Misalignment production of vector boson dark matter from axion-SU(2) inflation

Kai Murai
Tohoku University

Based on a collaboration with Tomohiro Fujita (Ochanomizu Univ.), Kazunori Nakayama (Tohoku Univ.), and Wen Yin (Tokyo Metropolitan Univ.)

JCAP 04 (2024) 007, arXiv:2312.06889

SUSY24 @ Instituto de Física Teórica, Madrid
June 14th, 2024

Dark matter

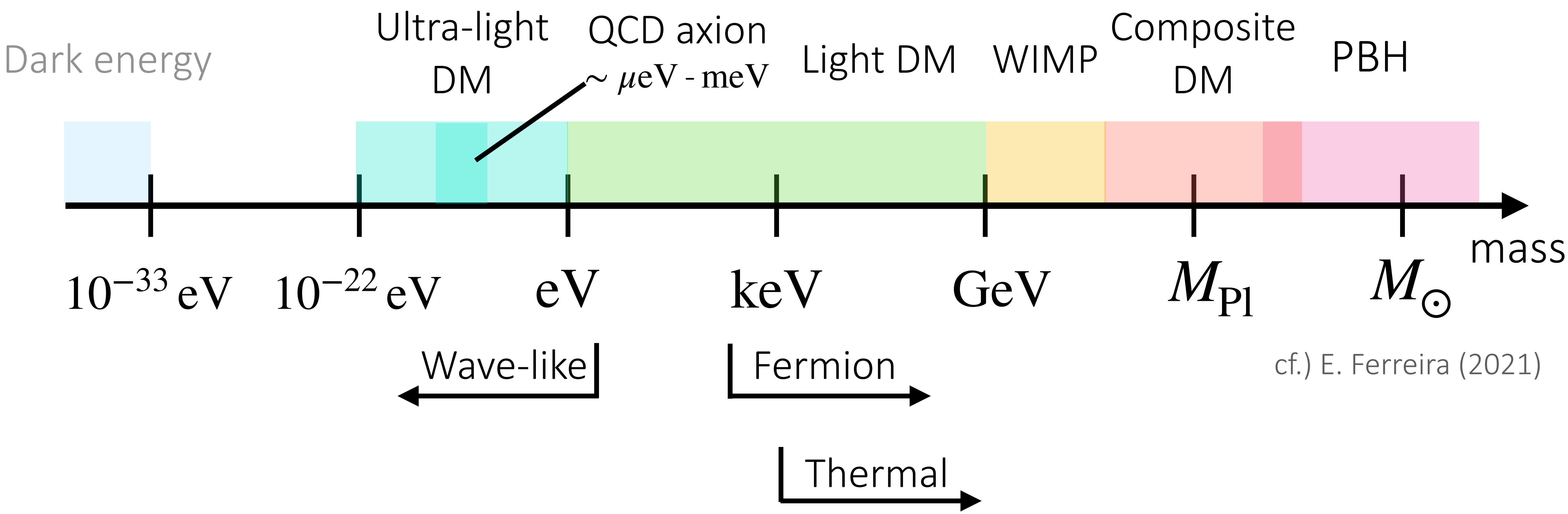
- DM mass range

We know the cold dark matter exists.

We don't know its mass, spin, interactions, boson or fermion, particle or wave.

Depending on its properties and origin, some mass regions are limited.

In this talk, we focus on vector boson dark matter.



cf.) E. Ferreira (2021)

Vector dark matter

- Vector dark matter

There are various known scenarios to produce dark photon **particles**:

- Gravitational production [Graham, Mardon, Rajendran (2015), Kolb & Long (2020), …]
- Through axion-like couplings [Agrawal et al. (2018), Co et al. (2018), Bastero-Gil et al. (2018), …]
- Thermal production with the bose enhancement [Yin (2023)]
- Decay of cosmic strings [Long & Wang (2019), Kitajima & Nakayama (2022)]

⋮

How about **condensate** of vector bosons?

In the case of scalars, we know the misalignment mechanism.

Vector dark matter

- Misalignment mechanism for vector?

Massive vector field: $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu \supset -\frac{m^2}{2a^2}A_iA_i$

It is convenient to define a "physical" field:

$$Q_i \equiv \frac{A_i}{a}$$

Then, the energy density and equation of motion become

$$\rho_A = \frac{1}{2}\left[\dot{Q}_i^2 + (m^2 + H^2)Q_i^2 + 2H\dot{Q}_iQ_i\right]$$

$$\ddot{Q} + 3H\dot{Q} + (m^2 + \dot{H} + 2H^2)Q = 0$$

Q is exponentially damped during inflation.

cf.) This damping can be compensated by a time-varying mass.

[Kaneta, Lee, Lee, Yi (2023)]

Vector dark matter

- How to avoid the damping

Non-minimal coupling [Arias et al. (2012)]

$$\mathscr{L} \supset \frac{1}{12} R A_\mu A^\mu$$ → The Hubble mass term is canceled.

This model suffers from a ghost instability of the vector field.

[Nakayama (2019)]

Coupling to the inflaton [Nakayama (2019)]

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \to -\frac{f^2(\phi)}{4} F_{\mu\nu} F^{\mu\nu}$$ $A_i(t)$ is generated during inflation

→
- Statistical anisotropy of adiabatic perturbations
- Isocurvature perturbations

We can avoid them by introducing a curvaton field.

[Kitajima & Nakayama (2023)]

Vector dark matter

- Our scenario

We consider the SU(2) gauge field: $A_i^a \ (a = 1,2,3)$.

→ An isotropic configuration is possible.

Free from anisotropies of the adiabatic perturbations

Such a configuration is **dynamically** realized in the axion-SU(2) inflation.

→ No dilution during inflation

This scenario can account for dark matter
with coherently oscillating vector field.

Axion-SU(2) inflation

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$
$$\tilde{F}^{a\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-\det[g_{\mu\nu}]}} F^a_{\rho\sigma},$$

- Chromo-natural inflation (CNI)

[Adshead & Wyman (2012)]

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

axion/inflaton — SU(2) gauge fields — Axion-gauge fields coupling

Slow-rolling axion

Source gauge fields

Effective friction

Homogeneous isotropic SU(2) gauge fields

$$A^a_i \propto \delta^a_i$$

This gauge field configuration is isotropic:

$$A^a_i \propto \delta^a_i \;\Rightarrow\; {}^\forall R, {}^\exists G : R_{ij}A^a_j = G^{ab}A^b_i \,.$$

spatial rotation (R), gauge transf. (G)

→ **Free from statistical anisotropy.**

Axion-SU(2) inflation

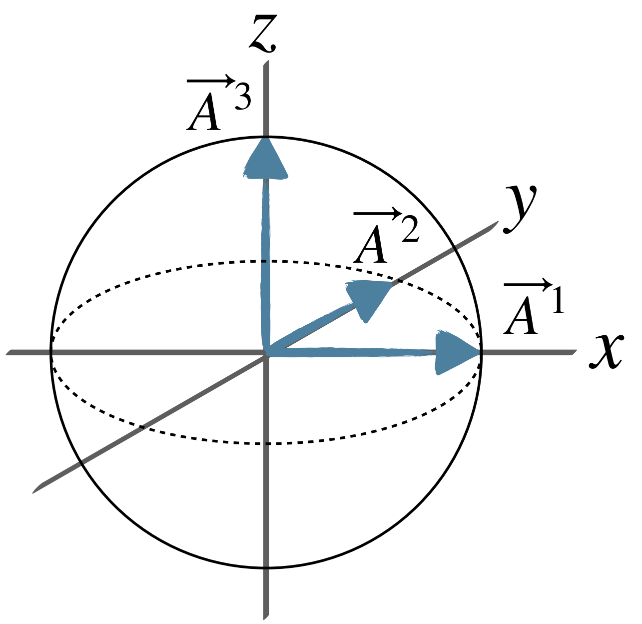
Equations of motion

Using the temporal gauge $A_0^a = 0$ and an ansatz $A_i^a(t) = \delta_i^a a(t) Q(t)$, we obtain the background EoMs:

gauge field amplitude

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f} Q^2 \left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right) Q + 2g^2 Q^3 = \frac{g}{f} Q^2 \dot{\phi}\,.$$



In the slow-roll limit, we have a solution of

$$m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f \partial_\phi V}{3H^4}\right)^{1/3}, \qquad \xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}\,.$$

This solution is an attractor solution. [I. Wolfson *et al.* (2021)]

Axion-SU(2) inflation

- End of inflation

We consider that Q becomes dark matter in the later universe.

The end of inflation is determined by

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \epsilon_\phi + \epsilon_E + \epsilon_B = 1$$

$$\text{with} \quad \epsilon_\phi \equiv \frac{\dot{\phi}^2}{2M_{\rm Pl}^2 H^2}, \quad \epsilon_E \equiv \frac{\left(\dot{Q} + HQ\right)^2}{M_{\rm Pl}^2 H^2}, \quad \epsilon_B \equiv \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2}$$

In our setup, ϵ_B makes the dominant contribution.

At the end of inflation,

$$\epsilon_B \equiv \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2} \simeq 1 \;\rightarrow\; m_{Q,\rm end} = \frac{g Q_{\rm end}}{H_{\rm end}} \simeq \sqrt{\frac{g M_{\rm Pl}}{H_{\rm end}}}$$

Evolution of vector fields

- Spontaneous breaking of SU(2)

As dark matter, Q should be massive.

We consider the SSB of SU(2) by introducing an SU(2) doublet Φ.

$$\mathscr{L}_{\rm SSB} = D_\mu \Phi^\dagger D^\mu \Phi - V_\Phi(\Phi)$$

$$D_\mu \Phi = \partial_\mu \Phi - ig A_\mu^a \frac{\sigma^a}{2} \Phi, \quad V_\Phi(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2$$

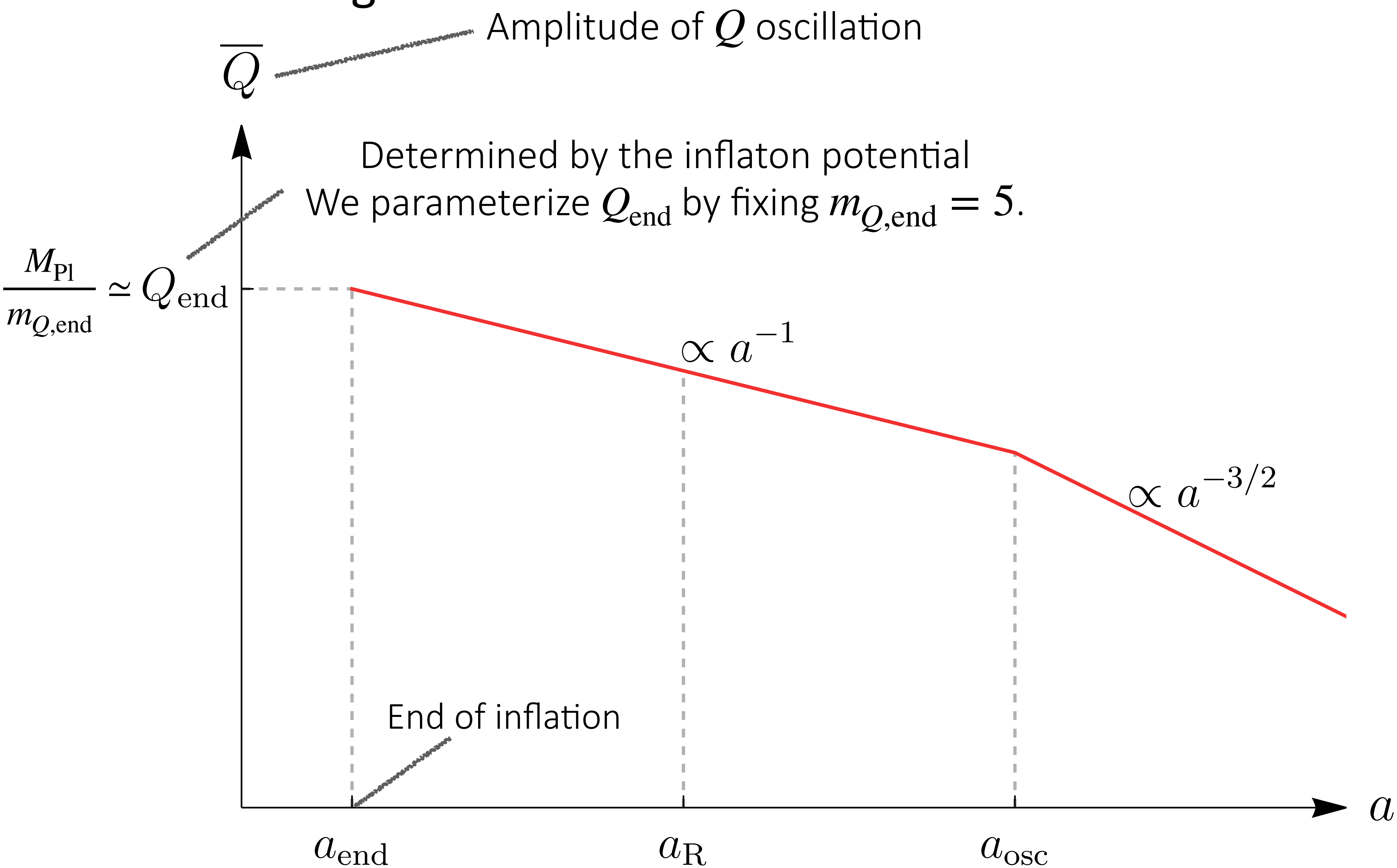
When Φ acquires a VEV of $\Phi^\dagger \Phi = v^2$,

$$\mathscr{L} \supset \frac{m^2}{2} A_\mu^a A^{a\mu}, \qquad m = gv/\sqrt{2}$$

If $m \ll H_{\rm inf}$, the inflationary dynamics is unchanged.

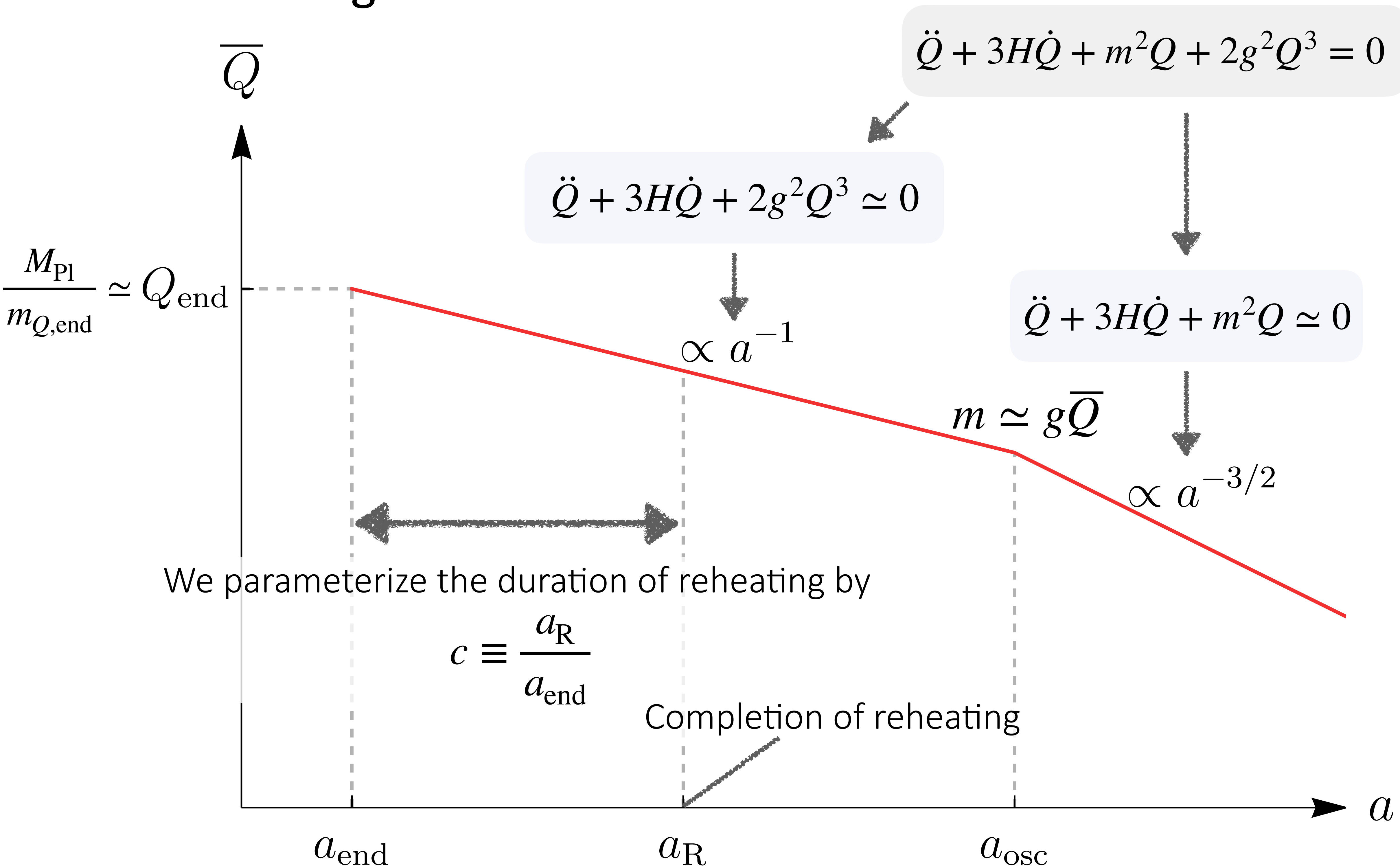
Evolution of vector fields

- After reheating



Evolution of vector fields

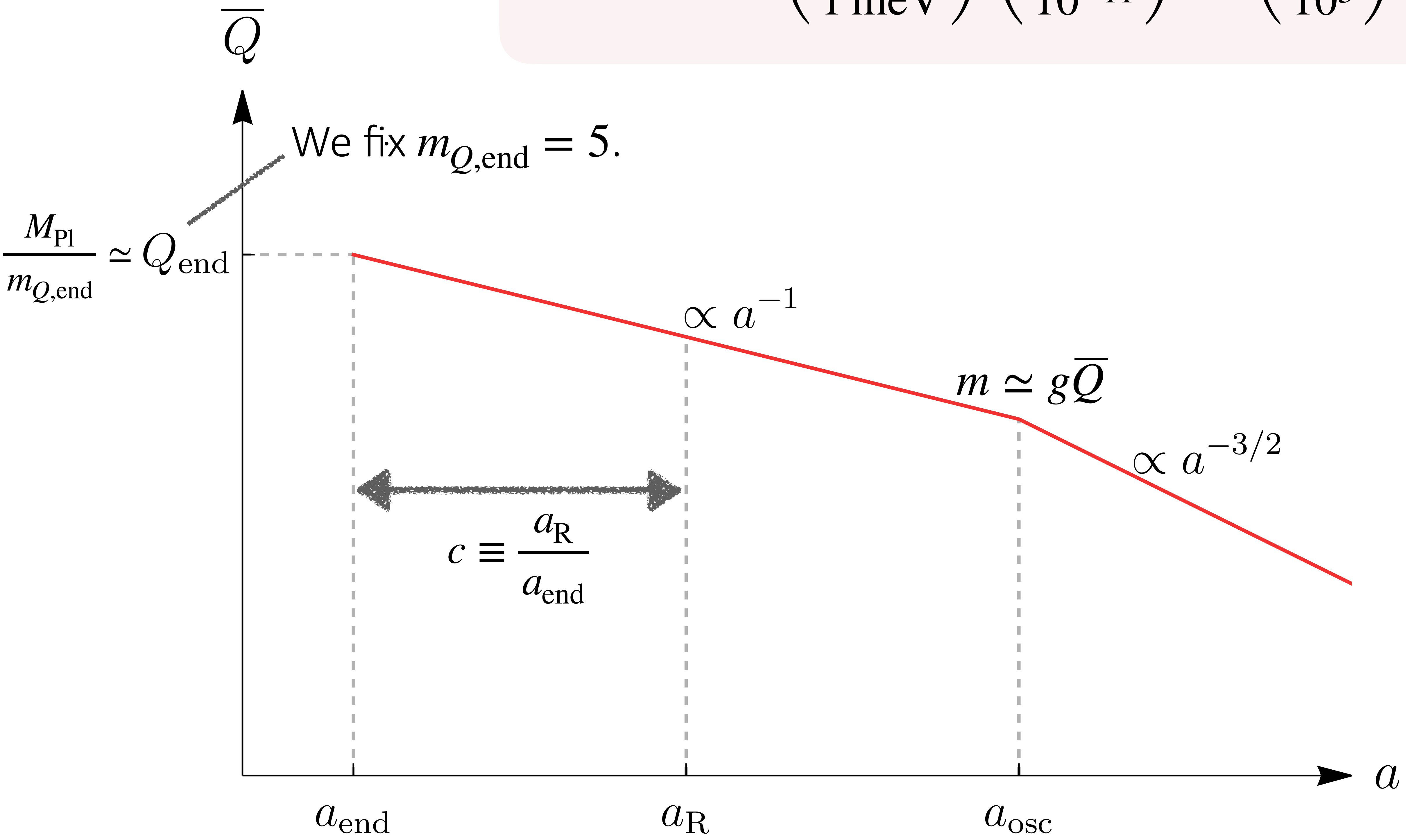
- After reheating



Evolution of vector fields

- DM abundance

$$\Omega_{Q,0}h^2 \simeq 0.1 \left(\frac{m}{1\,\mathrm{meV}}\right)\left(\frac{g}{10^{-11}}\right)^{-1/2}\left(\frac{c}{10^3}\right)^{-3/4}$$



Observational constraints

Self-interaction

The self-interaction of dark matter is constrained by the observations of galaxy clusters.

$$\frac{\sigma}{m} \lesssim \mathcal{O}(0.1 - 1)\,\mathrm{cm}^2/\mathrm{g}$$

The non-Abelian gauge field has a self-interaction of

$$\frac{\sigma}{m} \sim \frac{g^4}{m^3} \rightarrow m \gtrsim 60\,\mathrm{MeV} \times g^{4/3}$$

Big-bang nucleosynthesis

The reheating temperature should be larger than the BBN temperature:

$$T_\mathrm{R} > T_\mathrm{BBN} \simeq 10\,\mathrm{MeV}$$

Observational constraints

- Small-scale structures

In this scenario, A_i^a behaves as radiation when $g\overline{Q} \gtrsim m$.

In other words, dark matter is formed when $g\overline{Q} \sim m$.

Such a formation is constrained by the observations of small-scale structure:

$$z_{\rm T} > 5.5 \times 10^6$$

$$\rightarrow T_{\rm osc} > T_{\rm osc,min} \simeq 1.3 \times 10^{-6}\,{\rm GeV}$$

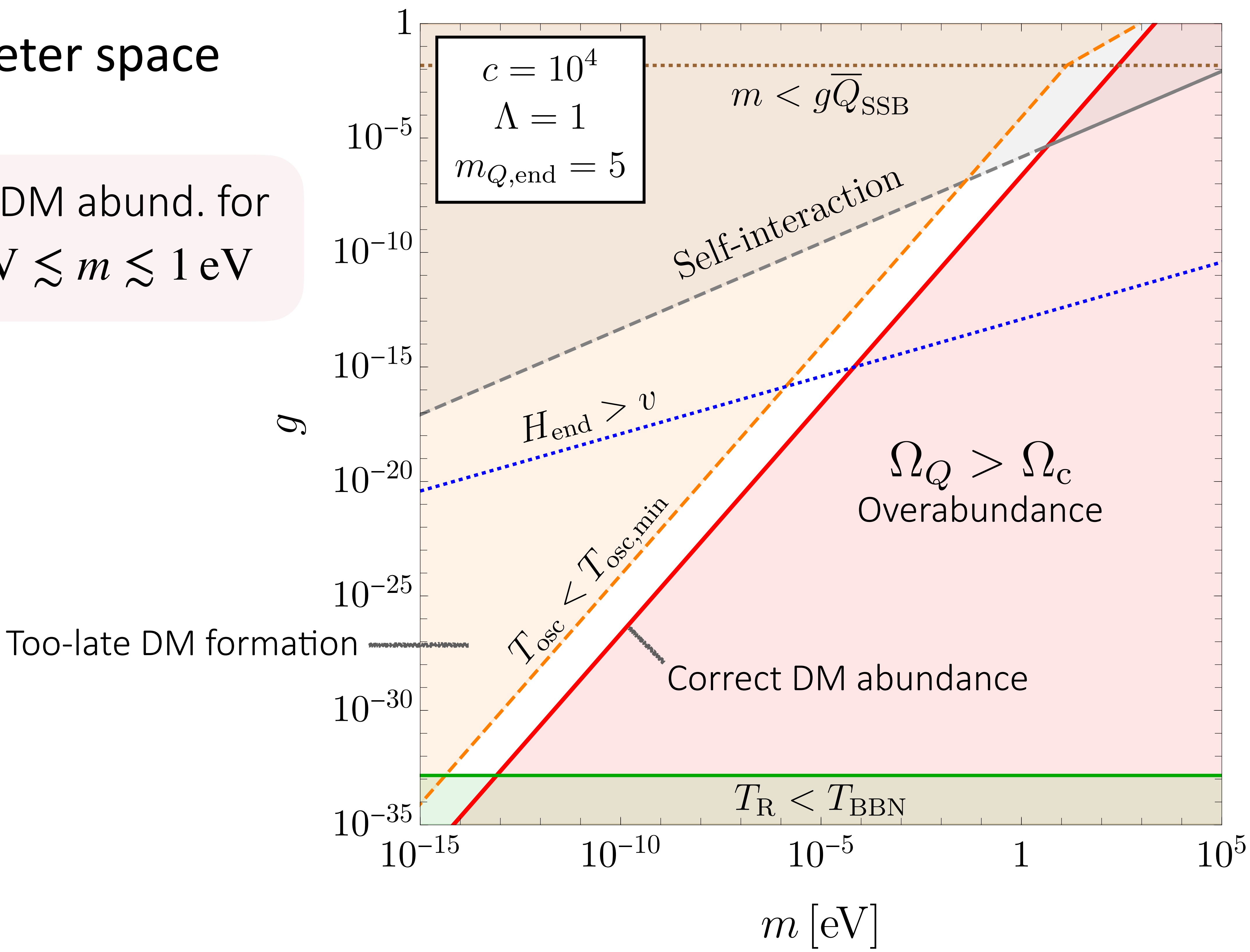
Once we fix $\Omega_{Q,0} = \Omega_{c,0}$, this limit is translated as

$$c > c_{\rm min} \simeq 1.1 \times 10^3$$

VDM from axion-SU(2) inflation

- Parameter space

Correct DM abund. for
$10^{-13}\,\mathrm{eV} \lesssim m \lesssim 1\,\mathrm{eV}$



Implications

Fate of dark Higgs

- Pre-inflationary SSB

Φ is decoupled from the visible sector. ρ_Φ is negligible.

- Post-inflationary SSB

Φ can be thermalized with the SM sector through the Higgs portal.

Thermal relic decoupled at the electroweak phase transition

The energy density of Φ is transferred into longitudinal modes of A_i^a.

$$\Delta N_{\rm eff} \simeq 0.12$$

Implications

- Kinetic mixing

In general, the SU(2) gauge field can couple with the SM photons as

$$\mathscr{L} \supset \frac{\kappa}{2f^2}\Phi^\dagger F^a_{\mu\nu}\sigma^a F^\gamma_{\mu\nu}\Phi$$

SM photon ($F^\gamma_{\mu\nu}$), dark SU(2) ($F^a_{\mu\nu}$)

When the SSB occurs, $\Phi = (v, 0)^{\mathrm{T}}$, it induces the kinetic mixing:

$$\mathscr{L} \supset \frac{\kappa m^2}{g^2 f^2} F^\gamma_{\mu\nu} F^{\mu\nu 3}$$

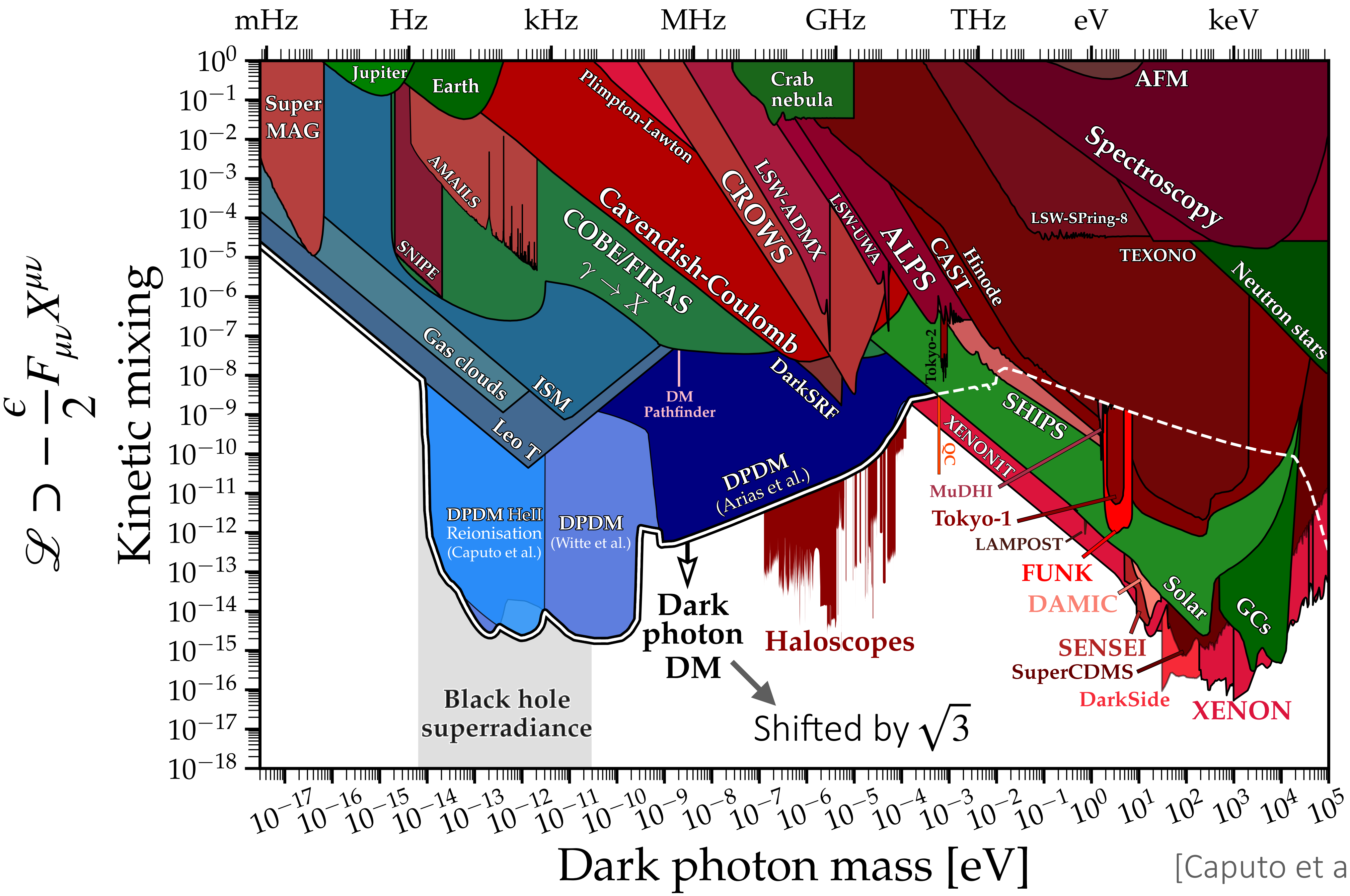
between the SM photon and A_i^3. → 1/3 of DM has a kinetic mixing. Fixed polarization

$$\mathscr{L} \supset -\frac{\epsilon}{2} F^\gamma_{\mu\nu} F^{\mu\nu 3}, \quad \epsilon \equiv -\frac{2\kappa m^2}{g^2 f^2}$$

Implications

- Kinetic mixing

The constraints on kinetic mixing of dark and SM photons:

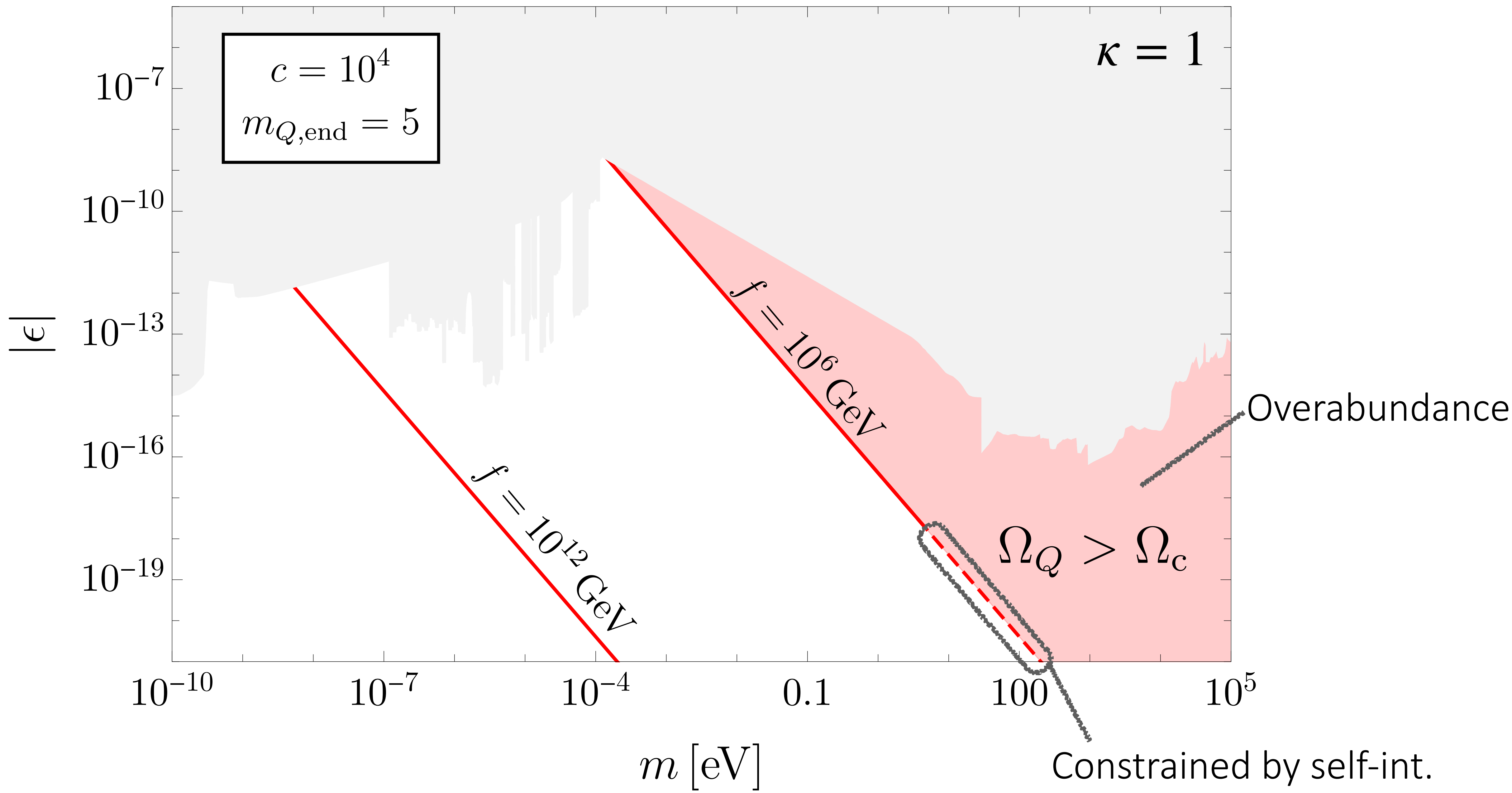


[Caputo et al. (2021)]

Implications

$$\mathscr{L} \supset \frac{\kappa}{2f^2}\Phi^\dagger F^a_{\mu\nu}\sigma^a F^\gamma_{\mu\nu}\Phi$$

- Kinetic mixing



Implications

- Possible extensions

  - SU(N) model

    We have considered the SU(2) gauge group.

    Since SU(N) has SU(2) subgroups, SU(N) models also have isotopic config.

    [Fujita et al. (2021)]

    Thus, we expect similar results for SU(N) models.

  - Other SSB setup

    We have considered an SU(2) doublet, Φ.

    For example, we can consider SU(2) triplets to break SU(2).

    In this case, the SU(2) gauge field can have a mass splitting.

    Existence of monopoles or strings? [Hindmarsh & Kibble (1985)]

Summary

We present a new mechanism to generate a coherently oscillating VDM using axion-SU(2) gauge field dynamics during inflation.

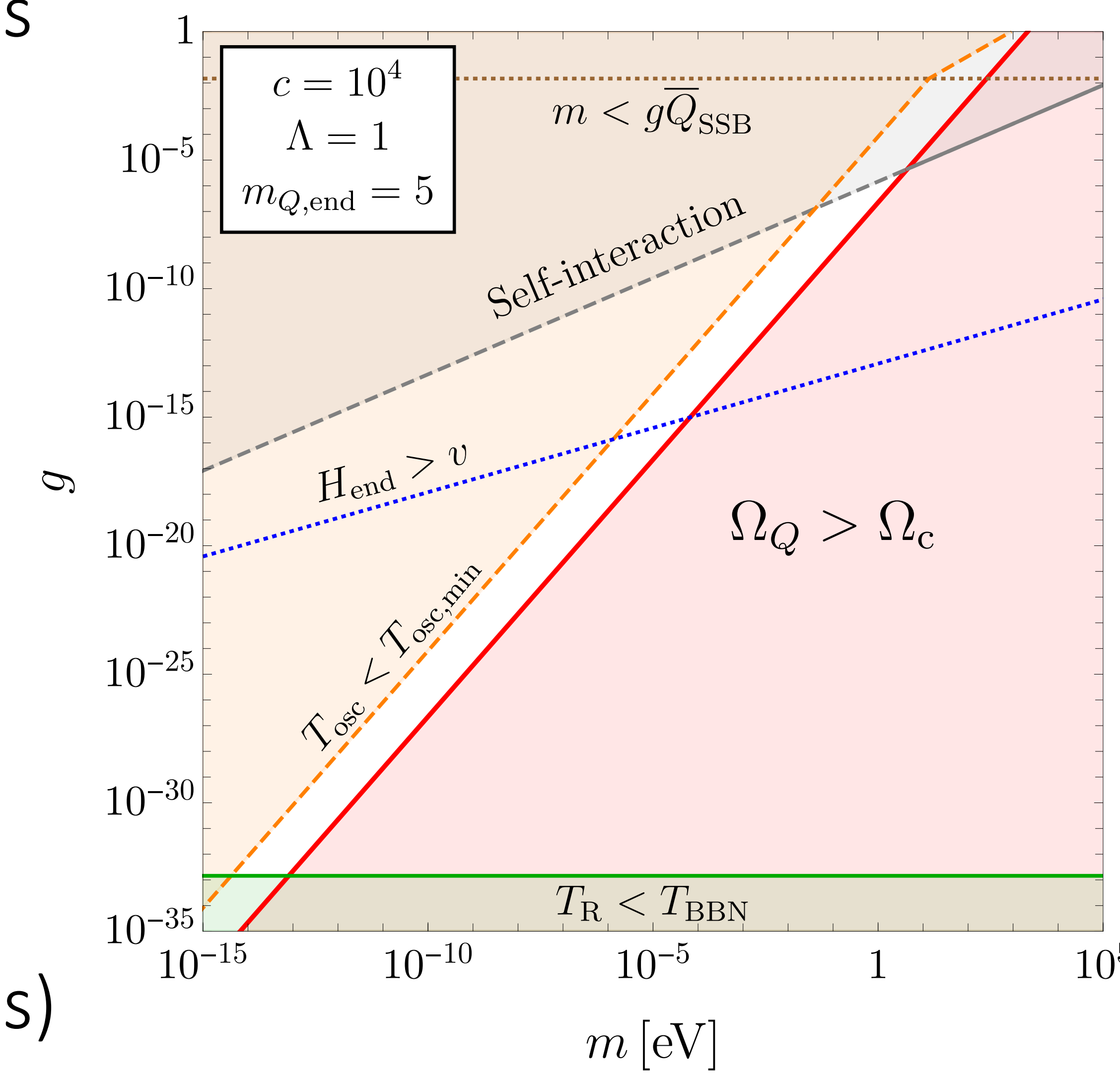
- Vector field is not damped during inflation
- Free from statistical anisotropies of the adiabatic perturbations.
- Free from isocurvature perturbations

Our scenario can be probed through

- kinetic mixing of 1/3 of VDM
- $\Delta N_{\rm eff}$
- Self-interactions

Our scenario can be extended into

- SU(N) models
- Other SSB models (e.g., SU(2) triplets)



Back up

Vector dark matter

- Misalignment mechanism for scalar

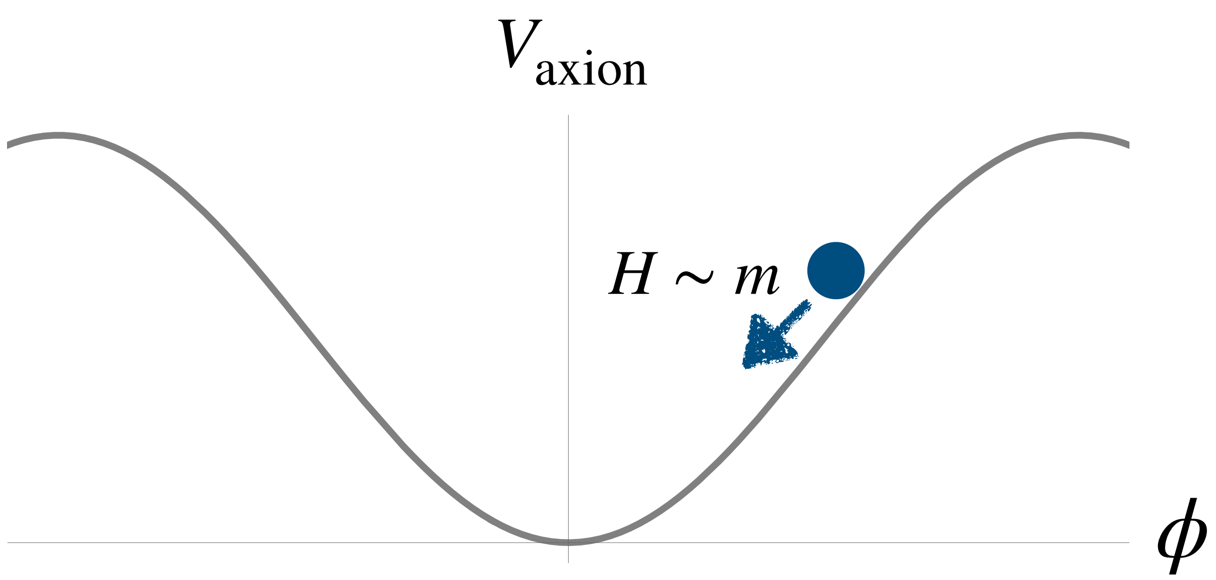
First, let us consider an example of coherently oscillating DM: misalignment production of axion/axion-like particle

During inflation, an axion ϕ becomes (almost) homogeneous:

$$\phi(t, \mathbf{x}) = \phi_0 \quad \text{(const.)}$$

When $H \sim m$, ϕ starts to oscillate:

$$\rho_\phi \propto a^{-3} \quad \text{(matter-like)}$$



In the case of axion, the coherent oscillation is realized in the misalignment mechanism.

Axion-SU(2) inflation

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu,$$

$$\tilde{F}^{a\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-\det[g_{\mu\nu}]}} F^a_{\rho\sigma},$$

- Chromo-natural inflation (CNI)

[Adshead & Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Due to the SU(2) gauge field background,
gauge field perturbations experience a tachyonic instability.

→ Chiral and non-Gaussian GWs are overproduced.
Chromo-natural inflation with a cosine-type potential fails.

[Adshead, Martinec, Wyman (2013)]

By considering general forms of the axion potential,
this constraint can be evaded. [Caldwell & Devulder (2017)]

Axion-SU(2) inflation

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

- Equations of motion

Let us consider the background dynamics:

$$\phi(t, \mathbf{x}) = \phi(t), \quad A^a_i(t, \mathbf{x}) = A^a_i(t)$$

The background metric is given by

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

$$\text{with} \quad H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_\phi + \rho_A}{3M_{\mathrm{Pl}}^2},$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad \rho_A = \frac{1}{2a^2}\dot{A}^a_i\dot{A}^a_i + \frac{g^2}{4a^4}\left[\left(A^a_iA^a_i\right)^2 - A^a_iA^b_iA^a_jA^b_j\right]$$

Axion-SU(2) inflation

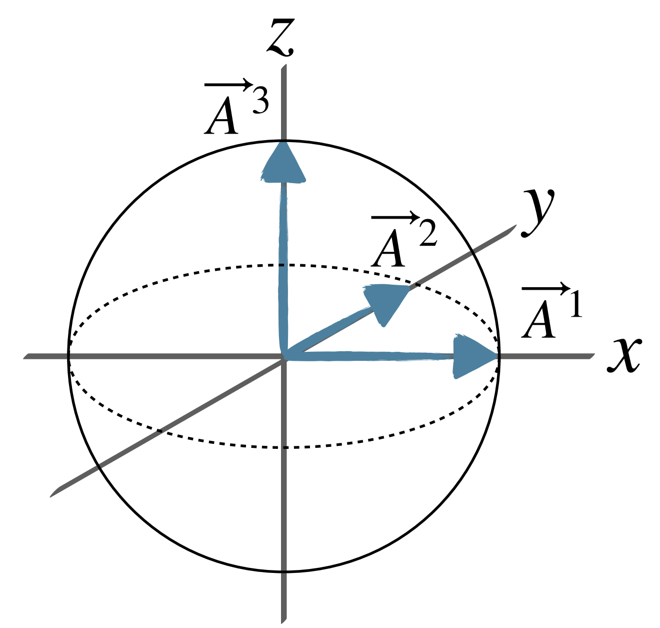
- Equations of motion

Using the temporal gauge $A_0^a = 0$ and an ansatz $A_i^a(t) = \delta_i^a a(t) Q(t)$,

we obtain the background EoMs: gauge field amplitude

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f} Q^2 \left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right) Q + 2g^2 Q^3 = \frac{g}{f} Q^2 \dot{\phi}\,.$$



This configuration is isotropic:

$$A_i^a \propto \delta_i^a \;\Rightarrow\; {}^\forall R, {}^\exists G : R_{ij} A_j^a = G^{ab} A_i^b\,.$$

spatial rotation   gauge transf.   (homomorphism of SU(2) to SO(3))

→ **Free from statistical anisotropy.**

Axion-SU(2) inflation

- Equations of motion

In the slow-roll limit, we parametrize

$$\xi \equiv \frac{\dot{\phi}}{2fH}, \quad m_Q \equiv \frac{gQ}{H}, \quad \Lambda_Q \equiv \frac{Q}{f}.$$

axion velocity — amplitude of A_i^a — strength of $\phi F\tilde{F}$

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = -\frac{3g}{f}Q^2\left(\dot{Q} + HQ\right),$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2\right)Q + 2g^2Q^3 = \frac{g}{f}Q^2\dot{\phi}\,.$$

$$\rightarrow m_Q - \xi m_Q^2 + m_Q^3 = 0$$

In the limit of $\Lambda_Q \gg 1$, the BG solution is

$$m_Q \simeq \left(\frac{-g^2 f\partial_\phi V}{3H^4}\right)^{1/3}, \qquad \xi \simeq m_Q + m_Q^{-1}\,.$$

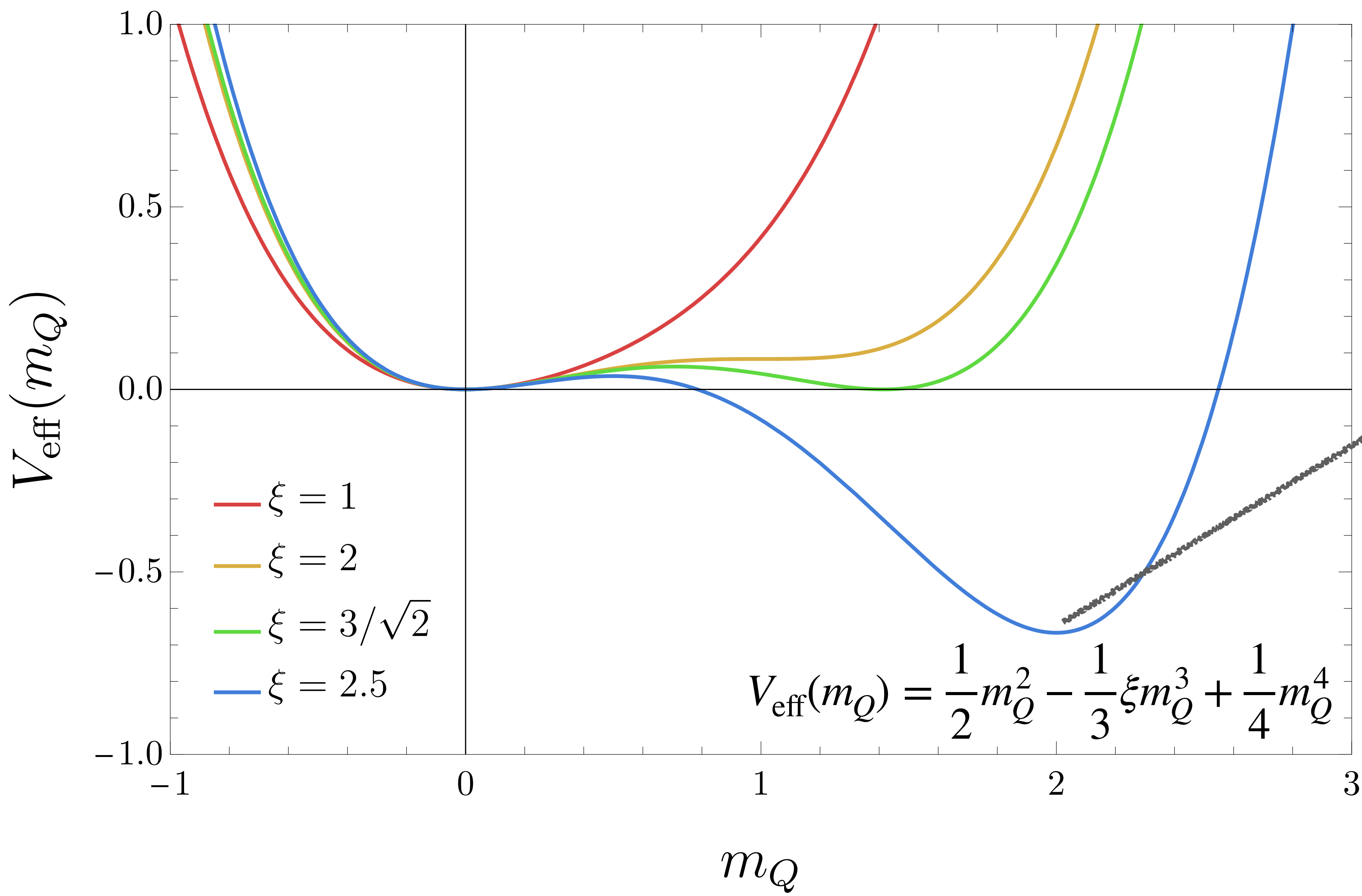
This solution is an attractor solution. [I. Wolfson *et al*. (2021)]

Axion-SU(2) inflation

- Stability

Let us consider the effective potential for m_Q.

→ $m_Q \geq \sqrt{2}$ is the true vacuum for $\xi > 3/\sqrt{2}$.



The potential min. is stabilized.

↓

Isocurvature pert. is suppressed.

- End of inflation

We consider that Q becomes dark matter in the later universe.

Thus, we need to evaluate Q at the end of inflation.

The end of inflation is determined by

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \epsilon_\phi + \epsilon_E + \epsilon_B = 1$$

with $\epsilon_\phi \equiv \dfrac{\dot{\phi}^2}{2M_{\rm Pl}^2 H^2}, \quad \epsilon_E \equiv \dfrac{\left(\dot{Q} + HQ\right)^2}{M_{\rm Pl}^2 H^2}, \quad \epsilon_B \equiv \dfrac{g^2 Q^4}{M_{\rm Pl}^2 H^2}$

In our setup, ϵ_B makes the dominant contribution.

At the end of inflation,

$$\epsilon_B \equiv \frac{g^2 Q^4}{M_{\rm Pl}^2 H^2} \simeq 1 \rightarrow m_{Q,\rm end} = \frac{g Q_{\rm end}}{H_{\rm end}} \simeq \sqrt{\frac{g M_{\rm Pl}}{H_{\rm end}}}$$

Evolution of vector fields

- Spontaneous breaking of SU(2)

As dark matter, Q should be massive.

We consider the SSB of SU(2) by introducing an SU(2) doublet Φ.

$$\mathscr{L}_{\rm SSB} = D_\mu \Phi^\dagger D^\mu \Phi - V_\Phi(\Phi)$$

$$D_\mu \Phi = \partial_\mu \Phi - igA_\mu^a \frac{\sigma^a}{2} \Phi$$

$$V_\Phi(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2$$

When Φ acquires a VEV of $\Phi^\dagger \Phi = v^2$,

$$\mathscr{L} \supset \frac{m^2}{2} A_\mu^a A^{a\mu}, \quad m = gv/\sqrt{2}$$

Evolution of vector fields

- Spontaneous breaking of SU(2)

We are interested in small m.

Even if Q is massive during inflation,

$m \ll H_{\mathrm{inf}}$ and the inflationary dynamics is unchanged.

The SSB depends on the potential and interactions of Φ.

In the following, we consider two scenarios:

- Pre-inflationary scenario
- Post-inflationary scenario

In fact, the evaluation of the dark matter abundance becomes the same in the two scenarios.

Pre-inflationary scenario

$$V_\Phi(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger\Phi - v^2\right)^2$$
$$\supset -\frac{\lambda v^2}{2}\Phi^\dagger\Phi$$

- Condition for pre-inflationary scenario

If $m_\Phi \sim \sqrt{\lambda}v \gg H_{\rm inf}$, the symmetry is broken during inflation.

If the dark sector is not reheated, the symmetry is kept broken.

In the following, we evaluate the amplitude of Q in order of time.

At the end of inflation,

$$\epsilon_B \simeq 1 \quad \rightarrow \quad Q_{\rm end} = \frac{M_{\rm Pl}}{m_{Q,{\rm end}}}$$

$m_{Q,{\rm end}}$ is determined by the coupling and axion potential.

Here, we treat $m_{Q,{\rm end}}\,(\geq \sqrt{2})$ as a parameter.

Considering the backreaction from perturbations,

$m_{Q,{\rm end}}$ should satisfy $m_{Q,{\rm end}} \lesssim O(10)$ for $g = 10^{-O(10)}$. [Fujita, Namba, Tada (2017)]

Pre-inflationary scenario

- During reheating

After inflation, the inflaton decays and stops to source A_i^a.

However, there is no reason for A_i^a to deviate from $\propto \delta_i^a$. → We follow Q.

Here, we assume that A_i^a decouples from ϕ soon after the end of inflation.

The EoM for Q is given by

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2 + m^2\right) Q + \underline{2g^2Q^3} = 0$$

If the mass term is dominant, $Q \propto a^{-3/2}$

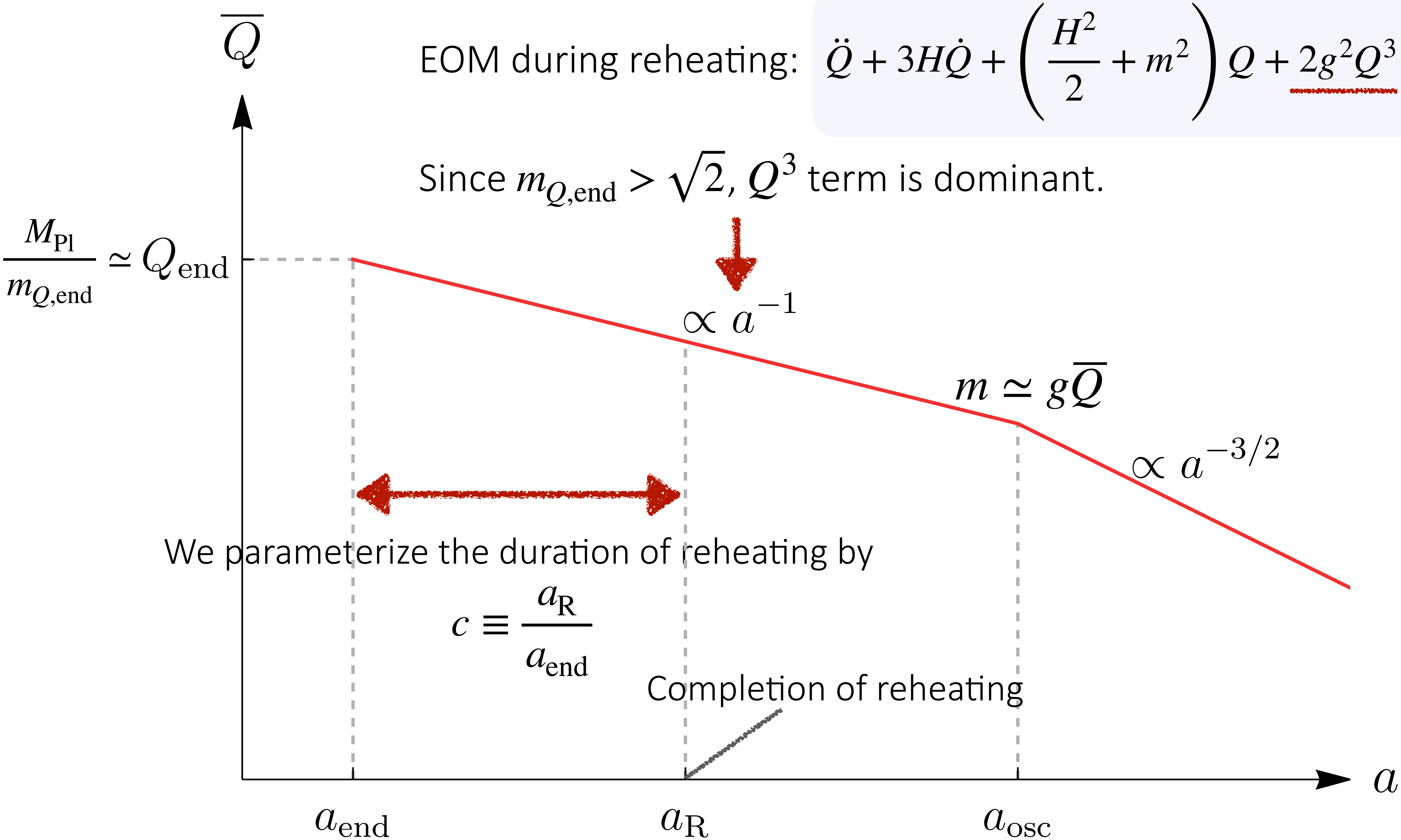
If the quartic term is dominant, $Q \propto a^{-1}$

Pre-inflationary scenario

- After reheating

EOM during reheating: $\ddot{Q} + 3H\dot{Q} + \left(\frac{H^2}{2} + m^2\right)Q + 2g^2Q^3 = 0$

Since $m_{Q,\mathrm{end}} > \sqrt{2}$, Q^3 term is dominant.



We parameterize the duration of reheating by

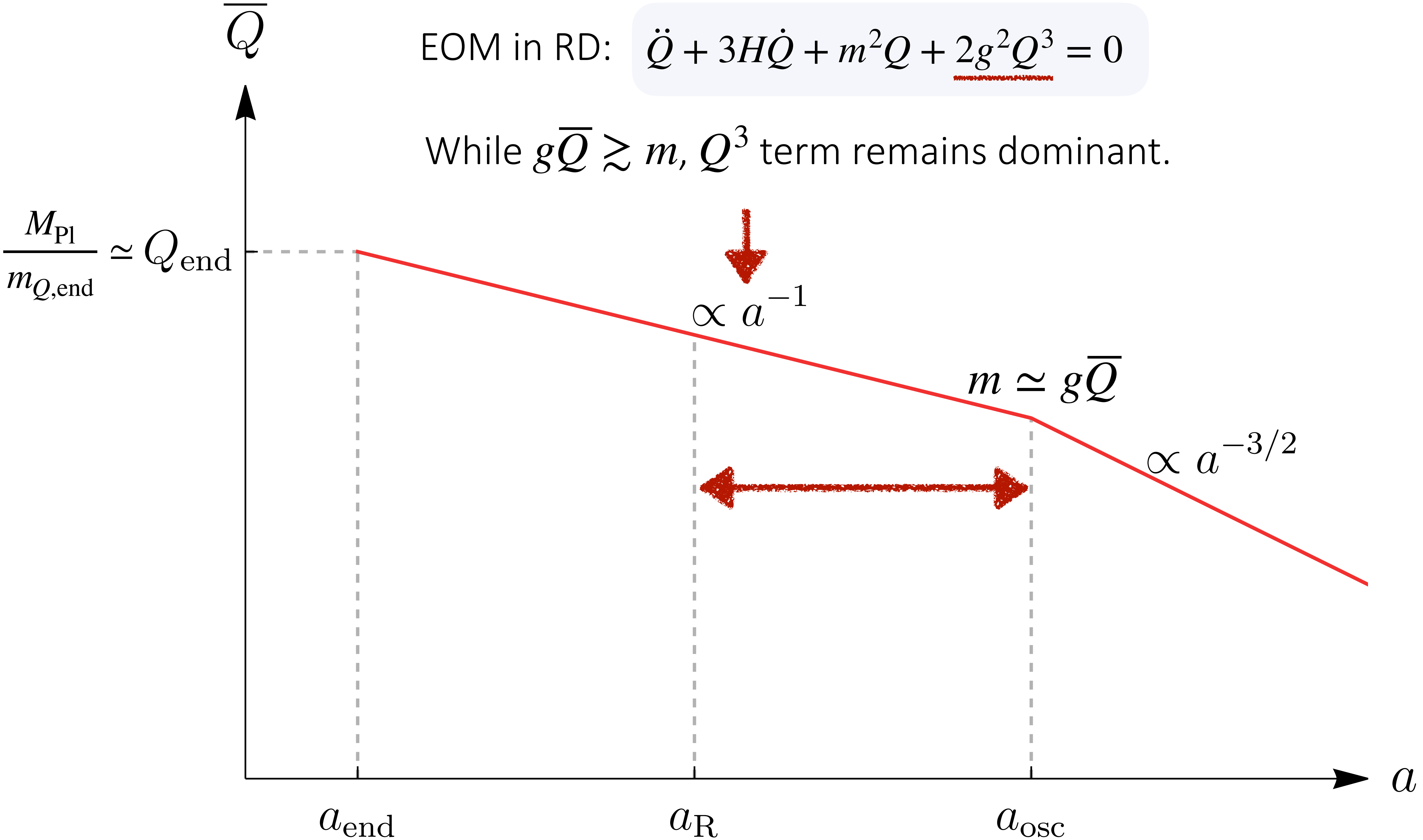
$$c \equiv \frac{a_{\mathrm{R}}}{a_{\mathrm{end}}}$$

Pre-inflationary scenario

- After reheating

EOM in RD: $\ddot{Q} + 3H\dot{Q} + m^2 Q + 2g^2Q^3 = 0$

While $g\overline{Q} \gtrsim m$, Q^3 term remains dominant.

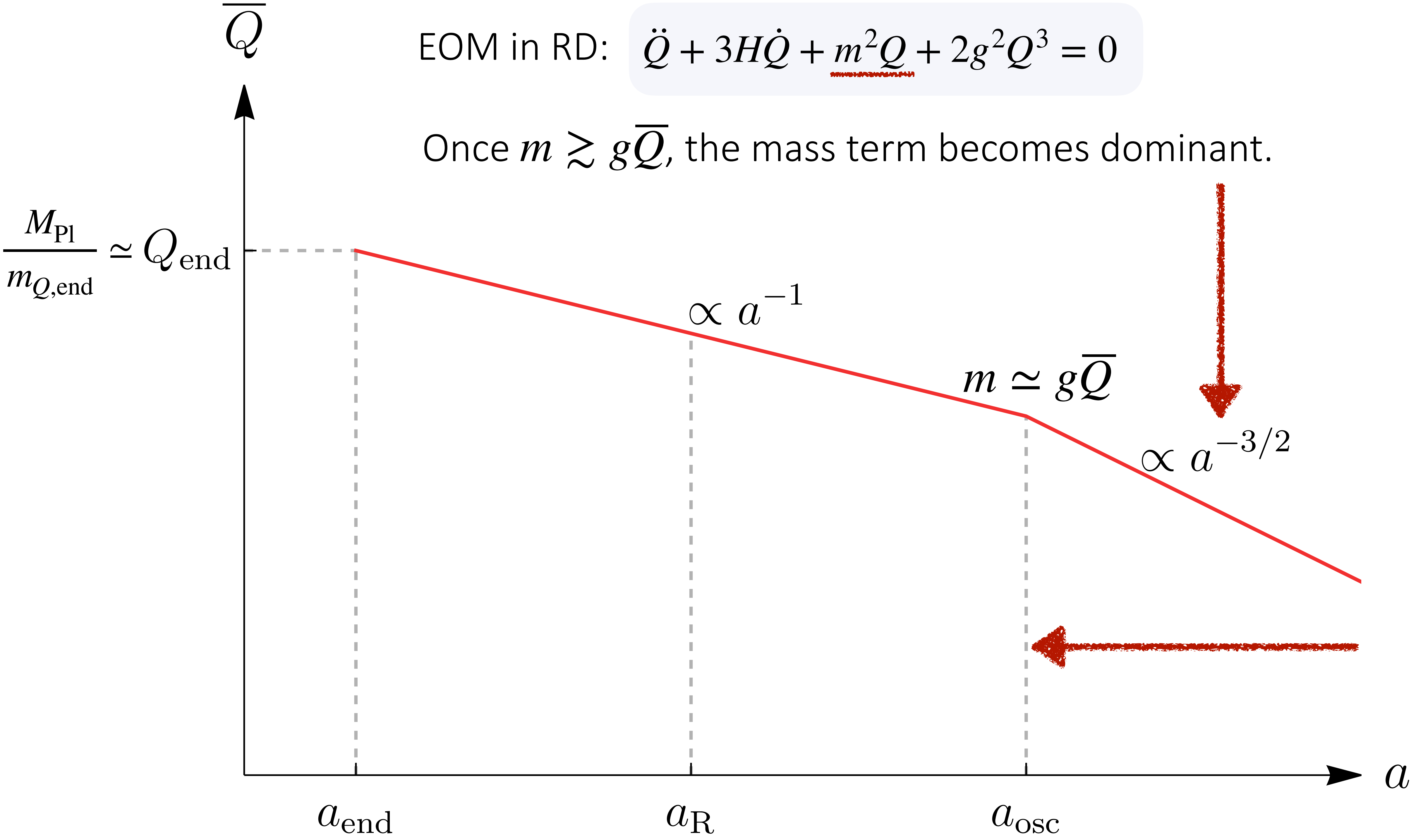


Pre-inflationary scenario

- After reheating

EOM in RD: $\ddot{Q} + 3H\dot{Q} + m^2 Q + 2g^2 Q^3 = 0$

Once $m \gtrsim g\overline{Q}$, the mass term becomes dominant.



Post-inflationary scenario

$$V_\Phi(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger\Phi - v^2\right)^2 \supset -\frac{\lambda v^2}{2}\Phi^\dagger\Phi$$

- Condition for post-inflationary scenario

Here, we consider that Φ is stabilized by the matter effects:

Self-interaction: $m_{\Phi,\mathrm{self}} \sim \sqrt{\lambda}T$

Higgs portal: $m_{\Phi,\mathrm{Higgs}} \sim \sqrt{\lambda_{\Phi H}}T$

$>$

V_Φ: $m_\Phi \sim \sqrt{\lambda}v$

We parameterize the SSB temperature by

$$T_{\mathrm{SSB}} = \sqrt{\Lambda}v = \frac{\sqrt{2\Lambda}m}{g}$$

($\Lambda \sim 1$ for self-interaction)

For $T > T_{\mathrm{SSB}}$, Q is massless, $m = 0$, and it arises at $T = T_{\mathrm{SSB}}$.

Post-inflationary scenario

$$\ddot{Q} + 3H\dot{Q} + m^2 Q + 2g^2 Q^3 = 0$$

- Dark matter abundance

The evolution of Q depends on whether m is relevant just after the SSB.

We consider two cases: $m < g\overline{Q}_{\rm SSB}$ or $m > g\overline{Q}_{\rm SSB}$.

Case 1: $m < g\overline{Q}_{\rm SSB}$

The mass term is subdominant just after the SSB.

The emergence of m at the SSB does not affect Q.

$\overline{Q}$ is the same as in the pre-inflationary scenario.

Post-inflationary scenario

$$\ddot{Q} + 3H\dot{Q} + m^2 Q + 2g^2 Q^3 = 0$$

- Dark matter abundance

Case 2: $m > g\overline{Q}_{\rm SSB}$

Just after the SSB, Q starts to oscillate by the mass term.

If the SSB occurs with a timescale longer than the Q oscillations,

the conservation of the adiabatic invariant leads to

$$\underset{\displaystyle a^{-3}}{\underset{\displaystyle \propto}{m\overline{Q}^2_{\rm aft}}} \simeq \underset{\displaystyle a^{-3}}{\underset{\displaystyle \propto}{g\overline{Q}^3_{\rm bef}}}$$

The adiabatic invariant evolves in the same way in the both cases.

→ The timing of the SSB does not affect $\overline{Q}_{\rm aft}$.

$\overline{Q}$ is the same as in the pre-inflationary scenario.

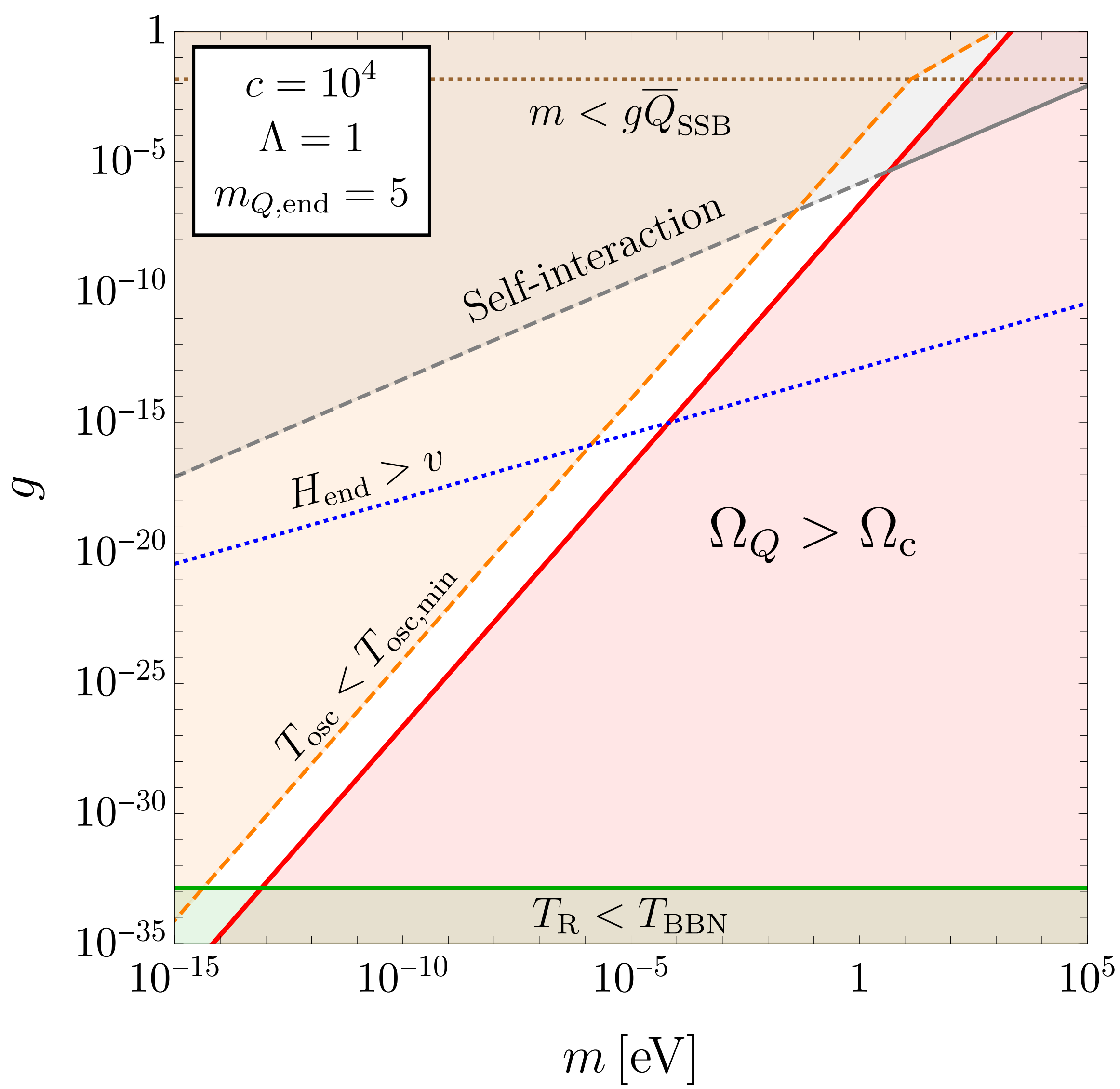
VDM from axion-SU(2) inflation

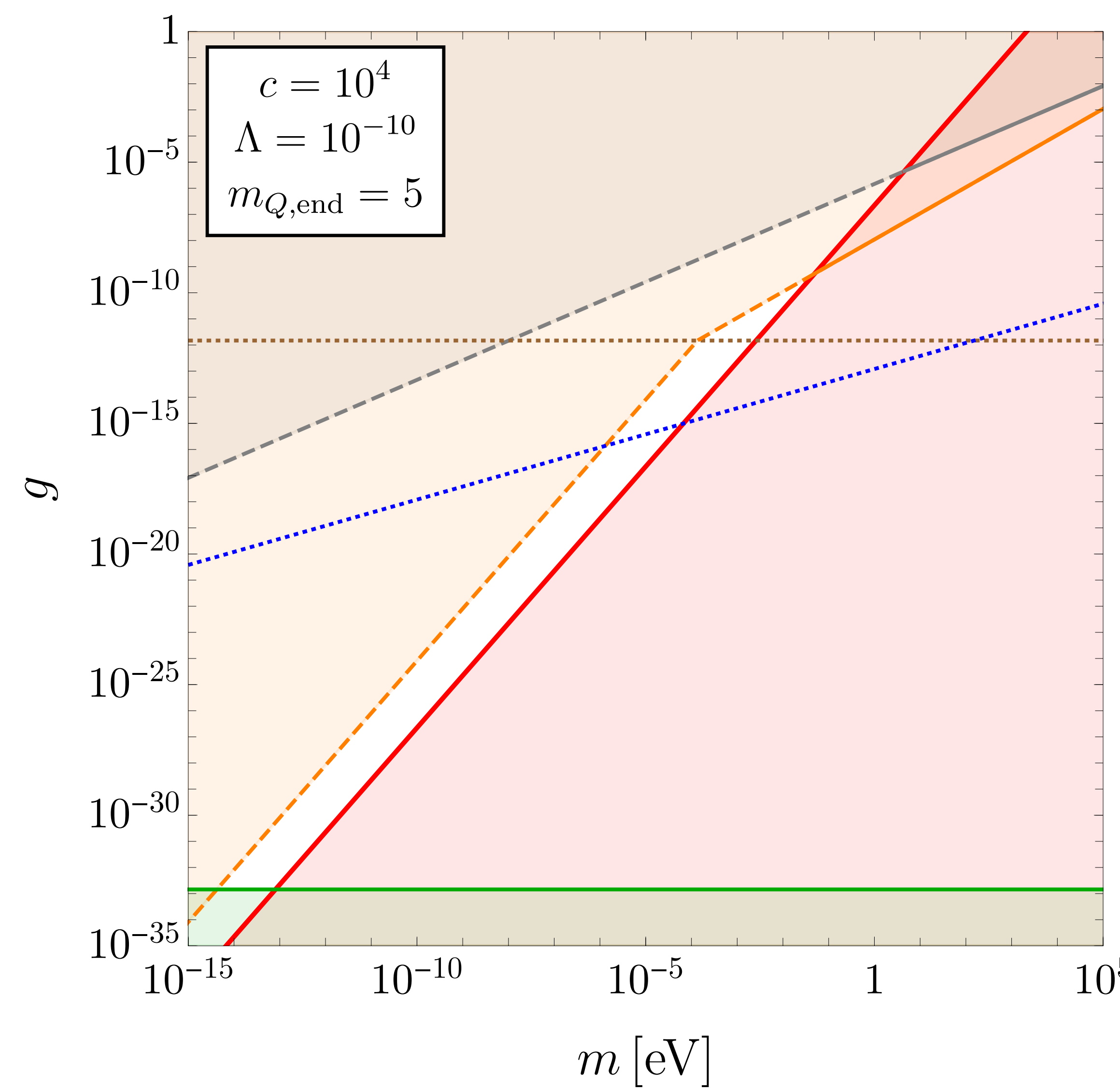
$$T_{\mathrm{SSB}} = \sqrt{\Lambda v}$$

- Parameter space

Smaller Λ → Delayed SSB and DM formation

→ Small-scale structure constraint becomes more severe.





VDM from axion-SU(2) inflation

$$c \equiv \frac{a_{\mathrm{R}}}{a_{\mathrm{end}}}$$

- Parameter space

Larger c → Abundance decreases. / Lower T_{R}