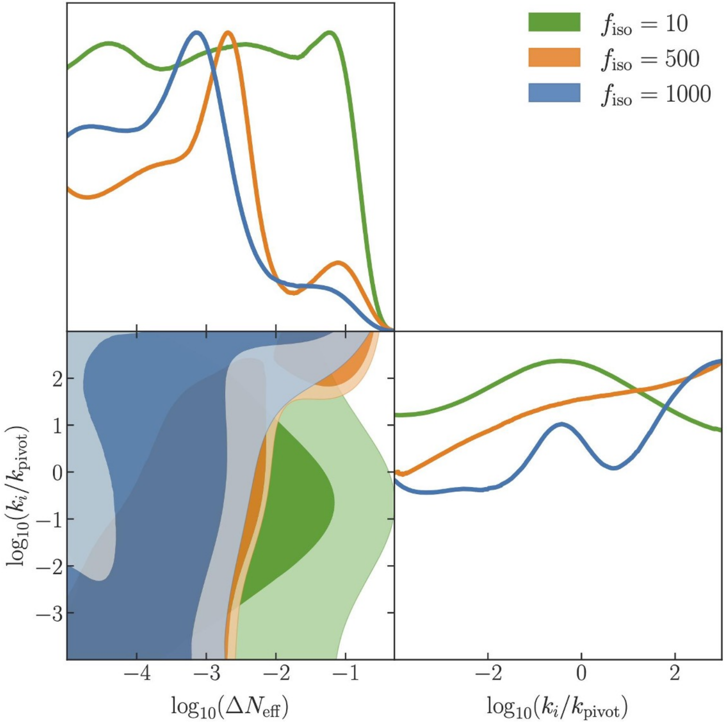
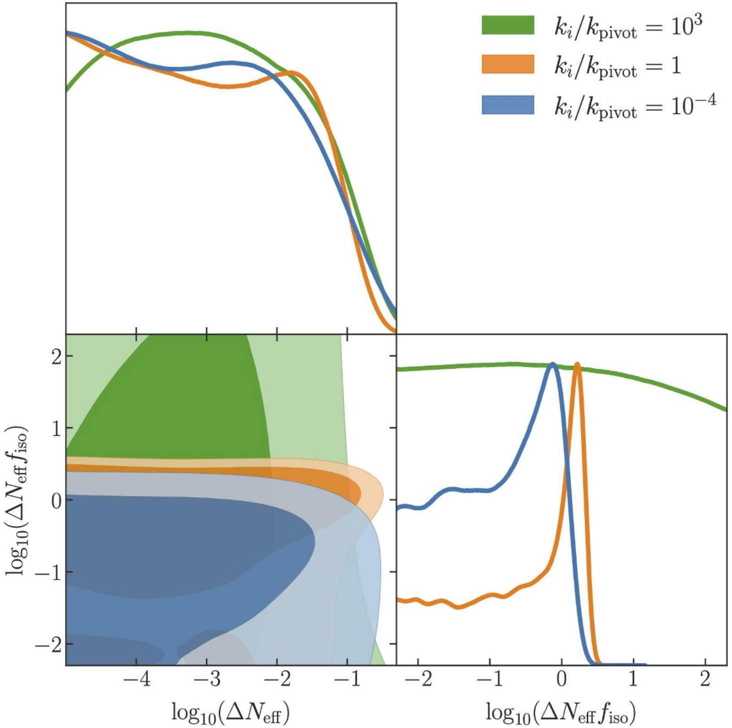
# MCMC Results with Planck+BAO





**$f_{\rm iso}$ Fixed (Left)**

-Small $f_{\rm iso}$ : preference for intermediate values of $k_i$

-large $f_{\rm iso}$ : limits weaken at large $k_i$

**$k_i$ Fixed (Right)**

-Limits depend on $\Delta N_{\rm eff} f_{\rm iso}$ for small $\Delta N_{\rm eff}$ and $k_i \lesssim k_{\rm pivot}$

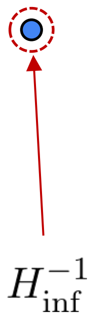
# Conclusion

- We set constraints on a class of FOPTs
    - PT starts during inflation and remain incomplete until after reheating
    - When the PT completes, bubble collisions produce dark radiation
    - Leads to DR isocurvature modes observable in CMB
- Associated signal: gravitational waves from when the PT completes
    - If a gravitational wave signal is ever observed, our constraints can be interpreted as constraints on the scale of inflation

# Thank You!

From Euler Lagrange Eqs.

$$\partial_t(\gamma_w v_w) + 3H_{\rm inf}(\gamma_w v_w) \approx \frac{\Delta V}{\sigma}$$

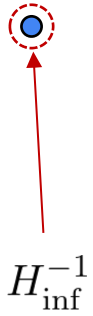


From Euler Lagrange Eqs.

$$\partial_t(\gamma_w v_w) + 3H_{\rm inf}(\gamma_w v_w) \approx \frac{\Delta V}{\sigma}$$

Bubble walls reach terminal velocity

$$(\gamma_w v_w)\,|_\infty = \frac{\Delta V}{3\sigma H_{\rm inf}} \gg 1$$



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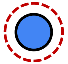
And quickly approach the comoving horizon $r_I = [a(t_I)H_{\text{inf}}]^{-1}$



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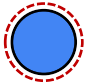


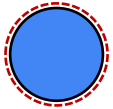
From Euler Lagrange Eqs.

$$\partial_t(\gamma_w v_w) + 3H_{\rm inf}(\gamma_w v_w) \approx \frac{\Delta V}{\sigma}$$

Bubble walls reach terminal velocity

$$(\gamma_w v_w)\,|_\infty = \frac{\Delta V}{3\sigma H_{\rm inf}} \gg 1$$

And quickly approach the comoving horizon $r_I = [a(t_I)H_{\rm inf}]^{-1}$



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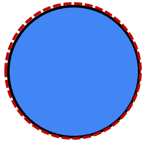

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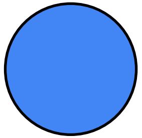
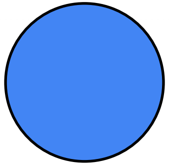


From Euler Lagrange Eqs.

$$\partial_t(\gamma_w v_w) + 3H_{\text{inf}}(\gamma_w v_w) \approx \frac{\Delta V}{\sigma}$$

$$\sigma = \int dn(\partial_n \phi)^2 = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}$$



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And quickly approach the comoving horizon $r_I = [a(t_I)H_{\text{inf}}]^{-1}$

# $\chi$ Power Spectrum (Details)

$$\bar{\rho}_\chi(t) = \Delta V p_{\text{false}}(t) = \Delta V e^{-t/\tau_{\text{PT}}}$$

$$\tau_{\text{PT}}^{-1} \equiv \frac{4\pi}{3} \gamma_{\text{PT}} H_{\text{inf}}$$

$$\langle \delta_\chi(\mathbf{k}) \delta_\chi(\mathbf{k}') \rangle = e^{2t_e/\tau_{PT}} \frac{(4\pi)^2}{k^3 k'^3} N \int d^4x \, p_1(x) e^{-i(\mathbf{k}+\mathbf{k}')\cdot \boldsymbol{x}} \mathcal{A}(kr(t)) \mathcal{A}(k'r(t))$$

$$p_1(x) = \frac{1}{\mathcal{V}} \left( \frac{1}{N} \frac{dN}{dt} \right)$$

$$\mathcal{A}(y) \equiv \sin y - y \cos y$$

$$\frac{dN}{dt} = \mathcal{V}_{\text{false}}(t) a(t)^3 \Gamma_{\text{PT}}$$

$$\mathcal{V}_{\text{false}}(t) = \mathcal{V} p_{\text{false}}(t) = \mathcal{V} e^{-t/\tau_{\text{PT}}}$$

# DR Isocurvature Power Spectrum

$$dt = a d\tau$$

The PT completes after reheating when $\frac{\Gamma_{\rm PT}}{H^4} = \gamma_{\rm PT}\left(\frac{T_{\rm rh}}{T}\right)^8 \approx 1$ at conformal time $\tau_*$

All remaining energy density in $\chi$ is quickly converted to DR

Working the gauge $\hat{\delta}_\gamma = 0$

$$\hat{\delta}_{\rm dr}(\mathbf{k}, \tau_*) \approx \delta_\chi(\mathbf{k}, t_*) \approx \delta_\chi(\mathbf{k}, t_e)$$

$$\mathcal{S}_{{\rm dr},\gamma} = \frac{3}{4}\left(\delta_{\rm dr} - \delta_\gamma\right) = \frac{3}{4}\hat{\delta}_{\rm dr}$$

$$P_{\rm iso}(k) = \frac{16}{9} P_{\mathcal{S}}(k) \approx P_\chi(k)$$

$\chi$ density is approximately gauge invariant as $\delta\hat{\rho}_a = \delta\rho_a + \rho_a' \delta\tau$

# DR Isocurvature and Adiabatic Modes Formalism

The perturbation variables $X \in [h, \eta, \delta_a, \theta_a, \sigma_a, F_{a,\ell} \cdots]$ can be written as

$$X(\mathbf{k}, \tau) = c^{\mathrm{ad}}(\mathbf{k}) X^{\mathrm{ad}}(k, \tau) + c^{\mathrm{iso}}(\mathbf{k}) X^{\mathrm{iso}}(k, \tau)$$

Where the initial conditions for the modes satisfy

$$\delta_{\mathrm{dr}}^{\mathrm{ad}} = \delta_{\nu}^{\mathrm{ad}} = \delta_{\gamma}^{\mathrm{ad}} \qquad \delta_{\mathrm{dr}}^{\mathrm{iso}} = 1, \quad \delta_{\gamma}^{\mathrm{iso}} = \delta_{\nu}^{\mathrm{iso}} = -\frac{R_{\mathrm{dr}}}{1 - R_{\mathrm{dr}}}$$

$$\zeta = \eta - \mathcal{H}\frac{\delta\rho}{\rho'} = c^{\mathrm{ad}}(\mathbf{k})$$

$$\mathcal{S}_{\mathrm{dr},\gamma} = -3\mathcal{H}\left(\frac{\delta\rho_{\mathrm{dr}}}{\rho'_{\mathrm{dr}}} - \frac{\delta\rho_{\gamma}}{\rho'_{\gamma}}\right) \approx \frac{3}{4} c^{\mathrm{iso}}(\mathbf{k})$$

And the power spectra are defined as $\langle c^A(\mathbf{k}) c^A(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3}(2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_A(k)$

$$P_{\mathrm{ad}} = P_{\zeta} \qquad P_{\mathrm{iso}} = \frac{16}{9} P_{\mathcal{S}}$$

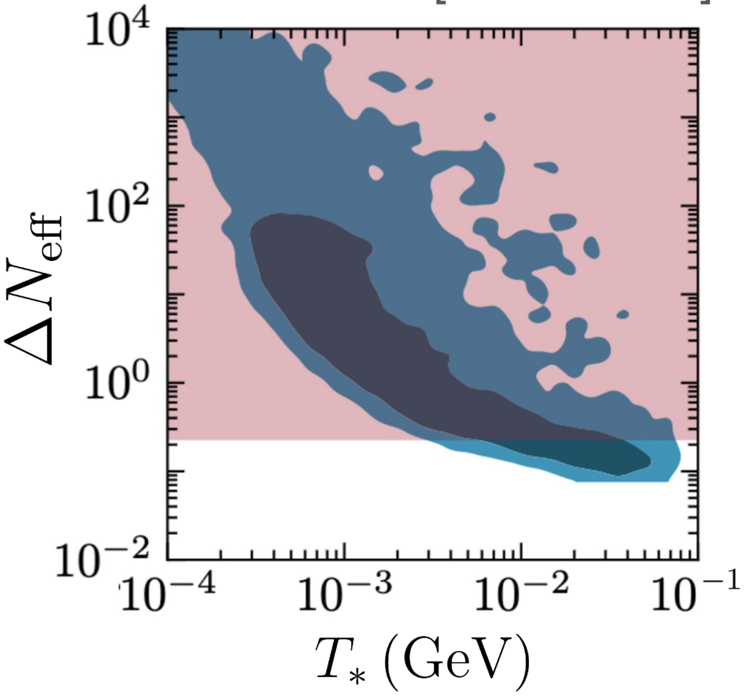
# NANOGrav Signal

Asymptotic limits for small $\Delta N_{\rm eff}$

$$\Delta N_{\rm eff}\, f_{\rm iso} < \beta(k_i) \sim \mathcal{O}(1)$$

$$\Delta N_{\rm eff} < 2.8 \times 10^{-5} \left(\frac{T_{\rm rh}}{T_*}\right)^4 \left(\frac{\beta(k_i)}{1.25}\right)$$

[2306.09411]



# Estimated Non-Gaussianity Constraints

$$\langle c_{\rm iso}(\mathbf{k_1}) c_{\rm iso}(\mathbf{k_2}) c_{\rm iso}(\mathbf{k_3})\rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) B_{\rm iso}(k_1+k_2+k_3)$$

$$B_{\rm iso}(k,k,k) \approx -0.97 \times f_{\rm iso}^2 A_s \frac{1}{k^6}$$

Naive mapping from neutrino density isocurvature suggests:

$$f_{\rm iso}^2 \Delta N_{\rm eff}^3 \lesssim 2 \times 10^{-4}$$

A dedicated search for non Gaussianity in the equilateral configuration is needed