Yukawa Couplings at Infinite Distance and Swampland Towers in Chiral Theories



Universidad Autónoma de Madrid Gonzalo F. Casas





Based on [2403.09775] + [240X.XXXX]

w/ Fernando Marchesano and Luis Ibáñez



Not every EFT can be embedded in quantum gravity —— there is a classification of such restrictions















Gonzalo F. Casas











ifł

Infinite distance limits in 4d N = 1,0 SUSY chiral theories are relatively unexplored. Most phenomenological interest

Such settings involve ingredients like e.g. perturbative trilinear Yukawa couplings Y_{ijk} . The natural question one could have is whether

$$Y_{ijk}
ightarrow 0$$

Cribiori-Farakos '23
Palti '20



Such settings involve ingredients like e.g. perturbative trilinear Yukawa couplings Y_{ijk} . The natural question one could have is whether

$$Y_{ijk} \to 0$$

Cribiori-Farakos '23 Palti '20



Such settings involve ingredients like e.g. perturbative trilinear Yukawa couplings Y_{ijk} . The natural question one could have is whether

$$Y_{ijk} \to 0$$

There is no obvious reason what goes wrong!

Cribiori-Farakos '23 Palti '20



Such settings involve ingredients like e.g. perturbative trilinear Yukawa couplings Y_{ijk} . The natural question one could have is whether

$$Y_{ijk} \to 0$$

There is no obvious reason what goes wrong!

Cribiori-Farakos '23 Palti '20

This limit MUST be explored!

In the SM $Y_{\nu} \sim 10^{-12}$



Compactify Type IIA string theory on a CY orientifold

SUSY is broken from N=2 to N=1





Compactify Type IIA string theory on a CY orientifold

SUSY is broken from N=2 to N=1









Compactify Type IIA string theory on a CY orientifold

SUSY is broken from N=2 to N=1

Mutual supersymmetric branes are related by a SU(3) rotation

$$\theta^1_{\alpha\beta} + \theta^2_{\alpha\beta} + \theta^3_{\alpha\beta} = 2\mathbb{Z}$$

How does it work?





Compactify Type IIA string theory on a CY orientifold

SUSY is broken from N=2 to N=1

How does it work?

Mutual supersymmetric branes are related by a SU(3) rotation

 $\theta^1_{\alpha\beta} + \theta^2_{\alpha\beta} + \theta^3_{\alpha\beta} = 2\mathbb{Z}$

Gonion masses may be understood as Fayet- Iliopoulos generated by the branes

Cremades-Ibañez-Marchesano '02

$$m_{\alpha\beta}^2 = q_{\alpha}^i g_{\alpha}^2 \xi_{\alpha} + q_{\beta}^i g_{\beta}^2 \xi_{\beta}$$





Compactify Type IIA string theory on a CY orientifold

SUSY is broken from N=2 to N=1

How does it work?

Mutual supersymmetric branes are related by a SU(3) rotation

 $\theta^1_{\alpha\beta} + \theta^2_{\alpha\beta} + \theta^3_{\alpha\beta} = 2\mathbb{Z}$

Gonion masses may be understood as Fayet- Iliopoulos generated by the branes

Cremades-Ibañez-Marchesano '02

$$m_{\alpha\beta}^2 = q_{\alpha}^i g_{\alpha}^2 \xi_{\alpha} + q_{\beta}^i g_{\beta}^2 \xi_{\beta}$$

It vanishes for SUSY configurations and a scalar partner of the fermion becomes massless

$$\ell_s^2 m_{\alpha\beta}^2 = \theta_{\alpha\beta}^1 \pm \theta_{\alpha\beta}^2 \pm \theta_{\alpha\beta}^3 = 0$$







YUKAWA COUPLINGS IN TOROIDAL COMPACTIFICATIONS

Perturbative Yukawas are generated by worldsheet disc instantons connecting three D6-brane intersections

$$Y_{ijk} = \frac{e^{\phi_4/2}}{\operatorname{Vol}_X^{1/2}} \prod_{r=1}^3 \left[2\pi \frac{\Gamma(1-|\chi_{ab}^r|)}{\Gamma(|\chi_{ab}^r|)} \frac{\Gamma(1-|\chi_{bc}^r|)}{\Gamma(|\chi_{bc}^r|)} \frac{\Gamma(1-|\chi_{ca}^r|)}{\Gamma(|\chi_{ca}^r|)} \right]^{1/4} W_{ijk}$$

$$\begin{pmatrix} & \swarrow \\ & \chi_{\alpha\beta}^r = |\theta_{\alpha\beta}^r| \text{ or } 1-|\theta_{\alpha\beta}^r| \end{pmatrix}$$



YUKAWA COUPLINGS IN TOROIDAL COMPACTIFICATIONS

Perturbative Yukawas are generated by worldsheet disc instantons connecting three D6-brane intersections

$$Y_{ijk} = \frac{e^{\phi_4/2}}{\operatorname{Vol}_X^{1/2}} \prod_{r=1}^3 \left[2\pi \frac{\Gamma(1-|\chi_{ab}^r|)}{\Gamma(|\chi_{ab}^r|)} \frac{\Gamma(1-|\chi_{bc}^r|)}{\Gamma(|\chi_{bc}^r|)} \frac{\Gamma(1-|\chi_{ca}^r|)}{\Gamma(|\chi_{ca}^r|)} \right]^{1/4} W_{ijk}$$
local geometry.

Depends on local geometry.

Yukawa couplings give information about the Kähler metric of chiral fields

$$Y_{ijk} = e^{K/2} \left(K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}} \right)^{-1/2} W_{ijk}$$

$$K_{i\bar{i}} = e^{\phi_4} (2\pi)^{-1/2} \prod_{r=1}^3 \left(\frac{\Gamma(|\chi_i^r|)}{\Gamma(1-|\chi_i^r|)} \right)^{1/2}$$





$$Y_{ijk} = \frac{W_{ijk}}{\text{Vol}_X^{1/2}} \ e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$



$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$



$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$

According to the SDC there should be a tower of states becoming massless



$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$

According to the SDC there should be a tower of states becoming massless

$$m_{D0}^2 \sim \frac{M_P^2}{\operatorname{Vol}_X}, \longrightarrow |Y| \simeq \frac{m_{D0}}{M_P}$$

$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Infinite distance in c.c moduli

$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$\Omega_c \equiv C_3 + i e^{-\phi} \text{Re}\Omega = (\xi^K + i u^K) \alpha_K$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$

According to the SDC there should be a tower of states becoming massless

$$m_{D0}^2 \sim \frac{M_P^2}{\operatorname{Vol}_X}, \longrightarrow |Y| \simeq \frac{m_{D0}}{M_P}$$



$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$

Infinite distance in c.s moduli

$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$\Omega_c \equiv C_3 + i e^{-\phi} \operatorname{Re}\Omega = (\xi^K + i u^K) \alpha_K$$

Take the infinite distance limit (at fixed t)

$$u^K = u_0^K + e^K \lambda, \quad \lambda \to \infty$$

According to the SDC there should be a tower of states becoming massless

$$m_{D0}^2 \sim \frac{M_P^2}{\operatorname{Vol}_X}, \longrightarrow |Y| \simeq \frac{m_{D0}}{M_P}$$



$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

Take the infinite distance limit (at fixed cc)

$$\operatorname{Vol}_X^{1/2} \to \infty$$

Infinite distance in c.s moduli

$$Y_{ijk} = \frac{W_{ijk}}{\operatorname{Vol}_X^{1/2}} e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

$$\Omega_c \equiv C_3 + i e^{-\phi} \operatorname{Re}\Omega = (\xi^K + i u^K) \alpha_K$$

Take the infinite distance limit (at fixed t)

$$u^K = u_0^K + e^K \lambda, \quad \lambda \to \infty$$

According to the SDC there should be a tower of states becoming massless

see also Castellano-Herraez-Ibañez '23

$$m_{D0}^2 \sim \frac{M_P^2}{\operatorname{Vol}_X}, \longrightarrow |Y| \simeq \frac{m_{D0}}{M_P}$$

$$|Y| \simeq e^{-\phi_4} \left(\frac{m_{\text{gon}}^i}{M_P} \frac{m_{\text{gon}}^j}{M_P} \frac{m_{\text{gon}}^k}{M_P} \right)^{1/2}$$

 \smile Tower of gonions



GFC-Ibañez-Marchesano '24

Consider the following stack of branes

 $\Pi_a = 2(1,1)(1,1)(1,-1)$ $\Pi_b = 2(0,1)(1,0)(0,-1)$ $\Pi_c = 2(0,1)(0,-1)(1,0)$



Toy model example



Toy model example

GFC-Ibañez-Marchesano '24

Consider the following stack of branes

 $\Pi_a = 2(1,1)(1,1)(1,-1)$ $\Pi_b = 2(0,1)(1,0)(0,-1)$ $\Pi_c = 2(0,1)(0,-1)(1,0)$

Take the complex structure limit

 $u^{(0)}, u^{(2)} \sim u \to \infty$

We find a gonion tower in sectors aa* and ab with masses



$$m_{
m gon}=e^{\phi_4}| heta_{ab}^2|^{1/2}M_P\simeq rac{M_P}{u}$$
 . KK modes at the same scale



Toy model example

GFC-Ibañez-Marchesano '24

Consider the following stack of branes

 $\Pi_a = 2(1,1)(1,1)(1,-1)$ $\Pi_b = 2(0,1)(1,0)(0,-1)$ $\Pi_c = 2(0,1)(0,-1)(1,0)$

Take the complex structure limit

 $u^{(0)},\,u^{(2)}\sim u\to\infty$

We find a gonion tower in sectors aa* and ab with masses

The Yukawa coupling involved takes the form



$$m_{
m gon}=e^{\phi_4}| heta_{ab}^2|^{1/2}M_P\simeq rac{M_P}{u}$$
 th

KK modes at ne same scale

$$Y_{abc}\simeq \left(rac{m_{
m gon}}{M_P}
ight)^{1/2}$$
 with $K_{ab}\simeq rac{M_P}{m_{
m gon}}$



Toy model example

GFC-Ibañez-Marchesano '24

Consider the following stack of branes

 $\Pi_a = 2(1,1)(1,1)(1,-1)$ $\Pi_b = 2(0,1)(1,0)(0,-1)$ $\Pi_c = 2(0,1)(0,-1)(1,0)$

Take the complex structure limit

 $u^{(0)}, u^{(2)} \sim u \to \infty$

We find a gonion tower in sectors aa* and ab with masses

The Yukawa coupling involved takes the form



$$Y_{abc} \simeq \left(rac{m_{
m gon}}{M_P}
ight)^{1/2}$$
 with $K_{ab} \simeq rac{M_P}{m_{
m gon}}$

$$Y_{abc} \sim g_a \sim g_b \longrightarrow$$
 Recovering global symmetry

а





GFC-Ibañez-Marchesano '24 (to appear)













 b^*

Anomaly FREE $y = \frac{2}{3}Q_a - \frac{1}{2}Q_b + Q_c$

Taking the limit $\theta \to 0$ modifies couplings $g_{\tilde{c}}, g_d$ but the hypercharge g_y







Taking the limit $\theta \to 0$ modifies couplings $g_{\tilde{c}}, g_d$ but the hypercharge g_y

This makes it possible to design a limit with suppressed neutrino yukawa coupling at the expense of having gonions and KK states around the corner.

$$Y_{\nu}^{\text{EXP}} \sim 6.9 \cdot 10^{-13} \longrightarrow m_{\text{gon},\nu} \simeq m_{KK} \simeq 500 eV$$

 $M_{\text{s}} = Y_{\nu} M_{P} = 700 \text{ TeV}$

 $Y_{\nu} \simeq \left(\frac{m_{\rm gon,\nu}}{M_P}\right)^{1/2}$

ifł

CONCLUSIONS AND OUTLOOK

We studied c.c limits in different set-ups

GFC-Ibañez-Marchesano '24

- 'STU' Type IIA orientifold models models, dual to magnetized Type I and SO(32) models with U(1) bundles Blumenhagen-Honecker-Weigand '05
- EFT string limits (anza-Marchesano-Martucci-Valenzuela '20' 21
- Type IIA toroidal orientifolds (e.g. Pati-Salam-like)

CONCLUSIONS AND OUTLOOK





CONCLUSIONS AND OUTLOOK





CONCLUSIONS AND OUTLOOK







THANK $Y_{\nu} O \bar{\nu} \nu$



SUSY CONDITION FOR BRANES



D6-branes wrapping three-cycles do not break SUSY if

 $\Pi_{\alpha} \in \text{Slag submanifolds}$

We can also characterize such calibration condition

 $\mathrm{Im}e^{-i\pi\varphi_{\alpha}}\Omega|_{\Pi_{\alpha}}=0$

At the intersection of two branes this translates to

$$\varphi = \varphi_{\alpha} - \varphi_{\beta} = 2\mathbb{Z}$$

Long story short, mutual supersymmetric branes are related by a SU(3) rotation

Berkooz-Douglas-Leigh '96

$$\theta^1_{\alpha\beta} + \theta^2_{\alpha\beta} + \theta^3_{\alpha\beta} = 2\mathbb{Z}$$

Gonion masses may be understood as Fayet- Iliopoulos generated by the branes

Cremades-Ibañez-Marchesano '02

 $m_{\alpha\beta}^2 = q_{\alpha}^i g_{\alpha}^2 \xi_{\alpha} + q_{\beta}^i g_{\beta}^2 \xi_{\beta}$

Vanishes in a SUSY configuration in which a complex scalar partner of the chiral fermion becomes massless

 $\ell_s^2 m_{\alpha\beta}^2 = \theta_{\alpha\beta}^1 \pm \theta_{\alpha\beta}^2 \pm \theta_{\alpha\beta}^3 = 0$





Swampland distance conjecture

At any infinite distance limit in moduli space there is a tower of states becoming exponentially light

$$m_n \sim m_0 e^{-\alpha \Delta \phi}$$

Weak gravity conjecture (and tWGC)

A gauge theory weakly coupled to gravity always contains a charged particle that satisfies

$$M \leq g M_P^{D/2-1}$$





At any infinite distance limit in moduli space there is a tower of states becoming exponentially light

$$m_n \sim m_0 e^{-\alpha \Delta \phi}$$

Weak gravity conjecture (and tWGC)

A gauge theory weakly coupled to gravity always contains a charged particle that satisfies

$$m_n \le g M_P^{D/2 - 1}$$