# Curvature Perturbations Protected Against One Loop

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# Main message

Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

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Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

**Even at one loop level**

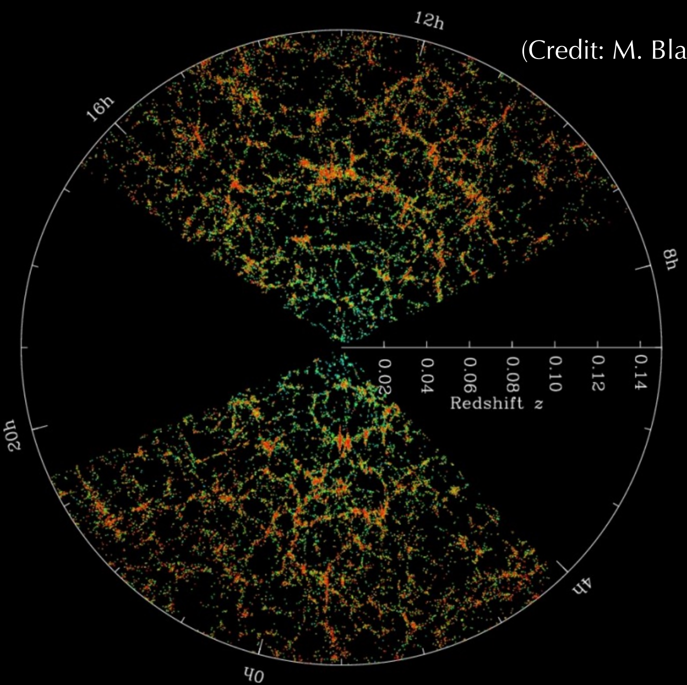
# Outline

- **Introduction of curvature perturbations**
- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- Summary

# Cosmological perturbations

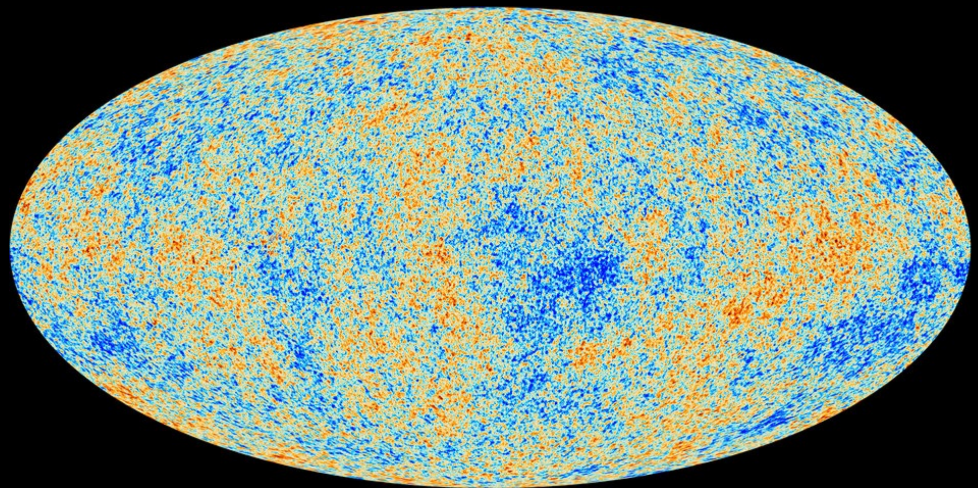
We have observed cosmological density perturbations in the Universe.

(Credit: M. Blanton and SDSS)




Large Scale Structure
(Galaxies are gathered in the bright regions.)

(Credit: ESA and the Planck Collaboration)



CMB anisotropies
(red: hot = dense, blue: cold = sparse )

$$\mathcal{P}_\zeta = 2.1 \times 10^{-9} \ \text{(Planck 2018)}$$

$$\to \delta\rho/\bar{\rho} \simeq 10^{-5}$$

$\zeta$: curvature perturbation

# What is curvature perturbation?

curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

(In the gauge where $g_{ij}|_{i\neq j} = 0$)

In the isotropic and homogeneous Universe (FLRW metric),

$$a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \to a^2\left(\frac{\mathrm{d}^2 r}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\Omega^2)\right)$$

$$R_3 = \frac{6K}{a^2} = 4\nabla^2\psi$$

3-dim. Ricci scalar

# What people call curvature perturbation

$$\mathrm{d}^2 s = g_{00}\mathrm{d}\eta^2 + 2g_{0i}\mathrm{d}\eta\mathrm{d}x^i + a^2 \mathrm{e}^{-2\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$\psi$ itself is gauge dependent (depends on the coordinate choice).

In Cosmology, the following gauge-invariant quantities are often used:

$$\zeta = -\psi + \frac{\delta\rho}{3(\rho + P)}$$

$\zeta$ coincides the curvature in uniform density gauge, $\delta\rho = 0$.

$$\mathcal{R} = -\psi - \frac{H\delta\phi}{\dot{\phi}}$$

$\mathcal{R}$ coincides the curvature in comoving gauge, $\delta\phi = 0$.

In the superhorizon limit,

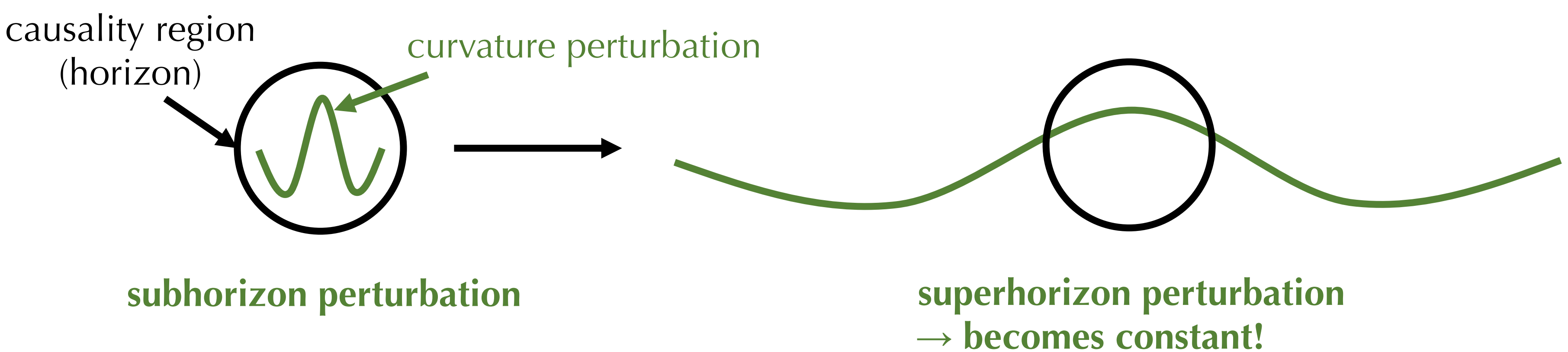
$$\zeta = \mathcal{R}$$

People often call $\zeta$ and $\mathcal{R}$ curvature perturbations.

# Why is curvature perturbation used?

$\zeta$ and $\mathcal{R}$ are conserved (constant) on superhorizon scales in single field inflation models.

**During inflation**

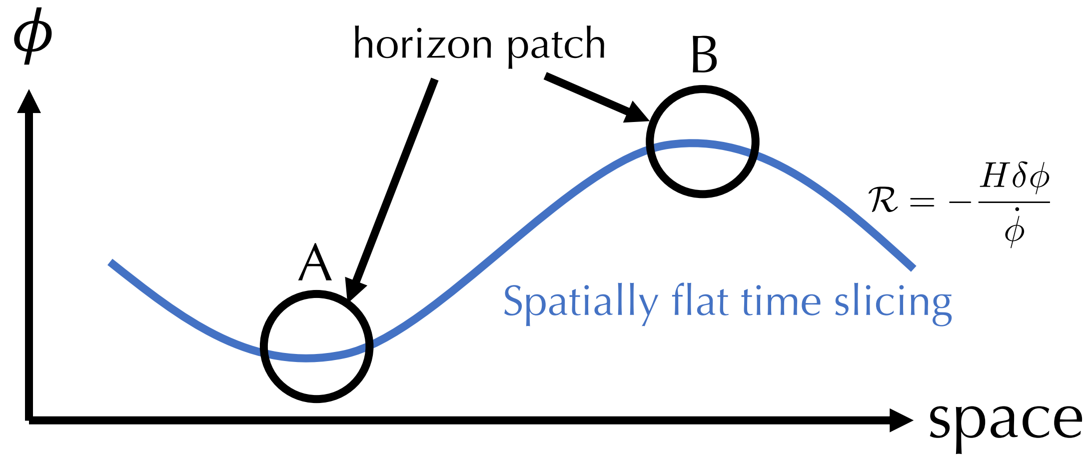


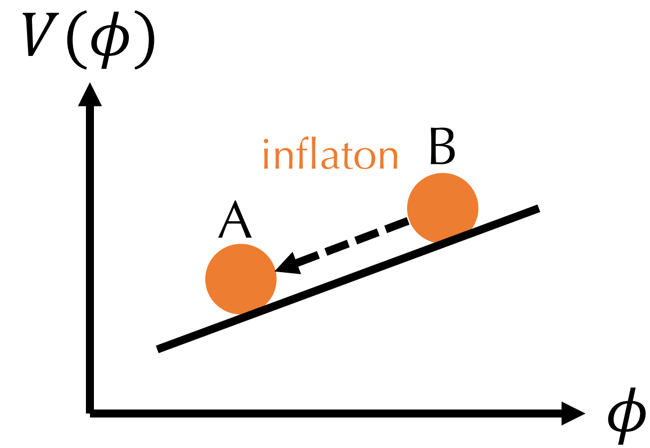

Useful in characterizing the amplitude of cosmological perturbations.

$$\mathcal{P}_\zeta(k < \mathcal{O}(1)\,\mathrm{Mpc}^{-1}) \simeq 2.1 \times 10^{-9} \;\; \text{(Planck 2018)}$$

# Separate Universe and curvature conservation

Focus on superhorizon-limit curvature perturbations. (neglect $\mathcal{O}((k/aH)^2)$ contributions)




**Separate Universe picture:**

For local Universes, superhorizon perturbations can be regarded as the background.

A local observer inside a horizon patch **cannot recognize the existence of the superhorizon-limit curvature perturbations**. To be consistent with this, curvature must be constant.

Q: What if superhorizon curvature is time dependent?

A: The local observer can recognize its existence through $e^{2\zeta(\eta)}a^2dx^2$

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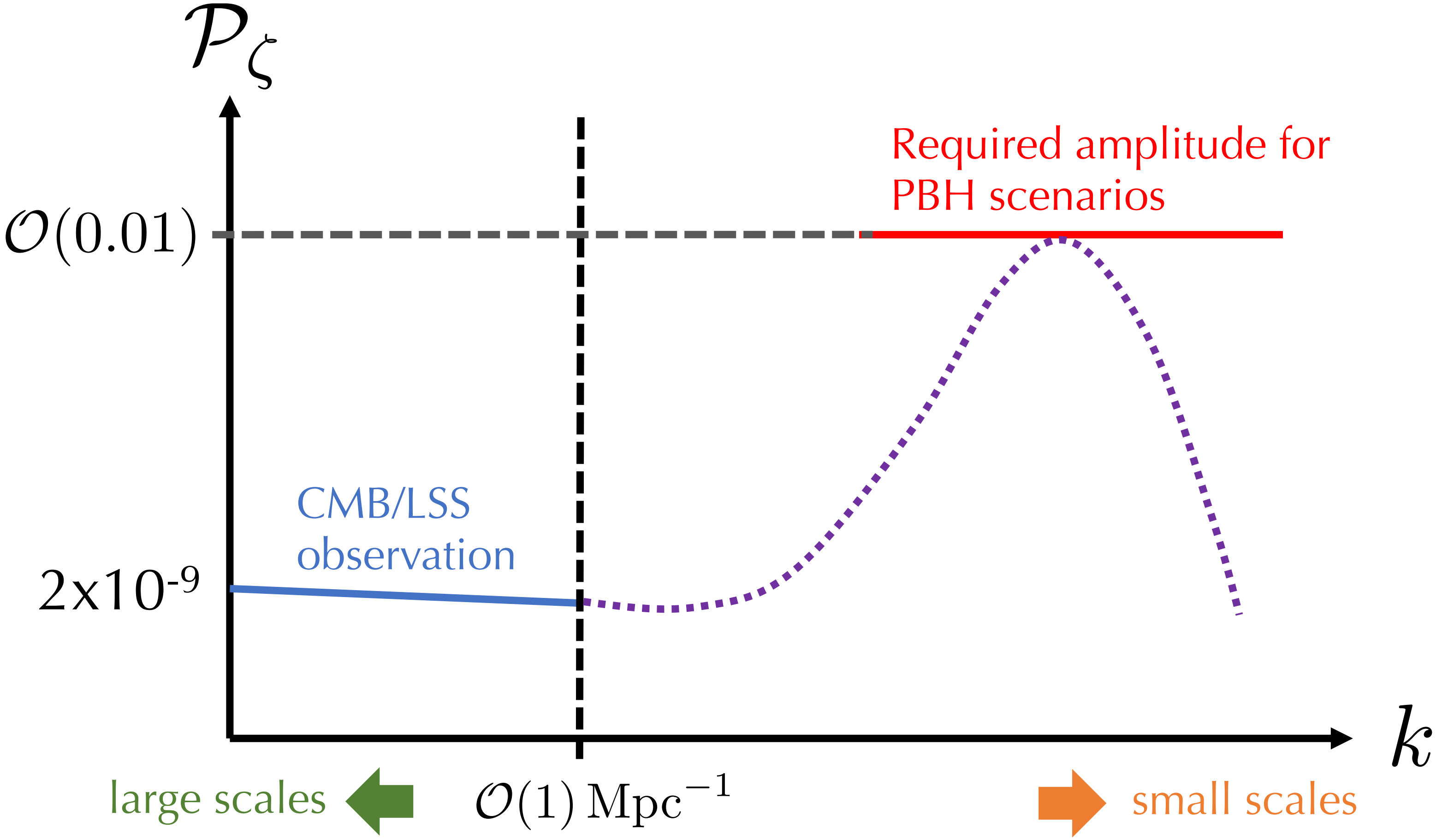
# Large perturbations for PBH scenarios

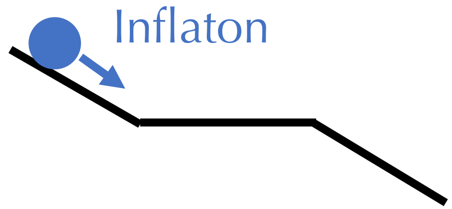
Primordial black holes are candidates of DM and BHs detected by LIGO-Virgo-KAGRA collaborations.



# Inflaton potentials for large amplification

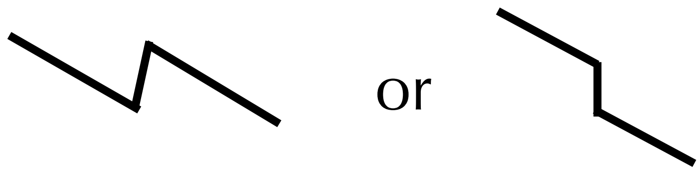
Single field models for large amplification of density perturbations:

flatter region



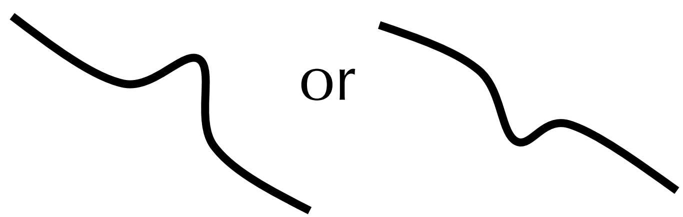

(Starobinsky 1992, Ivanov *et al*. 1994, Inoue and Yokoyama 2001, Kinney 2005)

step feature



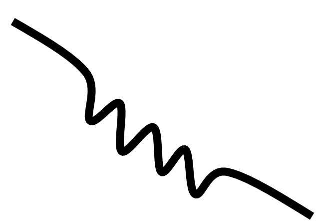

(Kefala et al. 2020, Inomata et al. 2021)

bump/dip feature




(Ozsoy et al. 2018, Mishra and Sahni 2019)

oscillatory feature



(R.G. Cai et al. 2019, Zhou et al. 2020, Peng et al. 2021)

# One loop corrections

Lagrangian: $$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi)$$

E.o.m. for the inflaton fluctuations: (slow-roll-parameter suppressed terms neglected)

$(V_{(n)} \equiv \partial^n V / \partial \phi^n)$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2 \delta\phi + a^2 \frac{\partial^2 V}{\partial \phi^2}\delta\phi = -a^2 \sum_{n>2} \frac{1}{(n-1)!} V_{(n)} (\delta\phi)^{n-1}$$

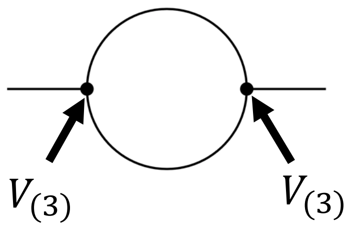
beyond liner order corrections

**In-in formalism:** (Jordan 1986, Calzetta and Hu 1987, Weinberg 2005)

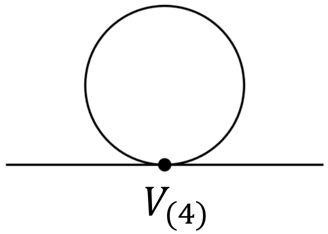
$$\langle \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta)\rangle = \langle 0| \left(T e^{-i\int_{-\infty}^{\eta} \mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi_{\mathbf{k}}(\eta)\delta\phi_{\mathbf{k}'}(\eta) \left(T e^{-i\int_{-\infty}^{\eta} \mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$\left(H_{{\rm int},n} \equiv \int \mathrm{d}^3 x\ a^4 \mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!} V^{(n)}(\phi)\delta\phi^n\right)$$

**two vertices**




**one vertex**




The lowest order corrections to linear power spectrum appear as one loops.

# Superhorizon curvature evolves?

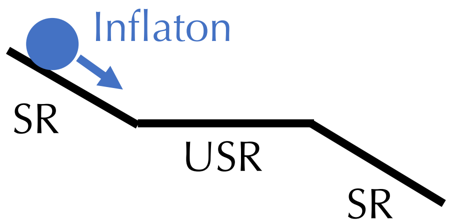
(Note: the conservation of linear $\zeta$ is well known.)

**Recent claim:**
Superhorizon-**limit** curvature perturbations **are not conserved at one-loop level** in the transitions of slow-roll (SR) → ultra-slow roll (USR) → SR.

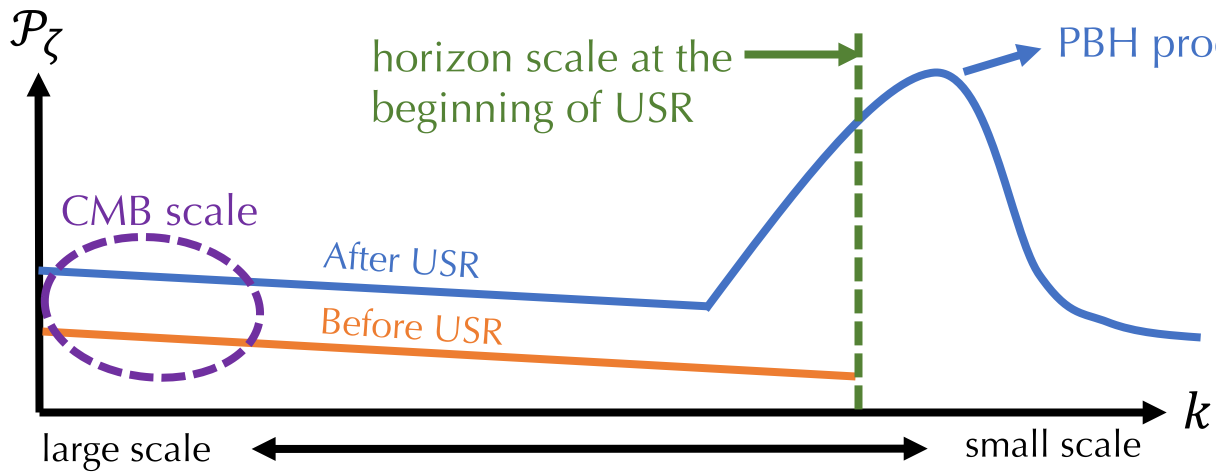
e.o.m.

$$3H\dot{\phi} + V'(\phi) = 0 \text{ (SR)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0 \text{ (USR)}$$




Kristiano & Yokoyama (2022), followed by
Riotto, Choudhury et al., Firouzhahi, Motohashi & Tada, Franciolini et al., Gianmassimo, Cheng et al., Maity et al., Davies et al. (2023), Saburov & Ketov, Guillermo & Egea (2024)



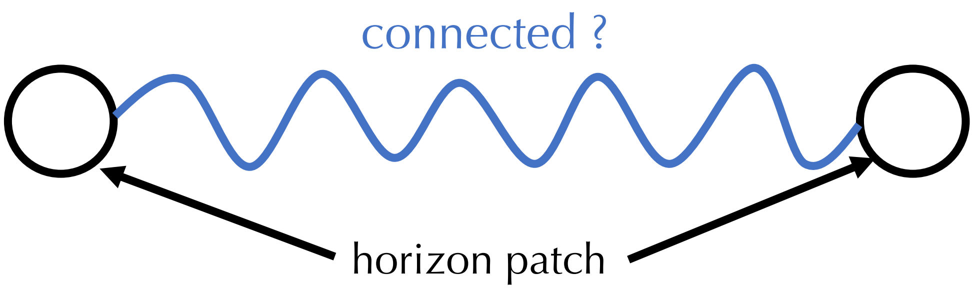

The one loop corrections can be comparable to the tree-level power spectrum in some PBH models. → Break down of perturbation theory? → models constrained?

**However, this is inconsistent with the separate Universe picture...**

# Universes connected?

The violation of the separate Universe means the Universes connected through the distance larger than the horizon.

→ The causality is violated?




**I am going to show the conservation of superhorizon curvature at one loop level.**
**(separate universe picture is valid, causality is satisfied)**

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# Ongoing debates

The higher order action is needed for one loop calculation.

**Comoving gauge ($\delta\phi$=0)** is often taken, where $\zeta$ appears as a metric perturbation.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

astro-ph/0210603, Maldacena

$$\begin{aligned} S_3 = \int & \frac{1}{4}\frac{\dot\phi^4}{\dot\rho^4}[e^{3\rho}\dot\zeta^2\zeta + e^\rho(\partial\zeta)^2\zeta] - \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\partial_i\chi\partial_i\zeta + \\ & -\frac{1}{16}\frac{\dot\phi^6}{\dot\rho^6}e^{3\rho}\dot\zeta^2\zeta + \frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\dot\zeta\zeta^2\frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi}{\dot\phi\dot\rho} + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}\right] + \frac{1}{4}\frac{\dot\phi^2}{\dot\rho^2}e^{3\rho}\partial_i\partial_j\chi\partial_i\partial_j\chi\zeta \\ & + f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1 \end{aligned}$$

arXiv:0709.2708, Jarnhus & Sloth

$$\begin{aligned} S^{(4)} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x a^3 \Big\{ & -\frac{1}{3}\zeta^3\partial^2\zeta - 2\alpha^{(1)}(\zeta\partial_i\zeta\partial^i\zeta + \zeta^2\partial^2\zeta) + \dot\phi_c^2\alpha^{(1)2}\left[\frac{9}{2}\zeta^2 - 3\zeta\alpha^{(1)} + \alpha^{(1)2}\right] \\ & \left[\frac{1}{2}\zeta^2 + \zeta\alpha^{(1)} + \alpha^{(1)2}\right]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(1)} - \partial^2\chi^{(1)}\partial^2\chi^{(1)}\right] + (6H^2 - \dot\phi^2)\alpha^{(2)2} \\ & - 2[\zeta + \alpha^{(1)}]\left[\partial_i\partial_j\chi^{(1)}\partial^i\partial^j\chi^{(2)} - \partial^2\chi^{(1)}\partial^2\chi^{(2)} - 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(1)}\partial^j\zeta\right] \\ & - 2\left[2\partial_i\partial_j\chi^{(2)}\partial^i\chi^{(1)}\partial^j\zeta + 2\partial_i\partial_j\chi^{(1)}\partial^i\chi^{(2)}\partial^j\zeta - \partial_j\chi^{(1)}\partial_i\zeta\partial^i\chi^{(1)}\partial^j\zeta\right] \\ & + \frac{1}{2}\partial_i\beta_j{}^{(2)}\partial^i\beta^{j\,(2)} - 2\alpha^{(1)}\partial_i\partial_j\chi^{(1)}\partial^i\beta^{j\,(2)} \Big\} \end{aligned}$$

However, these expressions neglect the boundary terms.

e.g.
$$\int \mathrm{d}t A\dot{B} = -\int \mathrm{d}t\dot{A}B + \underline{\int \mathrm{d}t\frac{\mathrm{d}}{\mathrm{d}t}(AB)}$$

boundary term

Ongoing debates: the missing boundary terms lead to the curvature conservation?

Fumagalli (2023), Tada et al. (2023), Firouzhahi (2023), Braglia & Pinol (2024), Kawaguchi et al. (2024)

# Strategy of this work

In this work, **spatially-flat gauge ($\psi = 0$)** is taken, where $\delta\phi$ is the basic quantity.

The metric perturbations are suppressed by slow-roll parameter $\epsilon$, compared to $V_{(n)}$ terms.

($\epsilon \to 0$ is known as the decoupling limit in effective field theory of inflation.)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

$$\downarrow \qquad \left(V_{(n)} \equiv \frac{\partial^n V(\phi)}{\partial\phi^n}\right)$$

$$S_n = -\int \mathrm{d}^4x\, a^4 \frac{V_{(n)}(\bar{\phi})}{n!}\delta\phi^n$$

**Simple!**

**Advantage:** The higher order action can be easily obtained.
$\rightarrow$ no need to worry about boundary terms!

**Strategy:** We first calculate the one-loop power spectrum of $\delta\phi$ .
Then, we connect it to the one loop-power spectrum of $\zeta$.

# One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \cdots)$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi})$$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k\delta\phi^{(2)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}},$$

$$\hat{\mathcal{N}}_k\delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}(\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}) - \frac{a^2}{6}V_{(4)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

**In-in formalism = equation of motion approach:** (Musso (2013), Inomata et al. (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\mathrm{int}}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\mathrm{int}}(\eta'')}\right)|0\rangle$$

$$= \dots$$

$$\left(H_{\mathrm{int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle$$

# One loop calculation

Equation of motion:

$$\delta\phi'' + 2\mathcal{H}\delta\phi' - \nabla^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\sum_{n>2}\frac{1}{(n-1)!}V_{(n)}(\delta\phi)^{n-1}$$

$(\delta\phi = \delta\phi^{(1)} + \delta\phi^{(2)} + \delta\phi^{(3)} + \cdots)$

$$\hat{\mathcal{N}}_k\delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2V_{(2)}(\bar{\phi})$$

$$\hat{\mathcal{N}}_k\delta\phi^{(2)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}},$$

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**In-in formalism = equation of motion approach:** (Musso (2013), Inomata et al. (2022))

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = \langle 0|\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta' H_{\rm int}(\eta')}\right)^\dagger \delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}\left(T e^{-i\int_{-\infty}^{\eta}\mathrm{d}\eta'' H_{\rm int}(\eta'')}\right)|0\rangle$$

$$= \ldots \qquad \left(H_{{\rm int},n} \equiv \int \mathrm{d}^3x\, a^4\mathcal{H}_n,\ \mathcal{H}_{n(>2)} = \frac{1}{n!}V^{(n)}(\phi)\delta\phi^n\right)$$

$$= \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{tree}} + \underbrace{\langle 0|\delta\phi^{(2)}_{\mathbf{k}}\delta\phi^{(2)}_{\mathbf{k}'}|0\rangle}_{\text{Poisson fluctuations}} + \underbrace{\langle 0|\delta\phi^{(1)}_{\mathbf{k}}\delta\phi^{(3)}_{\mathbf{k}'}|0\rangle + \langle 0|\delta\phi^{(3)}_{\mathbf{k}}\delta\phi^{(1)}_{\mathbf{k}'}|0\rangle}_{\text{nonzero even in superhorizon limit}}$$

tree

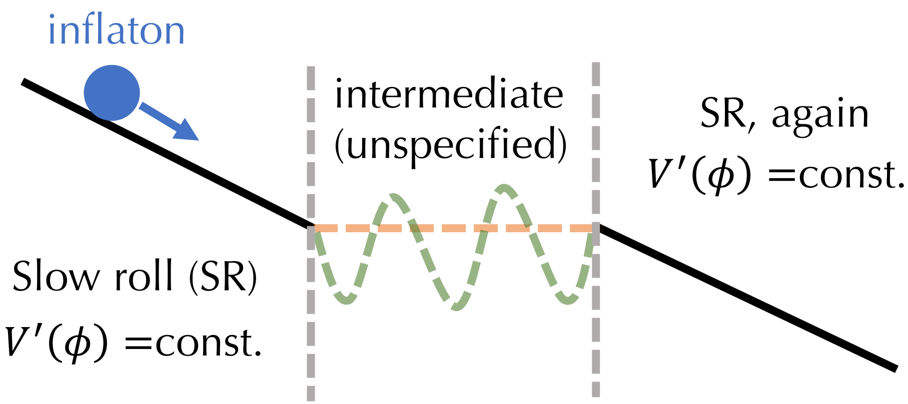
Poisson fluctuations
→ 0 in superhorizon limit

nonzero even in superhorizon limit

Superhorizon curvature perturbations evolve at one loop? → **No. There is a trick!**

# Conservation of curvature

The trick lies in the relation between $\delta\phi$ and $\zeta$.
Consider the simplest case:




During the intermediate period,
$\zeta$ is enhanced.
→ Peak power spectrum

We assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

From $\delta N$ formalism, curvature perturbations during the SR periods are:

$$-\zeta\big|_{\leq 1\text{-loop}} = \left.\frac{H\delta\phi}{\dot{\bar{\phi}}}\right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} \quad \text{(SR)}$$

On the other hand, $\frac{H\delta\phi}{\dot{\bar{\phi}}}$ is **always** constant:

$$\left.\frac{H\delta\phi}{\dot{\bar{\phi}}}\right|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

**Point:** $\dot{\bar{\phi}}$ gets the one-loop backreaction, $\dot{\bar{\phi}}^{(2)}$, which cancels $\delta\phi^{(3)}$.

$\zeta$ during the fist and the second SR periods coincide. → $\zeta$ is conserved!

# One loop backreaction

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{(1)}(\bar{\phi}) = -\frac{1}{2}V_{(3)}(\bar{\phi}) \left\langle (\delta\phi^{(1)})^2 \right\rangle$$

Take time derivative

$$\left( \bar{\Pi} \equiv \dot{\bar{\phi}},\ \hat{\mathcal{N}}_k \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + k^2 + a^2 V_{(2)}(\bar{\phi}) \right)$$

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(0)} = 0,$$

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2}\left( V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right)$$

Recap:

$$\hat{\mathcal{N}}_k \delta\phi^{(1)}_{\boldsymbol{k}} = 0,$$

$$\hat{\mathcal{N}}_k \delta\phi^{(3)}_{\boldsymbol{k}} = -\frac{a^2}{2}V_{(3)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} (\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(2)}_{\boldsymbol{p}} + \delta\phi^{(2)}_{\boldsymbol{k}-\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}}) - \frac{a^2}{6}V_{(4)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} \delta\phi^{(1)}_{\boldsymbol{p}}\delta\phi^{(1)}_{\boldsymbol{p}'}\delta\phi^{(1)}_{\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}'}$$

After some (easy) calculation, we find

$$\hat{\mathcal{N}}_q \delta\phi^{(3)}_{\boldsymbol{q}} \Big|_{q\to 0} = -\frac{a^2}{2}\left( V_{(3)} \left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)} \left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)} \right) \frac{\delta\phi^{(1)}_{\boldsymbol{q}}}{\bar{\Pi}^{(0)}}$$

We finally obtain

$$\delta\phi^{(3)}_{\boldsymbol{q}} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}}\delta\phi^{(1)}_{\boldsymbol{q}} \quad \Rightarrow \quad \frac{H\delta\phi}{\dot{\bar{\phi}}}\Bigg|_{\leq 1\text{-loop}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$

**This relation is always satisfied.**

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- Recent claim: Curvature is not conserved?
- Conservation of curvature at one loop
- **Summary**

# Summary

Non-conservation of superhorizon curvature perturbations at one-loop level has recently been claimed.

However, the claim is inconsistent with the separate Universe picture.

I have taken the spatially-flat gauge and focus on $\delta\phi$ evolution at one loop level.

I have found that the superhorizon curvature is conserved if we carefully consider the one-loop backreaction.

# Main message

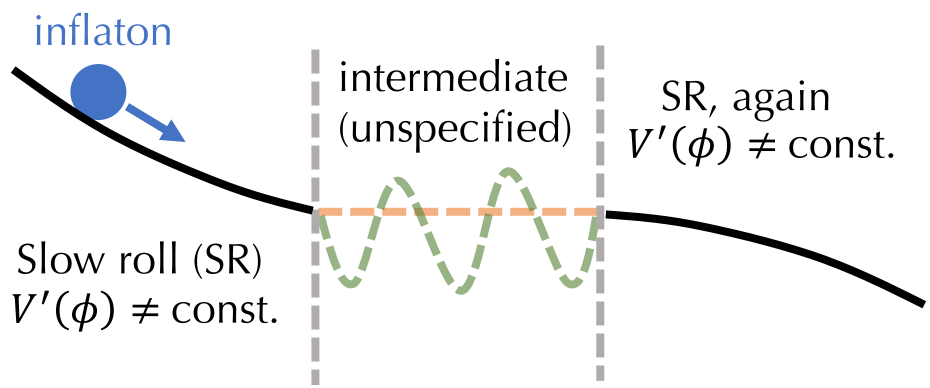
Superhorizon curvature perturbations are constant.

Separate Universe picture is valid.

**Even at one loop level**

# Backup

# In general SR potentials




Again, we assume the separate Universe satisfied at least during the SR periods **(do not assume that during the intermediate period)**.

When the separate Universe is satisfied, the curvature perturbations are conserved even at non-perturbative (including one-loop) level. (Lyth, Malik, and Sasaki, 2004)

This means that, if we find one concrete SR potential for the conservation of $\zeta$, the conservation is secured for any types of SR potential.

**One concrete example:** the SR potentials that have region of $V'(\phi) =$ const.

# Renormalization

In general, the loop contributions have divergence, which must be cancelled out by counter terms.

$$\hat{\mathcal{N}}_0 \bar{\Pi}^{(2)} = -\frac{a^2}{2}\left(V_{(3)}\left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)}\left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)}\right)$$

$$\hat{\mathcal{N}}_q \delta\phi_{\boldsymbol{q}}^{(3)}\Big|_{q\to 0} = -\frac{a^2}{2}\left(V_{(3)}\left\langle \delta\phi^2 \right\rangle^{\cdot} + V_{(4)}\left\langle \delta\phi^2 \right\rangle \bar{\Pi}^{(0)}\right)\frac{\delta\phi_{\boldsymbol{q}}^{(1)}}{\bar{\Pi}^{(0)}}$$

Counter terms are introduced in the same way for $\bar{\Pi}^{(2)} (= \dot{\bar{\phi}}^{(2)})$ and $\delta\phi_{\boldsymbol{q}}^{(3)}$ through $\hat{\mathcal{N}}_0$ .

$$\hat{\mathcal{N}}_q \equiv \frac{\partial^2}{\partial\eta^2} + 2\mathcal{H}\frac{\partial}{\partial\eta} + q^2 + a^2 V_{(2)}(\bar{\phi}) + \underline{a^2 m_{\text{ct}}^2}.$$

counter term

The introduction of the counter terms does not break the following relations:

$$\delta\phi_{\boldsymbol{q}}^{(3)} = \frac{\dot{\bar{\phi}}^{(2)}}{\dot{\bar{\phi}}^{(0)}}\delta\phi_{\boldsymbol{q}}^{(1)}$$

$$-\zeta = \frac{H\delta\phi}{\dot{\bar{\phi}}} = \frac{H(\delta\phi^{(1)} + \delta\phi^{(3)})}{\dot{\bar{\phi}}^{(0)} + \dot{\bar{\phi}}^{(2)}} = \frac{H\delta\phi^{(1)}}{\dot{\bar{\phi}}^{(0)}} = \text{const.}$$