Euclidean flows, wormholes and holography

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Why Euclidean Solutions?

$$Z_{\mathrm{QM}} = \int dq \, \langle q| \, e^{-\beta H} \, |q\rangle \, , \quad \langle q_F| \, e^{-i\,t\,H} \, |q_I\rangle \;\longrightarrow\; \begin{cases} t \to -i\tau \\ q_I = q_F \\ q(\tau+\beta) = q(\tau) \end{cases}$$

$$\text{Lorentz Path integral} \xrightarrow[\textbf{Wick rotation}]{\tau\sim\tau+\beta} \text{Thermodynamics}$$

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Lorentz Path integral $\xrightarrow[\text{Wick rotation}]{\tau \sim \tau+\beta}$ Thermodynamics

Why not with gravity?

$$Z[\beta] = \int [dg] e^{-S_E}$$

- Black hole thermodynamics [Hawking, 1974]
- Saddle-point approximation $\rightarrow$ Euclidean wormholes [see Hebecker, Mikhail and Soler, 2018]

Euclidean Subtleties

- We seeking (Euclidean) supergravity solutions with **scalars**
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  - Dual 2-form $\implies$ Potential?
  - **Euclidean SUGRA**: **SUSY** $\implies$ **Double d.o.f.**

From Lorentzian to Euclidean

- **Lorentzian**: $SO(1,3)$
  - Two independent spinorial irreps: χ, $\tilde{\chi} \longrightarrow \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}$
  - **Complex** but **mutually conjugated**: $\chi \overset{*}{\longleftrightarrow} \tilde{\chi}$
  - Left and right are **not independent** (Majorana condition)

- **Euclidean**: $SO(4) \sim \mathrm{SU}(2)_l \times \mathrm{SU}(2)_r$
  - Two independent spinorial irreps: χ, $\tilde{\chi}$
  - Real representations $\Rightarrow$ **self-contained**: $\begin{cases} \chi \overset{*}{\circlearrowleft} \\ \tilde{\chi} \overset{*}{\circlearrowleft} \end{cases}$
  - Left and right are **independent**

- **Lorentzian** chiral multiplet: $(Z, \hat{\chi} = \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}, F)$

$$\delta Z = i\epsilon^T (i\sigma_2)\chi$$
$$\delta \bar{Z} = i\tilde{\epsilon}^T (i\sigma_2)\tilde{\chi} \xrightarrow[Majorana]{} = (\delta Z)^*$$

- **Euclidean** chiral multiplet: $(Z, \tilde{Z}, \chi, \tilde{\chi}, F)$

$$\tilde{Z} \neq \bar{Z} \longrightarrow \textbf{double d.o.f}$$

(Euclidean) STU model

$\mathcal{N} = 2$ **STU-model** from $\mathrm{U}(1)^4$ inv. sector of 11D SUGRA on S^7

$g_{\mu\nu}$, $2\ \psi_\mu$, $4\ A_\mu$, $3\ z_i$

$\downarrow$

$$S_{\mathrm{Lor}} = \frac{1}{2\kappa^2}\int d^4x\, e\left[\frac{R}{2} - \frac{\partial_\rho z_i \partial^\rho \bar{z}_i}{(1 - z_i\bar{z}_i)^2} + g^2\left(3 - \frac{2}{1 - z_i\bar{z}_i}\right)\right]$$

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$$S_{\mathrm{Eucl}} = \frac{1}{2\kappa^2}\int d^4x\, e\left[-\frac{R}{2}+\frac{\partial_\rho z_i \partial^\rho \tilde{z}_i}{(1-z_i\tilde{z}_i)^2}+g^2\left(3-\frac{2}{1-z_i\tilde{z}_i}\right)\right]$$

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- Solutions with **non-backreacted** metric but scalar profiles.

Holographic dual: ABJM

- **ABJM**: U(N)×U(N) Superconformal Chern–Simons matter theory.

Field	$U(N) \times U(N)$	$U(1)_R$
$(A_\mu, \sigma, \lambda, D)$	$(\text{adj}, \mathbf{1})$	$(0, 0, 1, 0)$
$(\tilde{A}_\mu, \tilde{\sigma}, \tilde{\lambda}, \tilde{D})$	$(\mathbf{1}, \text{adj})$	$(0, 0, 1, 0)$
(Z^a, χ^a, F^a)	$(\bar{\mathbf{N}}, \mathbf{N})$	$(1/2, -1/2, -3/2)$
(W_a, η_a, G_a)	$(\mathbf{N}, \bar{\mathbf{N}})$	$(1/2, -1/2, -3/2)$

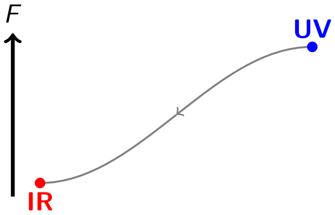
$$\mathcal{F}_{\rm ABJM} = S^{\rm EAdS}_{\text{on-shell}} = \frac{2\pi^2}{\kappa^2 g^2}$$

Scalars $\Longrightarrow$ **ABJM deformations**

Holographic renormalization $\oplus$ Legendre transform $\longrightarrow J_{\text{on-shell}}$

[Bianchi, Freedman and Skenderis, 2022]

c-theorem in 2D
[Zamolodchikov , 1986]
↓
conjectured **F-theorem** in 3D



F-theorem

For three-dimensional field theories, the finite part of the free energy **on** $\mathbf{S}^3$ decreases along RG trajectories and is stationary at the fixed points.
[Jafferis , Klebanov, Pufu and Safdi, 2011]

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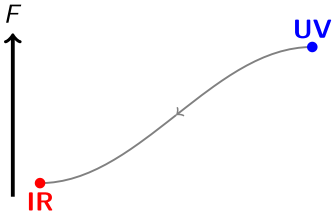

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Ansatz: S^3 boundary and $SO(4) \sim SU(2)_l \times SU(2)_r$ bulk isometry

$$ds_4^2 = d\mu^2 + g^{-2} e^{2A(\mu)} d\Sigma^2_{S^3}$$

Non-backreacted Solutions

$$ds^2 = ds^2_{EAdS_4} \quad \text{and} \quad z_2 = \tilde{z}_2 = z_3 = \tilde{z}_3 = 0$$

Singular solution: $z_1 = 0 \ , \ \tilde{z}_1 = \frac{\tilde{c}_1}{\sinh^2\left(\frac{1}{2} g\mu\right)}$

Regular solution: $z_1 = \frac{c_1}{\cosh^2\left(\frac{1}{2} g\mu\right)} \ , \ \tilde{z}_1 = 0$

- Most general **SUSY** non-backreacted solutions.

F-theorem

$$J_{\text{on-shell}}\Big|_{\text{non-back}} = J^{\text{EAdS}_4}_{\text{on-shell}}\left(1 - c_1^2\right)$$

Dual to **real mass deformations of ABJM** on S^3

↓

Equivalent to a **reassignment of $\mathbf{U(1)}_R$ charges** of Z_a and W_a.

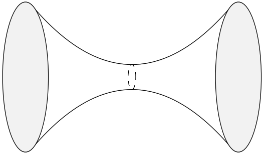
[Freedman and Pufu, 2014]

$$R[Z^1] = R[Z^2] = \tfrac{1}{2} + \delta_1\ ,\ R[W_1] = R[W_2] = \tfrac{1}{2} - \delta_1\ .$$

$$\mathcal{F} = \mathcal{F}_0\sqrt{16\,R[Z^1]\,R[Z^2]\,R[W_1]\,R[W_2]} = \mathcal{F}_0\left(1 - (2\delta_1)^2\right)$$

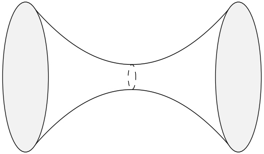
[Jafferis, Klebanov, Pufu and Safdi, 2011]

Wormholes



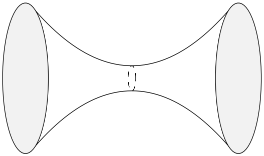
$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Wormholes



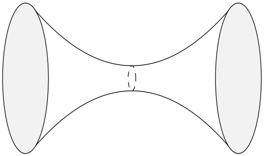
$$ds^2 = -dt^2 + dr^2 + (r^2 + l^2)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Wormholes



$$ds^2_{\mathrm{EAdS}} = d\mu^2 + g^{-2}\sinh^2(g\mu)\, d\Sigma^2_{\mathrm{S}^3}$$

Wormholes



$$ds^2_{\mathrm{WH}} = d\mu^2 + g^{-2}\frac{\sinh^2(g\mu)+1}{k^2}d\Sigma^2_{\mathrm{S}^3}$$

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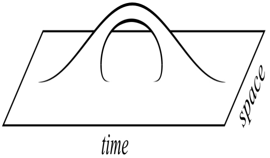

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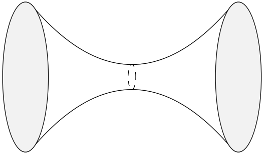

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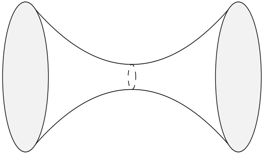

$$ds^2_{\mathrm{WH}} = d\mu^2 + g^{-2}\frac{\sinh^2(g\mu)+1}{k^2}d\Sigma^2_{\mathrm{S}^3}$$

$$z_i = \frac{\lambda_i\sqrt{k_i^2+1}}{k_i-\sinh(g\mu)} \quad , \quad \tilde{z}_i = \frac{\lambda_i^{-1}\sqrt{k_i^2+1}}{k_i+\sinh(g\mu)} \quad \text{with} \quad \sum_i k_i^2 = k^2-2$$

- **NO SUSY**
- **Complex** nature **when uplifted to 11D SUGRA**.

Free Energy

$$S_{\text{on-shell}}\Big|_{\text{WH}} = J_{\text{on-shell}}\Big|_{\text{WH}} = 0 \ .$$

Independent of renormalization scheme

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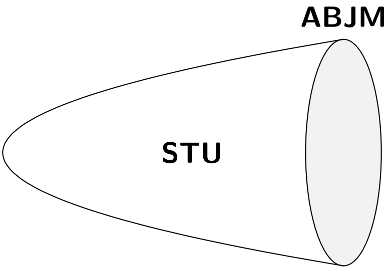

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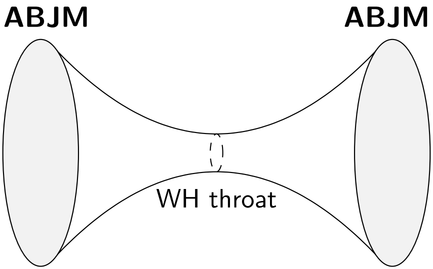

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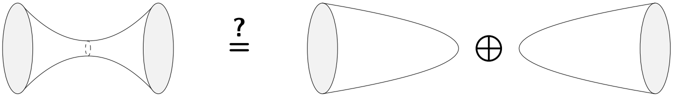

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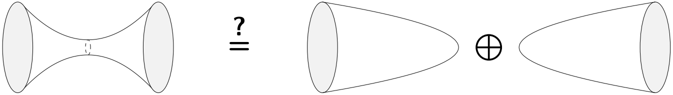

- The absence of supersymmetry makes a holographic test of the zero gravitational free energy difficult.

Conclusions

We constructed new classes of $U(1)^4$-invariant Euclidean solutions in the four-dimensional STU-model.

- Running scalar solutions with a **non-backreacted EAdS4**
  - ∗ Dual to **real mass deformations of ABJM**
  - ∗ **Free energy** decreases along the flow and has a maximum at the UV fixed point.
- Euclidean **wormhole** solutions
  - ∗ Complex nature when uplifted to 11D SUGRA
  - ∗ Zero gravitational free energy (independent of renormalization scheme)
  - ∗ Holographic understanding of WH remains unclear

Thanks!