WZW terms via Bordism

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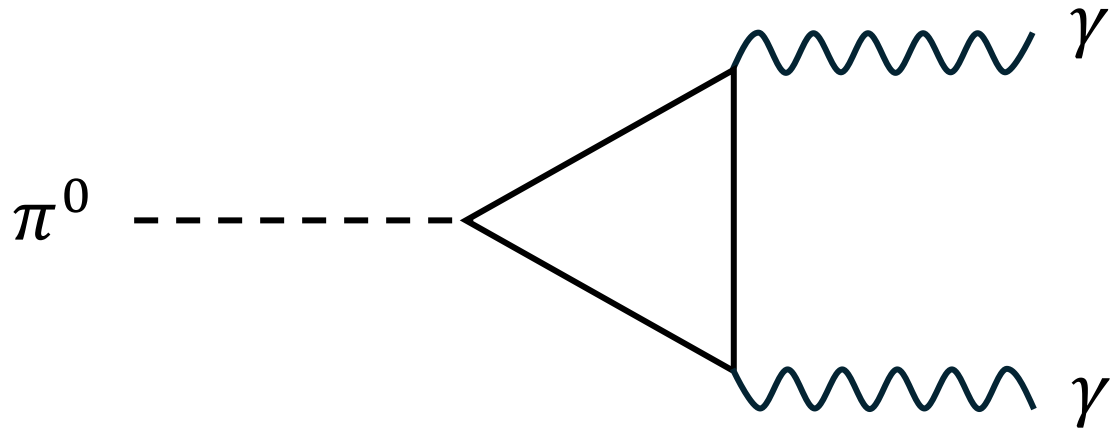
Based on arXiv:2404.06185, S. Saito

Contents

1. Introduction to WZW terms

Neutral Pion Decay

- Decay mode $\pi^0 \to 2\gamma$ is dominant.
- Realized by 1-loop diagram in chiral Lagrangian.



- This is contribution from $U(1)_A$ violation.

Considering symmetries

- Chiral Lagrangian has both P_0 and $(-1)^{N_\pi}$ symmetries.

  $P_0 : x \mapsto -x$

  $(-1)^{N_\pi} : \pi \mapsto -\pi$

- Anomaly term $\pi^0 \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}$ does not belong to Chiral Lagrangian.

- WZW term can satisfy invariance of parity $P_0(-1)^{N_\pi}$

  and can explain anomaly term.

WZW term as 5-dim action

- It is difficult to write down the action including both $\epsilon^{\alpha\beta\gamma\delta}$ and $tr(\cdots)$.

- WZW term is described in 5-dim action.

$$S_{WZW} \propto \int_{D^5} d^5x\, \epsilon^{\alpha\beta\gamma\delta\varepsilon} tr\{U^\dagger(\partial_\alpha U)U^\dagger(\partial_\beta U)U^\dagger(\partial_\gamma U)U^\dagger(\partial_\delta U)U^\dagger(\partial_\varepsilon U)\}$$

our 4-dim spacetime S^4 is boundary of D^5

- Gauged WZW term contains anomaly $\pi^0\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$

and describes neutral pion decay $\pi^0 \longrightarrow 2\gamma$

Motivation and Summary

- Anomalies of QCD with fundamental matter are
  - studied in SU and SO theories by Freed, Lee, Ohmori, and Tachikawa
  - not studied in Sp and exceptional Lie group theories
- Sp QCD is motivated in dark matter study, lattice study…

- We studied anomalies of Sp QCD in general case
  - perturbative and non-perturbative anomalies
  - topological structure of spacetime
  - spacetime equipped with spin structure

2. WZW terms via bordism

Anomaly as phase

- Anomaly is change of partition function under gauge transformation.

$$Z[A^g] = JZ[A] , \qquad J \neq 1$$

- Vector theories are known not to have anomalies.

$$Z[A^g]\overline{Z[A^g]} = Z[A]\overline{Z[A]}$$

- Anomalies are described by J where $|J| = 1$.

Anomaly Inflow

- Invertible field theory is QFT with partition function Z_{inv} which has one absolute value $|Z_{inv}| = 1$.

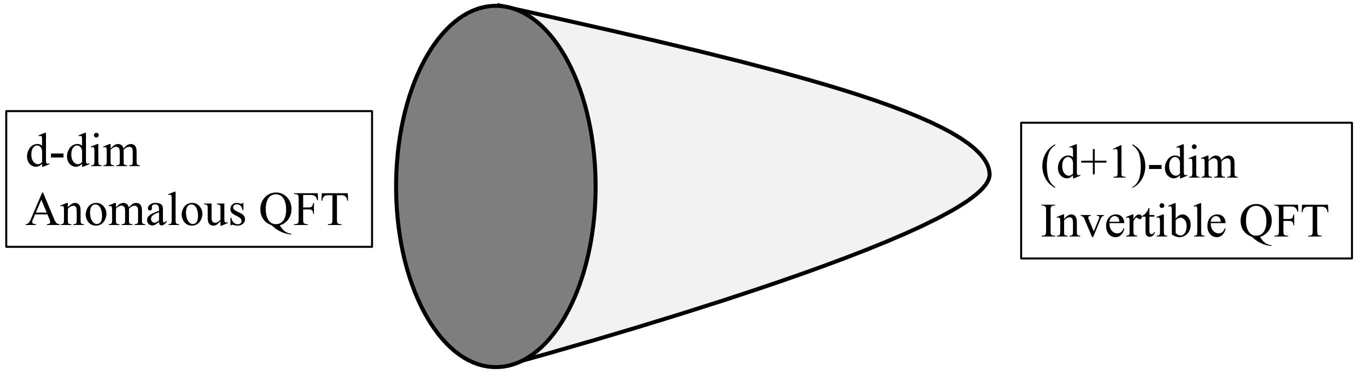
ANOMALY INFLOW :

Anomalous d-dim QFT is realized by boundary of invertible $(d + 1)$-dim QFT.

- When combined, phase change of partition function $Z \times Z_{inv}$ under gauge transformation does not occur.

Classification of invertible field theories

- Studying invertible field theories is studying anomalies.



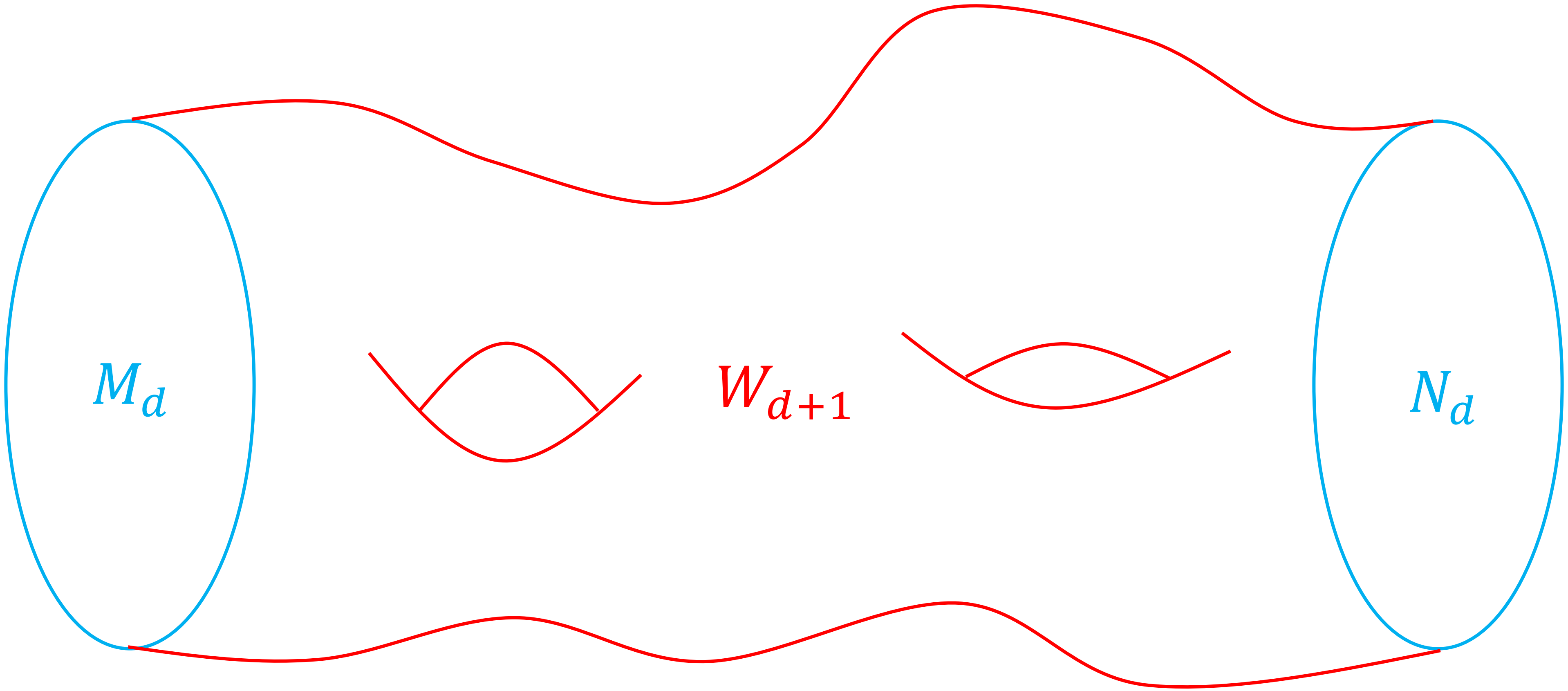
Picture of anomaly inflow

- We classify invertible field theories.

- Bordism theory is sometimes useful.

Before bordism, what is bordant?

Closed manifolds M_d and N_d are bordant if …



Bordism

- Bordism is group defined as

$$\Omega_n = \frac{\text{closed n}-\text{dim manifolds}}{\text{bordant}}$$

elements are equivalent class (bordant) of closed n-dim manifolds
operation is disjoint union
identity is equivalent class of empty manifolds
inverse is equivalent class of orientation-reversal

Classification of anomalies

- Invertible phase is described by bordism

$$0 \to Ext_{\mathbb{Z}}(\Omega_{d+1}^{spin}(BG), \mathbb{Z}) \to Inv_{spin}^{d+1}(BG) \to Hom_{\mathbb{Z}}(\Omega_{d+2}^{spin}(BG), \mathbb{Z}) \to 0$$

- One can find anomalies

perturbative anomaly : $Hom_{\mathbb{Z}}(\Omega_{d+2}^{spin}(BG), \mathbb{Z})$

non-perturbative anomaly : $Ext_{\mathbb{Z}}(\Omega_{d+1}^{spin}(BG), \mathbb{Z})$

WZW terms via bordism

- WZW term is described by bordism

$$0 \to Ext_{\mathbb{Z}}(\Omega_d^{spin}(X), \mathbb{Z}) \to Inv_{spin}^d(X) \to Hom_{\mathbb{Z}}(\Omega_{d+1}^{spin}(X), \mathbb{Z}) \to 0$$

⇧ 5-dim action

- Hom part is 5-dim acion explained in introduction.

- Ext part is discrete WZW term.

3. Application to Sp QCD

Quark condensate

- Assume fundamental quarks condensate in the most attractive channel.

- For $Sp(N_c) \times SU(2N_f)$ QCD, quarks condensate as

$$\langle \bar{\psi}_i \psi_j \rangle = v^3 \Sigma_{ij} \in SU(2N_f)/Sp(N_f)$$

- We study target space SU/Sp
  to investigate anomalies and WZW terms.

Result in bordism language

- WZW term is given by

$$Hom_{\mathbb{Z}}(\Omega_5^{spin}\big(SU(2N_f)/Sp(N_f)\ \big), \mathbb{Z}) \cong \mathbb{Z}$$

- Discrete WZW term does not exist

$$Ext_{\mathbb{Z}}(\Omega_4^{spin}\big(SU(2N_f)/Sp(N_f)\ \big), \mathbb{Z}) \cong 0$$

- WZW term reproduces anomaly

$$Hom_{\mathbb{Z}}(\Omega_{d+2}^{spin}\big(BSU(2N_f)\big), \mathbb{Z}) \to Hom_{\mathbb{Z}}(\Omega_5^{spin}\big(SU(2N_f)/Sp(N_f)\ \big), \mathbb{Z})$$

- No residual global anomaly and WZW term completely describes anomaly

$Ext_{\mathbb{Z}}(\Omega_5^{spin}(BSp), \mathbb{Z}) \cong \mathbb{Z}_2 \ni 0$ (pull-back of anomaly)

4. Summary

Summary

- WZW terms describe anomalies at low energies.
- WZW terms are written as invertible phase via bordism
- We study WZW terms of Sp QCD and get them in general forms.

Thank you for listening