Borel resummation of secular divergences in stochastic inflation

Ryusuke Jinno (Kobe Univ.)

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w/ Masazumi Honda, Lucas Pinol, Koki Tokeshi

SUSY2024@Madrid, June 10-14, 2024

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- *Stochastic approach to inflation*
- *Correlator & divergence*
- *Borel resummation*
- *Summary*

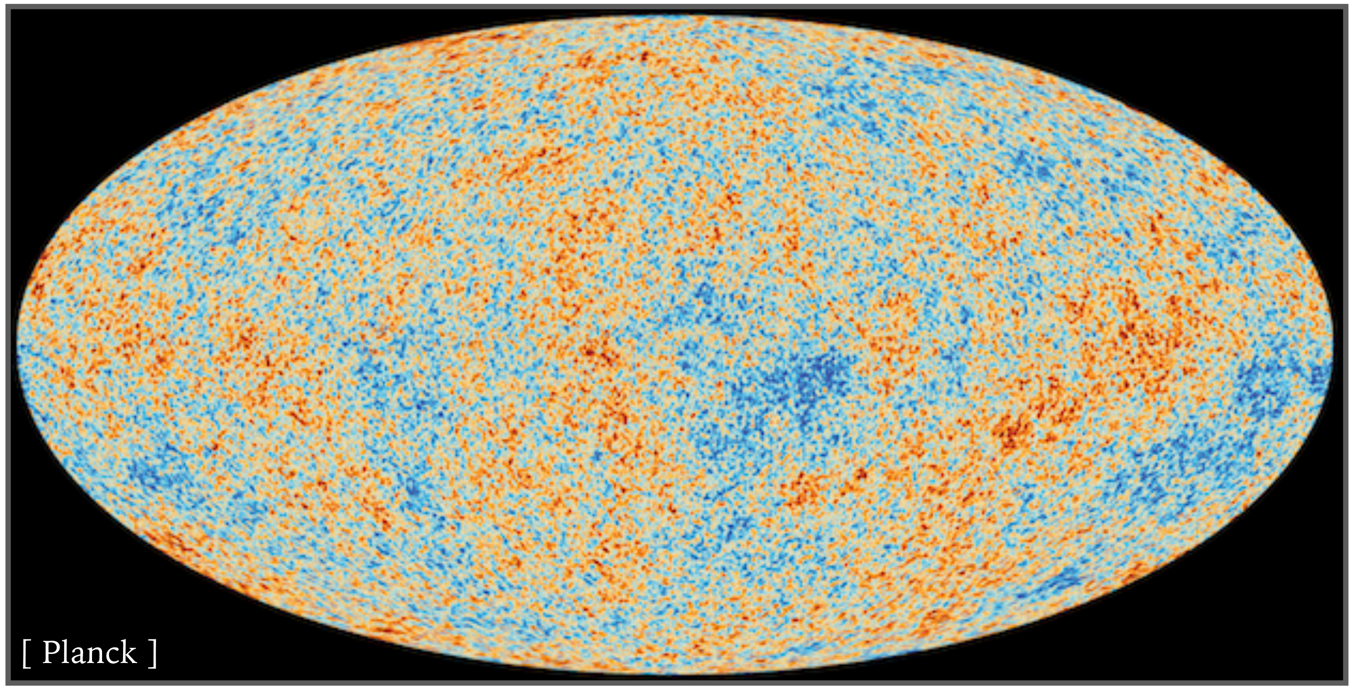
INFLATION AND STOCHASTIC APPROACH

➤ Inflation [Guth '81, Linde '82, '83, Albrecht & Steinhardt '82, Starobinsky '80, Sato '81]

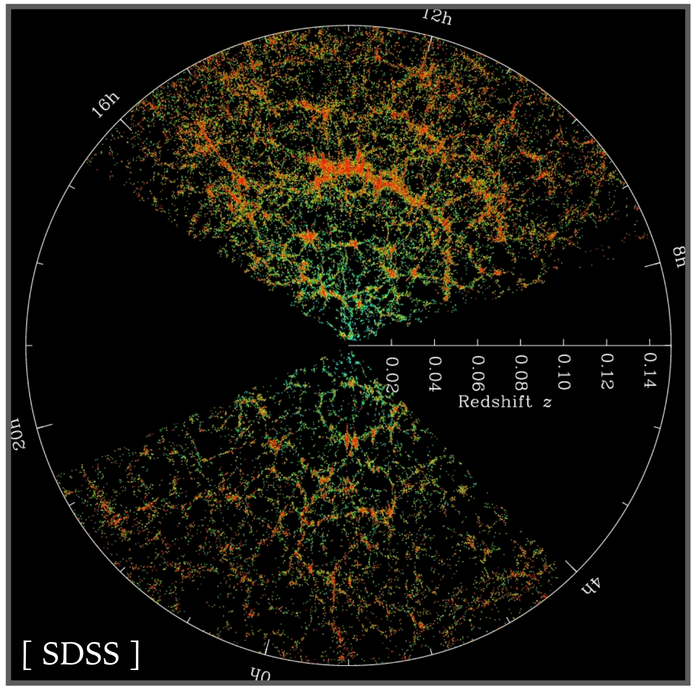
- A period of time when the Universe undergoes exponential expansion driven by a hypothetical field called "inflaton"

- Solves the flatness and horizon problems, as well as dilutes unwanted relics

- Generates primordial seeds for late-time structures [Mukhanov & Chibisov '81]



[Planck]

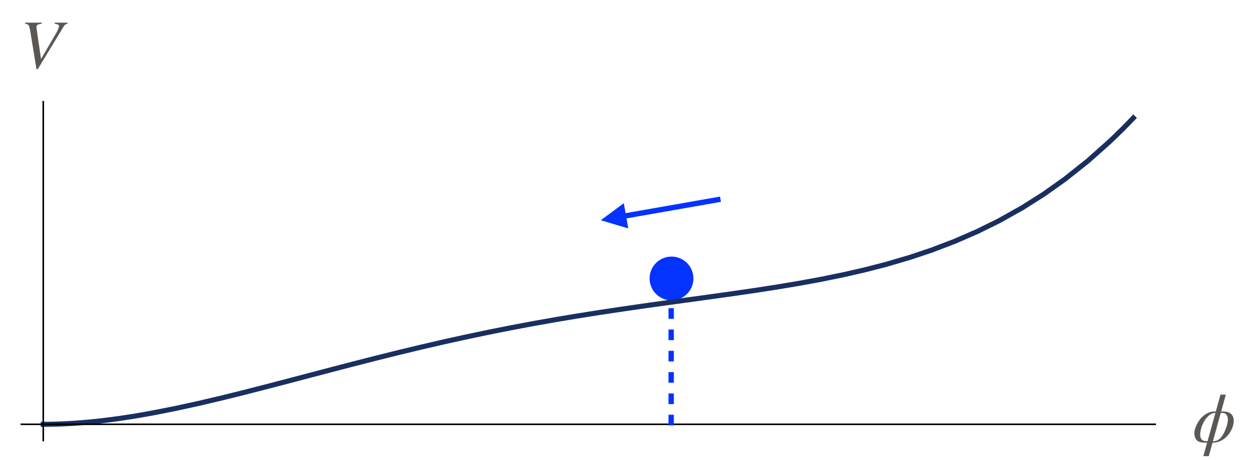


[SDSS]

INFLATION AND STOCHASTIC APPROACH

- Inflation and primordial seeds

  - Time evolution of the (coarse-grained value of) inflaton is important in estimating the amount of the primordial seeds



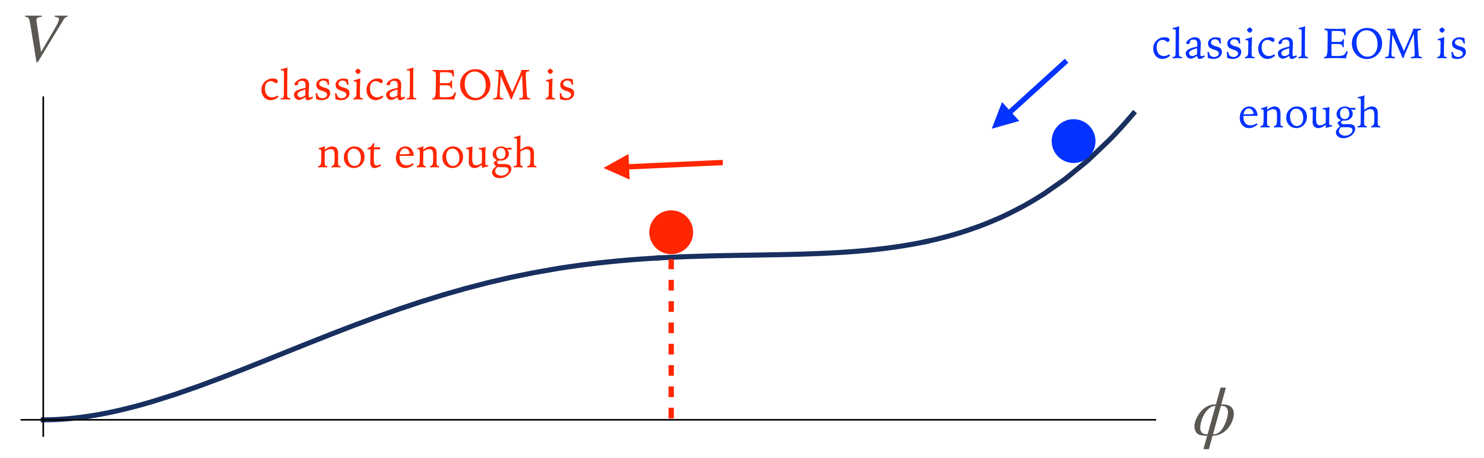
(averaged within the de Sitter Horizon)

  - For slow-roll, classical EOM is enough to track the inflaton value: the seeds can be calculated around the classical inflaton trajectory

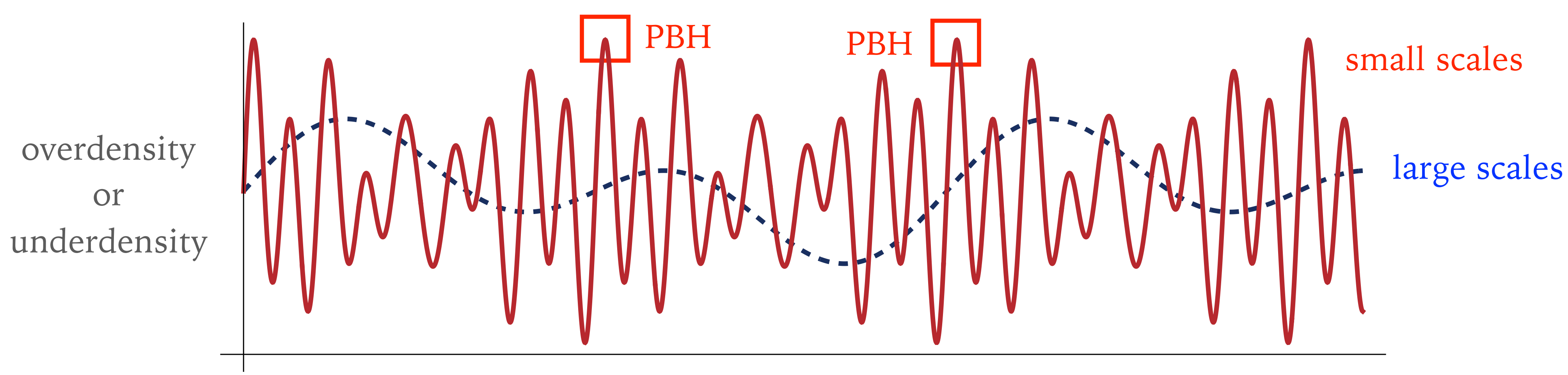
INFLATION AND STOCHASTIC APPROACH

➤ Inflation and primordial seeds

- However, the classical EOM is not enough when the potential is extremely flat:



- Such scenarios are often studied in the context of primordial black holes



INFLATION AND STOCHASTIC APPROACH

- Stochastic approach [Starobinsky '86]

  - Effective description of the inflaton/spectator long-wave modes

σ : small constant

$$\phi_{\text{coarse-grained}}(t, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3} \underbrace{\Theta(\sigma a H - k)}_{\text{takes only small } k \text{ (= coarse-grained modes) into account}} \tilde{\phi}(t, \boldsymbol{k})\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

  - During inflation, $H \simeq$ const and $a \sim e^{Ht}$, so the Heaviside Θ behaves as

$\Theta(\underbrace{\sigma a H}_{\text{exponentially increasing}} - k)$ → more and more k modes contribute at late times

  - The newly contributing modes work as Gaussian white noise to $\phi_{\text{coarse-grained}}$

$$\partial_t \phi_{\text{coarse-grained}} = -\frac{V'}{3H} + \xi$$

INFLATION AND STOCHASTIC APPROACH

- From Langevin to Fokker-Planck equation

- Langevin eq.: $$\partial_t \phi = -\frac{V'}{3H} + \xi$$ with $$\langle \xi(t,\boldsymbol{x})\xi(t',\boldsymbol{x}')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')\,\delta_{\boldsymbol{x},\boldsymbol{x}'}$$

classical force   stochastic noise

↓

- Fokker-Planck eq.: $$\partial_N P = \frac{1}{3H^2}\partial_\phi(V'P) + \frac{H^2}{8\pi^2}\partial_\phi^2 P$$

classical force   stochastic noise

N : rescaled time variable $N = Ht$ (e-folding)

$P(\phi, N)$: probability distribution of the coarse-grained inflaton ϕ

- We consider spectator ϕ in the following for simplicity

Stochastic approach to inflation

Correlator & divergence

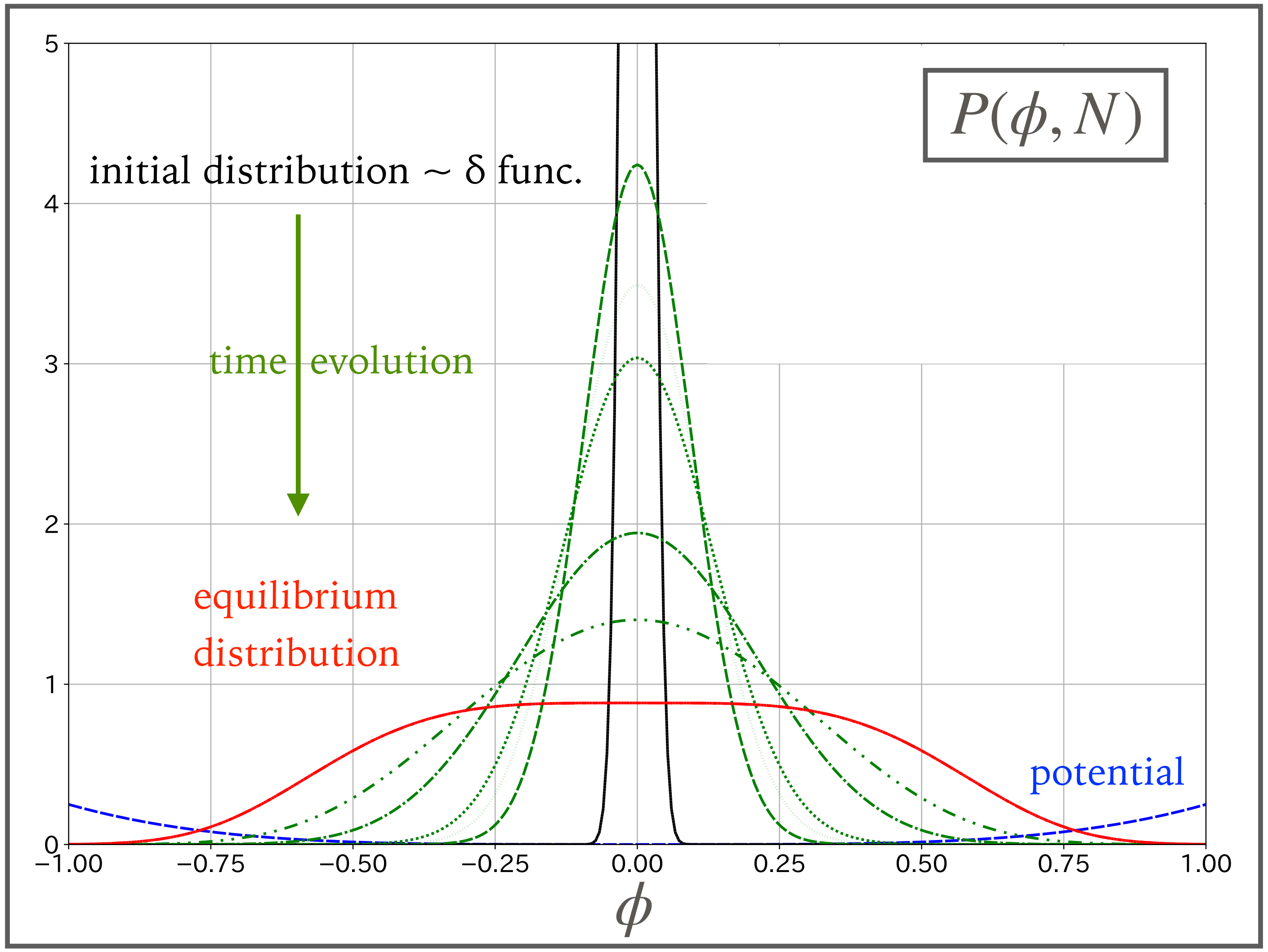
Borel resummation

Summary

BEHAVIOR OF THE PROB. DISTRIBUTION FUNCTION

➤ At first sight, Fokker-Planck eq. seems to be well behaved

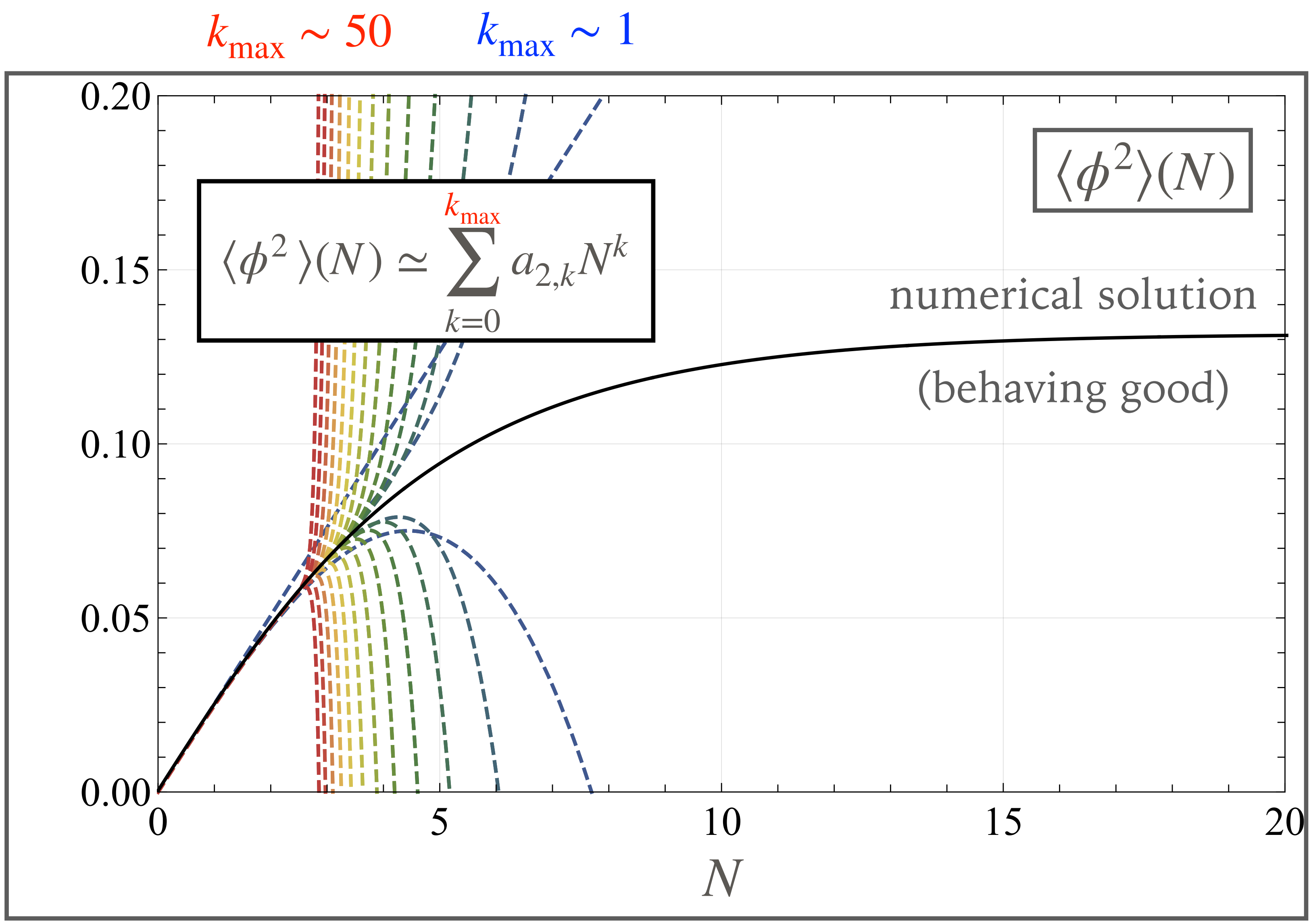
- Example : numerical solution for $\partial_N P = \frac{1}{3H^2}\partial_\phi(V'P) + \frac{H^2}{8\pi^2}\partial_\phi^2 P$ with $V = \frac{\lambda}{4}\phi^4$



BEHAVIOR OF THE CORRELATOR

➤ However, once analytically expanded, correlators behave badly

- Example : two-point correlator $\langle \phi^2 \rangle(N) = \int d\phi \; \phi^2 P(\phi, N)$ with $V = \frac{\lambda}{4}\phi^4$



DIVERGENT BEHAVIOR OF THE CORRELATOR

➤ Coefficients in the 2-point correlator $\langle \phi^n \rangle(N) = \sum_0^\infty a_{n,k} N^k$ with $V = \frac{\lambda}{4}\phi^4$

$a_{n,k}$

	$n \backslash k$	0	1	2	3	4	5	6	$\cdots$
0-pt.	0	1	0	0	0	0	0	0	$\cdots$
	1	0	0	0	0	0	0	0	$\cdots$
2-pt.	2	0	$1/4\pi^2$	0	$-1/24\pi^4$	0	$1/80\pi^6$	0	$\cdots$
	3	0	0	0	0	0	0	0	$\cdots$
4-pt.	4	0	0	$3/16\pi^4$	0	$-3/32\pi^6$	0	$53/960\pi^8$	$\cdots$
	5	0	0	0	0	0	0	0	$\cdots$
6-pt.	6	0	0	0	$15/64\pi^6$	0	$-15/64\pi^8$	0	$\cdots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

→

converging? NO!

DIVERGENT BEHAVIOR OF THE CORRELATOR

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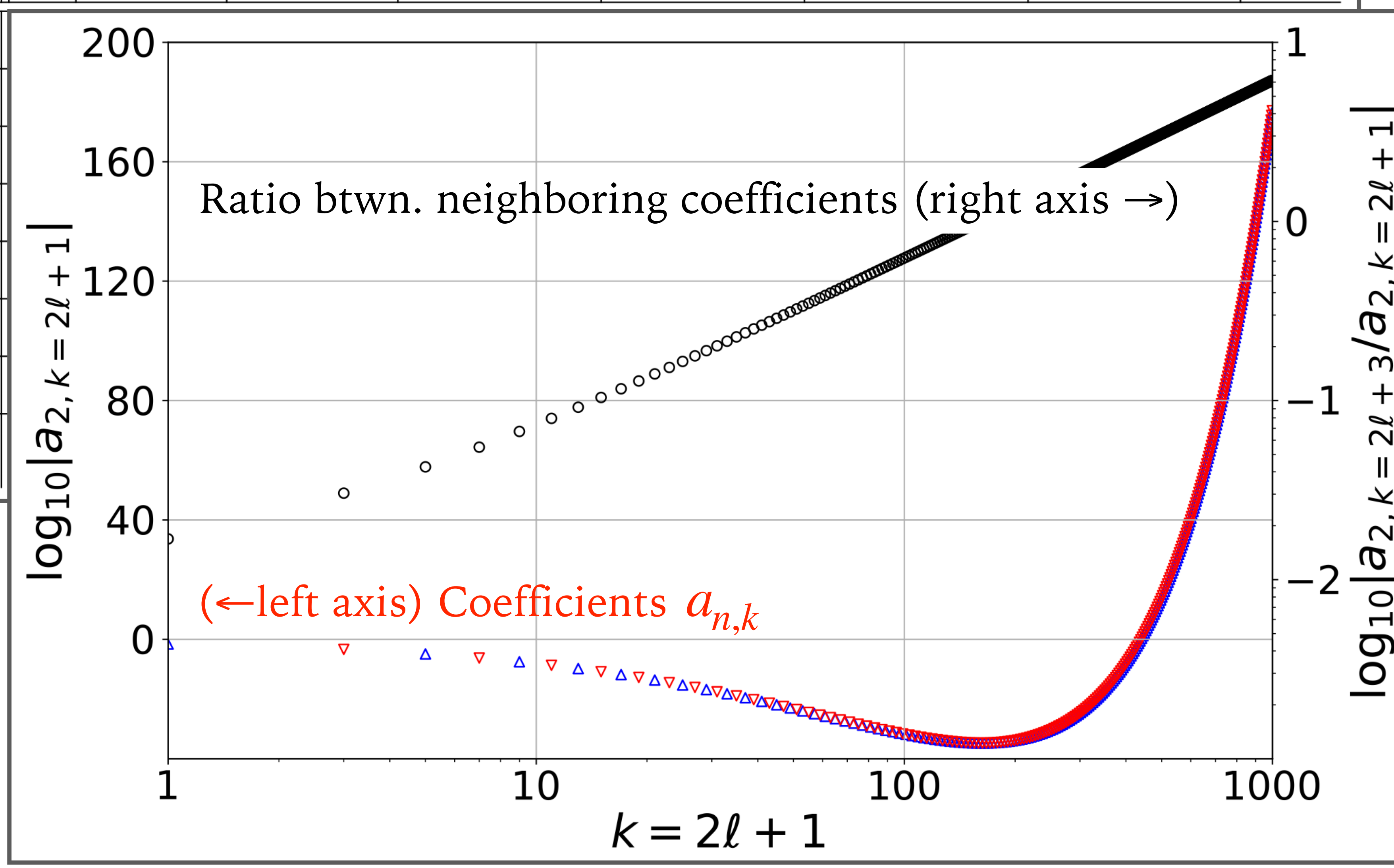
0-pt.	0								
	1								
2-pt.	2								
	3								
4-pt.	4								
	5								
6-pt.	6								
	$\vdots$								



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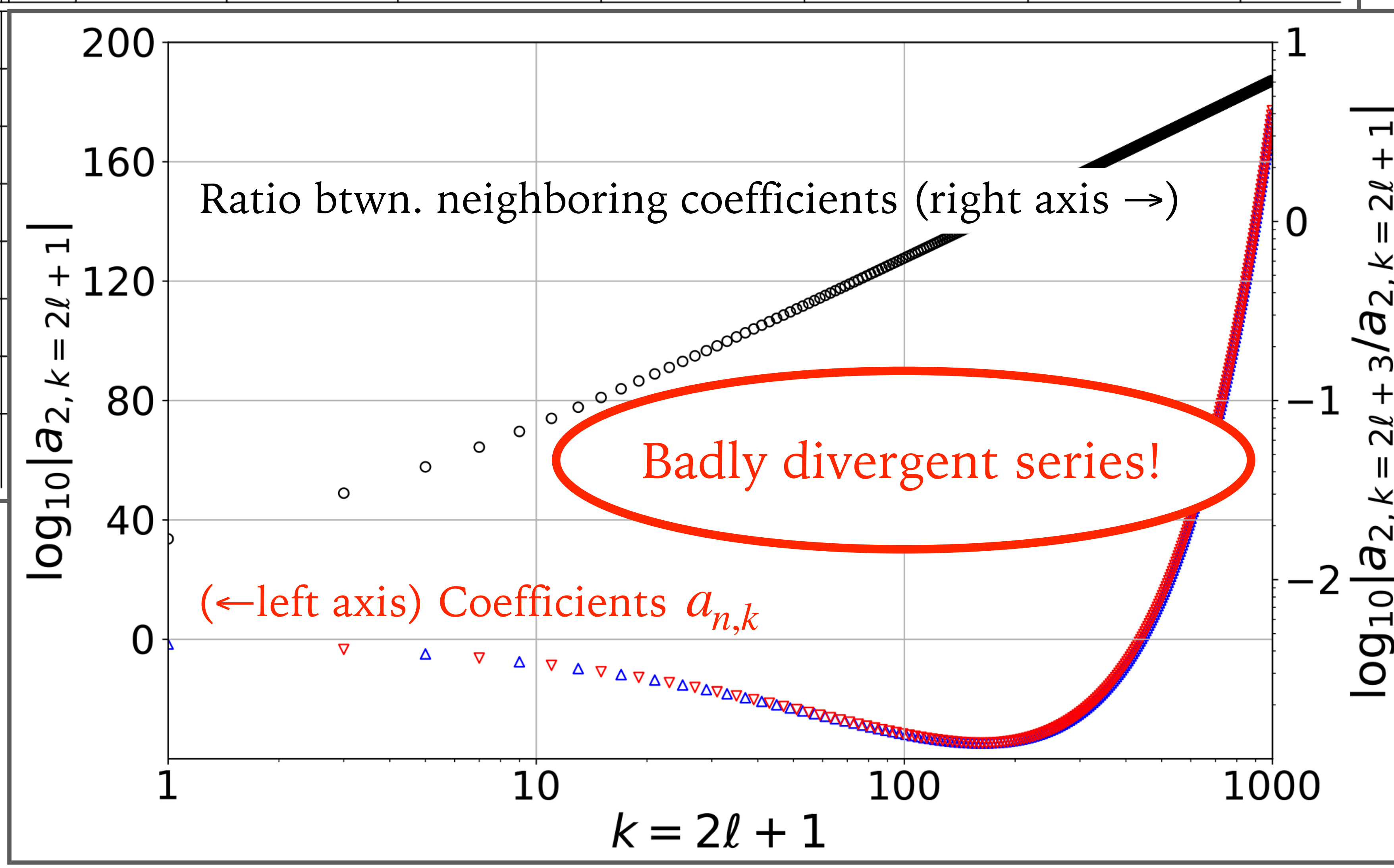


QUESTIONS

- Truncated correlators $\langle \phi^2 \rangle(N) \simeq \sum_{k=0}^{k_{\rm max}} a_{2,k} N^k$ are badly divergent

- Q1: Is it because of our poor way of looking at the series?

  Or is it truly badly divergent?

- Q2: Any nonperturbative effects in the correlators?

Stochastic approach to inflation

Correlator & divergence

Borel resummation

Summary

BOREL RESUMMATION

- Borel resummation [Borel 1899]

  - Method to construct, from a formally divergent series,

    an analytic function that gives the same expansion as the original series

- Example in 3 steps

  - Step1: Divergent series $\mathcal{O}(N) = \sum_{k=0}^{\infty} \underbrace{(-1)^k k!}_{a_k} N^{k+1} = N - N^2 + 2N^3 - 6N^4 + \cdots$

    ↓

  - Step2: Borel transf. $\mathcal{B}\mathcal{O}(t) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k+1)} t^k = 1 - t + t^2 - t^3 + \cdots = \frac{1}{1+t}$

    ↓

  - Step3: Borel resum. $\mathcal{S}\mathcal{O}(N) = \int_0^\infty dt\ e^{-\frac{t}{N}} \mathcal{B}\mathcal{O}(t) = \int_0^\infty dt\ \frac{e^{-\frac{t}{N}}}{1+t} = -\, e^{\frac{1}{N}} \mathrm{Ei}\left(-\frac{1}{N}\right)$

BOREL RESUMMATION

➤ Borel resummation [Borel 1899]

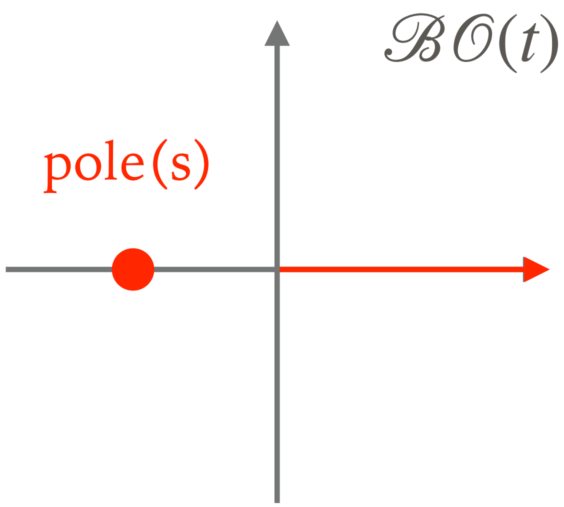
- Method to construct, from a formally divergent series,

  an analytic function that gives the same expansion as the original series

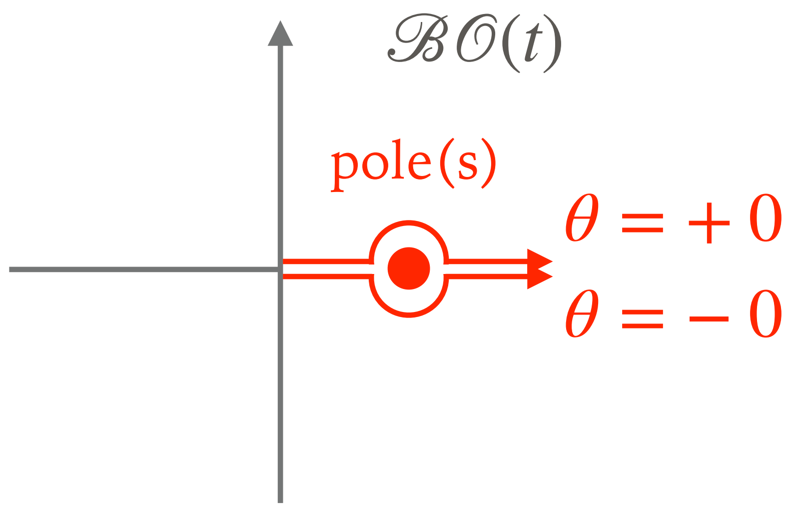
- Moreover, the ambiguity of Borel resummation ($\mathscr{S}\mathscr{O}(N) = \int_0^{e^{i\theta}\infty} dt\ e^{-\frac{t}{N}} \mathscr{B}\mathscr{O}(t)$),

  if exists, tells us about nonperturbative properties of the system ("resurgence")

No nonperturbative effects for real N



Nonperturbative effects for real N



PADÉ APPROXIMATION

- Padé approximation to Borel transformation [Padé 1892]

  - Practically, it is often difficult to obtain the coefficients a_k or their asymptotic form $a_k \simeq a_k^{(\text{asympt.})}$

  - Even in such cases, the system can be analyzed with Padé approximation

$$\mathscr{B}\mathscr{O}(t) = \sum_{k=0}^{k_{\max}} \frac{a_k}{\Gamma(k+1)} t^k \underset{\text{approximated}}{\simeq} \frac{\sum_{n=0}^{m} c_n t^n}{1 + \sum_{n=1}^{l} d_n t^n} \equiv P_{l,m}(t)$$

  - Poles of the Padé approximant $P_{m,n}(t)$ tells us about nonpert. effects in the system

BOREL-PADÉ APPLIED TO CORRELATORS IN STOCHASTIC INFLATION

- Naive truncated series

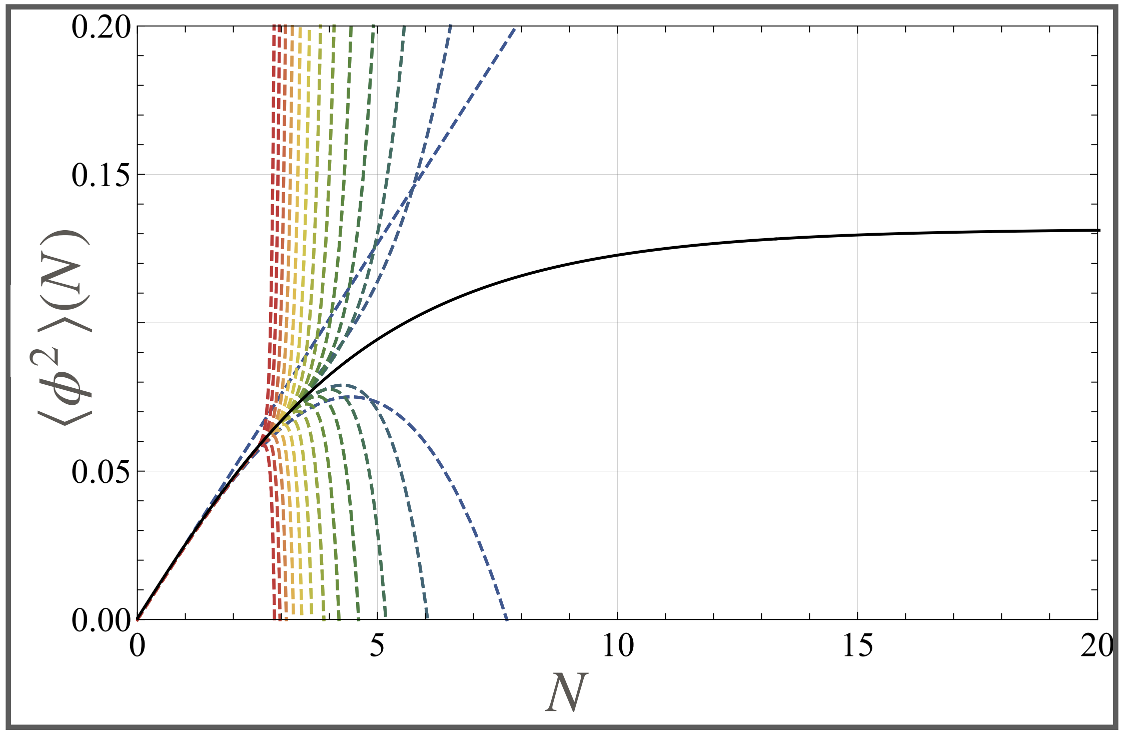
$$\langle \phi^2 \rangle(N) \simeq \sum_{k=0}^{k_{\max}} a_k N^{k+1}$$

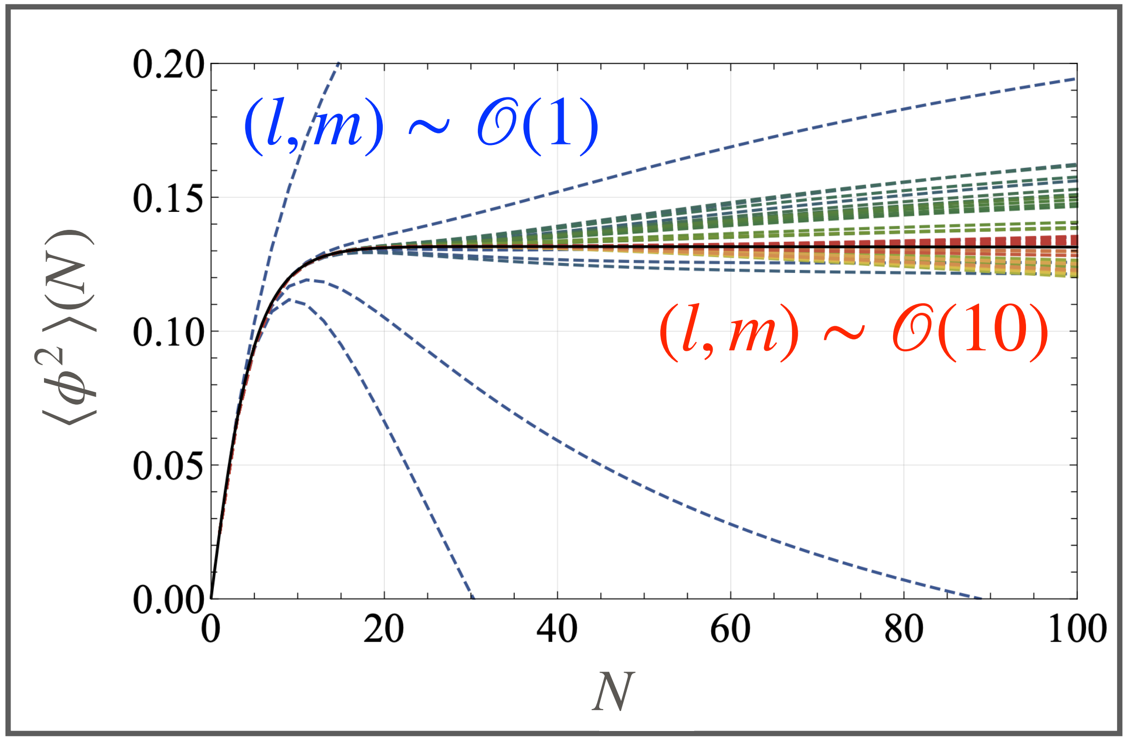
- Borel-Padé approx.

Step1: Divergent series

Step2: Borel transf. & Padé approx.
(with $(l, m) \sim \mathcal{O}(10)$)

Step3: Borel resum.

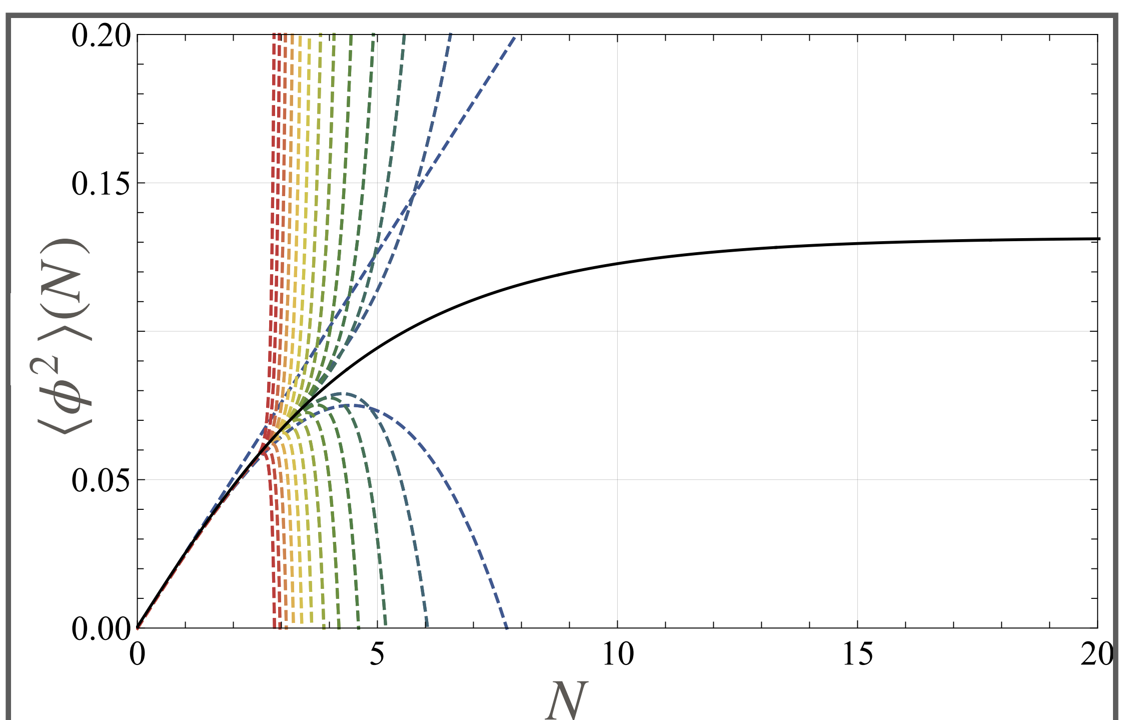




BOREL–PADÉ APPLIED TO CORRELATORS IN STOCHASTIC INFLATION

- Naive truncated series

$$\langle \phi^2 \rangle(N) \simeq \sum_{k=0}^{k_{\max}} a_k N^{k+1}$$



- Borel-Padé approx.

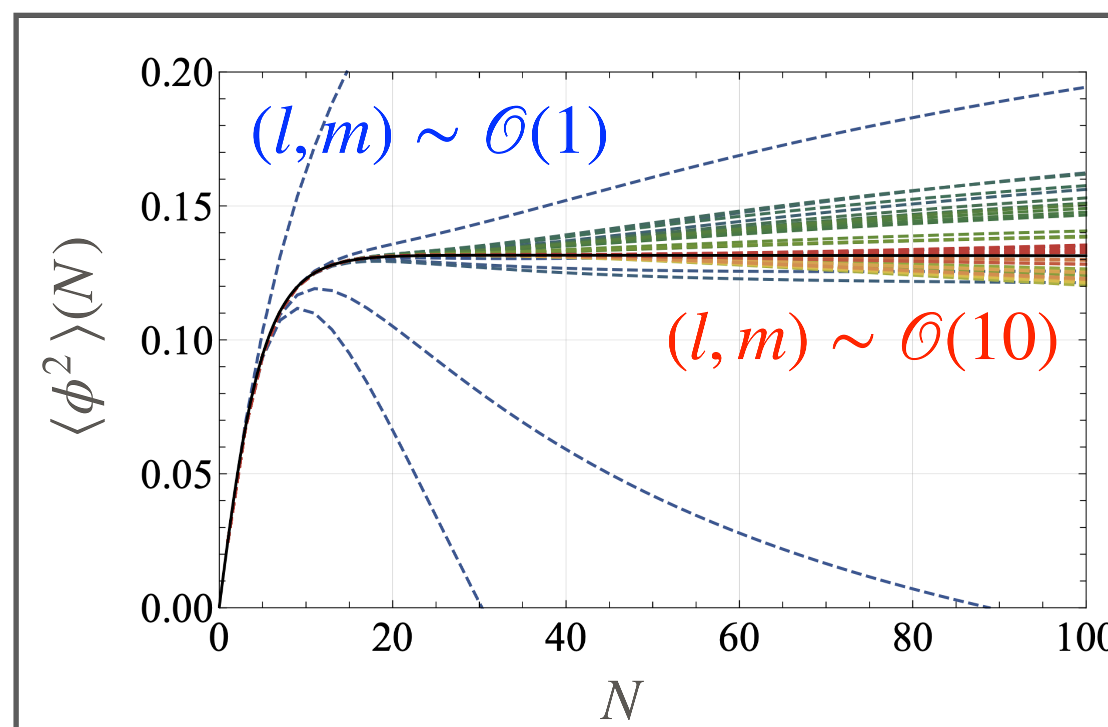
Step1: Divergent series

Ste ox.

initial → transient → stationary

correctly reproduced

Ste



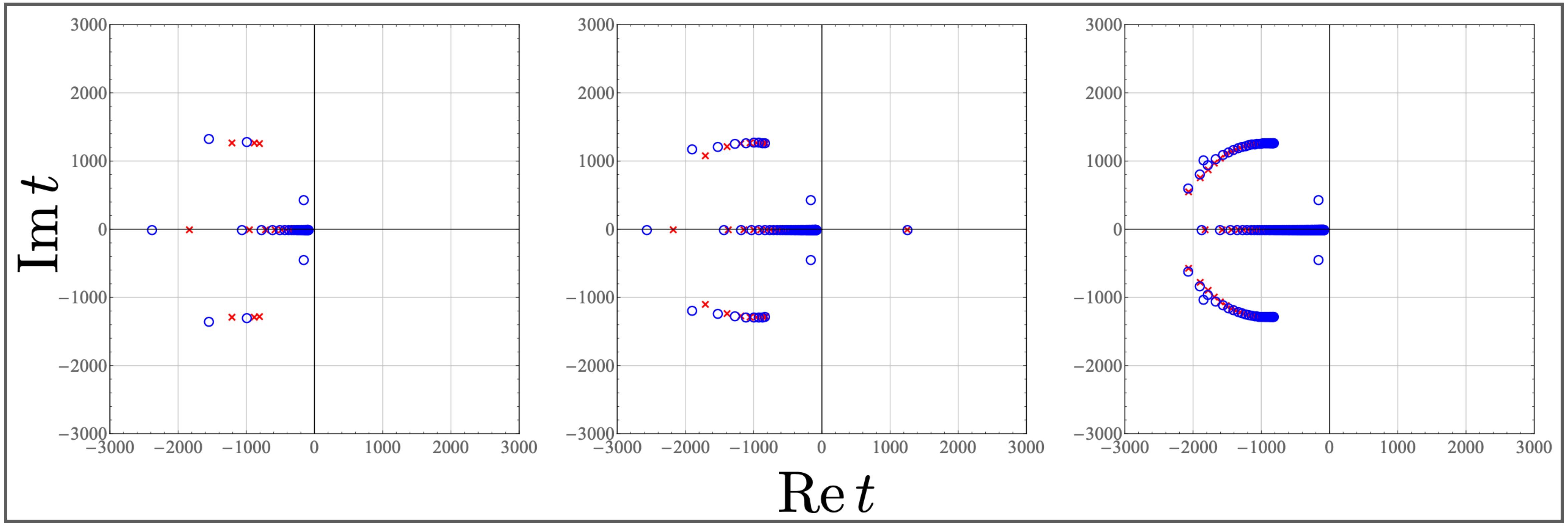
BOREL-PADÉ APPLIED TO CORRELATORS IN STOCHASTIC INFLATION

- Any signal of nonperturbative effects?

  - Zeros (blue circles) and poles (red crosses)

    in the Borel transform $\mathscr{B}\mathcal{O}(t)$ of $N \times \langle \phi^2 \rangle (N)$

$(l, m) \sim \mathcal{O}(10)$ $(l, m) \sim 100$ $(l, m) \sim \mathcal{O}(100)$



BOREL–PADÉ APPLIED TO CORRELATORS IN STOCHASTIC INFLATION

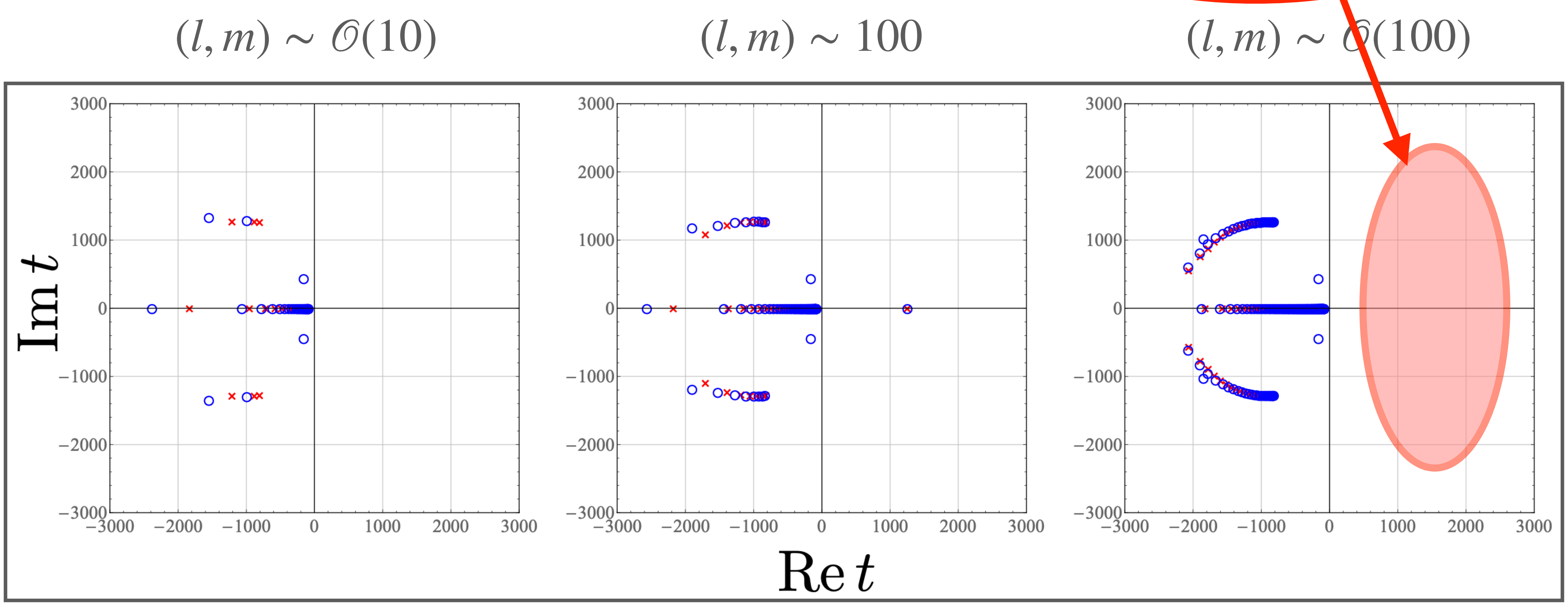
➤ Any signal of nonperturbative effects?

- Zeros (blue circles) and poles (red crosses) in the Borel transform $\mathscr{B}\mathscr{O}(t)$ of $N \times \langle \phi^2 \rangle(N)$

No poles in positive t

||

Signals the absence of nonperturbative effects



Self introduction

Part I

Quartic Gradient Flow

Part II

Borel Resummation

Summary

SUMMARY

- In stochastic inflation, correlators are often said to be divergent series

- Using Borel resummation (& Pade approximation),

  we have shown that the series can be brought back to converging ones

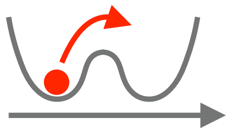
- From the Borel-plane (= t-plane) analysis,

  we confirmed no signs of nonperturbative effects in the system

- Further questions (~ random thoughts)

  - any cases where correlators have poles in positive t?

    spectator→inflaton?

     ?

  - relation to resummation of IR divergence in de Sitter?