Supersymmetric EFTs and the Swampland

SUSY 2024- Madrid
13th Jun 2024

Based on [arXiv:2310.07708]

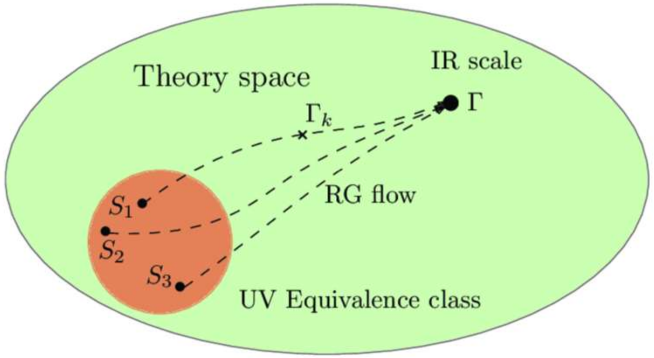
Work with L. Ibáñez and A. Herráez

EXCELENCIA SEVERO OCHOA
Instituto de Física Teórica UAM-CSIC
Universidad Autónoma de Madrid
CSIC

The Scale of QG

- Low energy physics is unavoidably described by Effective Field Theory (EFT) [Weinberg '95]

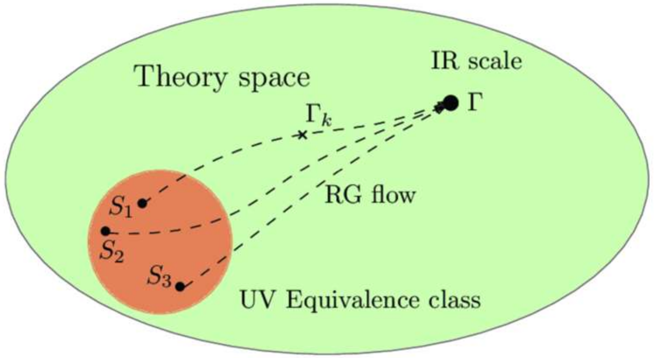
$$\mathcal{L}_{\mathrm{EFT},\,d} = \mathcal{L}_{\mathrm{renorm}} + \sum_{n\geq d} \frac{\mathcal{O}_n}{\Lambda_{\mathrm{UV}}^{n-d}}$$



The Scale of QG

- Low energy physics is unavoidably described by Effective Field Theory (EFT) [Weinberg '95]

$$\mathcal{L}_{\text{EFT},\,d} = \mathcal{L}_{\text{renorm}} + \sum_{n\geq d} \frac{\mathcal{O}_n}{\Lambda_{\text{UV}}^{n-d}}$$



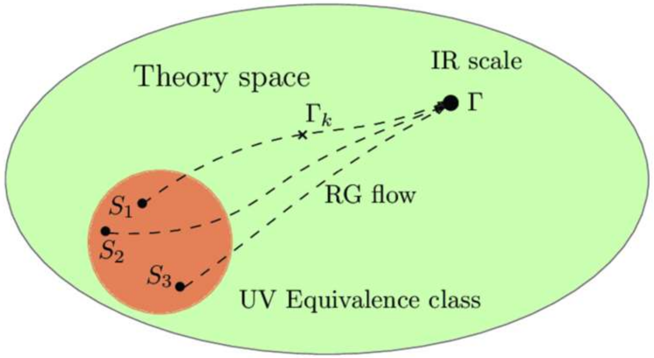
- For gravitational interactions, the same logic should apply [Donoghue '94]

$$\mathcal{L}_{\text{EFT},\,d} = \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \sum_n \frac{\mathcal{O}_n(\mathcal{R})}{\Lambda_{\text{QG}}^{n-2}}\right) + \text{(matter)} + \ldots$$

The Scale of QG

- Low energy physics is unavoidably described by Effective Field Theory (EFT) [Weinberg '95]

$$\mathcal{L}_{\mathrm{EFT},\,d} = \mathcal{L}_{\mathrm{renorm}} + \sum_{n\geq d} \frac{\mathcal{O}_n}{\Lambda_{\mathrm{UV}}^{n-d}}$$



- For gravitational interactions, the same logic should apply [Donoghue '94]

$$\mathcal{L}_{\mathrm{EFT},\,d} = \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \sum_n \frac{\mathcal{O}_n(\mathcal{R})}{\Lambda_{\mathrm{QG}}^{n-2}}\right) + \text{(matter)} + \ldots$$

- Q: Which energy scale should control the EFT expansion in (Quantum) Gravity?

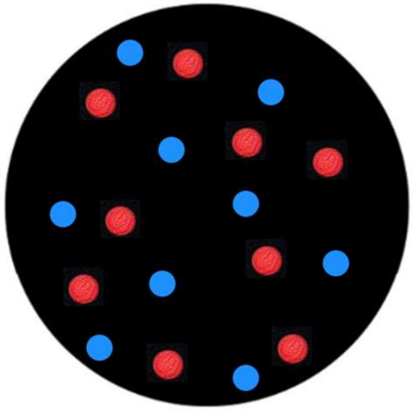
$$\kappa_d^2 = 8\pi G_N \Longrightarrow [G_N] = E^{2-d}$$

$$\boxed{M_{\mathrm{pl},\,d} \sim G_N^{\frac{1}{2-d}} \stackrel{?}{=} \Lambda_{\mathrm{QG}}}$$

The Scale of QG (cont'd)

- This naïve answer is essentially correct in the presence of a few number of dofs [AC, Herráez, Ibáñez '21-22]

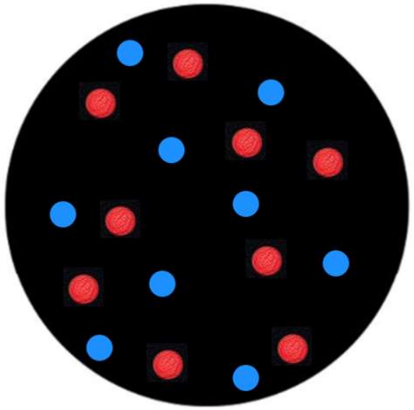
$N = \mathcal{O}(1)$



The Scale of QG (cont'd)

- This naïve answer is essentially correct in the presence of a few number of dofs [AC, Herráez, Ibáñez '21-22]

$N = \mathcal{O}(1)$

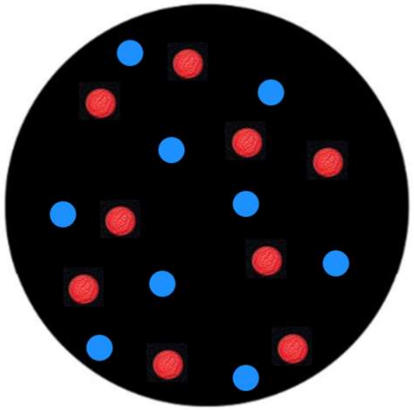


$$\begin{cases} E_{\text{gas}} \sim R^{d-1}T^{d} \\ S_{\text{gas}} \sim (RT)^{d-1} \end{cases}$$

The Scale of QG (cont'd)

- This naïve answer is essentially correct in the presence of a few number of dofs [AC, Herráez, Ibáñez '21-22]

$N = \mathcal{O}(1)$

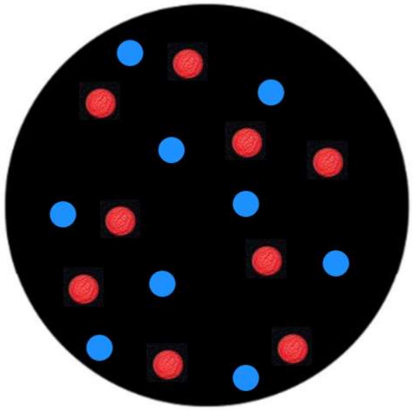


$$\begin{cases} E_{\text{gas}} \sim R^{d-1} T^d \\ S_{\text{gas}} \sim (RT)^{d-1} \end{cases} \xrightarrow{R \gtrsim \left(\frac{E}{M_{\text{pl},d}}\right)^{\frac{1}{d-3}}} \boxed{S_{\text{gas}} \sim A^{\frac{d-1}{d}} < A}$$

The Scale of QG (cont'd)

- This naïve answer is essentially correct in the presence of a few number of dofs [AC, Herráez, Ibáñez '21-22]

$N = \mathcal{O}(1)$



$$\begin{cases} E_{\text{gas}} \sim R^{d-1} T^{d} \\ S_{\text{gas}} \sim (RT)^{d-1} \end{cases} \overset{R \gtrsim \left(\frac{E}{M_{\text{pl},d}}\right)^{\frac{1}{d-3}}}{\Longrightarrow} \boxed{S_{\text{gas}} \sim A^{\frac{d-1}{d}} < A}$$

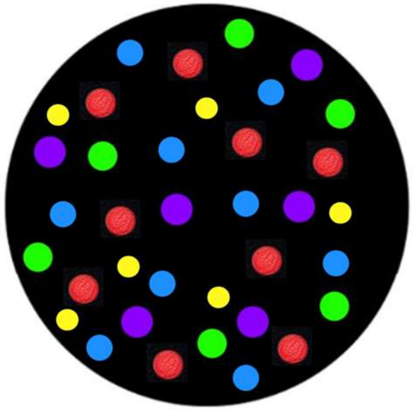
- Everything is consistent with the holographic principle [Bekenstein, Hawking, Susskind, Bousso, etc.]

- The minimal size of the box is reached when $A \sim 1$ (in Planck units)

The Scale of QG (cont'd)

- But what if N is very large? [AC, Herráez, Ibáñez '21-22]

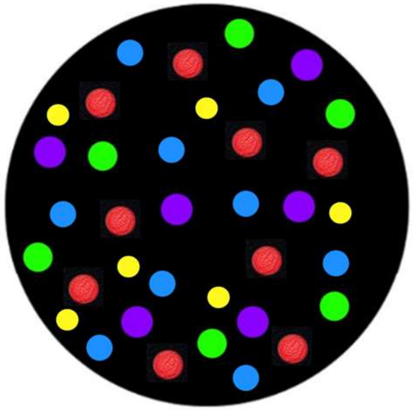
$N \gg 1$



The Scale of QG (cont'd)

- But what if N is very large? [AC, Herráez, Ibáñez '21-22]

$N \gg 1$

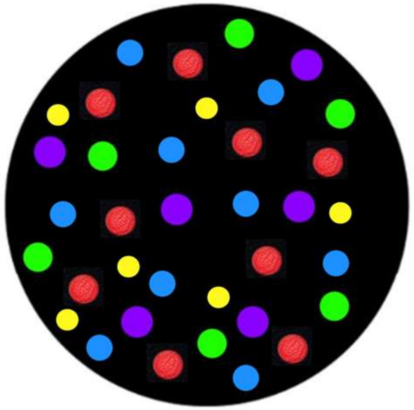


$$\begin{cases} E_{\text{gas}} \sim N R^{d-1} T^d \\ S_{\text{gas}} \sim N (RT)^{d-1} \end{cases} \xRightarrow{R \gtrsim \left(\frac{E}{M_{\text{pl},d}}\right)^{\frac{1}{d-3}}} \boxed{S_{\text{gas}} \sim \left(N A^{d-1}\right)^{\frac{1}{d}}}$$

The Scale of QG (cont'd)

- But what if N is very large? [AC, Herráez, Ibáñez '21-22]

$N \gg 1$



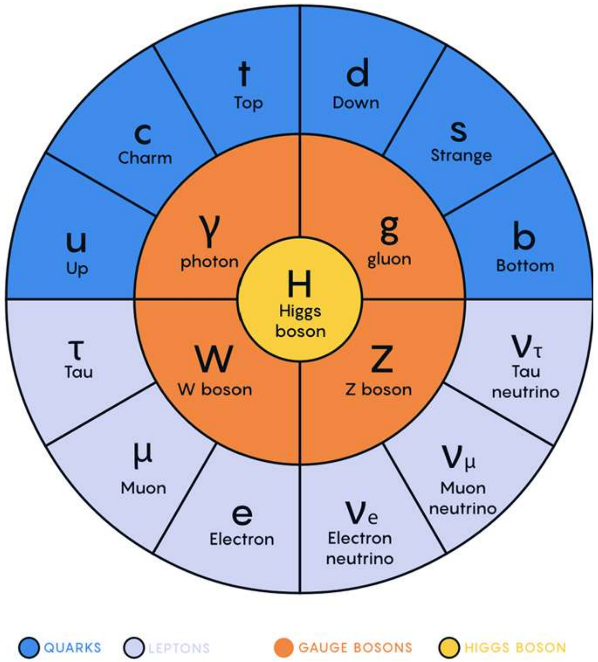
$$\begin{cases} E_{\text{gas}} \sim N\, R^{d-1} T^d \\ S_{\text{gas}} \sim N\, (RT)^{d-1} \end{cases} \xrightarrow{R \gtrsim \left(\frac{E}{M_{\text{pl},d}}\right)^{\frac{1}{d-3}}} \boxed{S_{\text{gas}} \sim \left(N\, A^{d-1}\right)^{\frac{1}{d}}}$$

- The holographic entropy seems to be violated if $N \gtrsim A$!!

The Scale of QG (cont'd)

- This is essentially the species problem [Sorkin, Wald, Zhang, Unruh, etc.]

- Is this really a problem? After all, in our Universe and at energies probed by LHC, we only see $\mathcal{O}(10^2)$ particle species

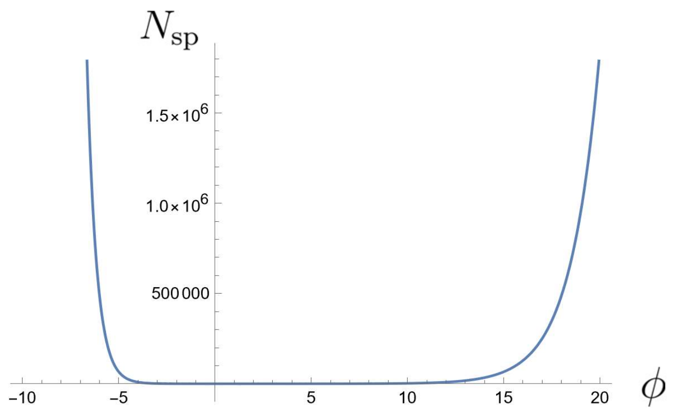


The Scale of QG (cont'd)

- This is known as the species problem [Sorkin, Wald, Zhang, Unruh, etc.]

- Is this really a problem? After all, in our Universe and at energies probed by LHC, we only see $\mathcal{O}(10^2)$ particle species

- Yes!! In quantum gravity we know that the number of species can change (e.g. decompactifications limits, tensionless string points, conifold loci, etc.)



e.g. 10d Type IIA

The Scale of QG (cont'd)

- This is known as the species problem [Sorkin, Wald, Zhang, Unruh, etc.]

- Is this really a problem? After all, in our Universe and at energies probed by LHC, we only see $\mathcal{O}(10^2)$ particle species

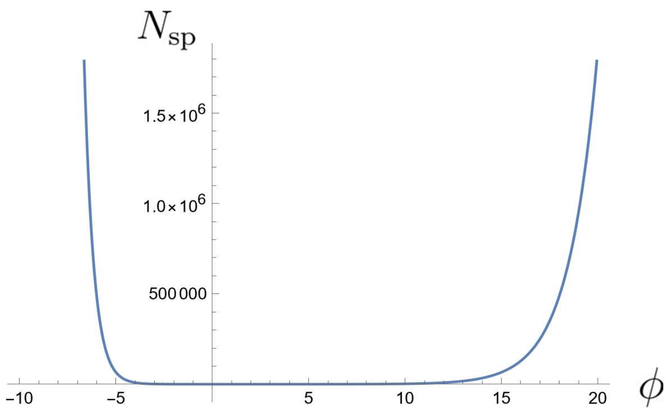
- Yes!! In quantum gravity we know that the number of species can change (e.g. decompactifications limits, tensionless string points, conifold loci, etc.)

- In fact, this must be the case (By No Global Symmetry Conjecture)

- Turning this logic around:

$$A \geq N \Longrightarrow \ell_{\min} = \ell_{\rm sp} := \ell_{\rm pl} N^{\frac{1}{d-2}}$$

[see also Vafa '24]



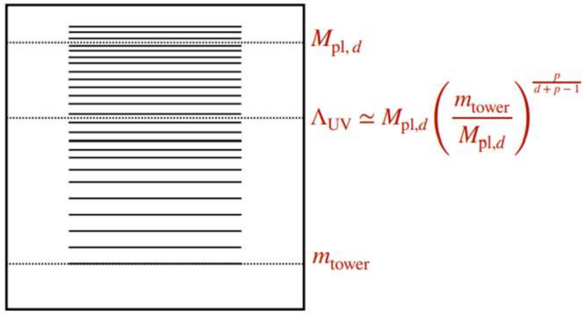
e.g. 10d Type IIA

The Species Scale

- The Species Scale is therefore defined (at least asymptotically) as [Dvali, Redi '07; Dvali, Gómez '10]

$$\Lambda_{\rm sp} \approx \frac{M_{{\rm pl},\, d}}{N^{\frac{1}{d-2}}} \leq M_{{\rm pl},\, d}$$

- In the presence of infinite towers of states there can be parametric decoupling! [AC, Herráez, Ibáñez '21-22]



$$\begin{cases} \Lambda_{\rm sp} = M_{{\rm pl},\, d+p} & \text{decompactification limits} \\ \Lambda_{\rm sp} = \sqrt{T_s} & \text{emergent string limits} \end{cases}$$

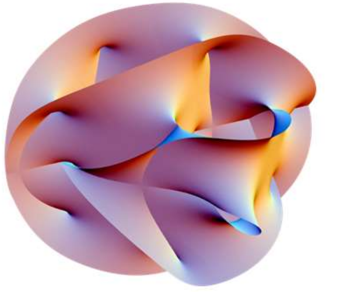
- This can be nicely encoded into the formula shown in the first slide [v. d. Heisteeg, Vafa, Wiesner, Wu '22-23] [AC, Herráez, Ibáñez '23]

$$\Lambda_{\rm QG} = \Lambda_{\rm sp} \Longrightarrow \qquad \mathcal{L}_{{\rm EFT},\, d} \supset \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \sum_{n>2} \frac{\mathcal{O}_n(\mathcal{R})}{\Lambda_{\rm sp}^{n-2}}\right)$$

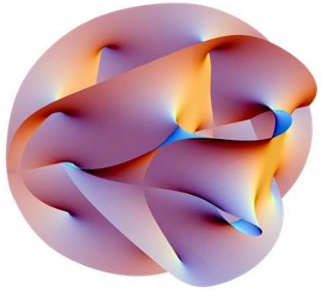
String Theory Checks

- How can we check this in explicit QG/string theory constructions?

- We need full control over certain higher-dimensional corrections.



String Theory Checks



- How can we check this in explicit QG/string theory constructions?
- We need full control over certain higher-dimensional corrections.
- This is where supersymmetry enters to our rescue!

SUSY 2024

- Basic strategy: Look into F-like terms or BPS operators in the theory

[AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

$$\text{e.g.,} \quad \mathcal{L}_{4\text{d}} \supset \int d^2\theta\, W(\Phi) + \text{h.c.}$$

String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory!

[AC, Herráez, Ibáñez ‘23, v. d. Heisteeg, Vafa, Wiesner, Wu ‘23]

10d Type IIB string theory

- Bosonic action

$$S_{\rm IIB}^{\rm 10d} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{1}{2}(\partial\phi)^2\right) - \frac{1}{4\kappa_{10}^2}\int e^{-\phi} H_3 \wedge \star H_3$$
$$- \frac{1}{4\kappa_{10}^2}\int \left[e^{2\phi} F_1 \wedge \star F_1 + e^{\phi}\tilde{F}_3 \wedge \star\tilde{F}_3 + \frac{1}{2}\tilde{F}_5 \wedge \star\tilde{F}_5 + C_4 \wedge H_3 \wedge F_3\right] ,$$

String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

10d Type IIB string theory

- Bosonic action

$$S_{\text{IIB}}^{10\text{d}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{1}{2}(\partial\phi)^2\right) - \frac{1}{4\kappa_{10}^2}\int e^{-\phi}H_3\wedge\star H_3$$
$$-\frac{1}{4\kappa_{10}^2}\int\left[e^{2\phi}F_1\wedge\star F_1 + e^{\phi}\tilde{F}_3\wedge\star\tilde{F}_3 + \frac{1}{2}\tilde{F}_5\wedge\star\tilde{F}_5 + C_4\wedge H_3\wedge F_3\right],$$

String Theory Checks

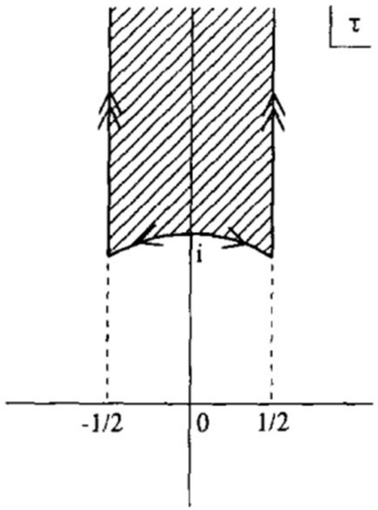
- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

10d Type IIB string theory

- Scalar-gravity sector

$$S_{\rm IIB}^{\rm 10d} \supset \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{\partial\tau\cdot\partial\bar{\tau}}{2({\rm Im}\,\tau)^2}\right)\,,$$

$$\text{under } \mathsf{SL}(2,\mathbb{Z}): \tau \to \frac{a\,\tau+b}{c\,\tau+d}\,, \qquad g_{\mu\nu}\to g_{\mu\nu}$$



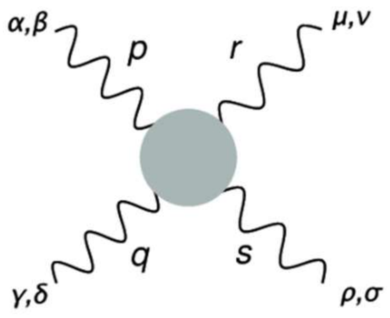
String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

10d Type IIB string theory

- The $\mathcal{R}^4$ operator

$$S_{\text{IIB}}^{\text{10d}} \supset \frac{1}{\ell_{10}^2} \int d^{10}x \sqrt{-g}\, E_{3/2}^{sl_2}(\tau, \bar{\tau})\, t_8 t_8 \mathcal{R}^4$$



where $t_8 t_8 \mathcal{R}^4 \equiv t^{\mu_1 \ldots \mu_8} t_{\nu_1 \ldots \nu_8} \mathcal{R}^{\nu_1 \nu_2}_{\mu_1 \mu_2} \ldots \mathcal{R}^{\nu_7 \nu_8}_{\mu_7 \mu_8}$

and $E_{3/2}^{sl_2} = 2\zeta(3)\tau_2^{3/2} + 4\zeta(2)\tau_2^{-1/2} + \mathcal{O}\left(e^{-2\pi\tau_2}\right)$

String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

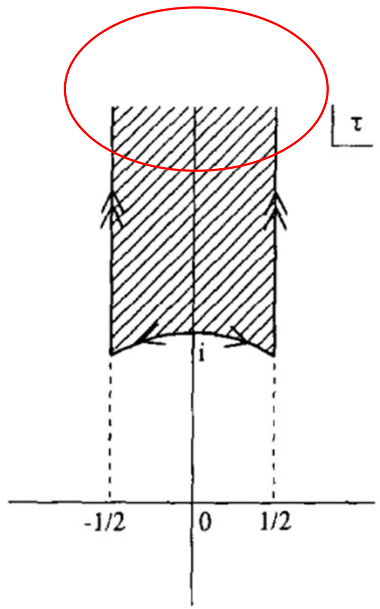
10d Type IIB string theory

- The $\mathcal{R}^4$ operator

$$S_{\text{IIB}}^{\text{10d}} \supset \frac{1}{\ell_{10}^2} \int d^{10}x \sqrt{-g}\, E_{3/2}^{sl_2}(\tau, \bar{\tau})\, t_8 t_8 \mathcal{R}^4$$

$$\text{therefore} \quad E_{3/2}^{sl_2} \sim \left(\frac{\Lambda_{\text{sp}}}{M_{\text{Pl},\,10}} \right)^{-6}$$

$$\text{since} \quad \Lambda_{\text{sp}} \sim m_s = \frac{M_{\text{Pl},\,10}}{(4\pi\tau_2^2)^{1/8}}$$



String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory!

[AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

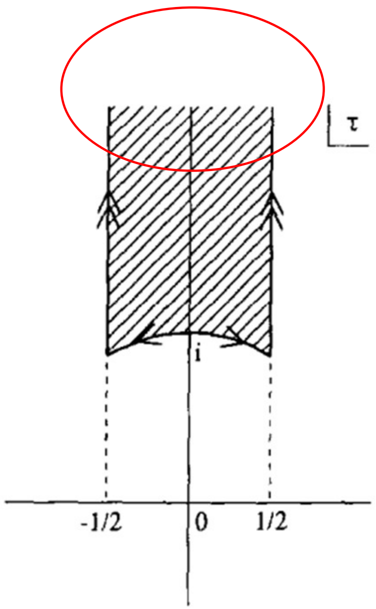
10d Type IIB string theory

- The $\mathcal{R}^4$ operator

$$S_{\rm IIB}^{\rm 10d} \supset \frac{1}{\ell_{10}^2} \int d^{10}x \sqrt{-g}\, E_{3/2}^{sl_2}(\tau,\bar\tau)\, t_8 t_8 \mathcal{R}^4$$

same for dual copies : $E_{3/2}^{sl_2}(\tau,\bar\tau) = \left(4\pi^{\frac{3}{4}}\right)^3 \sum'_{(p,q)\in\mathbb{Z}^2} \left(\frac{M_{\rm Pl,\,10}}{\sqrt{T_{p,q}}}\right)^6$

with $T_{p,q} = \frac{2\pi}{\ell_{10}^2} \frac{|p+q\tau|}{\sqrt{\tau_2}}$



String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

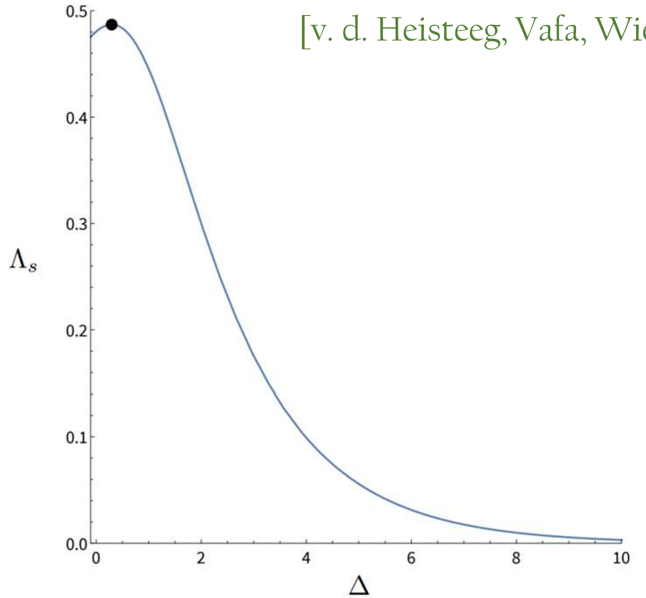
  - Theories with 32 supercharges in 7 < d < 11

  - Theories with 16 supercharges in 9d

  - Theories with 8 supercharges in 6d, 5d and 4d
    [see also AC, Herráez, Ibáñez '23]

M-theory on Calabi–Yau

$$\Lambda_{\rm sp} \sim \left(\frac{1}{12} \int c_2 \wedge J \right)^{-1/2}$$



[v. d. Heisteeg, Vafa, Wiesner, Wu '23]

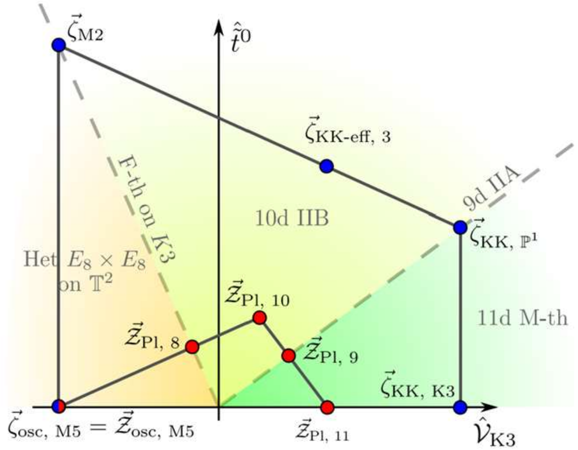
Summary & Outlook

- Supersymmetry allow us to study certain relevant quantities for QG in a controlled manner

- The underlying physics is not tied to SUSY, since there are numerous motivations

- It also allow us uncover new relations/dualities [AC, Ruiz, Valenzuela, '23]

- Many things yet to learn!



M-theory on K3

Thank you for your attention!

QUESTIONS?

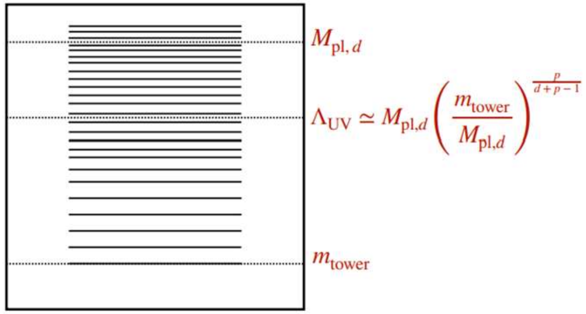
Contact: alberto.castellano@csic.es

The Species Scale: Definition

- The Species Scale is therefore defined (at least asymptotically) as [Dvali, Redi '07; Dvali, Gómez '10]

$$\Lambda_{\rm sp} \approx \frac{M_{\rm pl,\, d}}{N^{\frac{1}{d-2}}} \leq M_{\rm pl,\, d}$$

- In the presence of infinite towers of states there can be parametric decoupling! [AC, Herráez, Ibáñez '21-22]



$$\begin{cases} \Lambda_{\rm sp} = M_{\rm pl,\, d+p} & \text{decompactification limits} \\ \Lambda_{\rm sp} = \sqrt{T_s} & \text{emergent string limits} \end{cases}$$

- One can arrive at the same answer via two different kind of arguments

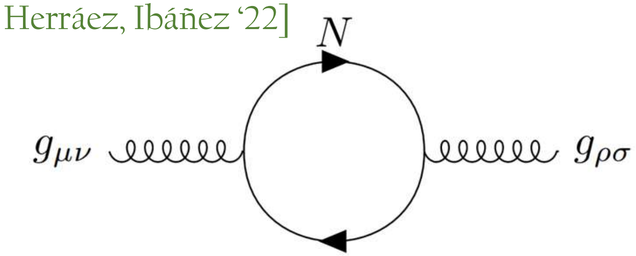
  i) Perturbative

  ii) Non-perturbative

A (species) cut-off for gravity

- The Species Scale is the scale at which the perturbative series of the graviton breaks down

[see e.g. AC, Herráez, Ibáñez '22]



$$i\,\Pi^{\mu\nu\rho\sigma} = i\left(P^{\mu\rho}P^{\nu\sigma} + P^{\mu\sigma}P^{\nu\rho} - P^{\mu\nu}P^{\rho\sigma}\right)\,\pi(p^2)$$

$$\pi^{-1}(p^2) = 2p^2\left(1 - \frac{Np^2}{120\pi M_{\text{pl},4}^2}\log(-p^2/\mu^2)\right)$$

- This can be nicely encoded into the formula shown in the first slide [v. d. Heisteeg, Vafa, Wiesner, Wu '22-23] [AC, Herráez, Ibáñez '23]

$$\Lambda_{\text{QG}} = \Lambda_{\text{sp}} \Longrightarrow \quad \mathcal{L}_{\text{EFT},\,d} \supset \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \sum_{n>2}\frac{\mathcal{O}_n(\mathcal{R})}{\Lambda_{\text{sp}}^{n-2}}\right)$$

- It also explains heuristically why the BH argument fails when $R \sim \ell_{\text{sp}}$

  - Important corrections to BH entropy!

[v. d. Heisteeg, Vafa, Wiesner, Wu '23 ; Calderón-Infante, Delgado, Uranga '23; Cribiori, Lust, Staudt '23]

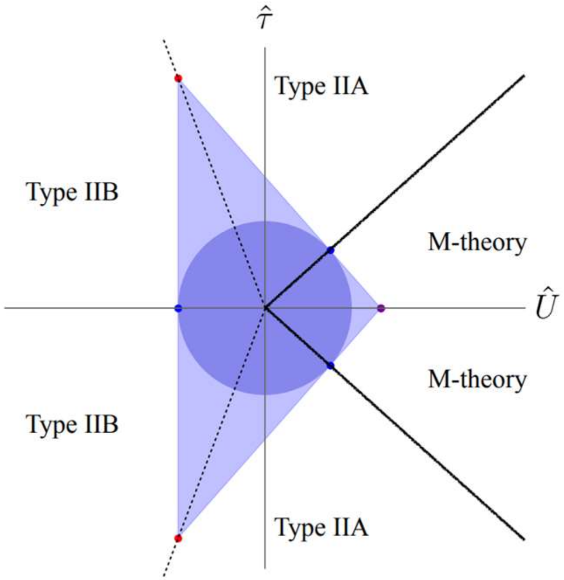
String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez '23, v. d. Heisteeg, Vafa, Wiesner, Wu '23]

  - Theories with 32 supercharges in 7 ‹ d ‹ 11

M-theory on $\mathbf{T}^2$

$$\Lambda_{\rm sp} \sim \left(\frac{2\pi^2}{3}\mathcal{V}_2^{6/7} + \mathcal{V}_2^{-9/14} E^{sl_2}_{3/2}(\tau)\right)^{-1/6}$$



[AC, Herráez, Ibáñez '23]

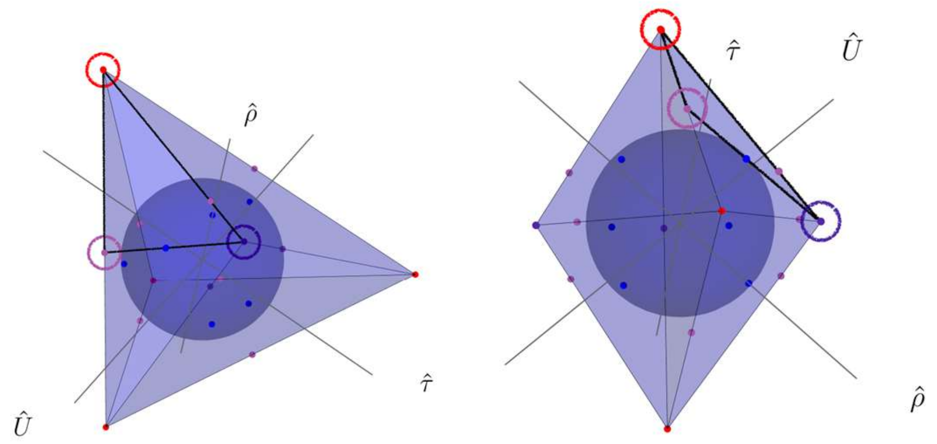
String Theory Checks

- Multiple (asymptotic) checks of this picture in string theory! [AC, Herráez, Ibáñez ‘23, v. d. Heisteeg, Vafa, Wiesner, Wu ‘23]

  - Theories with 32 supercharges in 7 ‹ d ‹ 11

M-theory on $\mathbf{T}^3$

$$\Lambda_{\rm sp} \sim \left(\hat{E}^{sl_3}_{3/2} + 2\hat{E}^{sl_2}_{1}\right)^{-1/6}$$



[AC, Herráez, Ibáñez ‘23]

Interesting examples (Part I)

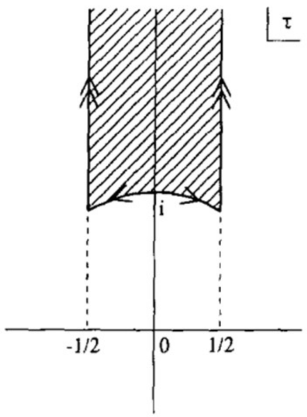
- One can go beyond the leading $\mathcal{R}^4$ term in e.g. 10d Type IIB [AC, Herráez, Ibáñez '23]

$$S_{\rm IIB}^{\rm 10d} \supset \int d^{10}x\sqrt{-g}\left(\frac{1}{\ell_{10}^2}E_{3/2}^{sl_2}(\tau,\bar\tau)\,t_8t_8\mathcal{R}^4+\frac{\ell_{10}^2}{2}E_{5/2}^{sl_2}(\tau,\bar\tau)\,\partial^4\mathcal{R}^4\right)$$

where [Green, Gutperle, Vanhove '97]

$$E_{3/2}^{sl_2}=2\zeta(3)\tau_2^{3/2}+4\zeta(2)\tau_2^{-1/2}+\mathcal{O}\left(e^{-2\pi\tau_2}\right)$$

$$E_{5/2}^{sl_2}(\tau,\bar\tau)=2\zeta(5)\tau_2^{5/2}+\frac{4\pi^4}{135}\tau_2^{-3/2}+\mathcal{O}(e^{-4\pi\tau_2})$$



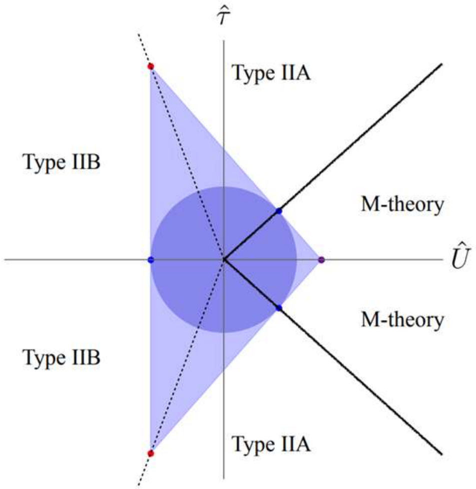
and

$$m_s=\frac{M_{\rm Pl;\,10}}{\left(4\pi\tau_2^2\right)^{1/8}}$$

Interesting examples (Part II)

- Certain higher-curvature corrections in M-theory compactified on tori seem to not follow the pattern

$$\mathcal{L}_{\text{EFT},\,d} \supset \frac{1}{2\kappa_d^2}\left(\mathcal{R} + \sum_{n>2} \frac{\mathcal{O}_n(\mathcal{R})}{\Lambda_{\text{sp}}^{n-2}}\right)$$



- E.g. M-theory on T² [AC, Herráez, Ibáñez '23]

$$S^{\text{9d}}_{\text{M-th}} \supset \ell_9^3 \int d^9x\sqrt{-g}\left(\frac{1}{2}\mathcal{V}_2^{-15/14} E^{sl_2}_{5/2}(\tau) + \frac{2\zeta(2)}{15}\mathcal{V}_2^{27/14} E^{sl_2}_{3/2}(\tau) + \frac{4\zeta(2)\zeta(3)}{15}\mathcal{V}_2^{-18/7}\right)\partial^4\mathcal{R}^4$$

but

$$\frac{M_{\text{Pl};\,11}}{M_{\text{Pl};\,9}} = (4\pi)^{-2/9}\,\mathcal{V}_2^{-1/7} \qquad \frac{M^{\text{IIA}}_{\text{Pl};\,10}}{M_{\text{Pl};\,9}} = (4\pi)^{1/56}\,\mathcal{V}_2^{-9/112}\tau_2^{-1/16} \qquad \frac{M^{\text{IIB}}_{\text{Pl};\,10}}{M_{\text{Pl};\,9}} = (4\pi)^{1/56}\,\mathcal{V}_2^{3/28}$$

Interesting examples (Part III)

- One can even find infinite families of grav. operators behaving this way
- E.g. 4d N=2 theories (say Type IIA on Calabi-Yau)

$$S_{\rm IIA}^{\rm 4d} \supset \int {\rm d}^4x \sqrt{-g} \int {\rm d}^4\theta \sum_{g\geq 1} \mathcal{F}_g(X^A)\, \mathcal{R}_+^2\, F_+^{2g-2} \;+\; {\rm h.c.}$$

- For M-theory limits one finds [Gopakumar, Vafa '98 (x2)]

$$\mathcal{F}_{g>1}^{\rm D0} = \chi(X_3)\frac{2(2g-1)\zeta(2g)\Gamma(2g-2)}{(2\pi)^{2g}}\frac{\zeta(2g-2)}{m_{\rm D0}^{2g-2}}$$

- And similarly for other inf. distance degenerations (e.g. F-theory/emergent string limits) [AC, Herráez, Ibáñez '23]

A simple explanation

- Consider KK thresholds for k-(super)graviton scattering in d dimensions

$$\mathcal{A}_{k,\,d} = \int_0^\infty \mathrm{d}^d p \int_0^\infty \frac{\mathrm{d}\tau}{\tau} \sum_n e^{-\tau\left(p^2+\frac{n^2}{R^2}\right)} \operatorname{tr}\left\langle \prod_{r=1}^{k} \left(\int_0^\tau \mathrm{d}t_r\, V_{g_{\mu\nu}}(t_r) \right) \right\rangle$$

- If corresponding operator is BPS then

$$\mathcal{A}_{k,\,d} = \tilde{K} \sum_{n\in\mathbb{Z}\setminus\{0\}} \int \mathrm{d}^d p \int_0^\infty \frac{\mathrm{d}\tau}{\tau}\, \tau^k\, e^{-\tau\left(p^2+m_n^2\right)}$$

- For irrelevant ops (i.e. 2k › d+1) the amplitude converges [AC, Herráez, Ibáñez ‘23]

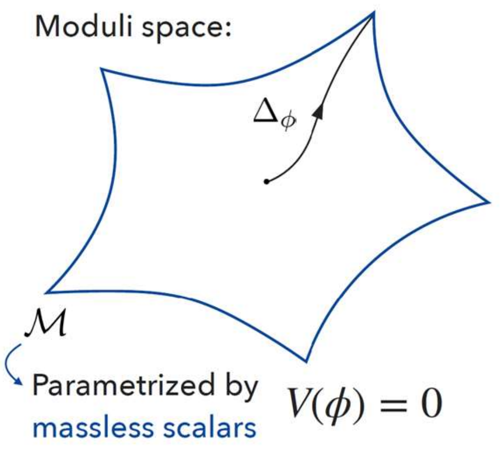
$$\mathcal{A}_{k,\,d} = 2\tilde{K}\,\Gamma\left(k-\frac{d}{2}\right) \sum_{n>0} \frac{1}{m_n^{2k-d}} = 2\tilde{K}\,\Gamma\left(k-\frac{d}{2}\right) \frac{\zeta(2k-d)}{m_{\mathrm{KK}}^{2k-d}}$$

Bounds on exponential rates

- It is interesting to study this scale systematically. Where?

$$\text{Distance Conj.} \quad \begin{cases} m_{\text{tower}} \sim e^{-\lambda \kappa_d \Delta_\phi}\,, & \lambda = \mathcal{O}(1) \\ N_{\text{sp}} \to \infty\,, & \Lambda_{\text{sp}} \sim e^{-\lambda_{\text{sp}} \kappa_d \Delta_\phi} \to 0 \end{cases}$$

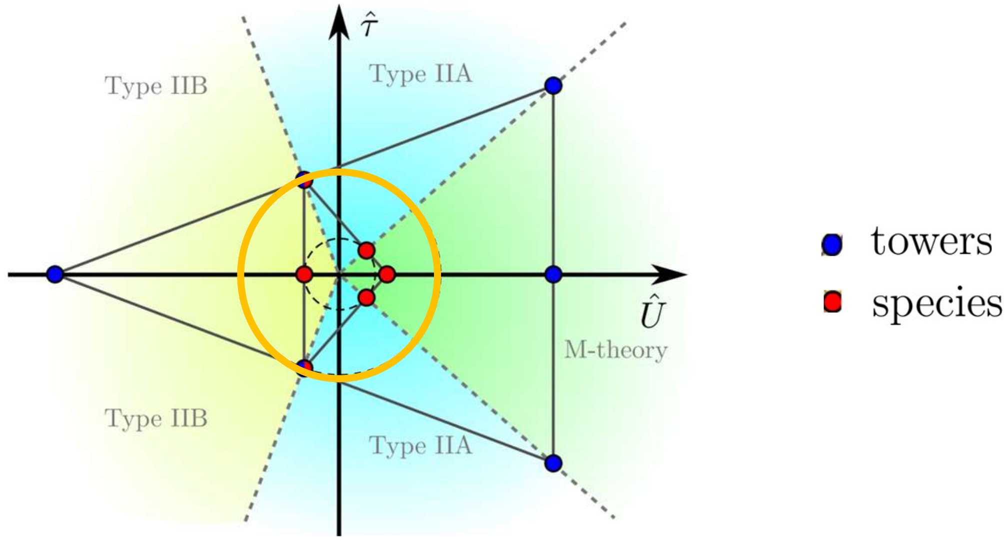
[Ooguri, Vafa '06]



- Can we say something universal about λ_{sp} ?
- ... As well as upper bounds

$$\lambda_{\text{sp}} \leq \frac{1}{\sqrt{d-2}}$$

[v. d. Heisteeg, Vafa, Wiesner, Wu '22]
[Calderón-Infante, AC, Herráez, Ibáñez '23]



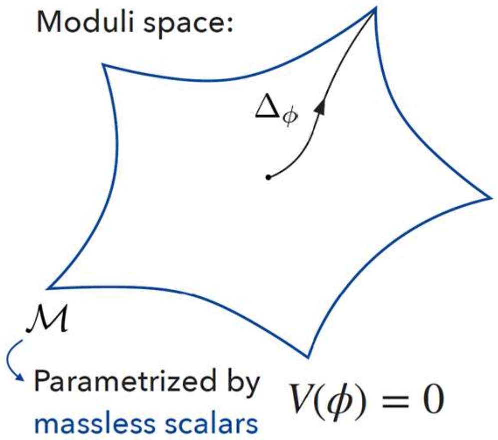
M-theory on T^2

Bounds on exponential rates

- It is interesting to study this scale systematically. Where?

Distance Conj.
$$\begin{cases} m_{\text{tower}} \sim e^{-\lambda \kappa_d \Delta_\phi}\,, \qquad \lambda = \mathcal{O}(1) \\ N_{\text{sp}} \to \infty\,, \quad \Lambda_{\text{sp}} \sim e^{-\lambda_{\text{sp}} \kappa_d \Delta_\phi} \to 0 \end{cases}$$

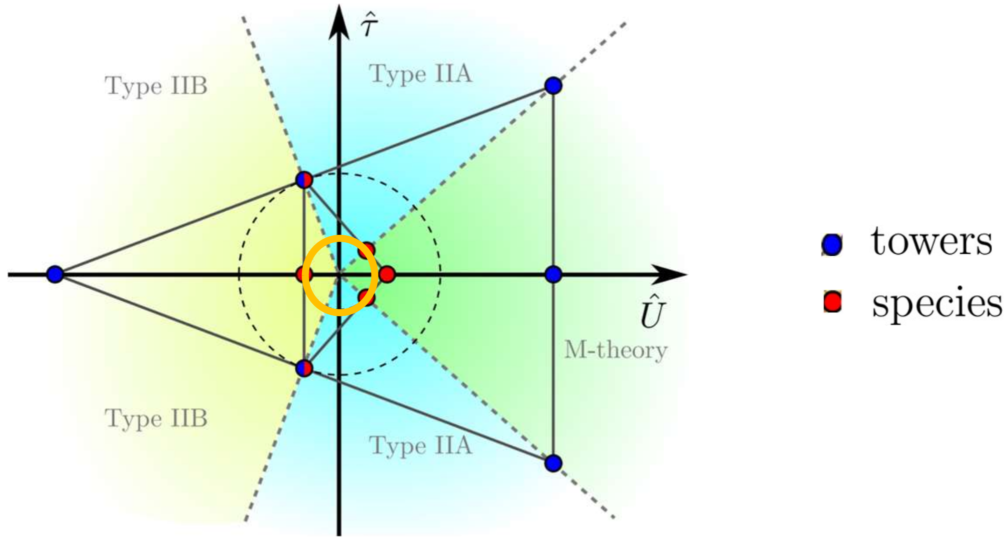
[Ooguri, Vafa '06]



- Can we say something universal about λ_{sp} ?
- Yes!! There seems to be lower bounds

$$\lambda_{\text{sp}} \geq \frac{1}{\sqrt{(d-1)(d-2)}}$$

[Calderón-Infante, AC, Herráez, Ibáñez '23]



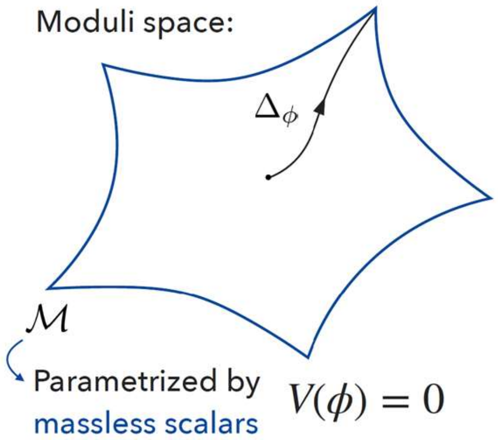
M-theory on T^2

Bounds on exponential rates

- It is interesting to study this scale systematically. Where?

Distance Conj. $\begin{cases} m_{\text{tower}} \sim e^{-\lambda \kappa_d \Delta_\phi}\,, & \lambda = \mathcal{O}(1) \\ N_{\text{sp}} \to \infty\,, \quad \Lambda_{\text{sp}} \sim e^{-\lambda_{\text{sp}} \kappa_d \Delta_\phi} \to 0 & \end{cases}$

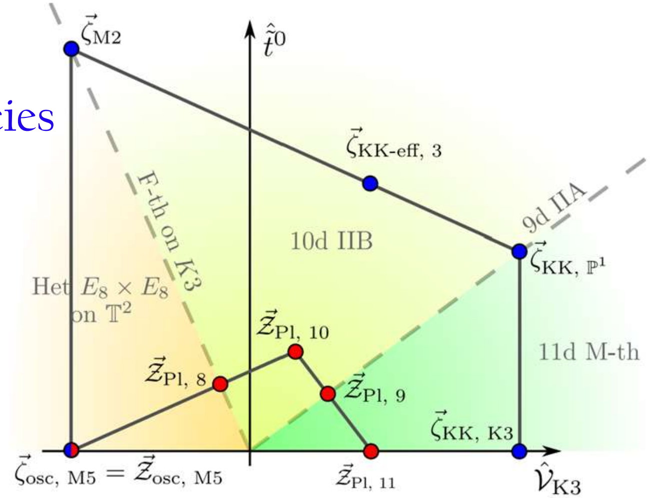
[Ooguri, Vafa '06]



- Can we say something universal about λ_{sp} ?
- ... And even some interesting pattern relating towers and species

$$\frac{\vec{\nabla} m_{\text{t}}}{m_{\text{t}}} \cdot \frac{\vec{\nabla} \Lambda_{\text{sp}}}{\Lambda_{\text{sp}}} = \frac{\kappa_d^2}{d-2}$$

[AC, Ruiz, Valenzuela, '23]



M-theory on K3