SUSY 2024

Catastrophic Decay of Kaluza-Klein Vacuum Mediated by Singular Instanton

Based on the study with Yutaka Ookouchi and Ryota Sato (arXiv: 2404.13917[hep-th]).

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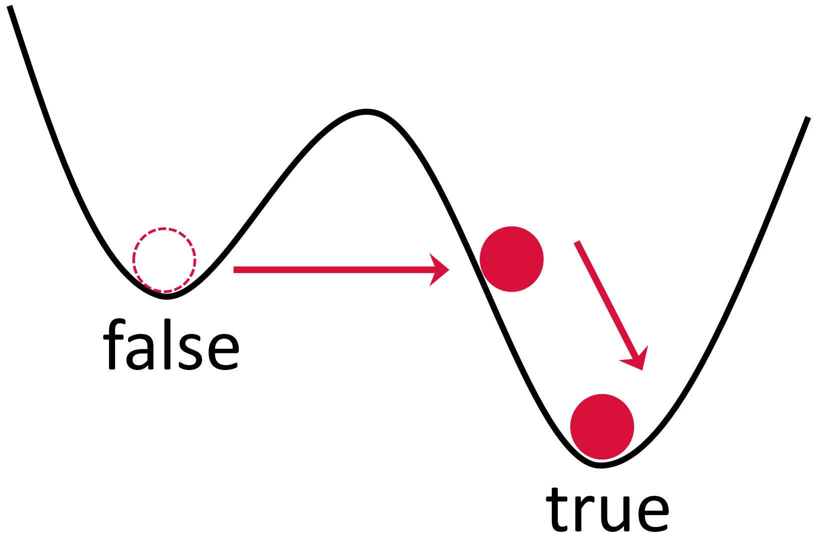
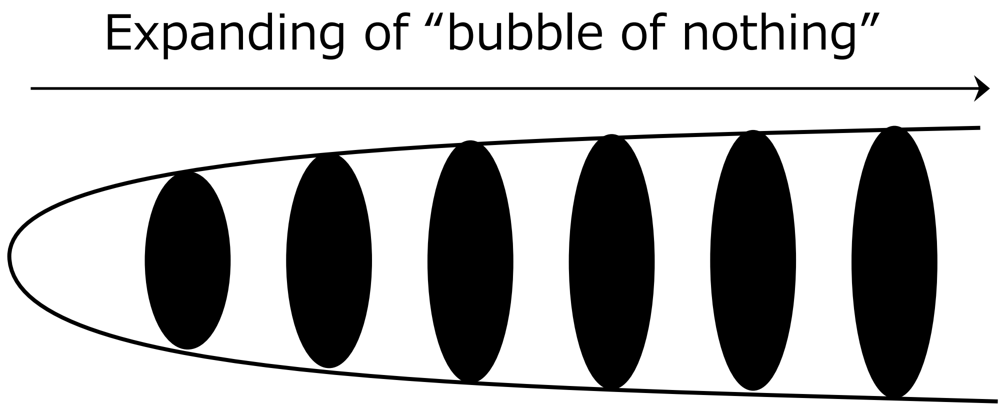
- Absence of guiding principle for compactification implies a huge number of (meta)stable states.

Eg. 1 ：metastable (anti) de Sitter



- ✓ Potential barriar.
- ✓ Stringy vacua are often metastable (can transit to a true vacuum with a lower vacuum energy).

Eg. 2 ：bubble of nothing



[E. Witten '81]

- ✓ The unique decay mode to compactified spacetime.
- ✓ Expanding bubble with no degree of freedom.

Current state of "Bubble of Nothing"

- Bubble of nothing (BoN) is attracting more and more attention because it would be the best candidate for a universal decay channel of non-supersymmetric string vacua.

  [I. Garcia Etxebarria, M. Montero, K. Sousa and I. Valenzuela, JHEP 12 (2020) 032 [arXiv: 2005.06494[hep-th]]]

  [G. Dibietto, N. Petri, and M. Schillo, JHEP 08 (2020) 040 [arXiv: 2002.01764[hep-th]]]

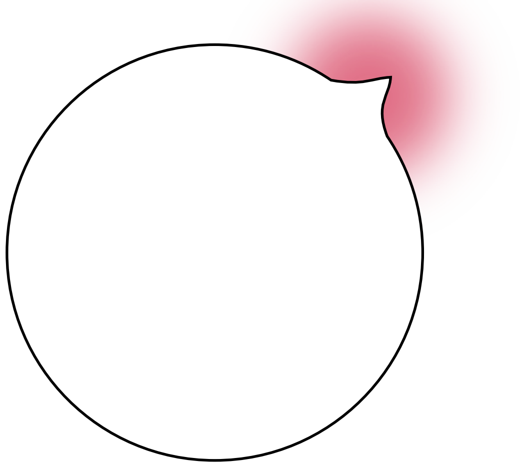
- While in conventional discussion, BoN is forbidden even for SUSY broken vacua if fermions with SUSY preserving boundary conditions exist, some counterexamples have been proposed in recent years.

  [J.J. Blanco-Pillado, B. Shlaer, K. Sousa and J. Urrestilla, JCAP 10 (2016) 002 [arXiv: 1606.03095[hep-th]]]

  [P. Draper, B. Lillard and C. Skye, JHEP 10 (2023) 049 [arXiv: 2305.17838[hep-th]]]

"Bubble of nothing" is more universal than you think.

- We considered a decay of Kaluza-Klein vacuum mediated by a singular instanton.
- We have evaluated the on-shell contribution of the singularity and find that it reduces a total bounce action.



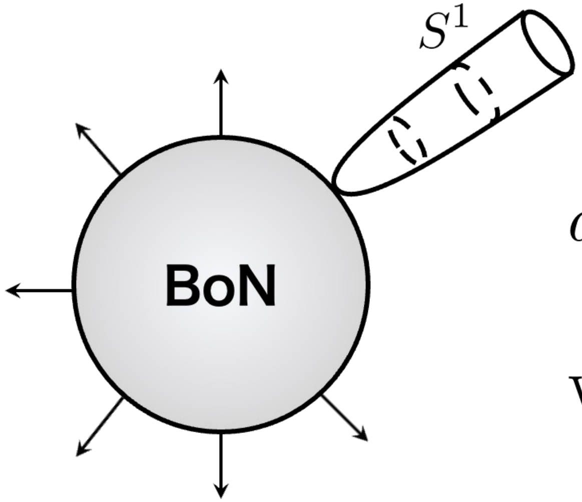
Take-home message

Decay via singular instanton may be more dominant channel in the context of BoN.

Contents

[E. Witten, Nucl.Phys.B 195 (1982) 481-492]

- "Bubble of Nothing" (often abbreviated as BoN) is a catastrophic decay phenomenon particular to compactified spacetime.
- Kaluza-Klein vacuum ($M_4 \times S^1$) can decay as the BoN expands at the speed of light.



$$ds_5^2 = \left(1 - \left(\frac{R}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{R}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{dS_3}^2,$$

where $ds_{dS_3}^2 = -d\tau^2 + \cosh^2 \tau ds_{S^2}^2$

Bubble of Nothing

[E. Witten, Nucl.Phys.B 195 (1982) 481-492]

- BoN instantons take the form of Euclidean black hole solutions.

$$ds_E^2 = \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right) d\phi^2 + \left(1 - \left(\frac{\sqrt{\alpha}}{r}\right)^2\right)^{-1} dr^2 + r^2 ds_{S^3}^2$$

- Euclidean black hole solutions have conical singularity at the position of event horizon.

  - Fix the periodicity of the imaginary time to appropriate value.

$$\alpha = R_{KK}^2 \quad \text{(Smoothness condition)}$$

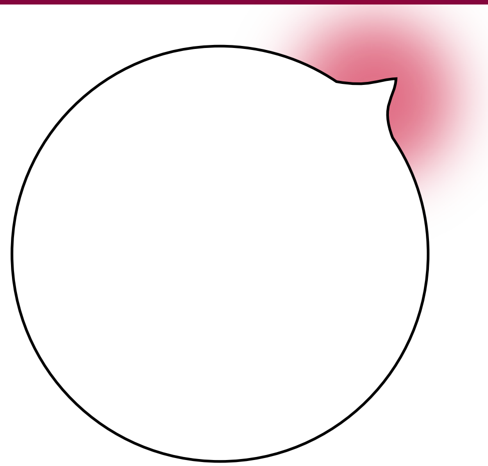
- if the contribution to the on-shell action from the singularity is finite, the condition may be relaxed.

Contents

Singular instanton and conical deficit regularization

- The study of singular instantons was initiated by Hawking and Turok and then explored mainly in the context of open universe.

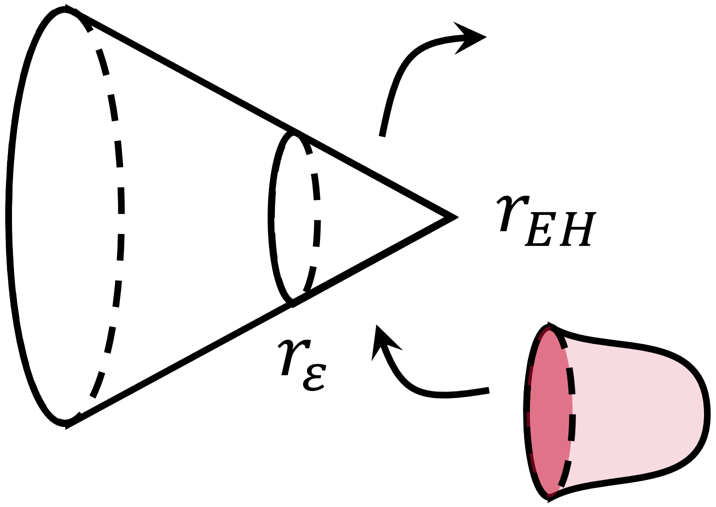
  [S.W. Hawking and N. Turok, Phys. Lett. B 425 (1998) 25 [hep-th/9802030]]
  [N. Turok and S.W. Hawking, Phys. Lett. B 432 (1998) 271 [hep-th/9803156]]



dS instanton + singularity

- Gregory, Moss and Withers have recently refined the geometrical technology for more precise treatment of singularities.

  [D. V. Fursaev, A. N. Solodukhin, Phys.Rev.D 52 (1995) 2133-2143 [arXiv:9501127[hep-th]]]
  [R. Gregory et. al., JHEP 03 (2014) 081 [arXiv:1401.0017[hep-th]]]



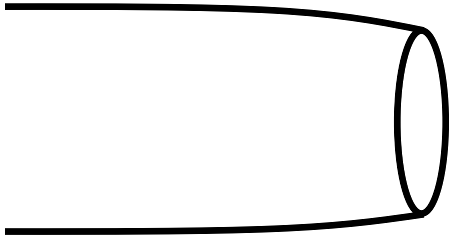
$$I_{\mathcal{B}} = -\frac{1}{16\pi G_n}\int_{\mathcal{B}} d^n x\sqrt{g}R - \frac{1}{8\pi G_n}\int_{\partial\mathcal{B}} d^{n-1}y\sqrt{h}\,(K - K_0)$$

$$= -\frac{\mathcal{A}_{EH}}{4G_n}$$

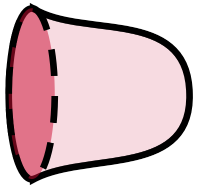
Entropy-like formula

Smooth cap

Split the manifold into two and calculate the Euclidean action for each.



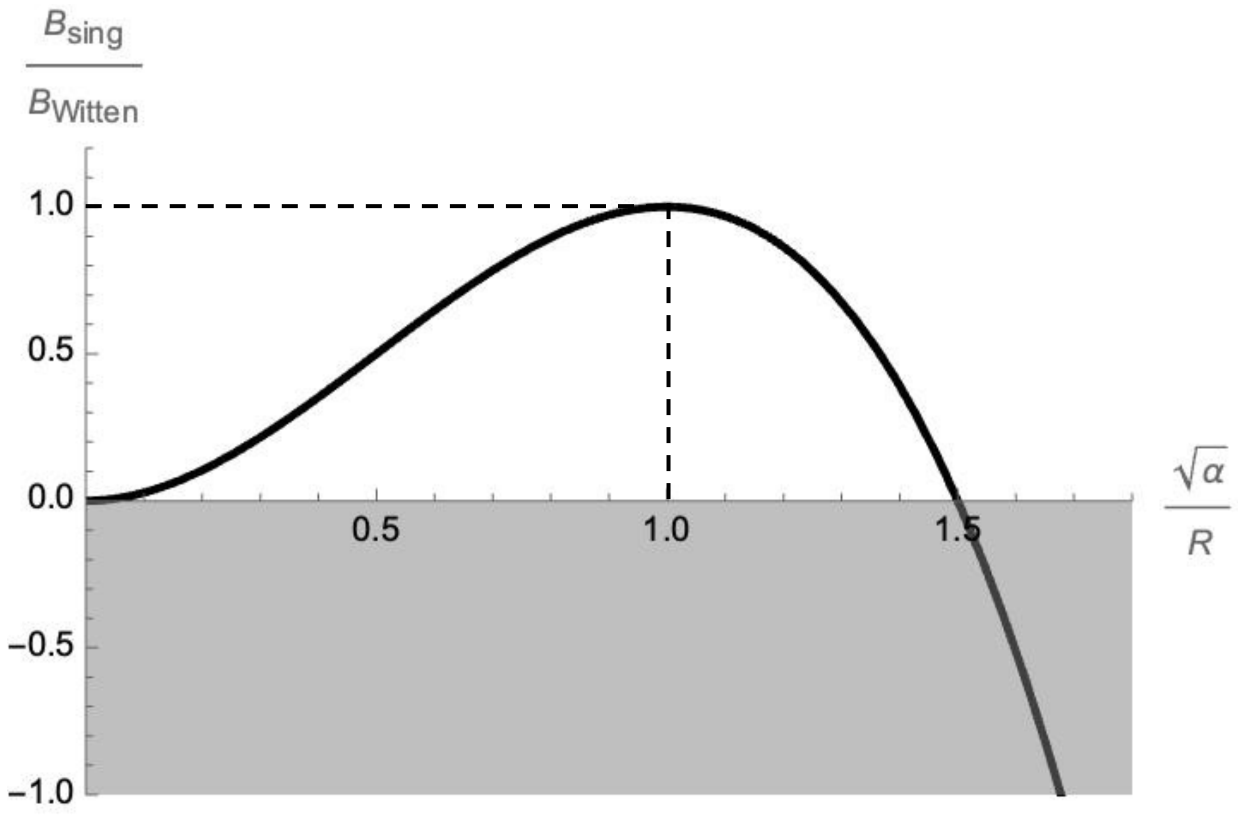
$$I_{\mathcal{M}-\mathcal{B}} = -\frac{1}{8\pi G_n}\int d^{n-1}y\sqrt{h}\,(K-K_0)\Big|_{r_\varepsilon, r_\infty} = \frac{3\pi\alpha}{8G_4}$$



$$I_{\mathcal{B}} = -\frac{2\pi^2\alpha^{3/2}}{4G_5} = -\frac{\pi\alpha^{3/2}}{4G_4 R}$$

$$B = I_{\mathcal{M}-\mathcal{B}} + I_{\mathcal{B}} = \frac{\pi R^2}{8G_4}\left(\frac{3\alpha}{R^2} - \frac{2\alpha^{3/2}}{R^3}\right)$$ (Bounce action)

- ✓ Conical singularities in Euclidean solutions play an important role as a catalyst which reduce bounce actions.
- ✓ Our semiclassical analysis is unreliable in the shaded region.



Perhaps we should pay more attention to “singular” BoN.

Contents

We can reproduce the bounce action with thermodynamic functions.

ADM energy
$$E = -\frac{1}{8\pi G_5}\oint_{S_{\phi r}} (k - k_0)\sqrt{\sigma} d^3\theta = \frac{3\pi\alpha}{8G_5}$$

Entropy
$$S = \frac{2\pi^2\alpha^{3/2}}{4G_5}$$

$$B = \frac{W}{T} = \frac{E - TS}{T} = \frac{3\pi\alpha}{8G_4} - \frac{\pi\alpha^{3/2}}{4RG_4}$$

Let us consider shifting α slightly from R^2 up to the second order.

$$\Delta E = \frac{3}{16 G_4 R} \Delta\alpha \ , \qquad \Delta S = \frac{3\pi}{8 G_4} \Delta\alpha + \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

$$B \simeq B_{\mathrm{witten}} + \frac{3\pi}{8 G_4} \Delta\alpha - \frac{3\pi}{8 G_4} \Delta\alpha - \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

$$= B_{\mathrm{witten}} - \frac{3\pi}{32 G_4 R^2} \Delta\alpha^2$$

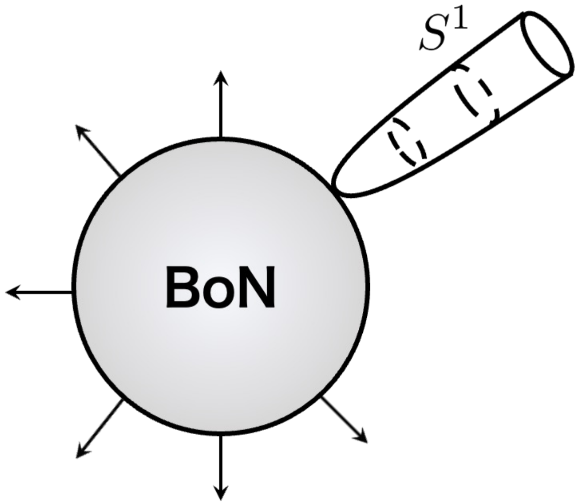
Negative contribution

Contents

Summary and future work

- Bubble of nothing is a catastrophic decay phenomenon which "nothing" overwhelms the spacetime.
- Singular instanton may play an important role in decays of higher-dimensional spacetime.
  - Conical singularity works as reducing the value of bounce action.
  - Our calculation is consistent with Witten's original argument.
  - We can reproduce the bounce action with thermodynamic functions and give an interpretation.

✓ Validity of the regularization method.
✓ Uniform flux?
✓ Embedding into stringy model?



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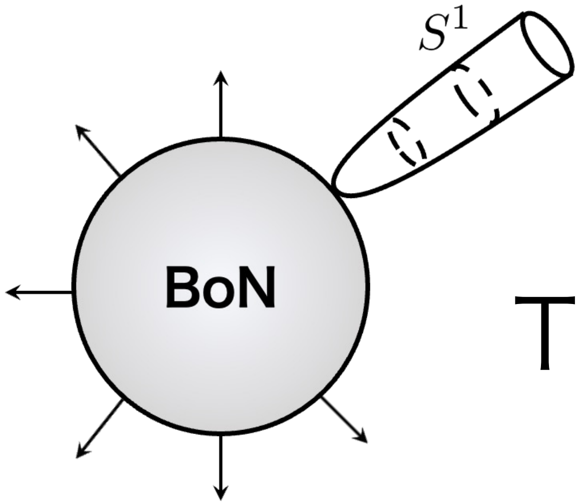
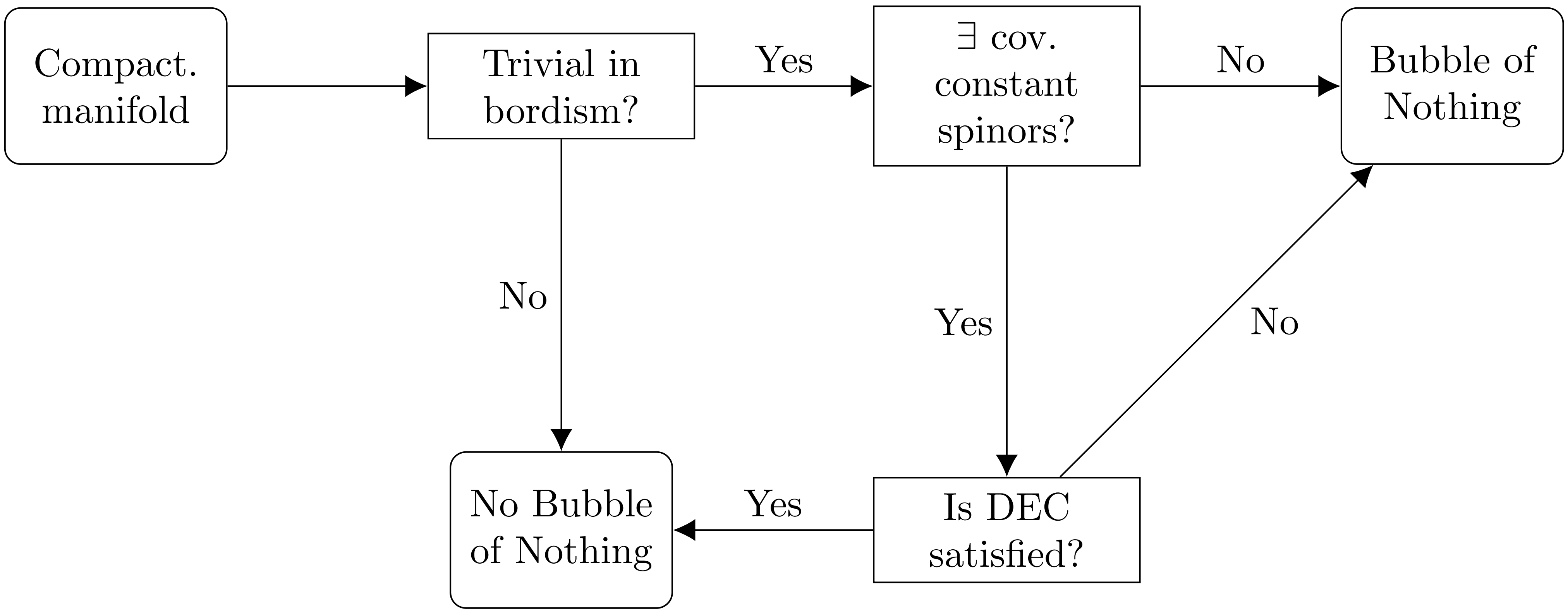


Thank you!

BACKUP

Flowchart



[I. G. Etxebarria, M. Montero, K. Sousa and I. Valenzuela, JHEP 12, (2020) 032]

Introducing $\rho \equiv r\sqrt{f(r)}$, we can rewrite the instanton solution as

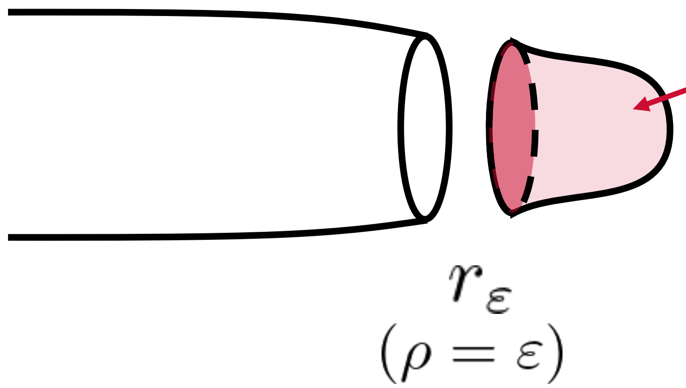
$$ds^2 = F(\rho)^2 \underline{d\chi^2} + d\rho^2 + r(\rho)^2 d\Omega_3^2, \quad F(\rho)^2 \equiv \left(1 - \left(\frac{\sqrt{\alpha}}{r(\rho)}\right)^2\right) R^2$$

2π periodic

$$ds^2 \simeq d\rho^2 + \rho^2 d(F'(0)\chi)^2 + r(0)^2 d\Omega_3^2$$ (near the singularity)

Since $F'(0) \neq 1$ in general, there would be a deficit angle defined as

$$2\pi\delta = 2\pi\left(1 - F'(0)\right) = 2\pi\left(1 - \frac{R}{\sqrt{\alpha}}\right)$$



$$ds^2 = \widetilde{F}(\rho)^2 d\chi^2 + d\rho^2 + r(\rho)^2 d\Omega_3^2$$

$\widetilde{F}'(0) = 1$, but $\delta \simeq 1 - \widetilde{F}'(\varepsilon)$

smooth

$$-\frac{1}{16\pi G_n}\int_{\mathcal{B}} \mathcal{R} = -\frac{\mathcal{A}}{4G_n}\delta \ , \qquad -\frac{1}{8\pi G_n}\oint_{\partial\mathcal{B}} (\mathcal{K} - \mathcal{K}_0) = -\frac{\mathcal{A}}{4G_n}(1-\delta)$$

$$I_{\mathcal{B}} = -\frac{\mathcal{A}}{4G_n}\delta - \frac{\mathcal{A}}{4G_n}(1-\delta) = -\frac{\mathcal{A}}{4G_n}$$

cancel each other out