Symmetric moduli spaces: boundaries, geodesics and the Distance Conjecture

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Upcoming work with S. Baines, B. Fraiman, M. Graña and D. Waldram

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June 10, 2024

The Swampland program: a web of conjectures

Consistent QFT + Einstein gravity ✓

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From string theory, we cannot get any effective field theory

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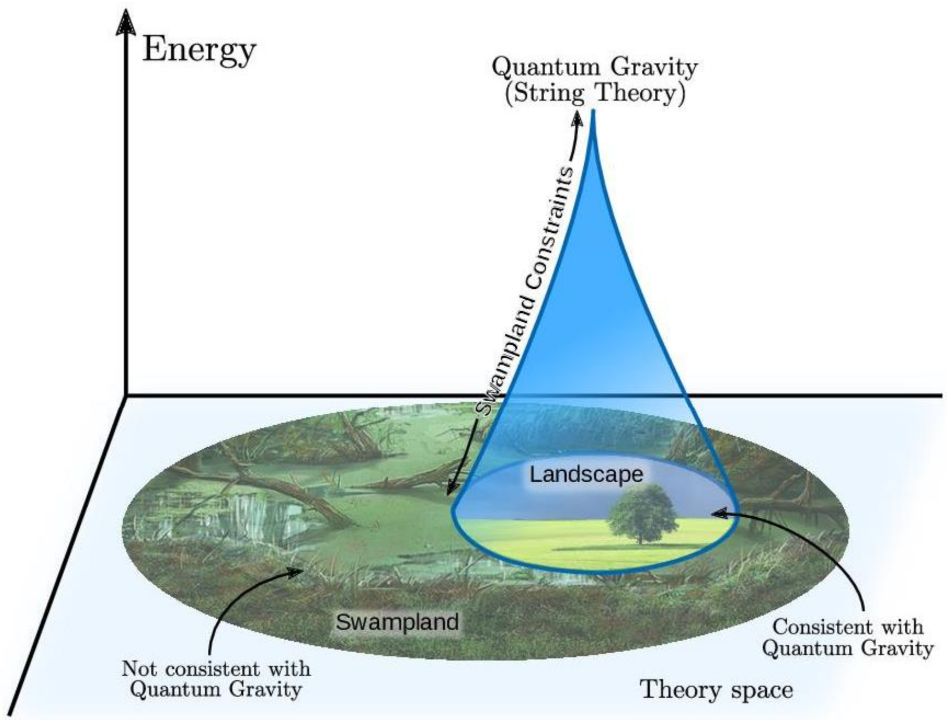

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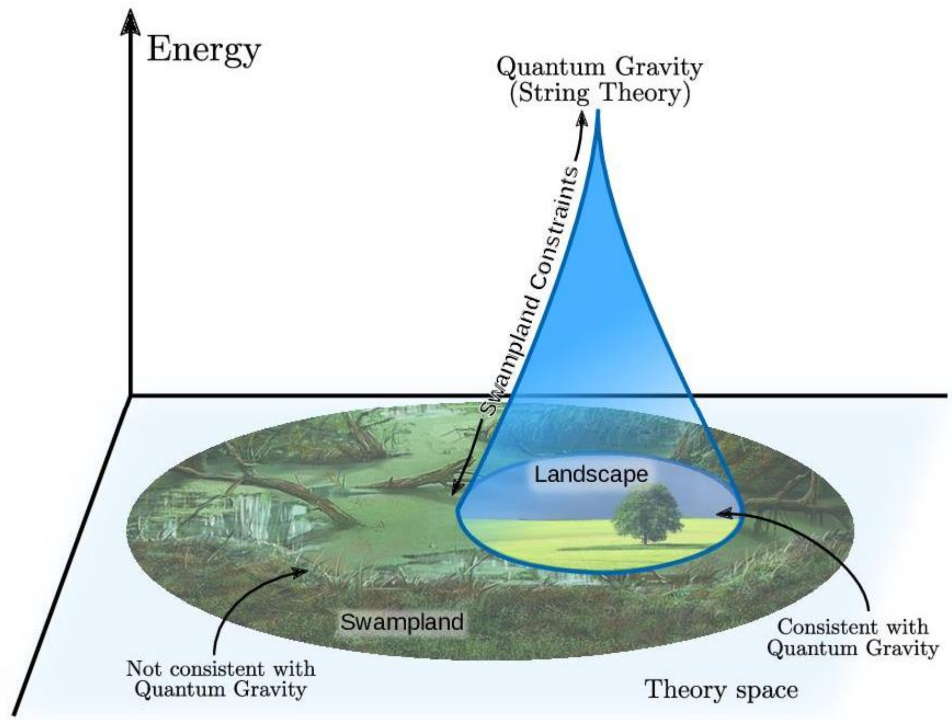
⟶ **Swampland program**

- Universal features from string compactifications
- Bottom-up arguments (eg. black hole physics)



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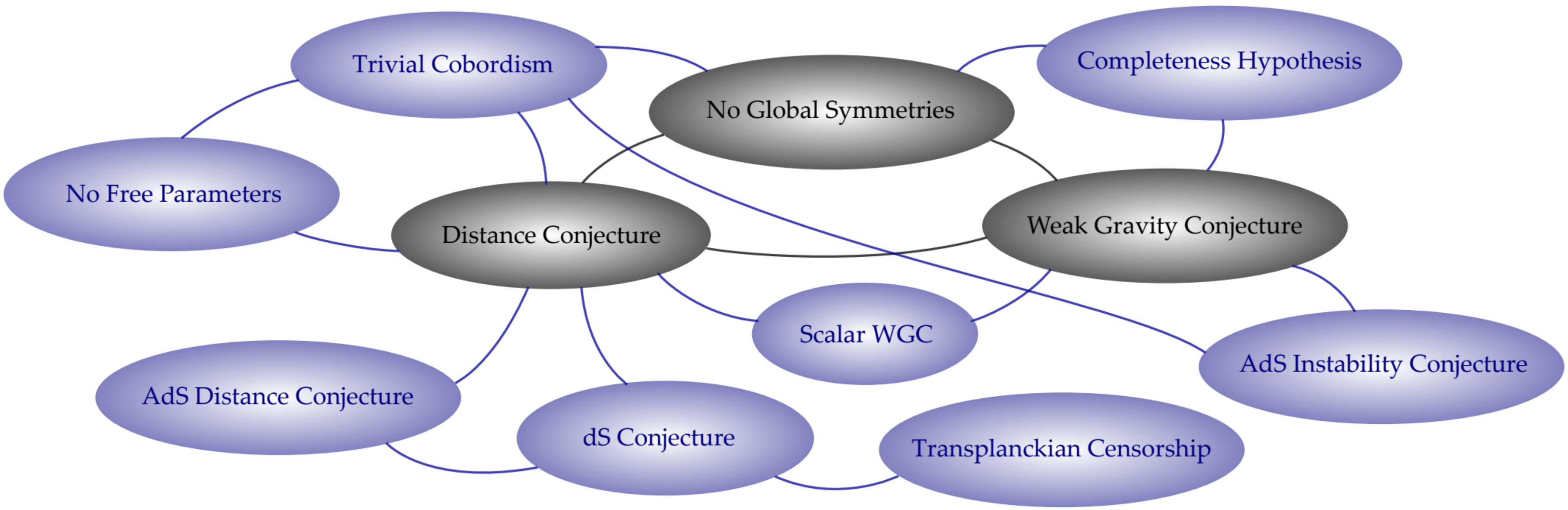

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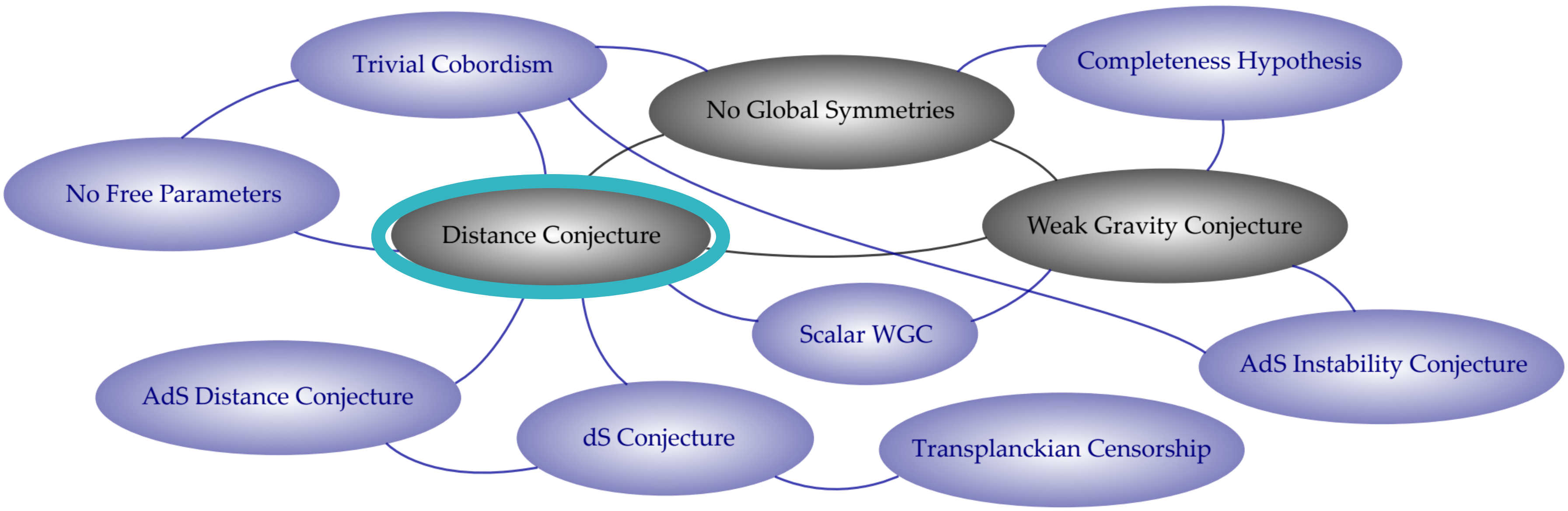

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The Distance Conjecture

$$S = \frac{M_{\rm pl}^{d-2}}{2} \int d^d x \sqrt{-g} (R - G_{ij} \partial_\mu \phi \partial^\mu \phi)$$

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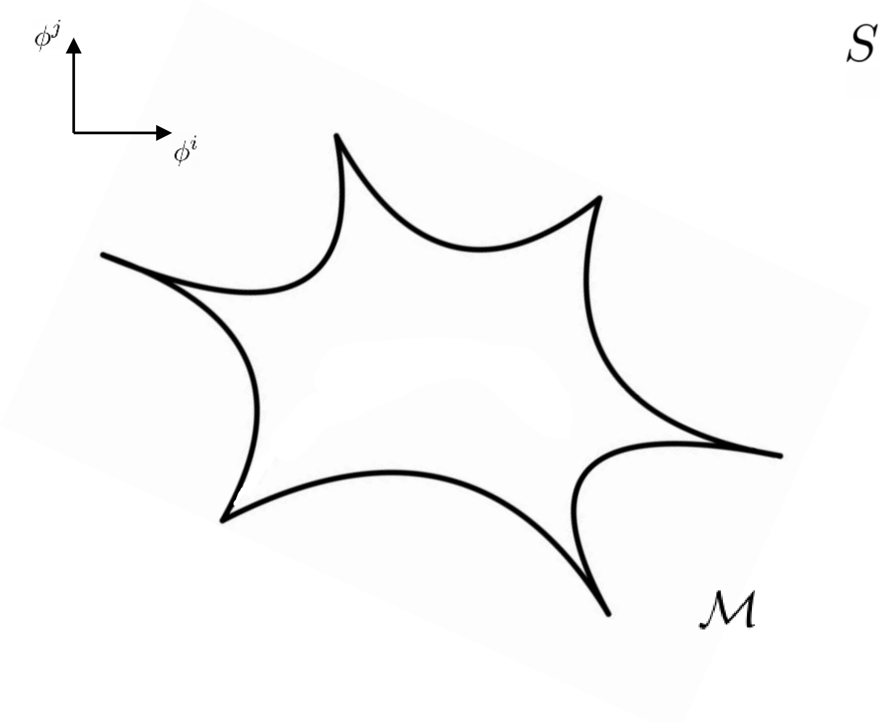

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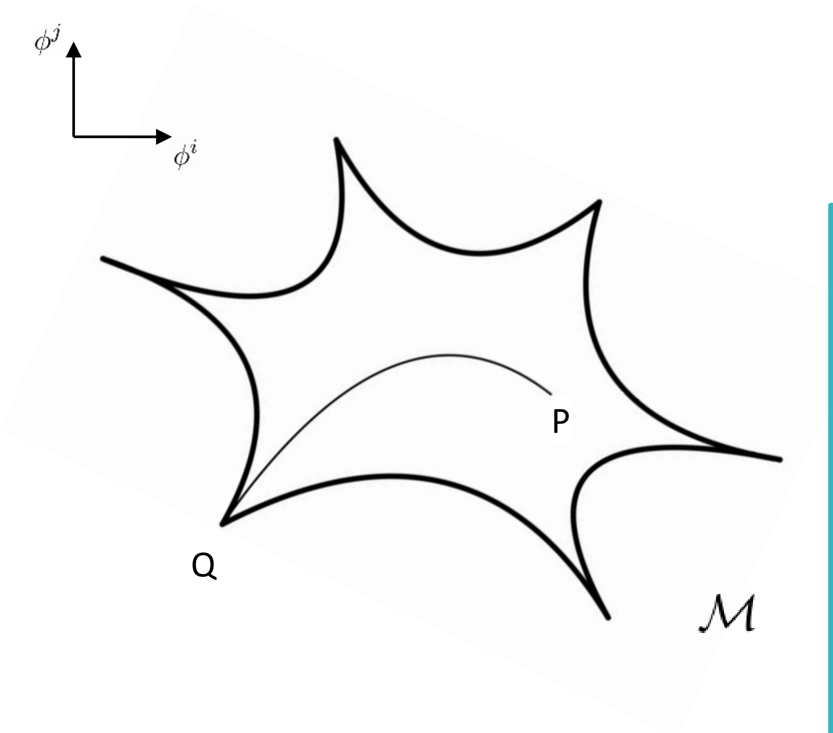

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Moving in moduli space from a point P towards a point Q an infinite geodesic distance away, an infinite tower of states becomes exponentially light (in Planck units) as

$$M(Q) \sim M(P) e^{-\alpha\, d_{P,Q}}$$

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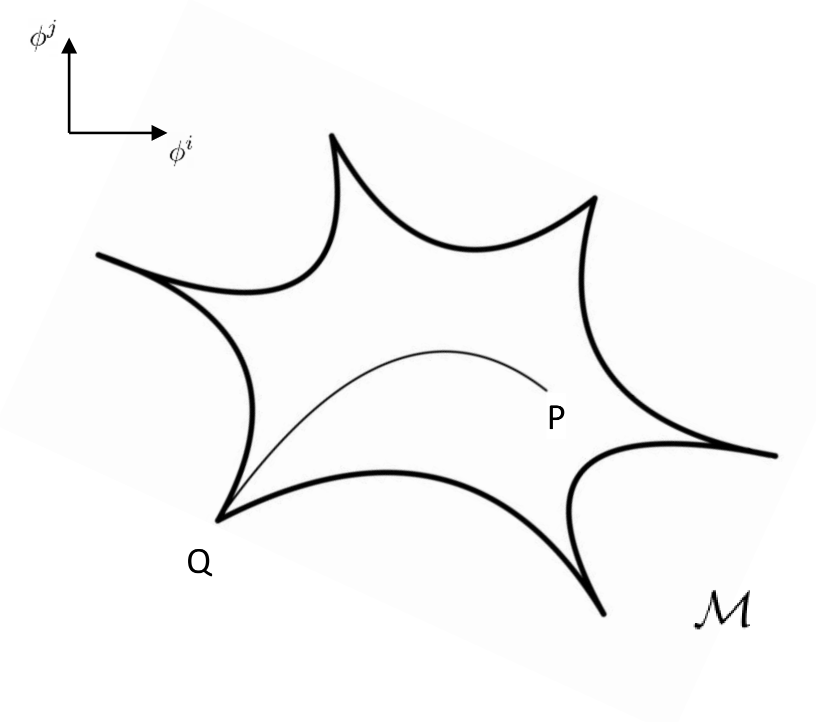

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The tower can be (**Emergent string conjecture**): [Lee, Lerche, Weigand '19]

- Kaluza-Klein states ⟶ decompactification
- Oscillators of a tensionless critical string

Motivation

Geometry of moduli spaces ⟷ **Spectrum** of the theory

- Geodesics
- Structure of the boundary

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Clear connection for symmetric moduli spaces:

$$\mathcal{M} \sim G(\mathbb{Z}) \Big\backslash \frac{G(\mathbb{R})}{K}$$

(Connected) group of isometries of $\mathcal{M}$ — $G(\mathbb{R})$

Duality group — $G(\mathbb{Z})$

Subgroup of isometries fixing one point, o — K

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- $G(\mathbb{R})$: (Connected) group of isometries of $\mathcal{M}$
- K: Subgroup of isometries fixing one point, o

From string theory:

- M theory on T^d : $G = E_{d(d)}$
- Heterotic on T^d: $G = O(d, d+8)$
- CHL string on T^d: $G = O(d, d+16)$
- Bosonic string on T^d : $G = O(d, d)$

…but also non supersymmetric strings

Toy model: SL(2,R)

Moduli space of Type IIB in 10 dimensions, or of T^2 at fixed volume.

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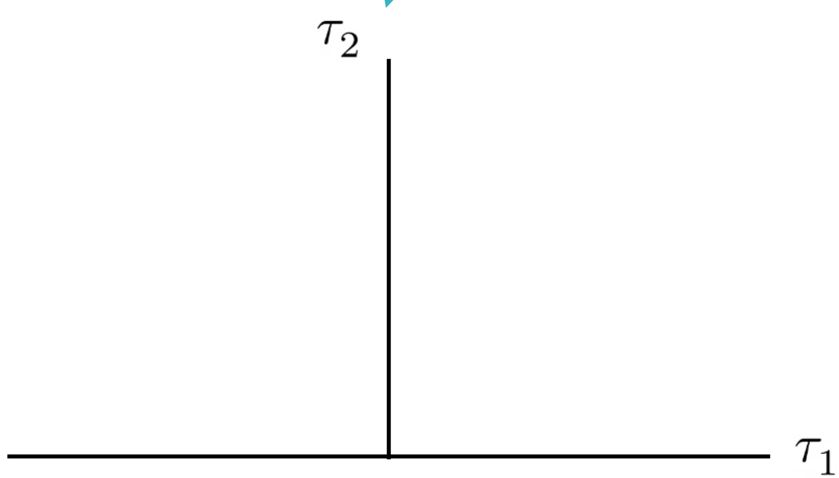

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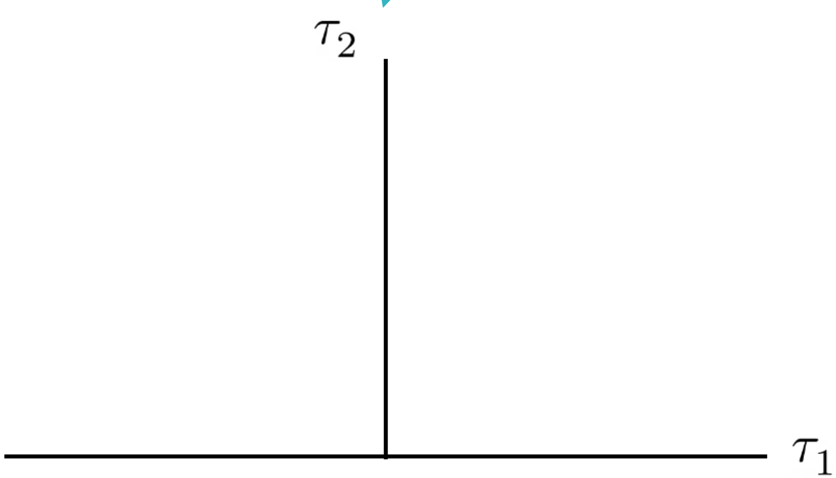

Boundary: $\mathbb{R} \cup \{i\infty\} \sim S^1$

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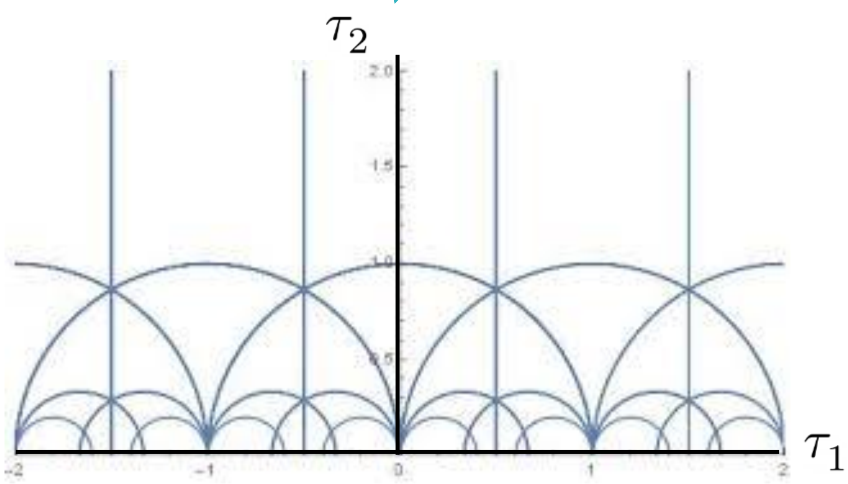

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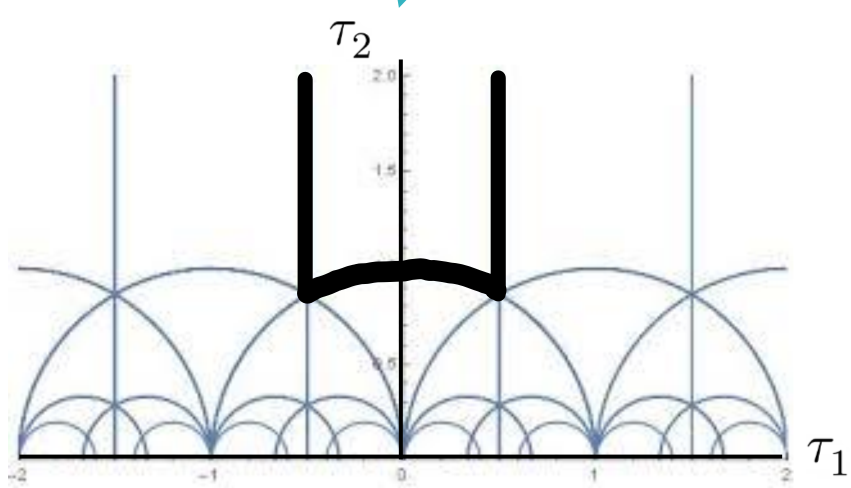

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$SL(2,\mathbb{Z})$: Restrict to one fundamental domain: one point at infinity

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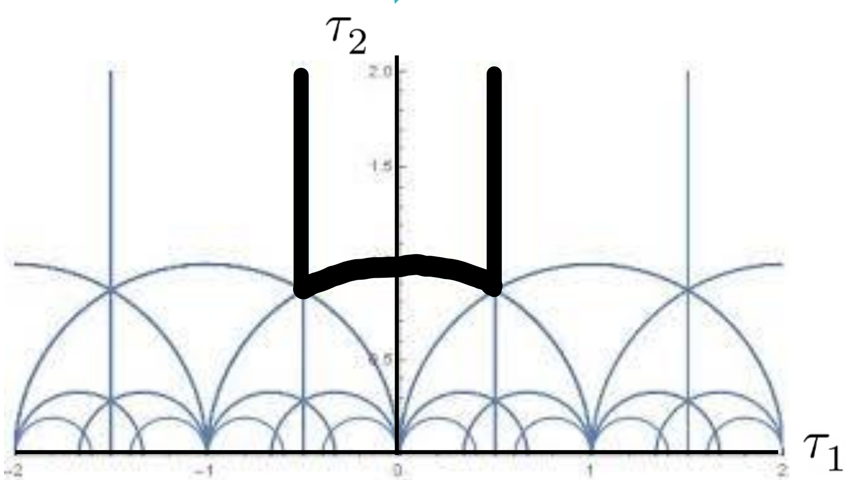

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$SL(2,\mathbb{Z})$: Restrict to one fundamental domain: one point at infinity

Geodesics on $\mathbb{H}^2$ either go to the boundary, or have an ergodic or periodic motion. [Keurentjes, '06]

Geodesics and boundaries

[Borel, Ji '06]

Use the geodesic flow to study the boundary of these spaces.

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Geodesics (distance induced from the Killing form on $\mathfrak{g}$)

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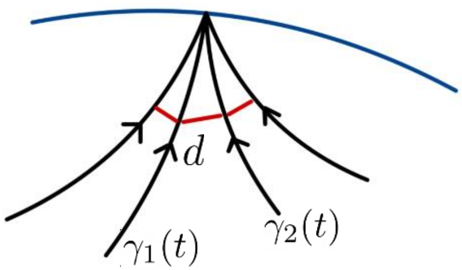
$$\lim_{t \to +\infty} d(\gamma_1(t), \gamma_2(t)) < \infty$$



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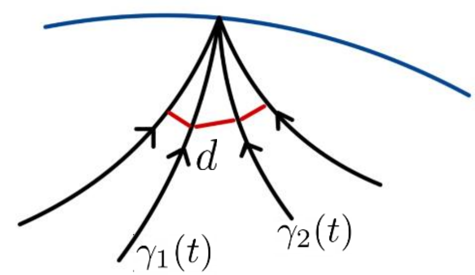

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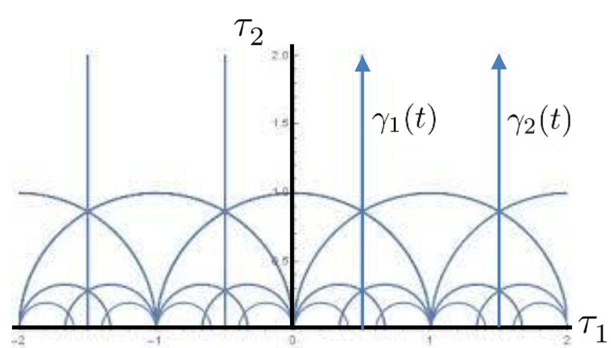

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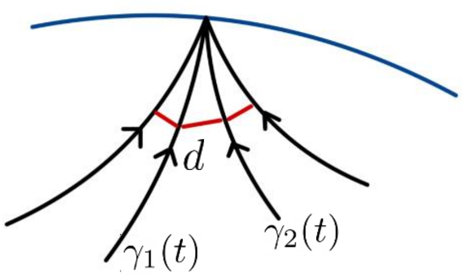

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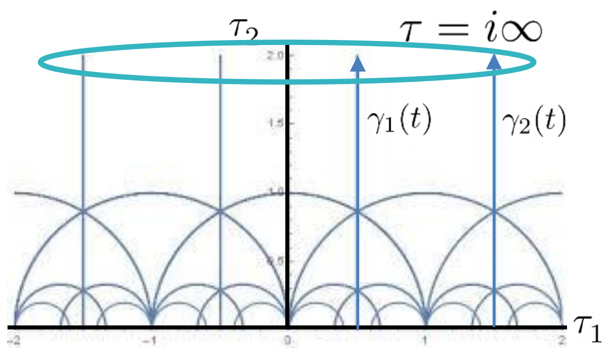

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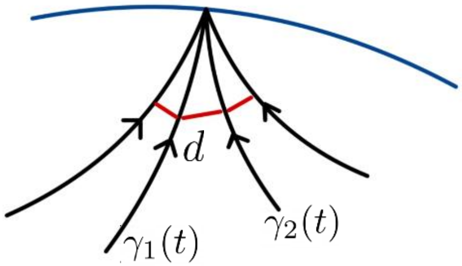

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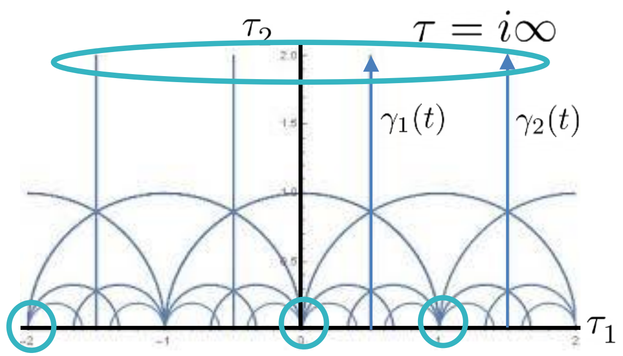

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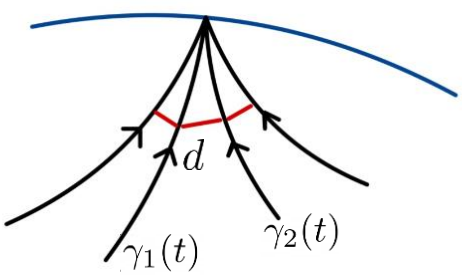

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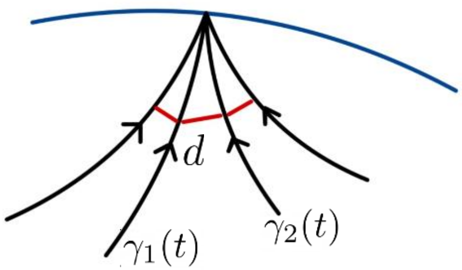

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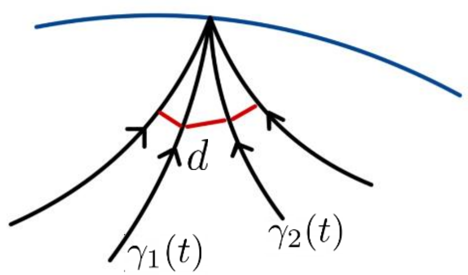

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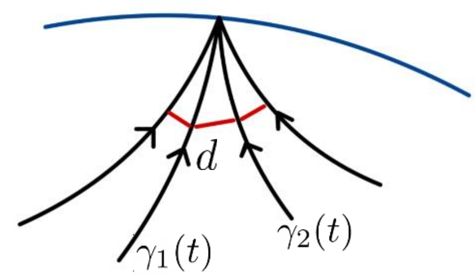

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- Considering the duality group, information of the boundary contained in $G(\mathbb{Q})$: only rational compact moduli.

Towards the Distance Conjecture

[Cecotti '15]

Assumptions: (motivated by string compactifications)

- Existence of a lattice of states $\Sigma \hookrightarrow V$ on which G acts

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$$M_q^2(t) = q^T \gamma(t)^T \gamma(t) q$$

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There is always a massless tower. ✓

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Thank you!