Instituto de Física Teórica
UAM-CSIC

Universidad Autónoma de Madrid

Infinite Distance Limits, Light Towers & Duality Frames

Ignacio Ruiz,

SUSY2024 , June 10th, 2024

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Infinite Distance Limits, Light Towers & Duality Frames

or:

“Three Supersymmetries for the prize of one shape”

Ignacio Ruiz,

SUSY2024 , June 10th, 2024

Based on:

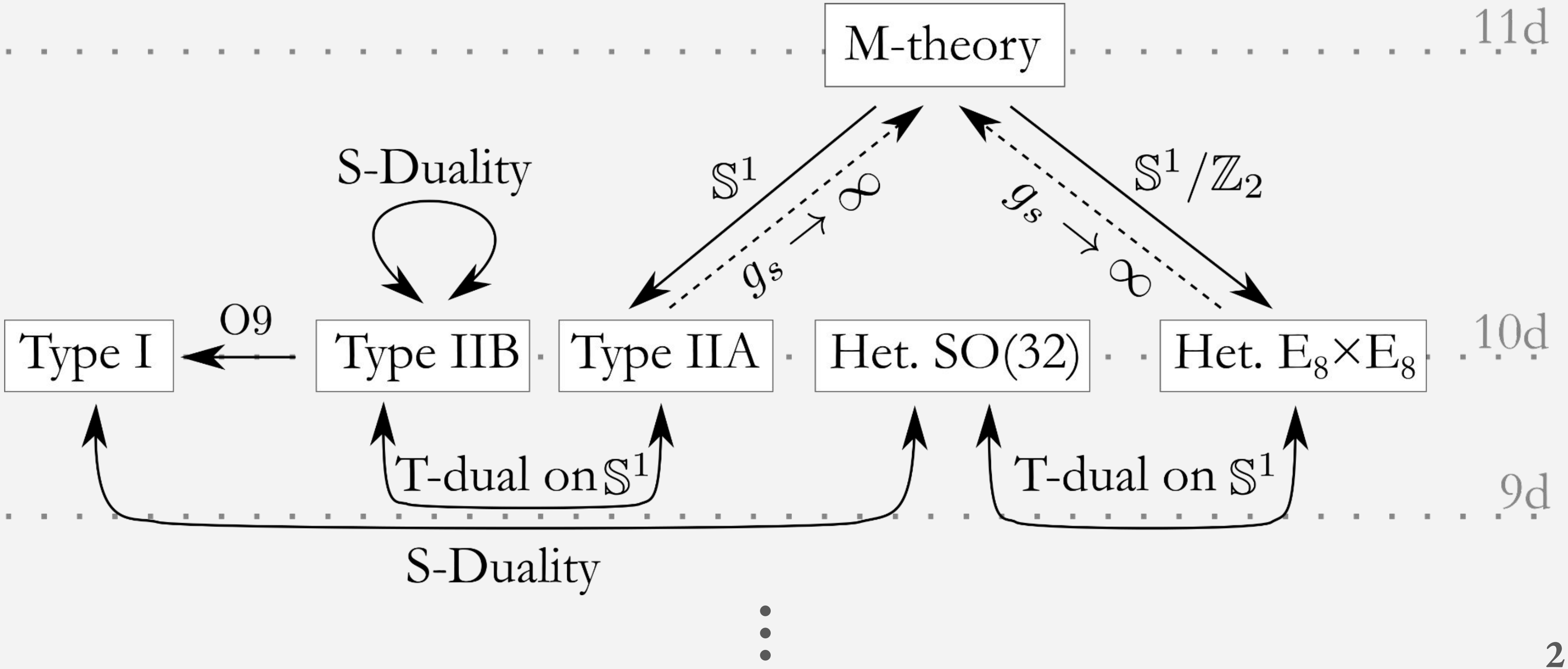
[2405.20332] Etheredge, Heidenreich, Rudelius, **I.R.**, Valenzuela

[2311.01501, 2311.01536] Castellano, **I.R.**, Valenzuela

[2306.16440] Etheredge, Heidenreich, McNamara, Rudelius, **I.R.**, Valenzuela

[WIP] Fraiman, **I.R.**, Valenzuela

String Theory and dualities



Swampland Program

c.f. Veronica's talk!

What are the conditions that any consistent theory of Quantum Gravity must follow?

Can it give us information about dualities?

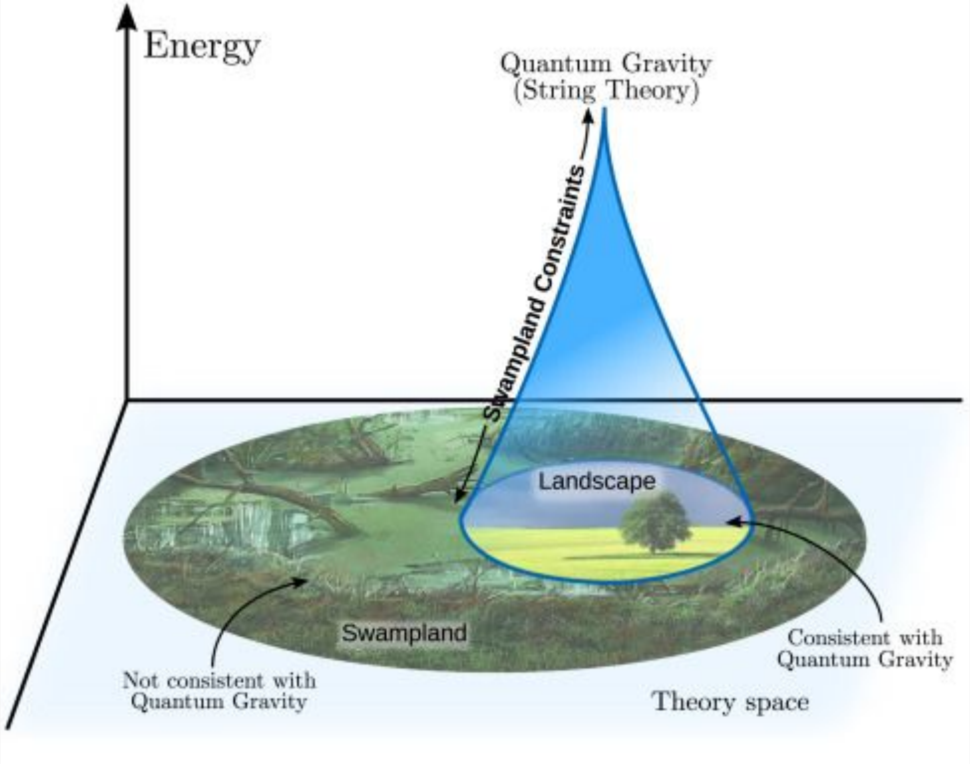


figure taken from [van Beest, Calderón-Infante,Mirfendereski,Valenzuela, '20]

1. Introduction

Swampland Program

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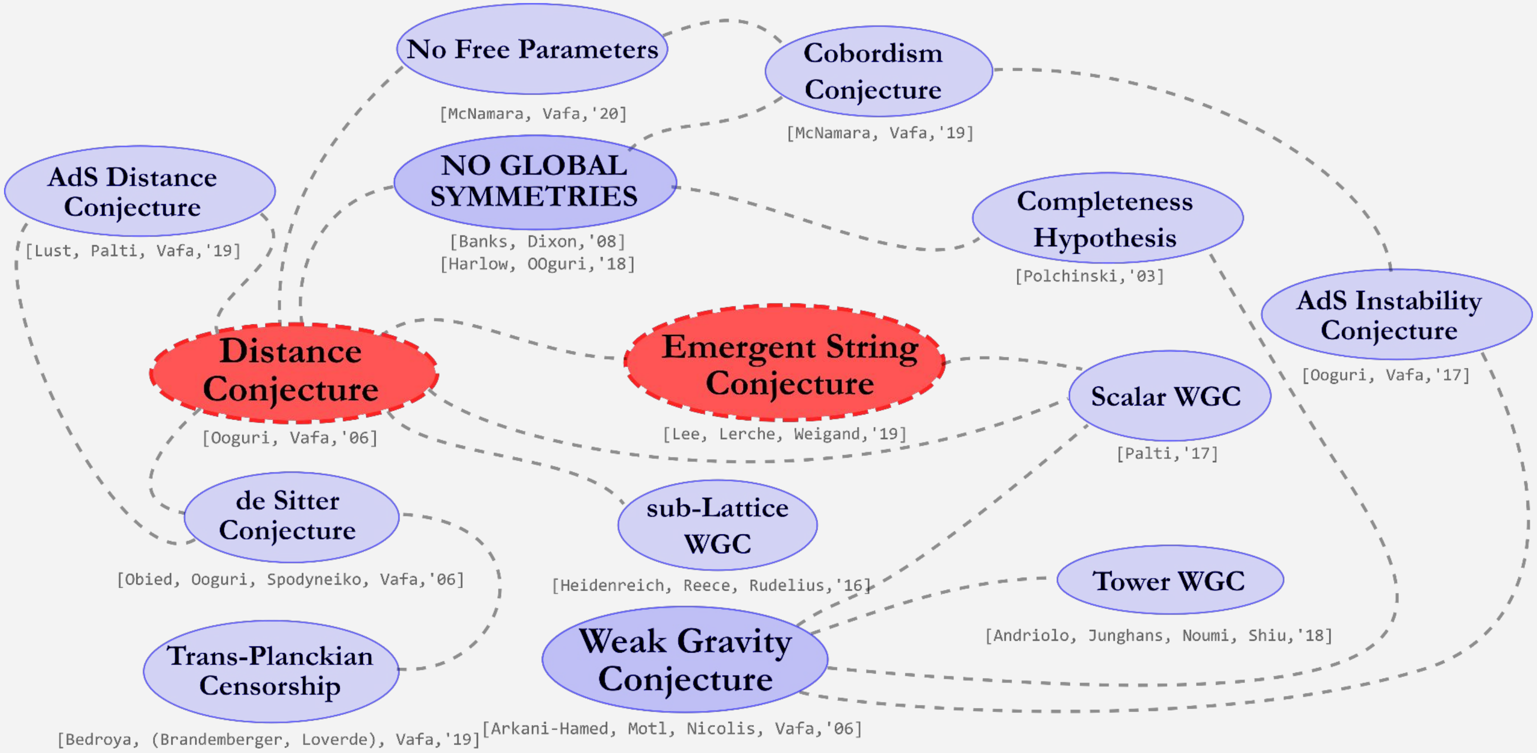

Swampland Distance Conjecture

[Ooguri, Vafa, '07]

c.f. Veronica's talk!

Given a d-dimensional EFT coupled to Einstein gravity with scalars taking value on some moduli space $\mathcal{M}$ we have the following terms in the effective action:

$$S_{\text{EFT}} \supseteq \frac{1}{2\kappa_d^2} \int \mathrm{d}^d x \sqrt{-g} \left(R_g - \mathsf{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j \right)$$

with G_{ij} the **moduli space metric**.

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with G_{ij} the **moduli space metric**.

We can use it to define angles and distances, e.g.:

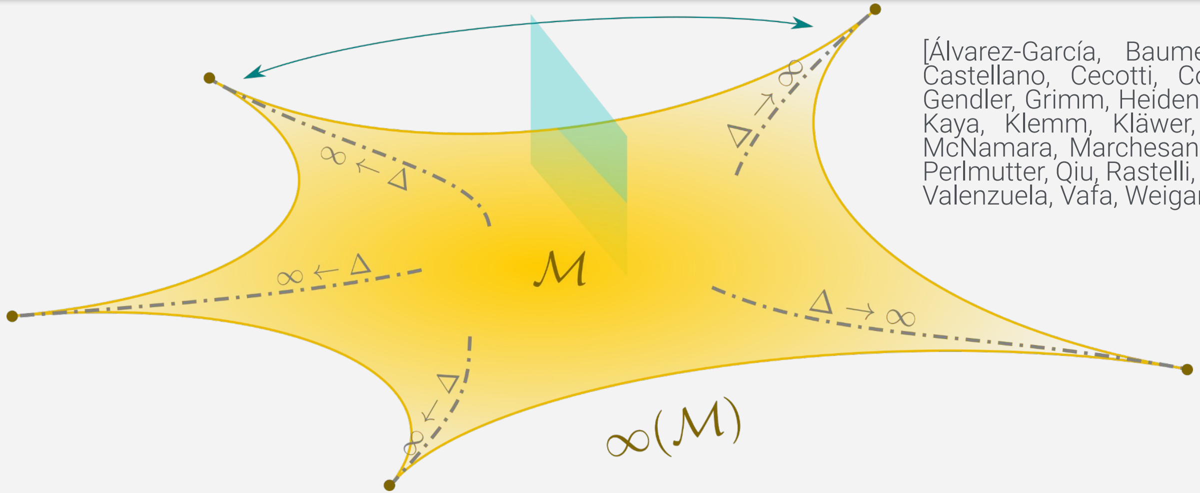
$$\Delta(\tau) = \int_0^\tau \sqrt{\mathsf{G}_{ij}\partial_t\phi^i\partial_t\phi^j}\,\mathrm{d}t$$

with $\vec{\phi}:\mathbb{R}\to\mathcal{M}$ some geodesic trajectory.

Swampland Distance Conjecture

As we move towards infinite distance limits, there is an **infinite tower of states** becoming **exponentially light**:

$$M(\Delta) \sim M(0)e^{-\alpha\Delta} \quad \text{as} \quad \Delta \to \infty \text{ with } \alpha = \mathcal{O}(1).$$



[Álvarez-García, Baume, Blumenhagen, Buratti, Calderón-Infante, Castellano, Cecotti, Corvilain, Cribiori, Etheredge, Erkinger, Font, Gendler, Grimm, Heidenreich,van de Heisteeg, Herráez, Ibáñez, Joshi, Kaya, Klemm, Kläwer, Knapp, Lanza, Lee, Lerche, Li, Lockhart, McNamara, Marchesano, Montella, Martucci, Montero, Ooguri, Palti, Perlmutter, Qiu, Rastelli, Reece, Rudelius, **IR**, Schlechter, Stout, Uranga, Valenzuela, Vafa, Weigand, Wiesner, Wolf…]

Emergent String Conjecture

[Lee, Lerche, Weigand, '19]

What are the possible tower one finds?

Any infinite distance limit is either a decompactification limit or a limit in which there is a weakly coupled string becoming tensionless.

[Álvarez-García, Aoufia, Basile, Baume, Calderón-Infante, Kläwer, Lanza, Lee, Leone, Lerche, Marchesano, Martucci, Perlmutter, Rastelli, Rudelius, Vafa, Valenzuela, Weigand, Wiesner, Xu...]

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Possible light towers:

KK states **String oscillator modes**

Different towers and limits will have different exponential rates α.

2. Type IIB on $\mathbb{S}^1$ (32 supercharges)

Duality Frames

We have two moduli: 10d dilaton $\phi_{\text{IIB}} = \log g_{\text{IIB}}$ and circle radius R_{IIB}.

Perturbative control is given in **large volume**, **small coupling** regime:

$$g_{\text{IIB}} \ll 1 \qquad\qquad R_{\text{IIB}} \gg \sqrt{\alpha'}$$

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Perturbative control is given in **large volume, small coupling** regime:

$$g_{\text{IIB}} \ll 1 \qquad\qquad R_{\text{IIB}} \gg \sqrt{\alpha'}$$

Out of it we perform T- or S-duality to go to there:

IIB-IIB S-duality

$$R_{\text{IIB}'} = R_{\text{IIB}}$$
$$g_{\text{IIB}'} = g_{\text{IIB}}^{-1}$$

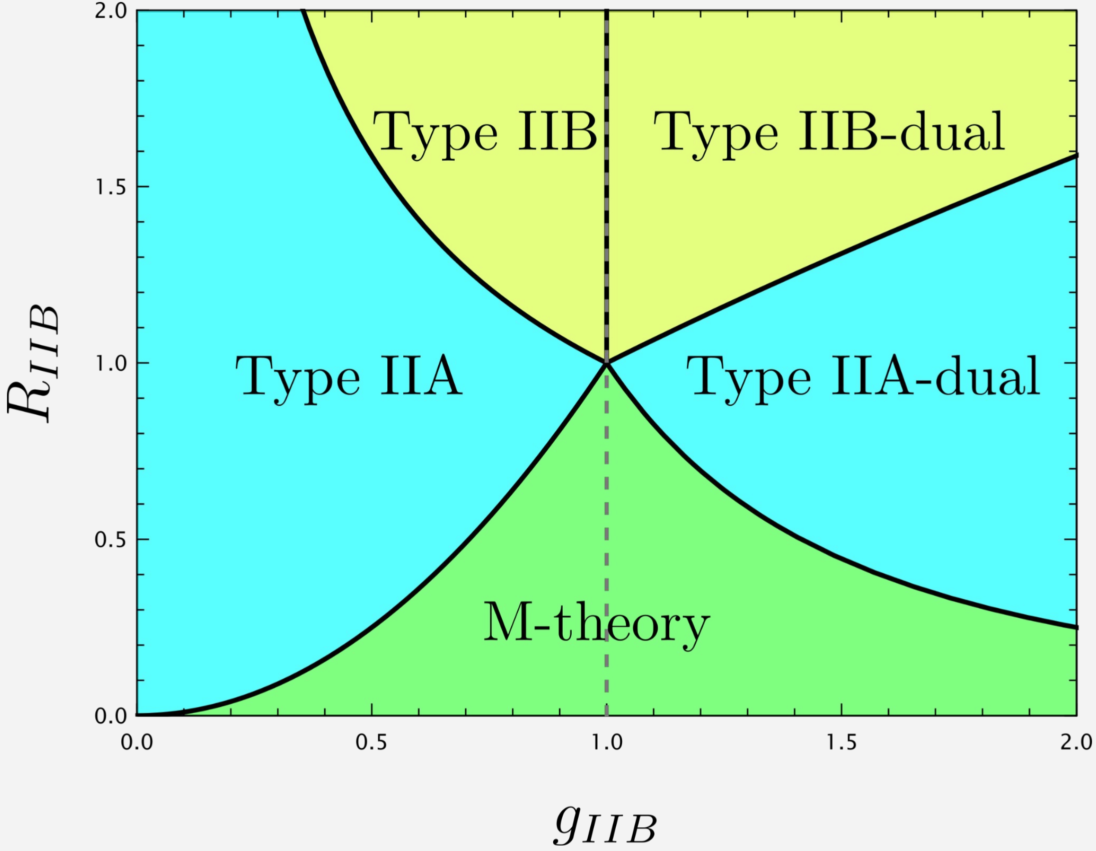
IIB-IIA T-duality

$$R_{\text{IIA}} = \frac{\alpha'}{R_{\text{IIB}}} \qquad g_{\text{IIA}} = g_{\text{IIB}} \frac{\sqrt{\alpha'}}{R_{\text{IIB}}}$$

IIA-Mth duality

$$R_{10} = g_{\text{IIA}}^{2/3}$$

Duality Frames



Canonical coordinates

The 9d EFT action has a kinetic term for the scalars

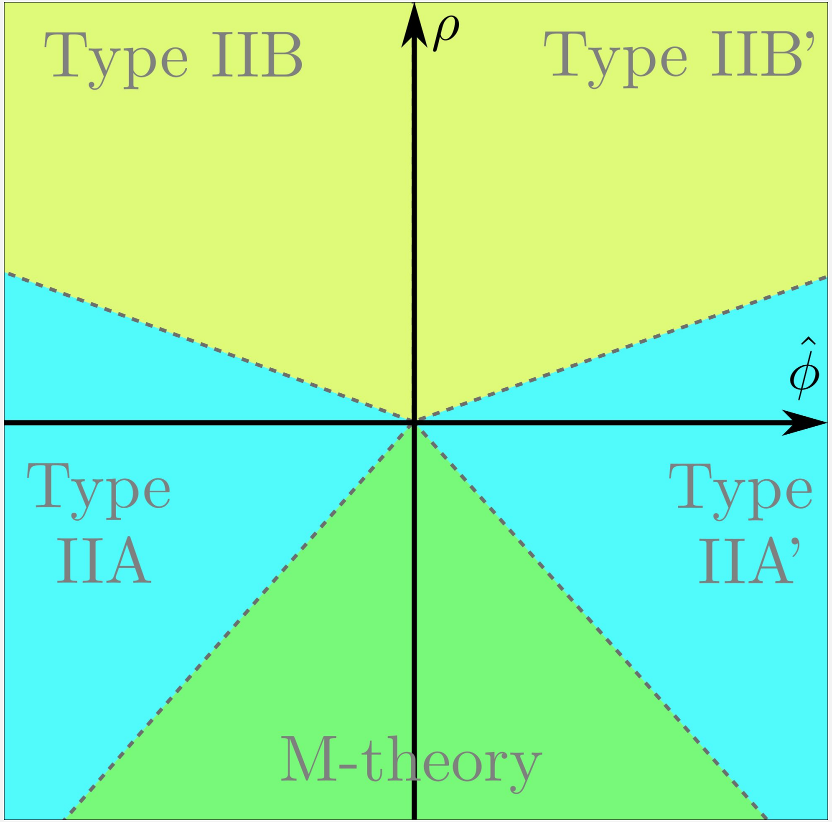
$$S_{\mathrm{IIB}}^{(9\mathrm{d})} \supset \frac{1}{2\kappa_9^2}\int \mathrm{d}^9x\sqrt{-g}\left\{R_g - \frac{1}{2}(\partial\phi)^2 - \frac{8}{7R_{\mathrm{IIB}}^2}(\partial R_{\mathrm{IIB}})^2\right\}$$

We can perform a change of coordinates to go to **flat coordinates**

$$\mathsf{G}_{ab} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{8}{7R_{\mathrm{IIB}}^2} \end{pmatrix} \longrightarrow \hat{\phi} = \frac{1}{\sqrt{2}}\phi \qquad \rho = \sqrt{\frac{8}{7}}\log R_{\mathrm{IIB}}$$

This allows identification between $\mathcal{M}$ and $T_p\mathcal{M}$.

Putting things together



Light towers of states

We can compute states becoming light in at least one limit:

$$T_{\rm F1} \sim M_{\rm Pl,10}^2 e^{-\phi/2} \Rightarrow \frac{m_{\rm F1}}{M_{\rm Pl,9}} \sim R_{\rm IIB}^{-1/7} e^{-\phi/4}$$

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$$m_{\mathrm{F-w}} \sim R_{\mathrm{IIB}} e^{-\phi/2} \Rightarrow \frac{m_{\mathrm{F-w}}}{M_{\mathrm{Pl},9}} \sim R_{\mathrm{IIB}}^{6/7} e^{-\phi/2}$$

$$m_{\mathrm{D1-w}} \sim R_{\mathrm{IIB}} e^{-\phi/2} \Rightarrow \frac{m_{\mathrm{D1-w}}}{M_{\mathrm{Pl},9}} \sim R_{\mathrm{IIB}}^{6/7} e^{\phi/2}$$

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ALL TOWERS

$$T_{\mathrm{D1}} \sim M_{\mathrm{Pl},10}^2 e^{+\phi/2} \Rightarrow \frac{m_{\mathrm{D1}}}{M_{\mathrm{Pl},9}} \sim R_{\mathrm{IIB}}^{-1/7} e^{\phi/4}$$

ARE BPS!

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Scalar charge to mass ratio vectors

For multifield moduli spaces asymptotic limits can be complicated: Exponential rate of towers also depends on the **direction** we are taking!

For it we define **scalar charge-to-mass ratio vectors**:

$$\vec{\zeta}_I = -\vec{\nabla} \log m_I(\vec{\varphi})$$

Given an asymptotic direction $\hat{\tau}$, then $\lambda_I = \hat{\tau} \cdot \vec{\zeta}_I$

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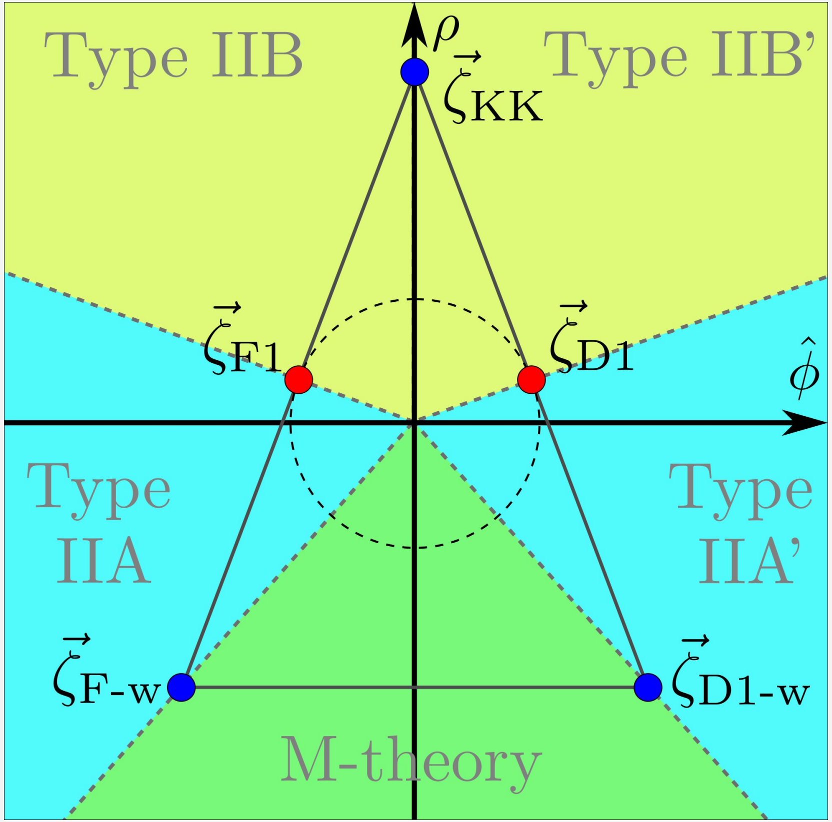
$$\vec{\zeta}_{\rm KK} = \left(0, \sqrt{\frac{8}{7}}\right) \qquad \vec{\zeta}_{\rm F1} = \left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{14}}\right) \qquad \vec{\zeta}_{\rm D1} = \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{14}}\right)$$

$$\vec{\zeta}_{\rm F-w} = \left(\frac{1}{\sqrt{2}}, -\frac{3}{\sqrt{14}}\right) \qquad \vec{\zeta}_{\rm D1-w} = \left(-\frac{1}{\sqrt{2}}, -\frac{3}{\sqrt{14}}\right)$$

Putting things together



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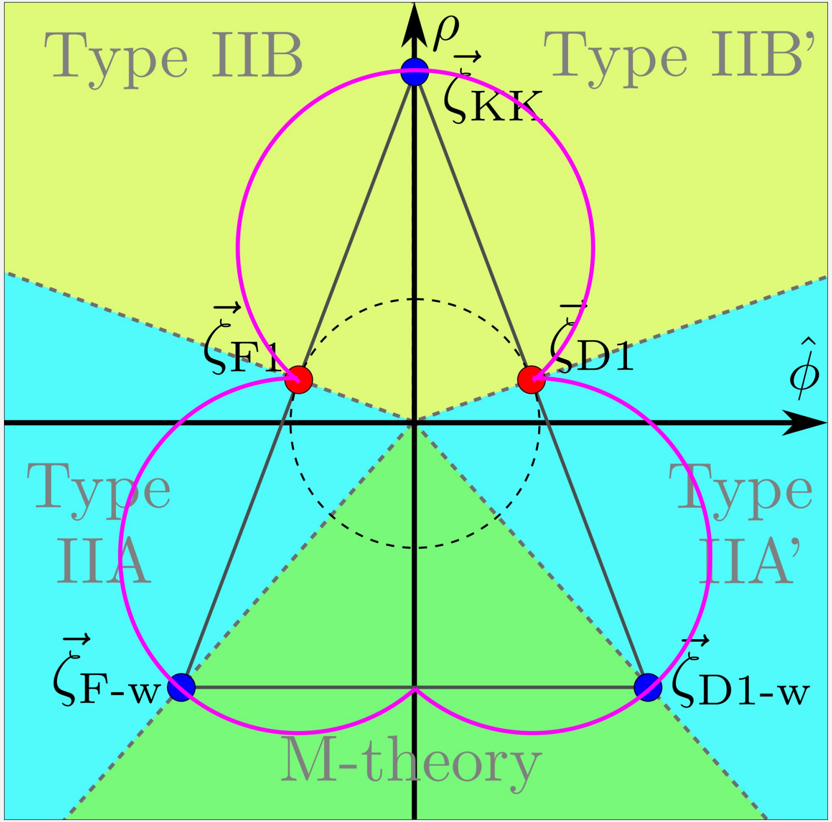

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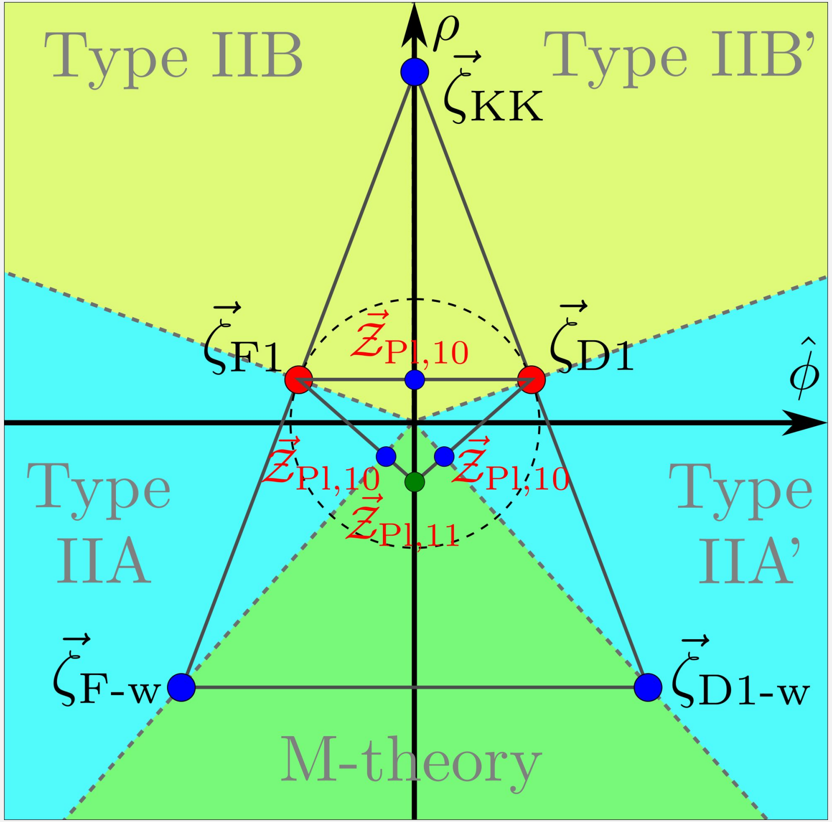

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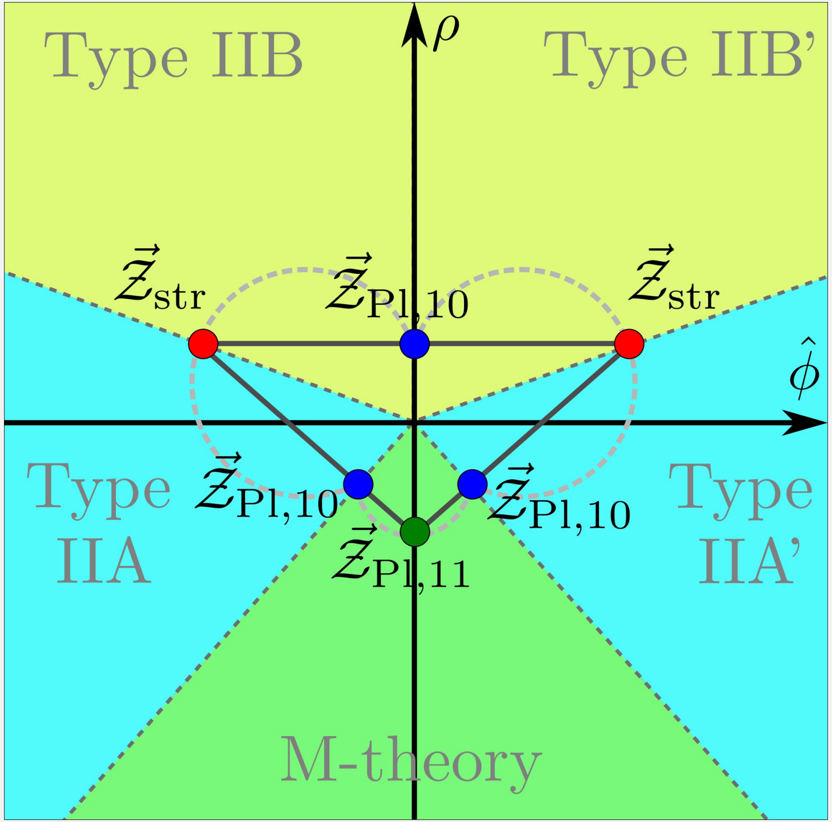
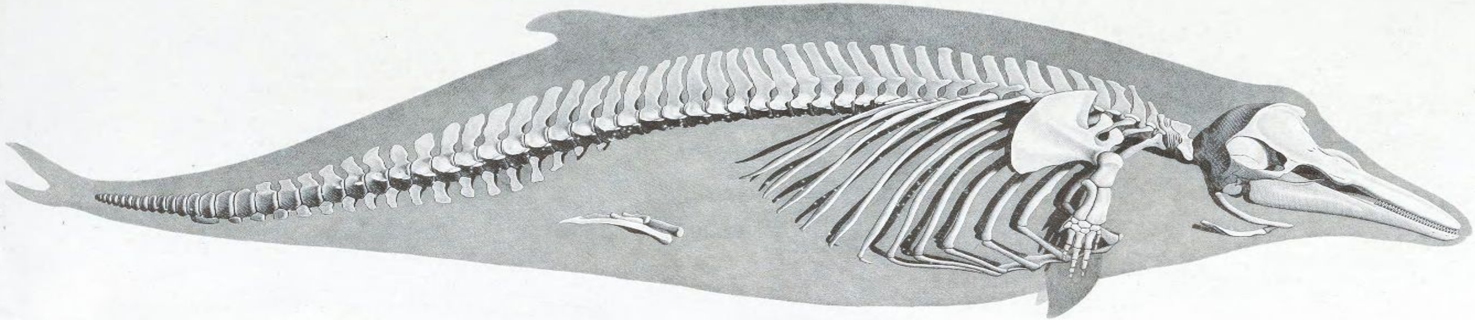
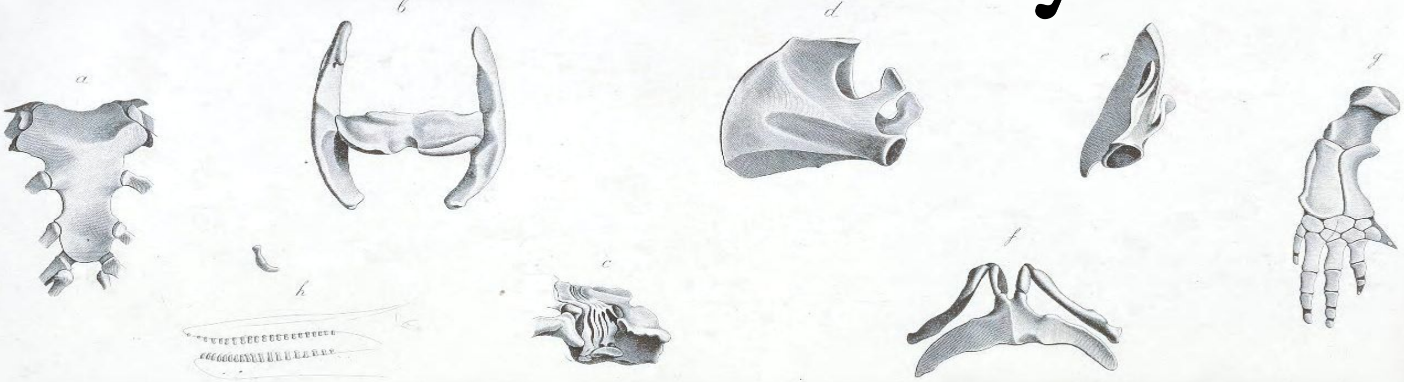




3. Some Taxonomy Rules



Dimensional reduction

Consider EFT action in D dimensions:

$$S_D \supseteq \int \mathrm{d}^D x \sqrt{-g_D} \left[\frac{1}{2\kappa_D^2} \mathcal{R}_D - \frac{1}{2} \left(\partial \hat{\phi} \right)^2 \right]$$

We reduce to $d = D - n$:

$$S_d \supseteq \int \mathrm{d}^d x \sqrt{-g_d} \left[\frac{1}{2\kappa_d^2} \left(\mathcal{R}_d - \frac{d+n-2}{n(d-2)} \left(\partial \log \mathcal{V}_n \right)^2 \right) - \frac{1}{2} \left(\partial \hat{\phi} \right)^2 \right]$$

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We define canonically normalized volume modulus:

$$\hat{\rho} = \frac{1}{\kappa_d} \sqrt{\frac{d+n-2}{n(d-2)}} \log \mathcal{V}_n$$

Dimensional reduction

This controls the KK scale:

$$m_{\mathrm{KK},\,n} \sim \mathcal{V}_n^{-1/n} \sim M_{\mathrm{Pl};\,d}\, e^{-\kappa_d \sqrt{\frac{d+n-2}{n(d-2)}}\hat{\rho}}$$

Dimensional reduction

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Higher dimensional tower:

$$m_0 \sim M_{\mathrm{Pl};\,D}\, e^{-\kappa_D \lambda_D \hat{\phi}} \sim M_{\mathrm{Pl};\,d}\, \exp\left\{-\kappa_d \lambda_D \hat{\phi} - \kappa_d \sqrt{\frac{n}{(d+n-2)(d-2)}}\hat{\rho}\right\}$$

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So that

$$\vec{\zeta}_{\mathrm{KK},n} = \left(0, \sqrt{\frac{d+n-2}{n(d-2)}}\right) \qquad \vec{\zeta}_0 = \left(\lambda_D, \sqrt{\frac{n}{(d+n-2)(d-2)}}\right)$$

Dimensional reduction

It is then evident that:

$$|\vec{\zeta}_{\mathrm{KK},n}|^2 = \frac{d-2+n}{n(d-2)} \qquad |\vec{\zeta}_{\mathrm{osc}}|^2 = \frac{1}{d-2}$$

While neighbouring towers have

$$\vec{\zeta} \cdot \vec{\zeta}' = \frac{1}{d-2}$$

Dimensional reduction

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Generally:

$$\boxed{\vec{\zeta}_a \cdot \vec{\zeta}_b = \frac{1}{d-2} + \frac{1}{n_a}\delta_{ab}}$$

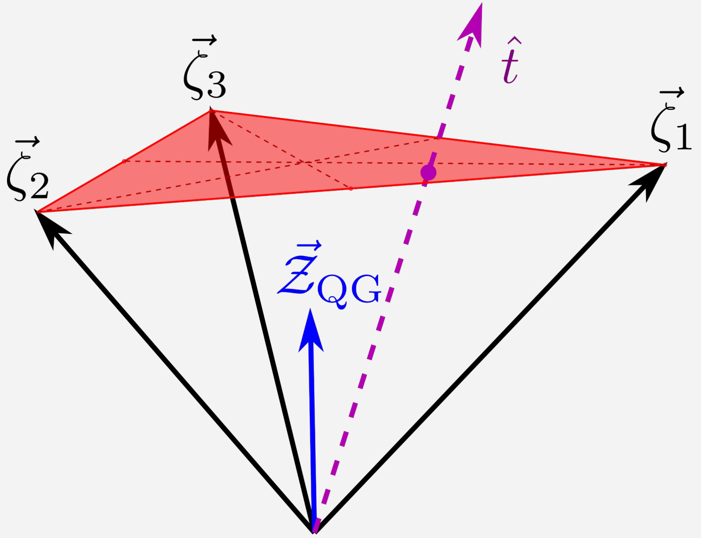
Some assumptions

We can use the above expressions if the following is true:

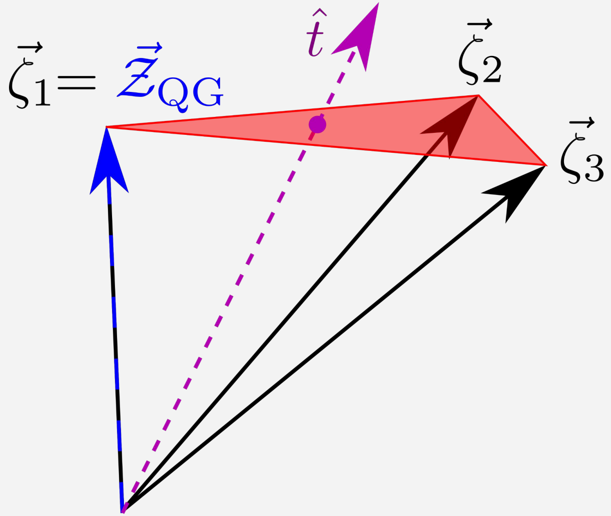
- The **Emergent String Conjecture** holds!
- In **decompactifications limits** the resulting spacetime manifold is **Ricci-flat** except in measure-zero regions (so **no defects** or **running solutions**).
- The above is true in the resulting EFT after decompactification: We can proceed in an **iterative manner**.

Putting things together

Neighboring tower vectors within a **duality frame** (same species scale) form a **frame simplex**:



"Planckian" phase



"Stringy" phase

Some assumptions

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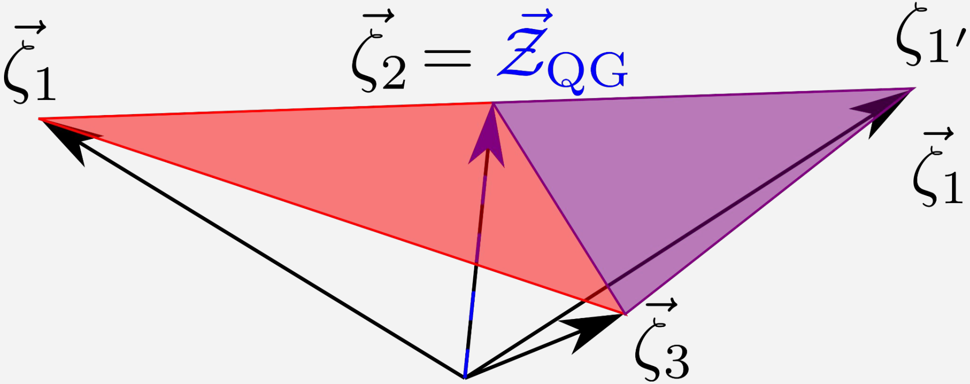
- The above is true in the resulting EFT after decompactification: We can proceed in an **iterative manner**.

In order to be able to glue together the different **frames** we will need

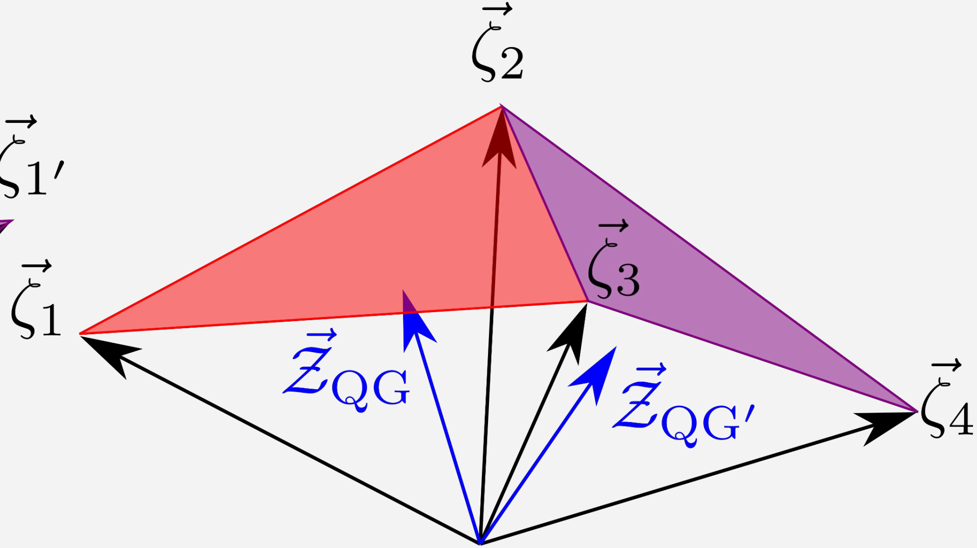
- There is an **asymptotically flat** slice of $\mathcal{M}$ to which the ζ-vectors are **tangent**.
- For generic limits the expression of the leading ζ-vectors is **constant** (so **no sliding**).

Putting things together

Under *relatively mild* assumptions we can **glue together** frame simplices into **frame complex**:



Generalized T-duality



Generalized S-duality

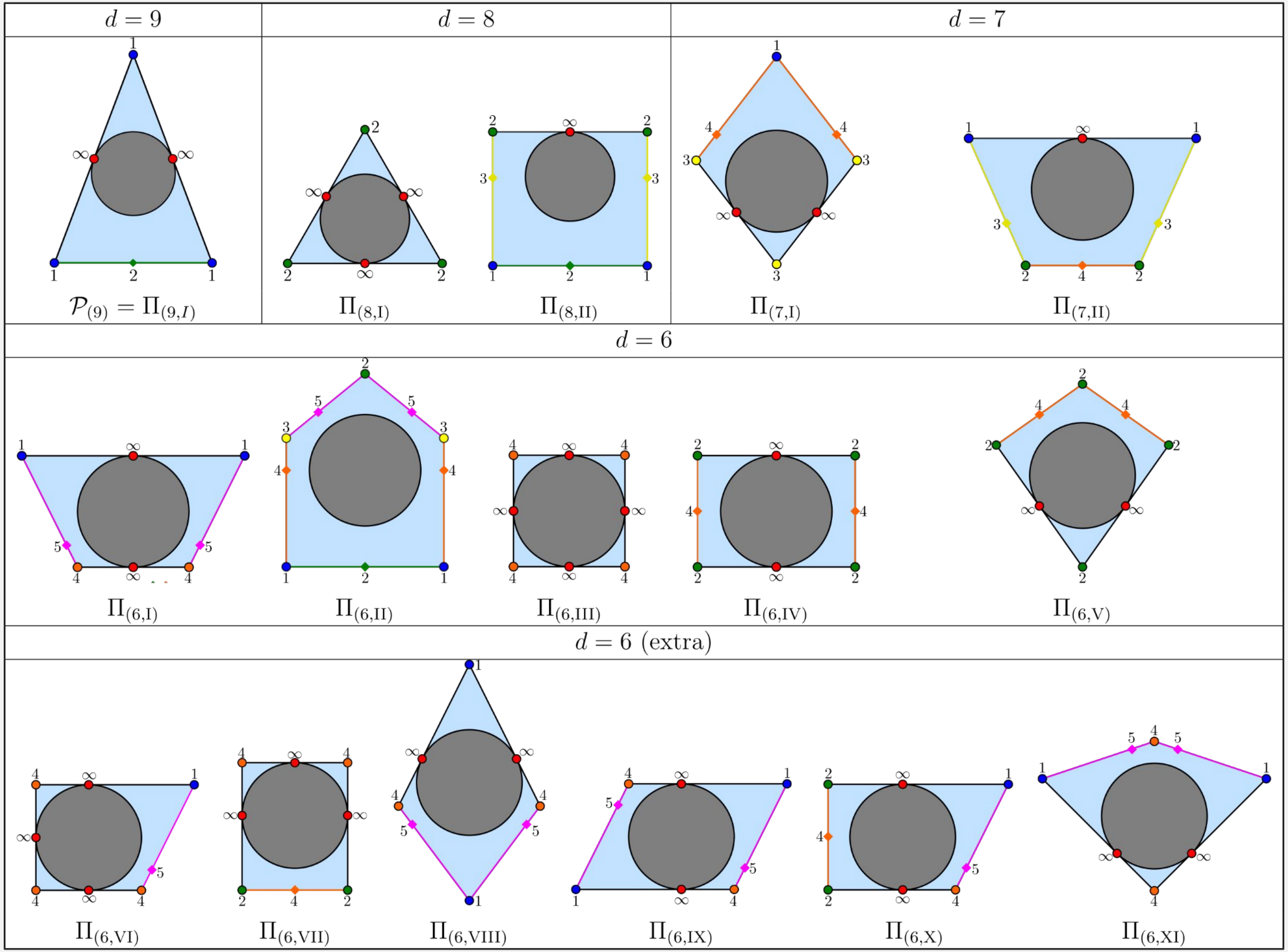
Putting things together

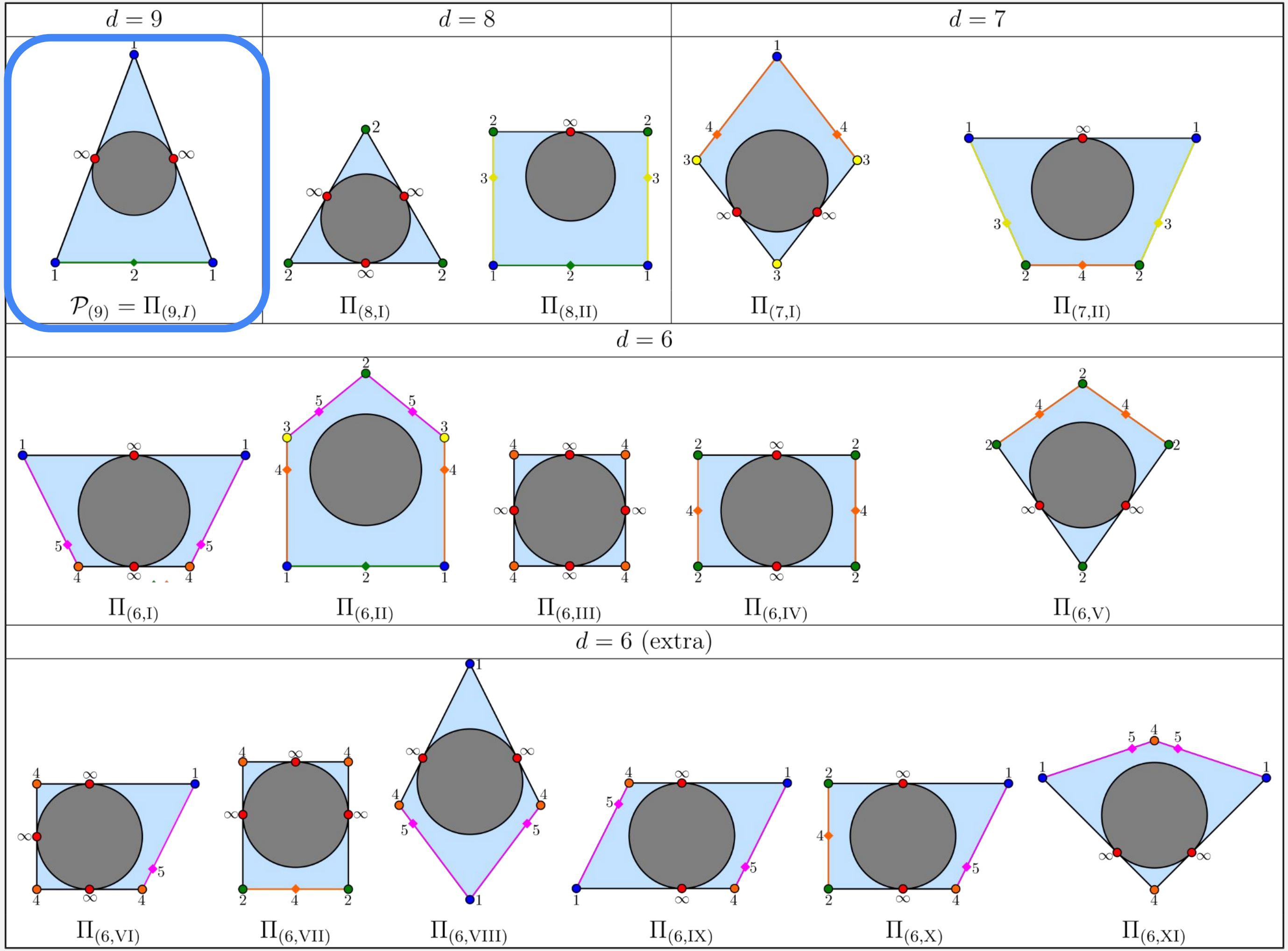
Under *relatively mild* assumptions we can **glue together** frame simplices into **frame complex**:

Finite number of ways of glueing frames together:
FINITE CLASSIFICATION OF FRAME COMPLEXES / GLOBAL DUALITY FRAMES!

Generalized T-duality

Generalized S-duality


$d = 9$
$d = 8$
$d = 7$
$\mathcal{P}_{(9)} = \Pi_{(9,I)}$
$\Pi_{(8,\text{I})}$
$\Pi_{(8,\text{II})}$
$\Pi_{(7,\text{I})}$
$\Pi_{(7,\text{II})}$
$d = 6$
$\Pi_{(6,\text{I})}$
$\Pi_{(6,\text{II})}$
$\Pi_{(6,\text{III})}$
$\Pi_{(6,\text{IV})}$
$\Pi_{(6,\text{V})}$
$d = 6$ (extra)
$\Pi_{(6,\text{VI})}$
$\Pi_{(6,\text{VII})}$
$\Pi_{(6,\text{VIII})}$
$\Pi_{(6,\text{IX})}$
$\Pi_{(6,\text{X})}$
$\Pi_{(6,\text{XI})}$



d = 9
d = 8
d = 7
P(9) = Π(9,I)
Π(8,I)
Π(8,II)
Π(7,I)
Π(7,II)
d = 6
Π(6,I)
Π(6,II)
Π(6,III)
Π(6,IV)
Π(6,V)
d = 6 (extra)
Π(6,VI)
Π(6,VII)
Π(6,VIII)
Π(6,IX)
Π(6,X)
Π(6,XI)

What about fewer supercharges?

What about fewer supercharges?

Laboratory: 9d Heterotic String Theories

Quick Review:
Heterotic string theory on $\mathbb{S}^1$

In d=10 there are 8 theories with rank 16.

Name	Gauge symmetry	$\mathcal{N}$	Tachyons
HE	$E_8 \times E_8 \ltimes \mathbb{Z}_2$	1	0
HO	$\frac{\text{Spin}(32)}{\mathbb{Z}_2}$	1	0
$O(16) \times O(16)$	$\frac{\text{Spin}(16) \times \text{Spin}(16)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	0
U(16)	$\frac{SU(15) \times U(1)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	2
$(E_7 \times SU(2))^2$	$\frac{(E_7 \times SU(2))^2}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	4
$O(24) \times O(8)$	$\frac{\text{Spin}(24) \times \text{Spin}(8)}{\mathbb{Z}_2}$	0	8
$E_8 \times O(16)$	$E_8 \times \text{Spin}(16)$	0	16
$O(32)$	$\text{Spin}(32)$	0	32

Quick Review: Heterotic string theory on $\mathbb{S}^1$

In d=10 there are 8 theories with rank 16.

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HO	$\frac{\mathrm{Spin}(32)}{\mathbb{Z}_2}$	1	0
$O(16) \times O(16)$	$\frac{\mathrm{Spin}(16) \times \mathrm{Spin}(16)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	0
U(16)	$\frac{SU(15) \times U(1)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	2
$(E_7 \times SU(2))^2$	$\frac{(E_7 \times SU(2))^2}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	4
$O(24) \times O(8)$	$\frac{\mathrm{Spin}(24) \times \mathrm{Spin}(8)}{\mathbb{Z}_2}$	0	8
$E_8 \times O(16)$	$E_8 \times \mathrm{Spin}(16)$	0	16
$O(32)$	$\mathrm{Spin}(32)$	0	32

Quick Review:
Heterotic string theory on $\mathbb{S}^1$

In d=10 there are 8 theories with rank 16.

Compactifying on $\mathbb{S}^1$ one obtains a 9d theory with rank 17, 18 moduli $(R, \phi, \vec{A})$ and

$$\mathcal{M}_{\text{SUSY}} = O(\Gamma_{17,1})\backslash O(17,1)/O(17) \times \mathbb{R}^+$$

$$\mathcal{M}_{\not{\text{SUSY}}} = O(\Upsilon_{17,1})\backslash O(17,1)/O(17) \times \mathbb{R}^+$$

$$\mathsf{G}_{\text{cl}} = \text{Diag}\left\{\frac{1}{2}, \frac{8}{7}R^{-2}, \frac{\alpha' e^{-\phi}}{2R^2}, \dots, \frac{\alpha' e^{-\phi}}{2R^2}\right\}$$

Non flat + Compact moduli!

Name	Gauge symmetry	$\mathcal{N}$	Tachyons
HE	$E_8 \times E_8 \ltimes \mathbb{Z}_2$	1	0
HO	$\frac{\text{Spin}(32)}{\mathbb{Z}_2}$	1	0
$O(16) \times O(16)$	$\frac{\text{Spin}(16) \times \text{Spin}(16)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	0
U(16)	$\frac{SU(15) \times U(1)}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	2
$(E_7 \times SU(2))^2$	$\frac{(E_7 \times SU(2))^2}{\mathbb{Z}_2} \ltimes \mathbb{Z}_2$	0	4
$O(24) \times O(8)$	$\frac{\text{Spin}(24) \times \text{Spin}(8)}{\mathbb{Z}_2}$	0	8
$E_8 \times O(16)$	$E_8 \times \text{Spin}(16)$	0	16
$O(32)$	$\text{Spin}(32)$	0	32

Quick Review: Heterotic string theory on $\mathbb{S}^1$

Worldsheet perturbative states:

$$\frac{m}{M_{\mathrm{Pl},9}} = \frac{|(R^2 e^{\phi/2} + \frac{1}{2}\vec{A}\cdot\vec{A})w - n + \vec{\pi}\cdot\vec{A}|}{R^{8/7}}$$

Particularly $\frac{m_{\mathrm{KK}}}{M_{\mathrm{Pl},9}} \sim \frac{|n - \vec{\pi}\cdot\vec{A}|}{R^{8/7}}$ (more complicated for winding modes).

Quick Review: Heterotic string theory on $\mathbb{S}^1$

Worldsheet perturbative states:

$$\frac{m}{M_{\mathrm{Pl},9}} = \frac{|(R^2 e^{\phi/2} + \frac{1}{2}\vec{A}\cdot\vec{A})w - n + \vec{\pi}\cdot\vec{A}|}{R^{8/7}}$$

Particularly $\frac{m_{\mathrm{KK}}}{M_{\mathrm{Pl},9}} \sim \frac{|n - \vec{\pi}\cdot\vec{A}|}{R^{8/7}}$ (more complicated for winding modes).

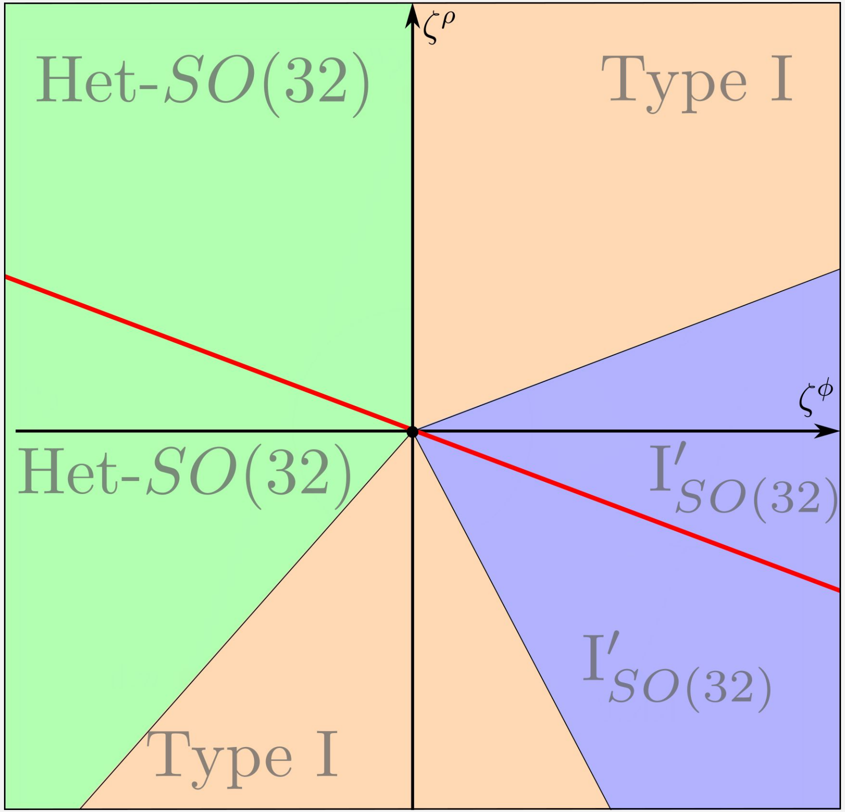
As well as string oscillator modes

$$\frac{m_{\mathrm{osc}}}{M_{\mathrm{Pl},9}} \sim e^{\phi/4} R^{-1/7}$$

4. SUSY Heterotic on $\mathbb{S}^1$ (16 Supercharges)

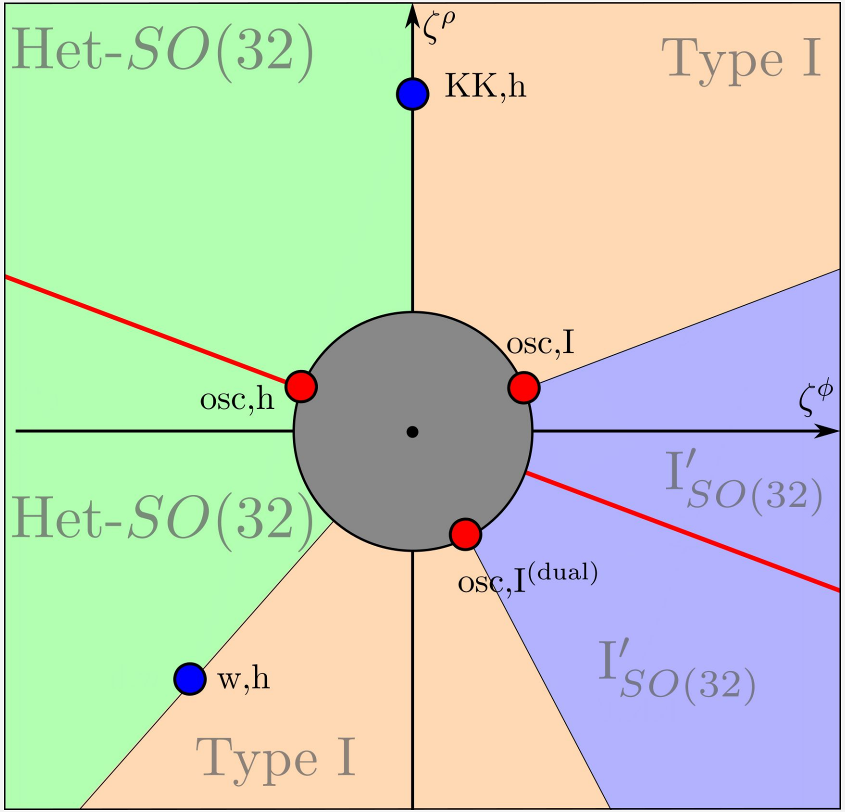
[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela, **2306.16440**]

$SO(32)$ heterotic on $\mathbb{S}^1$

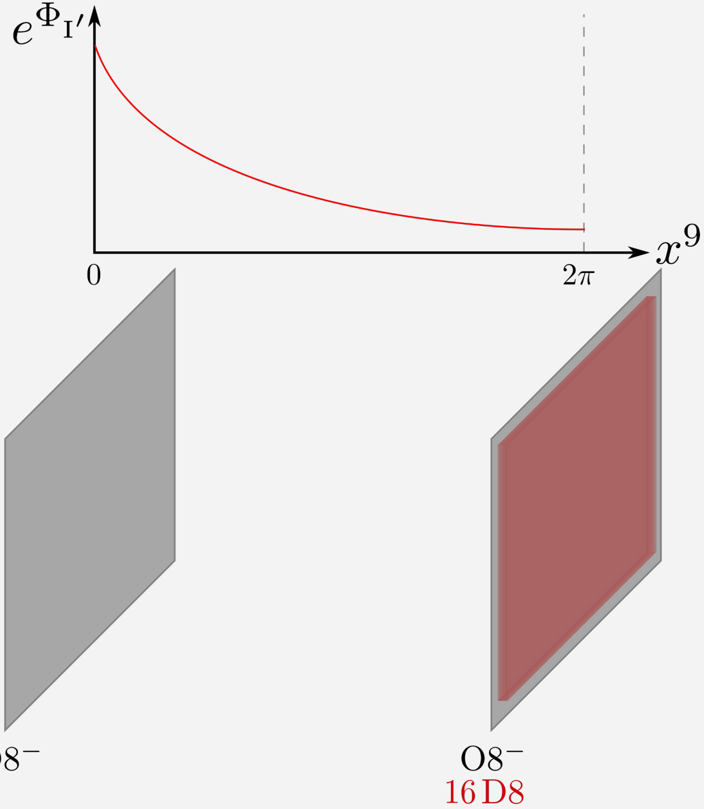


[Polchinski, Witten,'96]

$SO(32)$ heterotic on $\mathbb{S}^1$

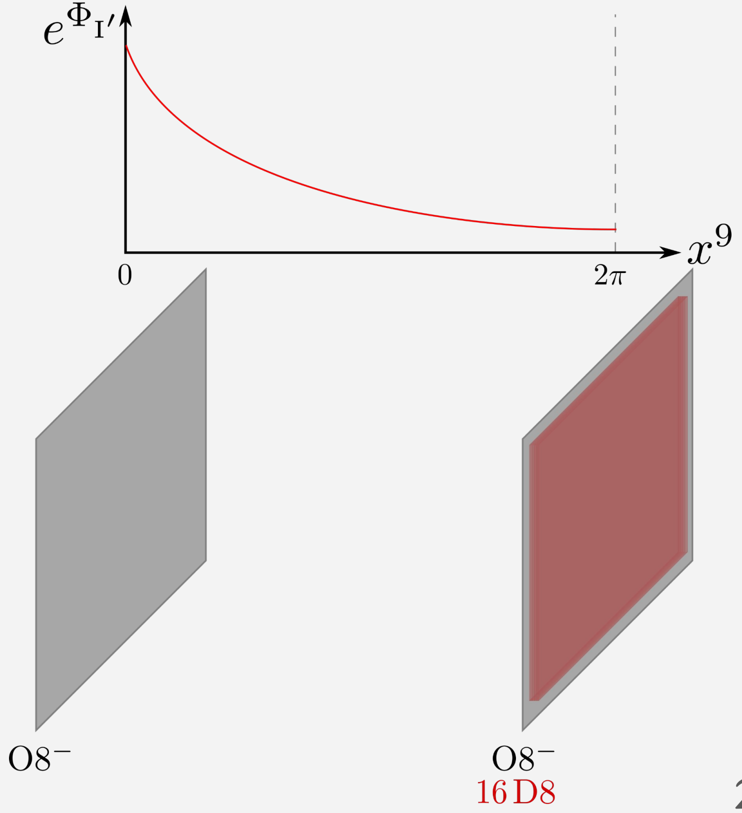


$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame



$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame

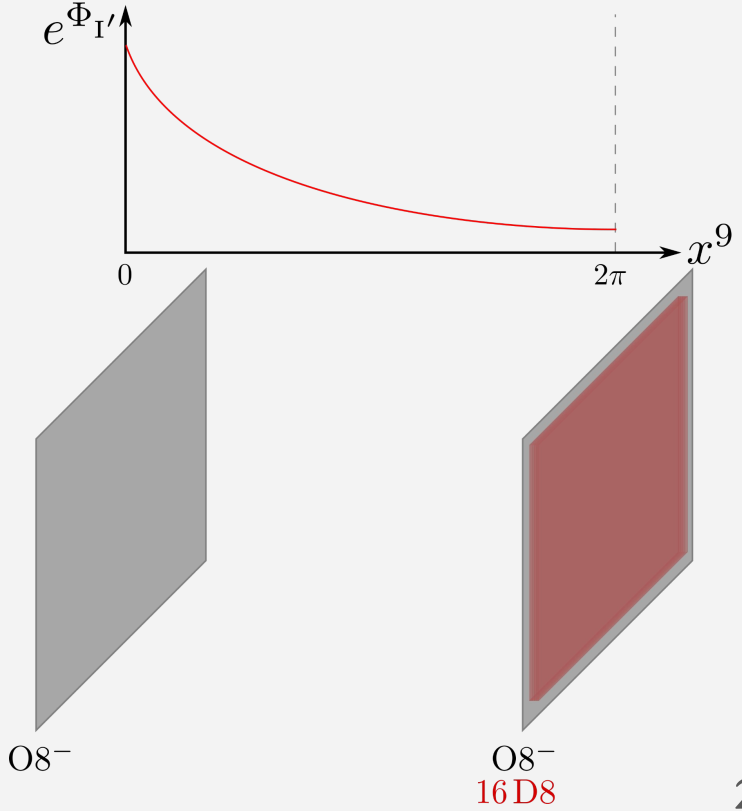
When moving **parallel** to the self-dual line we do **NOT** decompactify to a vacuum but to a running solution: **Massive Type IIA**.



$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame

When moving **parallel** to the self-dual line we do **NOT** decompactify to a vacuum but to a running solution: **Massive Type IIA**.

- The emergent string tower does not become light.



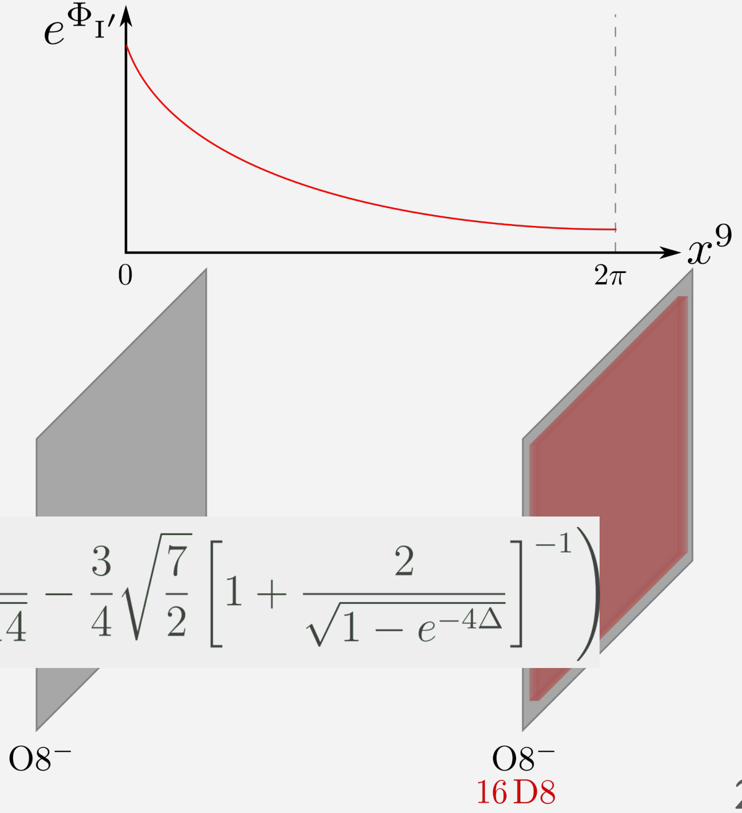
$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame

When moving **parallel** to the self-dual line we do **NOT** decompactify to a vacuum but to a running solution: **Massive Type IIA**.

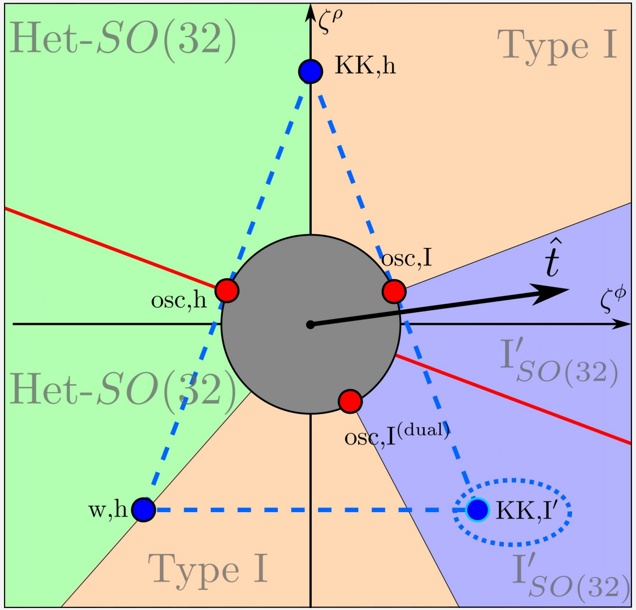
- The emergent string tower does not become light.
- The expression $m_{\mathrm{KK,I'}}(\vec{\phi})$ is not homogeneous:

$$\vec{\zeta}_{\mathrm{KK,I'}} = \left(\frac{5}{4\sqrt{2}} + \frac{3}{4\sqrt{2}} \left[1 + \frac{2}{\sqrt{1-e^{-4\Delta}}} \right]^{-1}, -\frac{5}{4\sqrt{14}} - \frac{3}{4}\sqrt{\frac{7}{2}} \left[1 + \frac{2}{\sqrt{1-e^{-4\Delta}}} \right]^{-1} \right)$$

depends on distance Δ to self-dual line.

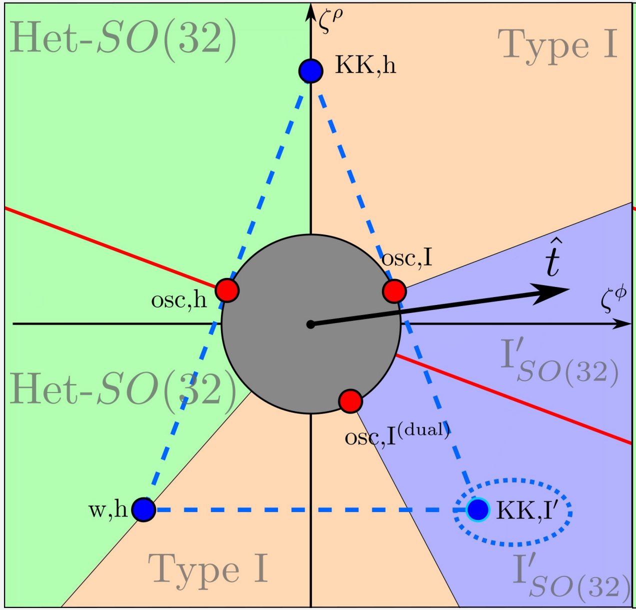


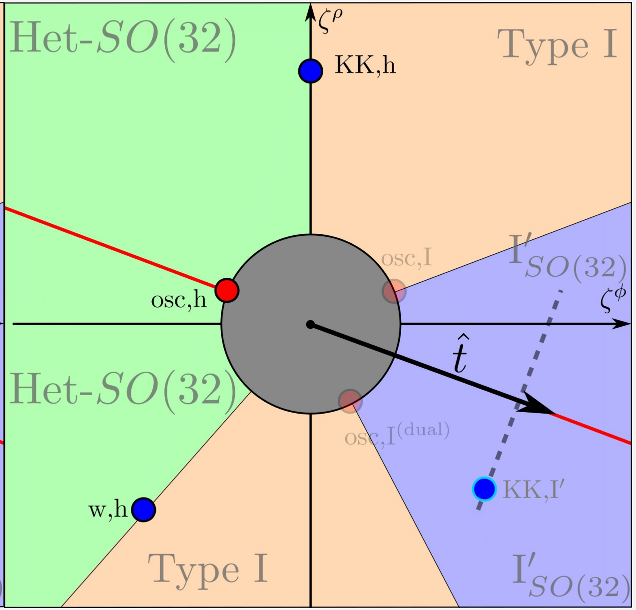
$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame



[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

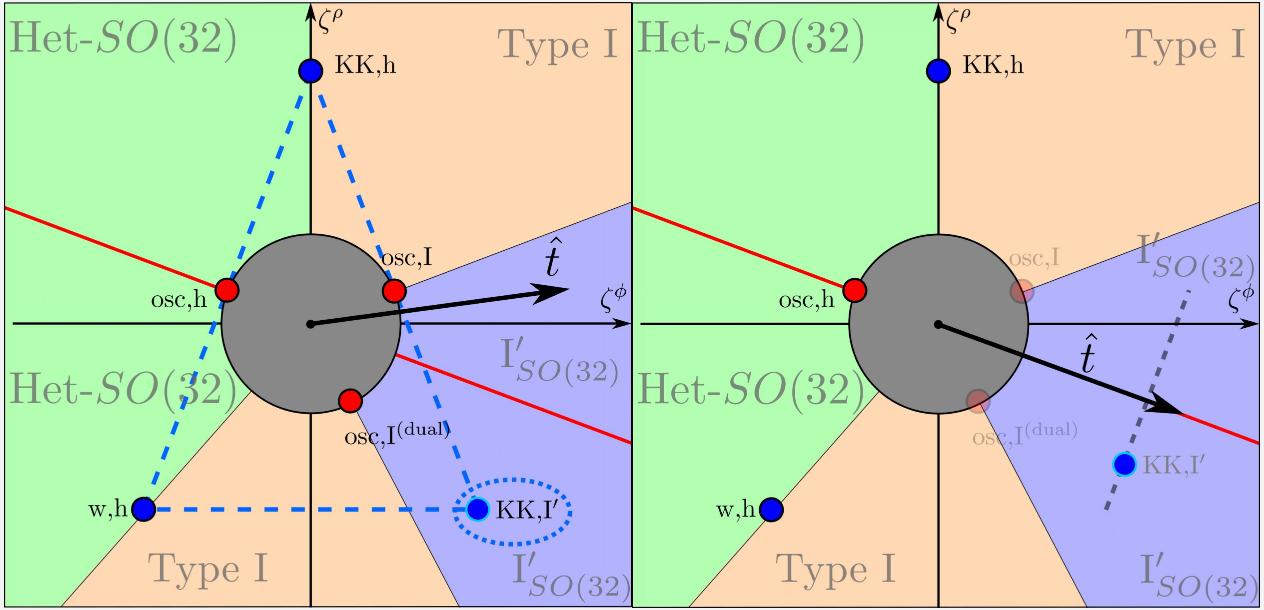
$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame





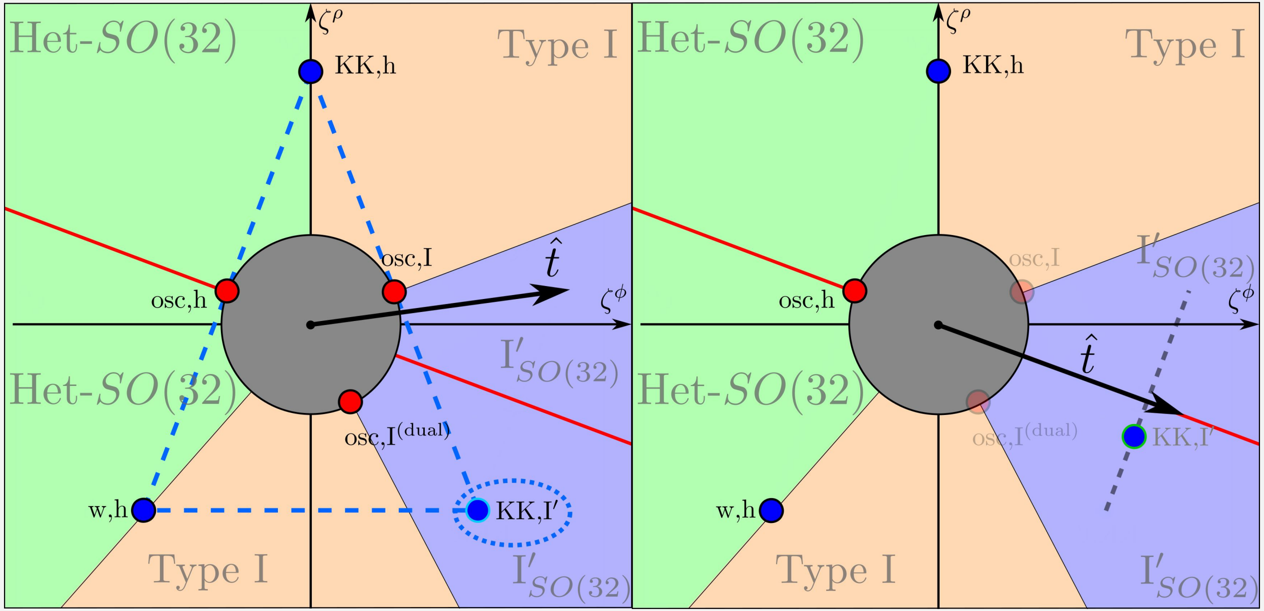
[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame



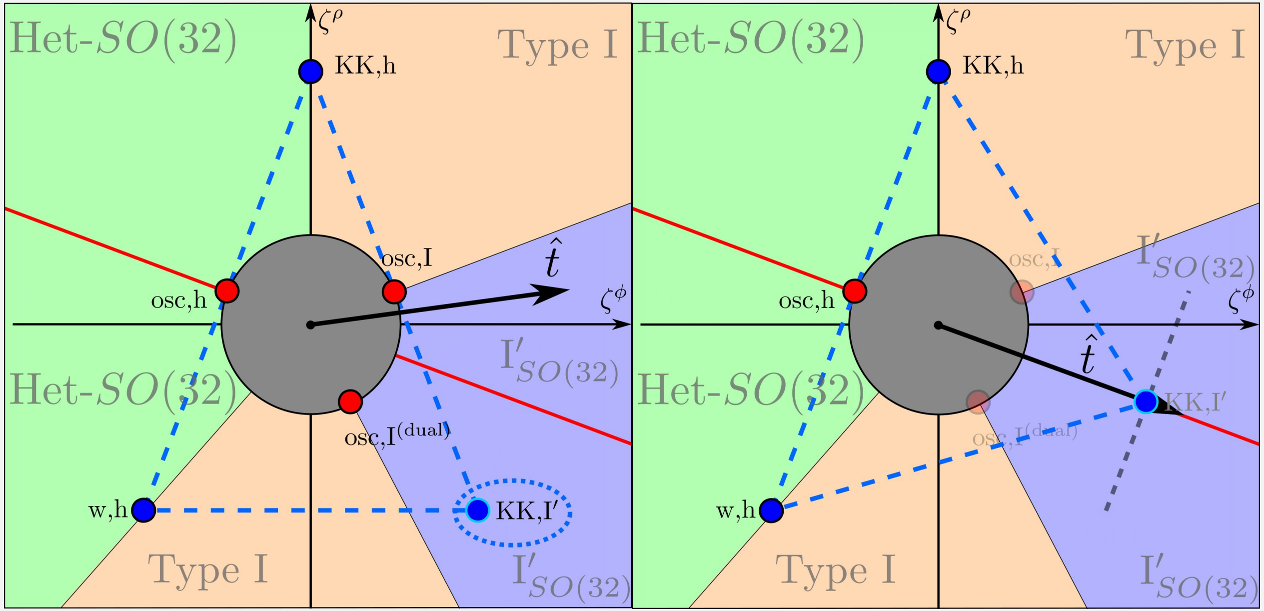
[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame

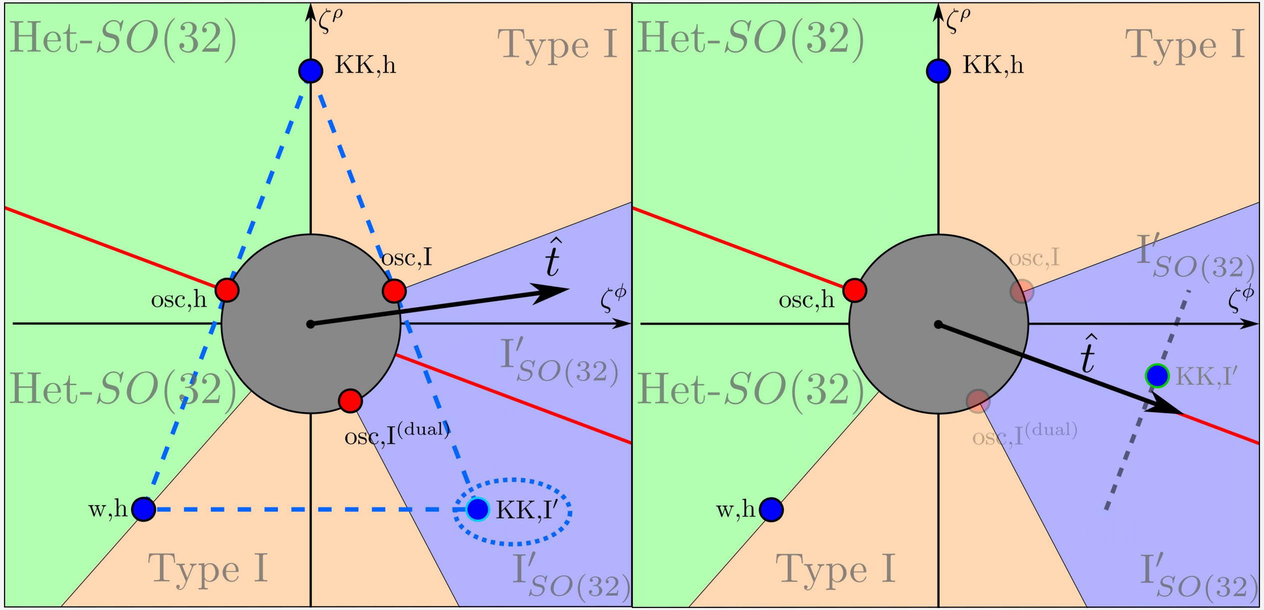


[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

$SO(32)$ heterotic on $\mathbb{S}^1$: Type I' frame

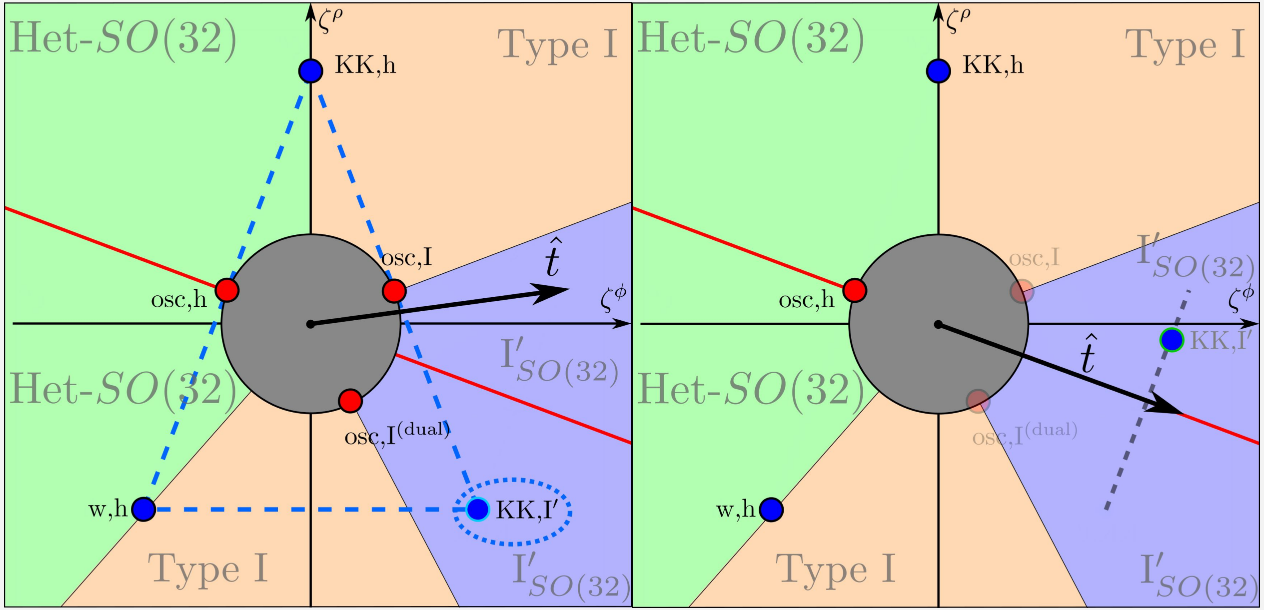


[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]



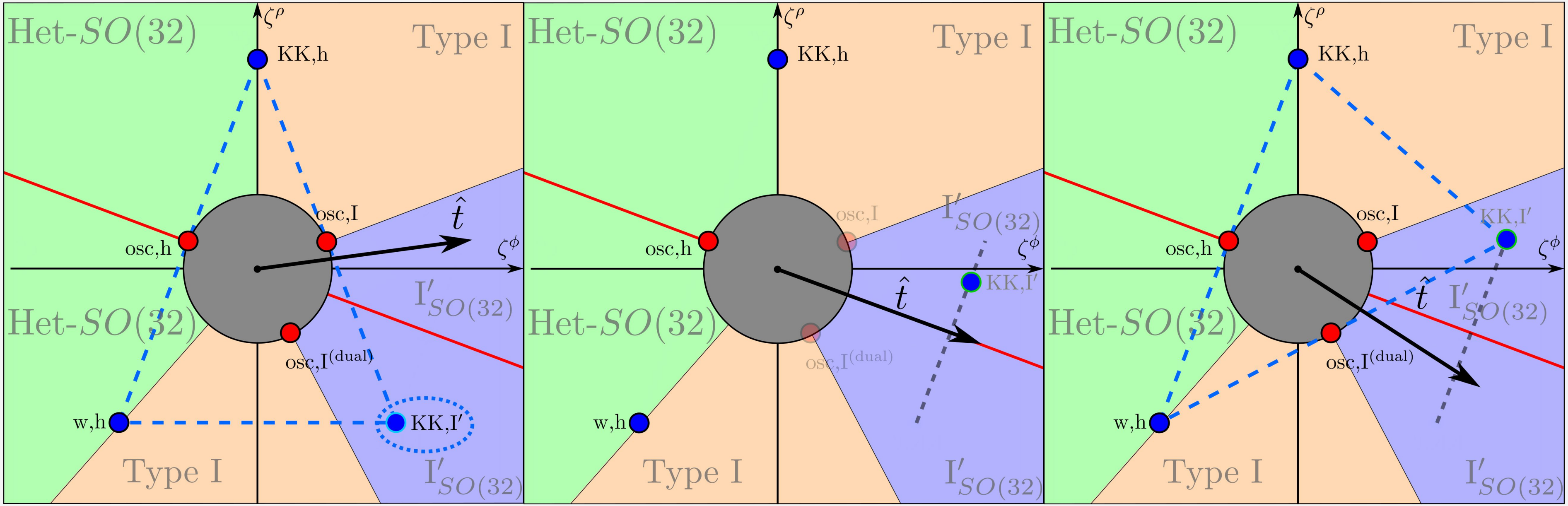
[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame



[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

$SO(32)$ heterotic on $\mathbb{S}^1$:Type I' frame

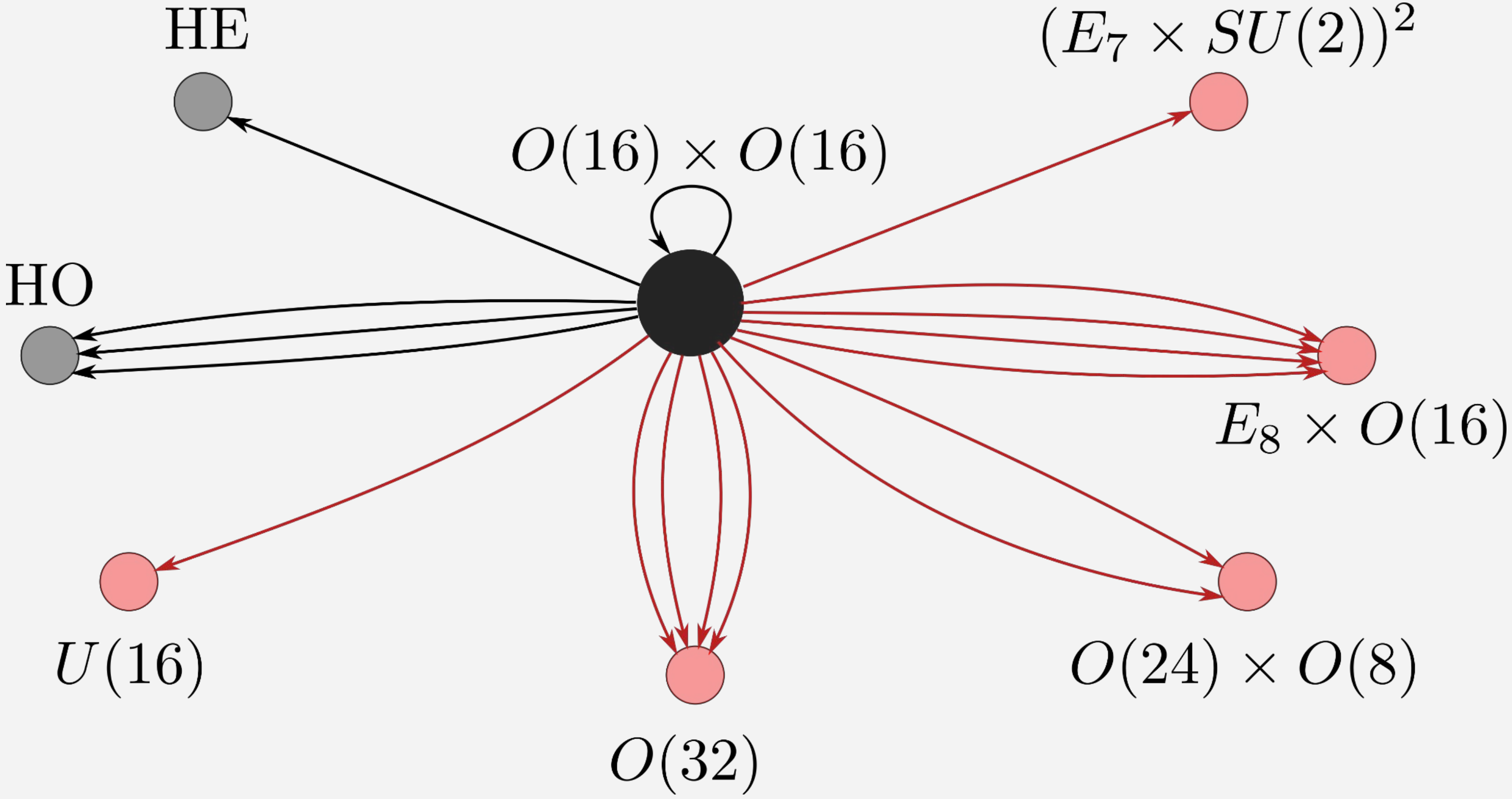


[Etheredge, Heidenreich, MacNamara, Rudelius, **I.R.**. Valenzuela,'23]

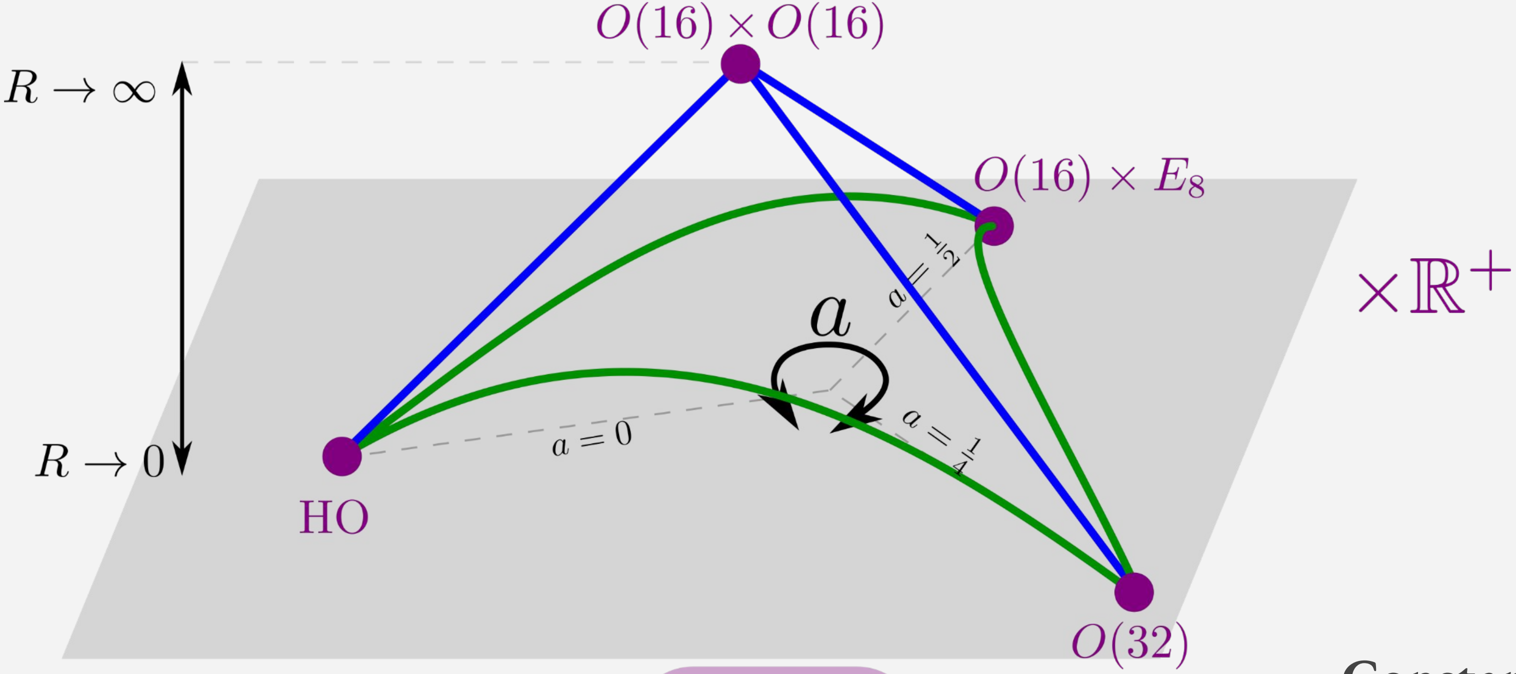
5. ~~SUSY~~ Heterotic on $\mathbb{S}^1$ (0 Supercharges)

[Fraiman, **I.R.**. Valenzuela, *WIP*]

Compactifying O(16)×O(16) on $\mathbb{S}^1$



Compactifying O(16)×O(16) on $\mathbb{S}^1$



$\infty(\mathcal{M})$

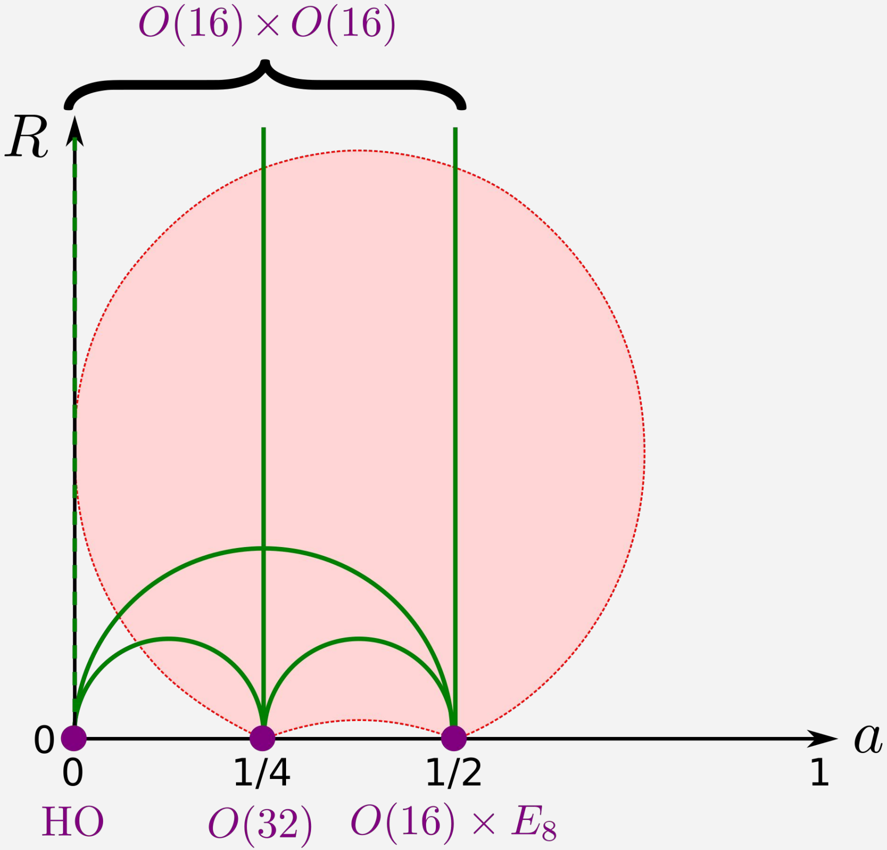
Constant ϕ and

$$\vec{A} = (0^7, 1, a^7, 1 - a),\ a \in [0, 1]$$

Compactifying O(16)×O(16) on $\mathbb{S}^1$

Slice with constant ϕ and

$$\vec{A} = (0^7, 1, a^7, 1 - a),\ \ a \in [0, 1]$$



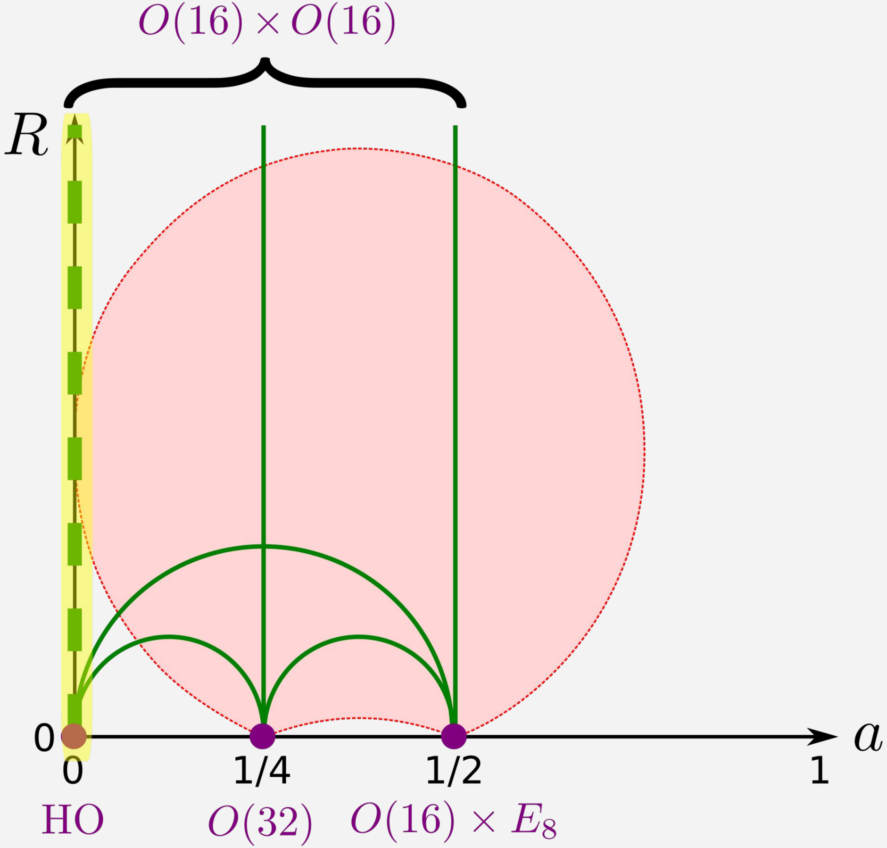
Compactifying O(16)×O(16) on $\mathbb{S}^1$

Slice with constant ϕ and

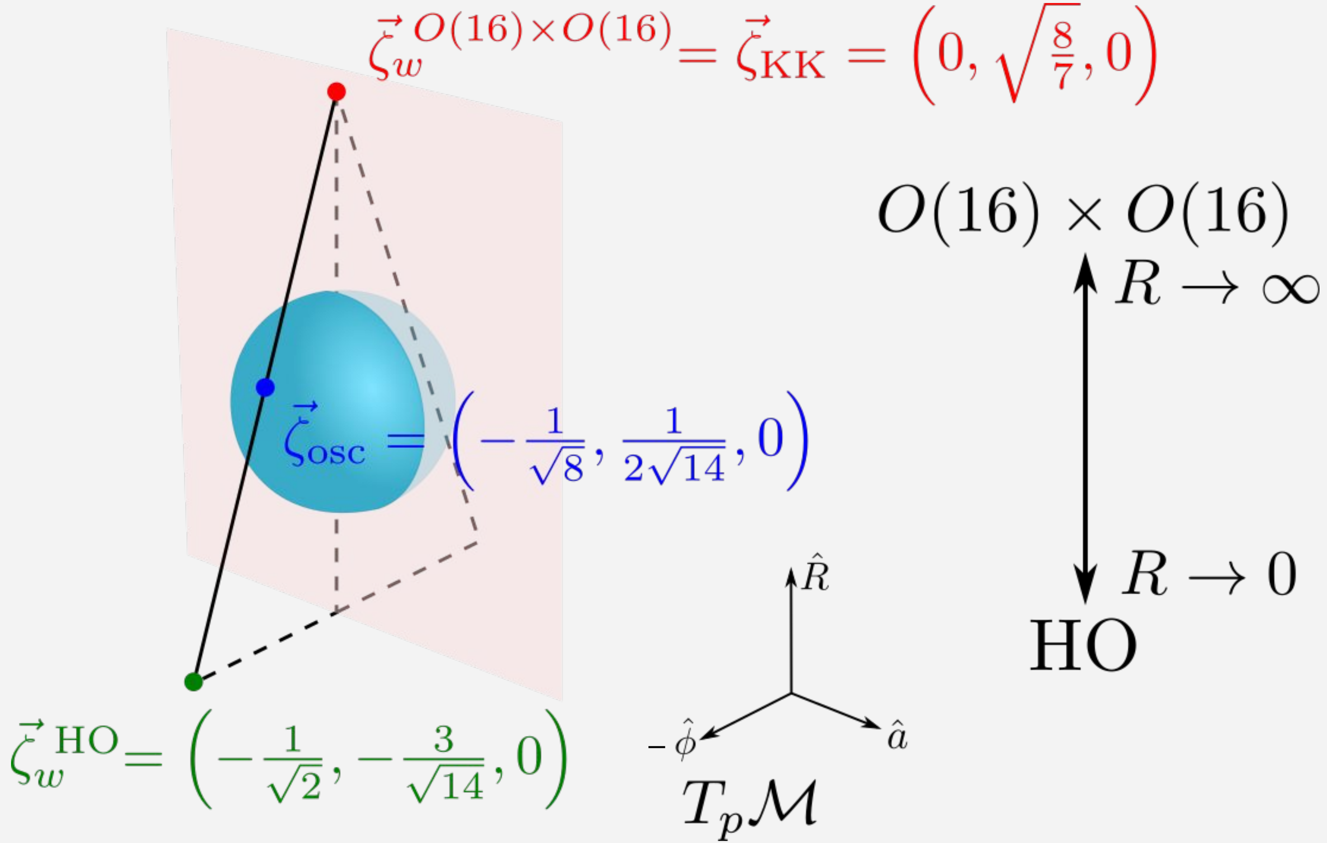
$$\vec{A} = (0^7, 1, a^7, 1-a), \;\; a \in [0,1]$$

Interpolation HO⇄ O(16)×O(16):

$$\gamma(a) = (\phi_0, R, 0), \;\; R \in [0, \infty]$$



Interpolating mode: HO⇄ O(16)×O(16)



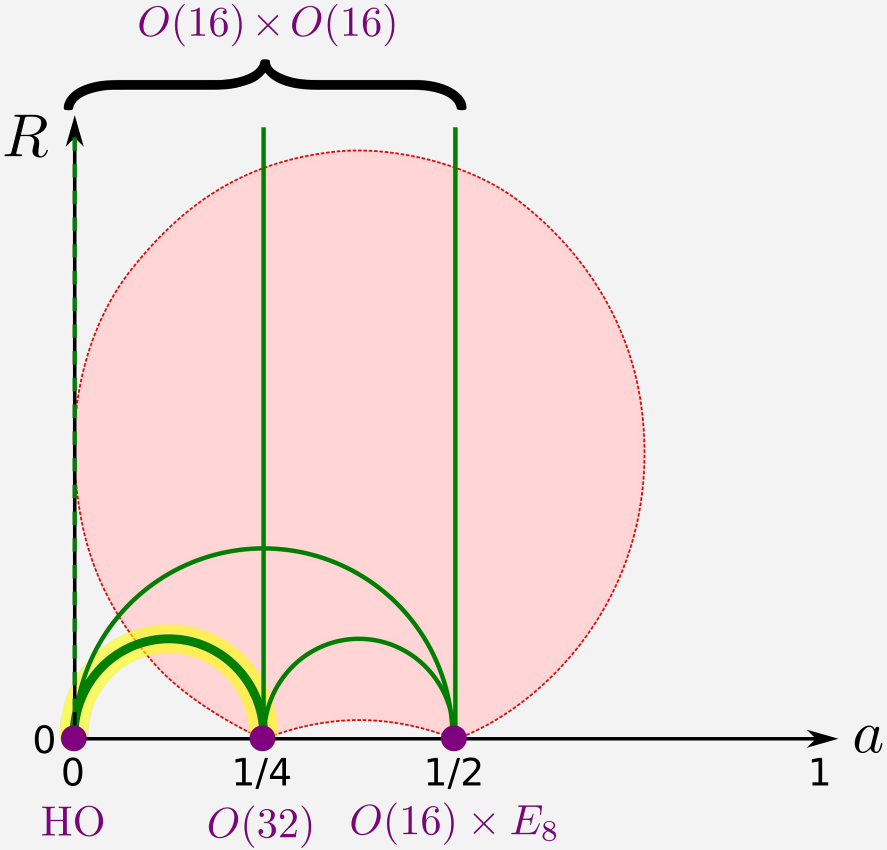
Compactifying O(16)×O(16) on $\mathbb{S}^1$

Slice with constant ϕ and

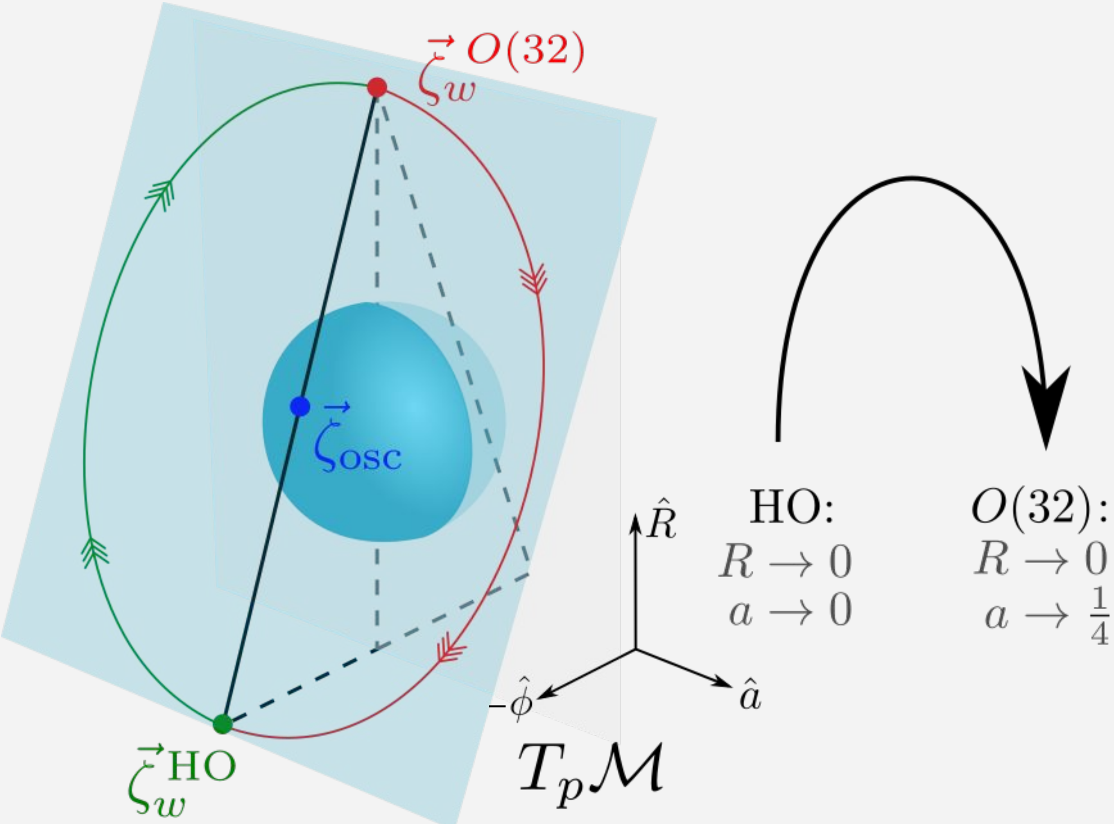
$$\vec{A} = (0^7, 1, a^7, 1-a),\ \ a \in [0,1]$$

Geodesic interpolating HO⇄O(32):

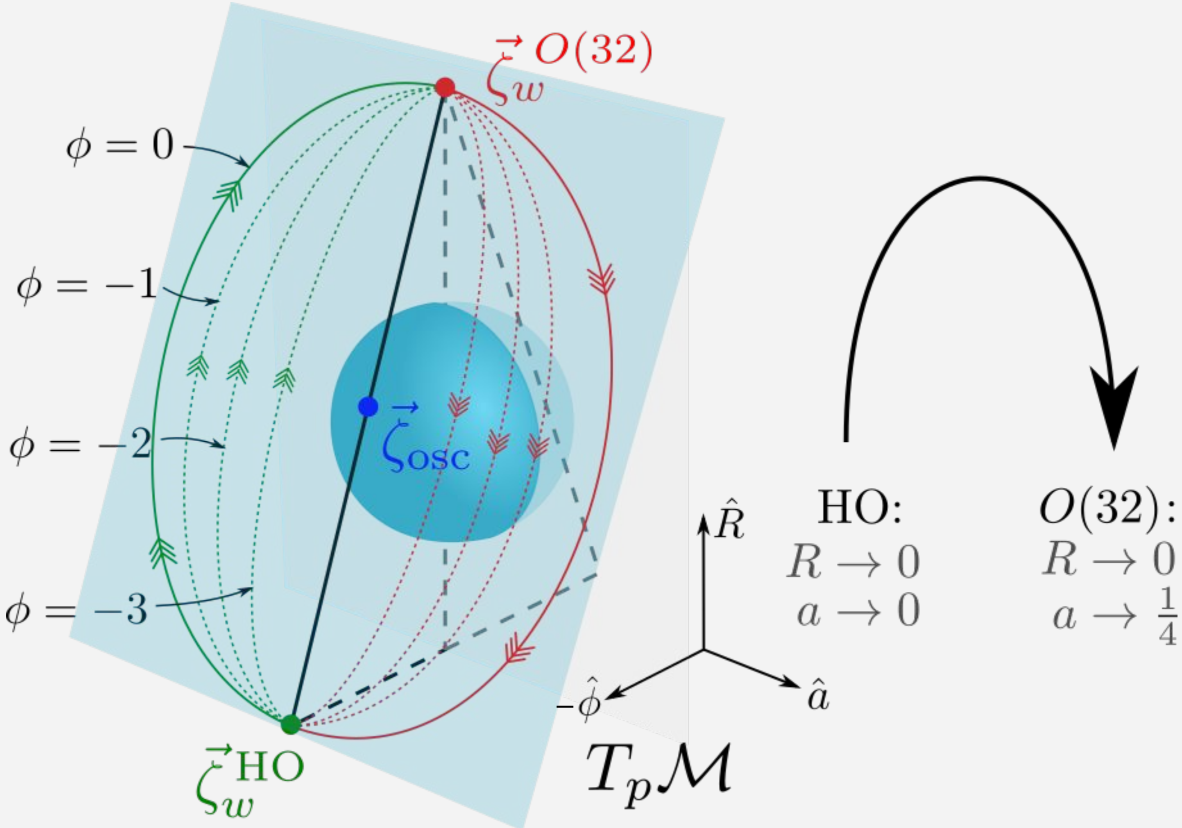
$$\gamma(a) = (\phi_0, \sqrt{a-4a^2}, a),\ \ a \in \left[0, \frac{1}{4}\right]$$

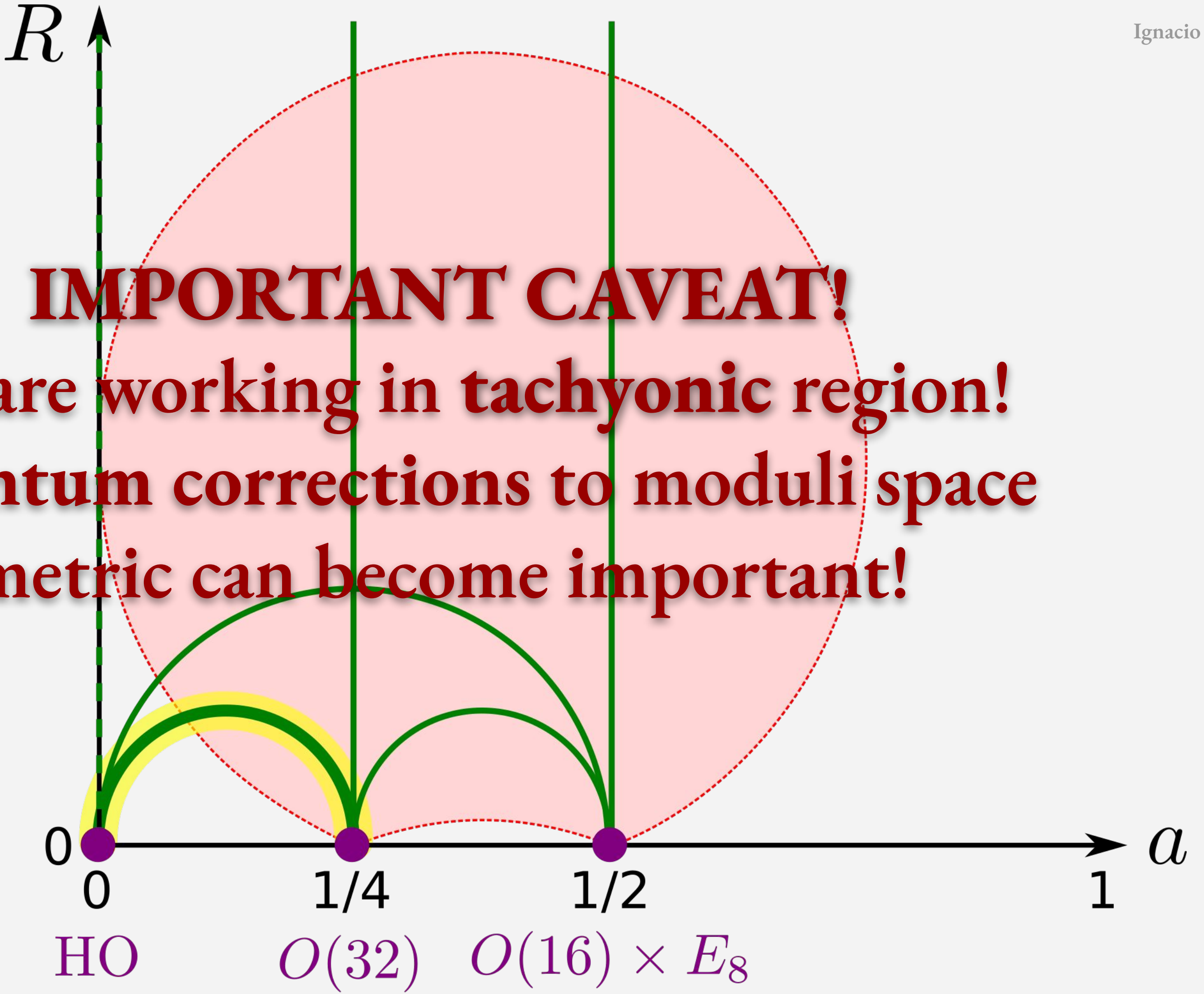


Interpolating mode: HO⇄O(32)



Interpolating mode: HO⇄O(32)





IMPORTANT CAVEAT!

We are working in **tachyonic** region!

Quantum corrections to moduli space metric can become important!

6. Conclusions & Outlook

Conclusions and Outlook

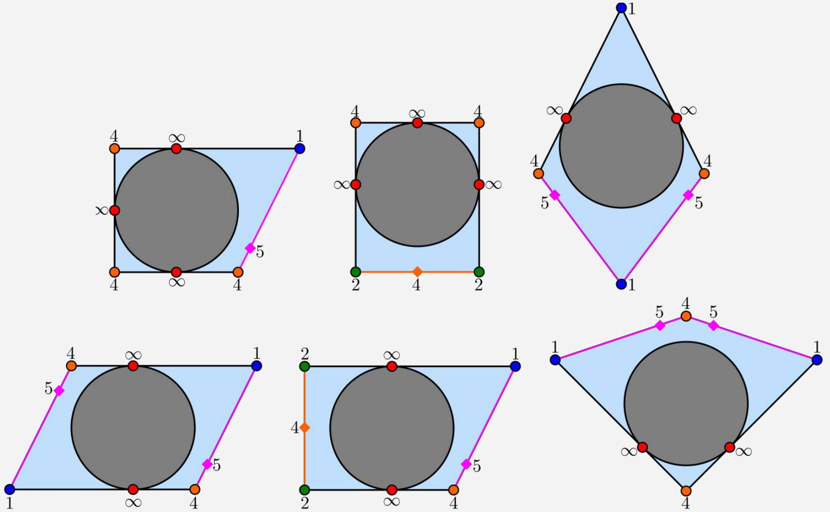
- Using **Emergent String Conjecture** (only KK or strings as leading towers) we can derive a set of rules that constrain the behavior of both towers of light states and species scale for generic **infinite distance limits**.
- Under additional assumptions this rules can be applied globally to produce a **systematic classification** of a **finite** list of possibilities of **duality frame/tower** arrangement.
- **Generalization** to broader settings with lower dimensions and less supercharges:
    - Possibility of **sliding**.
    - **Non-flat** moduli spaces → Local frames?
    - Inclusion of axions and other **compact scalars**.

} Can we fully understand all possibilities?

Open problems

Some examples not observed:

- Some featuring **asymmetric** KK vertices with respect to string oscillator modes: Some **worldsheet** CFT argument to **exclude** this?
- Others seem fine but have not been observed: **Unobserved corner of Landscape?**



Take home message:

There is much we can learn about **duality frames** and **global** geometry of moduli spaces from Swampland principles.

Emergent String Conjecture powerful enough to highly constrain possible limits, even in multi-field settings and less supercharges.

Continuous families of infinite distance limits can be sorted into a highly constrained **discrete set of duality frames** sharing a common perturbative limit of specific validity range.

Other work: [Bedroya, Hamada,'23], [van de Heisteeg,'24]

Thank you!