STRIPPER – **S**ec**T**o**R** **I**mproved **P**hase s**P**ac**E** for real **R**adiation

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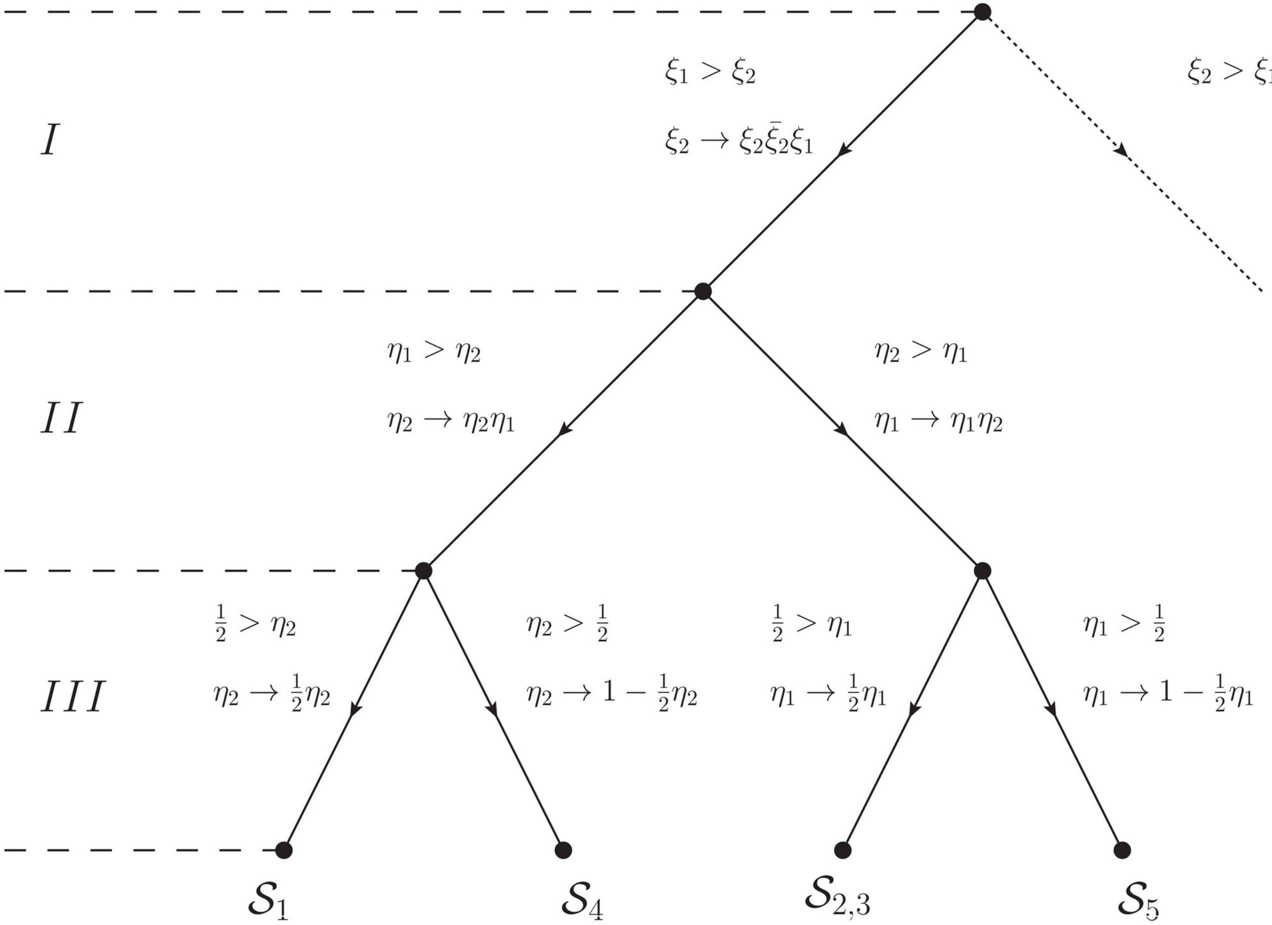
Based on:

A novel subtraction scheme for double-real radiation at NNLO
Czakon [1005.0274]

Four-dimensional formulation of the sector-improved residue subtraction scheme
Czakon, Heymes [1408.2500]

Single-jet inclusive rates with exact color at $O(\alpha_s^4)$
Czakon, Hameren, Mitov, Poncelet [1907.12911]

Triple-collinear sector decomposition



Phase space mappings triple-collinear

Initial-state reference

$$(P - r - u_1 - u_2)^2 = (P - \tilde{r})^2 \,,$$

$$x = \frac{P \cdot r}{(P - r) \cdot (r + u_1 + u_2) - u_1 \cdot u_2} \,,$$

$$x = \frac{P \cdot (\tilde{r} - u_1 - u_2) + u_1 \cdot u_2}{(P - u_1 - u_2) \cdot \tilde{r}} \,,$$

$$(u_1^0)_{\max} = \frac{P \cdot \tilde{r}}{P \cdot \hat{u}_1} \,,$$

$$(u_2^0)_{\max} = \frac{P \cdot (\tilde{r} - u_1)}{(P - u_1) \cdot \hat{u}_2} \,,$$

$$\mathcal{J} = \frac{x^{d-3}\, P \cdot \tilde{r}}{(P - u_1 - u_2) \cdot \tilde{r}} \,,$$

Initial-state reference

$$(r + p - u_1 - u_2)^2 = (\tilde{r} + p)^2 \,,$$

$$z = \frac{(r + p) \cdot (r - u_1 - u_2) + u_1 \cdot u_2}{p \cdot r} \,,$$

$$z = \frac{(p - u_1 - u_2) \cdot \tilde{r}}{p \cdot (\tilde{r} + u_1 + u_2) - u_1 \cdot u_2} \,,$$

$$(u_1^0)_{\max} = (1 - \tilde{x}) \frac{p \cdot \tilde{r}/\tilde{x}}{(\tilde{r}/\tilde{x} + p) \cdot \hat{u}_1} \,,$$

$$(u_2^0)_{\max} = \frac{(\tilde{r}/\tilde{x} + p) \cdot \big((1 - \tilde{x})\, \tilde{r}/\tilde{x} - u_1\big)}{(\tilde{r}/\tilde{x} + p - u_1) \cdot \hat{u}_2} \,,$$

$$\mathcal{J} = \frac{p \cdot \tilde{r}}{(p - u_1 - u_2) \cdot \tilde{r}} \,.$$

Phase space mappings double-collinear

final-final:

$$\left(P - r_1 - u_1 - r_2 - u_2\right)^2 = \left(P - \tilde{r}_1 - \tilde{r}_2\right)^2 ,$$

$$x_1 = \frac{P \cdot r_1}{(P - u_1) \cdot (r_1 + u_1)} ,$$

$$x_1 = \frac{P \cdot (\tilde{r}_1 - u_1)}{(P - u_1) \cdot \tilde{r}_1} ,$$

$$(u_1^0)_{\max} = \frac{P \cdot \tilde{r}_1}{P \cdot \hat{u}_1} ,$$

$$\mathcal{J} = \frac{x_1^{d-3}\, P \cdot \tilde{r}_1}{(P - u_1) \cdot \tilde{r}_1} \frac{x_2^{d-3}\, (P - \tilde{r}_1) \cdot \tilde{r}_2}{(P - x_1\tilde{r}_1 - u_1 - u_2) \cdot \tilde{r}_2} ,$$

$$\left(P - r_1 - u_1\right)^2 = \left(P - \tilde{r}_1\right)^2 ,$$

$$x_2 = \frac{(P - r_1/x_1) \cdot r_2}{(P - r_1 - u_1 - u_2) \cdot (r_2 + u_2)} ,$$

$$x_2 = \frac{(P - \tilde{r}_1) \cdot \tilde{r}_2 - (P - x_1\tilde{r}_1 - u_1) \cdot u_2}{(P - x_1\tilde{r}_1 - u_1 - u_2) \cdot \tilde{r}_2} ,$$

$$(u_2^0)_{\max} = \frac{(P - \tilde{r}_1) \cdot \tilde{r}_2}{(P - x_1\tilde{r}_1 - u_1) \cdot \hat{u}_2} ,$$

Phase space mappings double-collinear

final-initial:

$$(p + r_2 - r_1 - u_1 - u_2)^2 = (p + \tilde{r}_2 - \tilde{r}_1)^2 \,,$$

$$z_2 = \frac{(p - r_1 - u_1 - u_2) \cdot (r_2 - u_2)}{(p - r_1 - u_1) \cdot r_2} \,,$$

$$x_1 = \frac{(p + \tilde{r}_2) \cdot (\tilde{r}_1 - u_1)}{(p + \tilde{r}_2 - u_1) \cdot \tilde{r}_1} \,,$$

$$(u_1^0)_{\max} = \frac{(p + \tilde{r}_2) \cdot \tilde{r}_1}{(p + \tilde{r}_2) \cdot \hat{u}_1} \,,$$

$$\mathcal{J} = \frac{x_1^{d-3}\,(p + \tilde{r}_2) \cdot \tilde{r}_1}{(p + \tilde{r}_2 - u_1) \cdot \tilde{r}_1} \frac{(p - x_1\tilde{r}_1 - u_1) \cdot \tilde{r}_2}{(p - x_1\tilde{r}_1 - u_1 - u_2) \cdot \tilde{r}_2} \,.$$

$$(p + \tilde{r}_2 - r_1 - u_1)^2 = (p + \tilde{r}_2 - \tilde{r}_1)^2 \,,$$

$$x_1 = \frac{(p + z_2 r_2) \cdot r_1}{(p + z_2 r_2 - u_1) \cdot (r_1 + u_1)} \,,$$

$$z_2 = \frac{(p - x_1\tilde{r}_1 - u_1 - u_2) \cdot \tilde{r}_2}{(p - x_1\tilde{r}_1 - u_1) \cdot (\tilde{r}_2 + u_2)} \,,$$

$$(u_2^0)_{\max} = (1 - \tilde{x}_2)\frac{(p - x_1\tilde{r}_1 - u_1) \cdot \tilde{r}_2/\tilde{x}_2}{(p + \tilde{r}_2/\tilde{x}_2 - x_1\tilde{r}_1 - u_1) \cdot \hat{u}_2} \,,$$

Phase space mappings double-collinear

initial-final:

$$\big(r_1+p-u_1-r_2-u_2\big)^2 = \big(\tilde{r}_1+p-\tilde{r}_2\big)^2 \,, \qquad \big(r_1+p-u_1\big)^2 = \big(\tilde{r}_1+p\big)^2 \,,$$

$$z_1 = \frac{(r_1+p)\cdot(r_1-u_1)}{p\cdot r_1} \,, \qquad x_2 = \frac{(z_1 r_1+p)\cdot r_2}{(r_1+p-u_1-u_2)\cdot(r_2+u_2)} \,,$$

$$z_1 = \frac{(p-u_1)\cdot\tilde{r}_1}{p\cdot(\tilde{r}_1+u_1)} \,, \qquad x_2 = \frac{(\tilde{r}_1+p)\cdot\tilde{r}_2-(\tilde{r}_1/z_1+p-u_1)\cdot u_2}{(\tilde{r}_1/z_1+p-u_1-u_2)\cdot\tilde{r}_2} \,,$$

$$\big(u_1^0\big)_{\max} = (1-\tilde{x}_1)\frac{p\cdot\tilde{r}_1/\tilde{x}_1}{(\tilde{r}_1/\tilde{x}_1+p)\cdot\hat{u}_1} \,, \qquad \big(u_2^0\big)_{\max} = \frac{(\tilde{r}_1+p)\cdot\tilde{r}_2}{(\tilde{r}_1/z_1+p-u_1)\cdot\hat{u}_2} \,,$$

$$\mathcal{J} = \frac{p\cdot\tilde{r}_1}{(p-u_1)\cdot\tilde{r}_1}\,\frac{x_2^{d-3}\,(\tilde{r}_1+p)\cdot\tilde{r}_2}{(\tilde{r}_1/z_1+p-u_1-u_2)\cdot\tilde{r}_2} \,,$$

Phase space mappings double-collinear

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initial-initial:

$$(r_1 + r_2 - u_1 - u_2)^2 = (\tilde{r}_1 + \tilde{r}_2)^2 \,,$$

$$z_2 = \frac{(r_1 - u_1 - u_2)\cdot(r_2 - u_2)}{(r_1 - u_1)\cdot r_2} \,,$$

$$z_1 = \frac{(\tilde{r}_2 - u_1)\cdot\tilde{r}_1}{\tilde{r}_2\cdot(\tilde{r}_1 + u_1)} \,,$$

$$(u_1^0)_{\max} = (1 - \tilde{x}_1)\frac{\tilde{r}_2\cdot\tilde{r}_1/\tilde{x}_1}{(\tilde{r}_1/\tilde{x}_1 + \tilde{r}_2)\cdot\hat{u}_1} \,,$$

$$\mathcal{J} = \frac{\tilde{r}_2\cdot\tilde{r}_1}{(\tilde{r}_2 - u_1)\cdot\tilde{r}_1}\,\frac{(\tilde{r}_1/z_1 - u_1)\cdot\tilde{r}_2}{(\tilde{r}_1/z_1 - u_1 - u_2)\cdot\tilde{r}_2} \,.$$

$$(r_1 + \tilde{r}_2 - u_1)^2 = (\tilde{r}_1 + \tilde{r}_2)^2 \,,$$

$$z_1 = \frac{(z_2 r_2 - u_1)\cdot(r_1 - u_1)}{z_2 r_2\cdot r_1} \,,$$

$$z_2 = \frac{(\tilde{r}_1/z_1 - u_1 - u_2)\cdot\tilde{r}_2}{(\tilde{r}_1/z_1 - u_1)\cdot(\tilde{r}_2 + u_2)} \,,$$

$$(u_2^0)_{\max} = (1 - \tilde{x}_2)\frac{(\tilde{r}_1/z_1 - u_1)\cdot\tilde{r}_2/\tilde{x}_2}{(\tilde{r}_1/z_1 + \tilde{r}_2/\tilde{x}_2 - u_1)\cdot\hat{u}_2} \,,$$

Subtraction kinematics

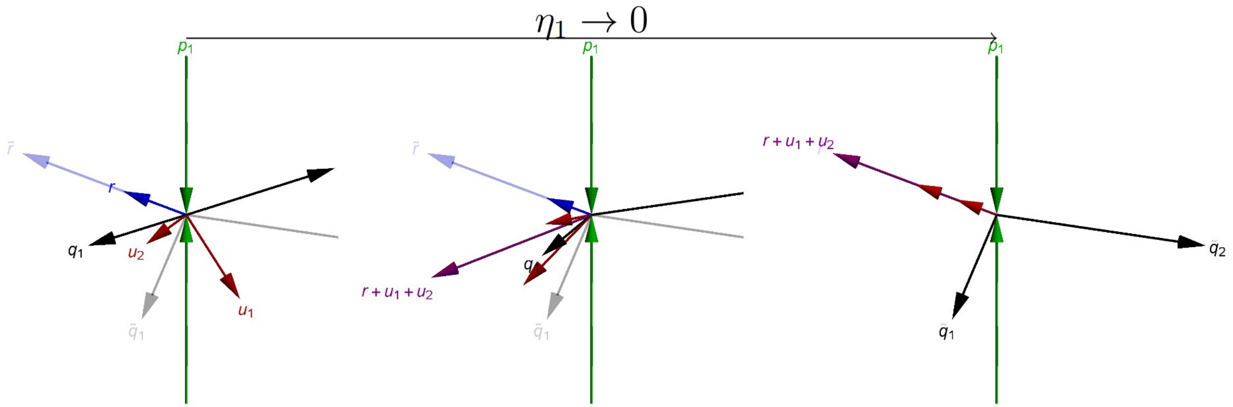
	Original parameterization	
$\mathcal{S}$	unresolved config.	number
single	$\{r\}, \{r+u\}$	2
triple	double unres.	
	$\{r\}, \{r+u_1\}, \{r+u_1+u_2\}$	3
	single unres.	
$\mathcal{S}_1$	$\{u_1, r\}, \{u_1, r+u_2\}$	2
$\mathcal{S}_2$	$\{u_1, r\}$	1
$\mathcal{S}_3$	$\{u_2, r+u_1\}$	1
$\mathcal{S}_4$	$\{u_1, r\}, \{u_1+u_2, r\}$	2
$\mathcal{S}_5$	$\{u_1, r\}, \{u_1+u_2, r\}$,	
	$\{u_1+\text{soft}u_2, r\}$	3
double	double unres.	
	$\{r_1, r_2\}, \{r_1+u_1, r_2\}$,	
	$\{r_1+u_1, r_2+u_2\}$	3
	single unres.	
	$\{u_1, r_1, r_2\}, \{u_1, r_1, r_2+u_2\}$,	
	$\{r_1+u_1, r_2, u_2\}$	3

	New parameterization	
$\mathcal{S}$	unresolved config.	number
single	$\{r+u\}$	1
triple	double unres.	
	$\{r+u_1+u_2\}$	1
	single unres.	
$\mathcal{S}_1$	$\{u_1, r+u_2\}$	1
$\mathcal{S}_{2,3}$	$\{u_1, r\}/\{u_2, r+u_1\}$	2
$\mathcal{S}_4$	$\{u_1+u_2, r\}$	1
$\mathcal{S}_5$	$\{u_1, r\}, \{u_1+u_2, r\}$	2
double	double unres.	
	$\{r_1+u_1, r_2+u_2\}$	1
	single unres.	
	$\{u_1, r_1, r_2+u_2\}$,	
	$\{u_2, r_1+u_1, r_2\}$,	2

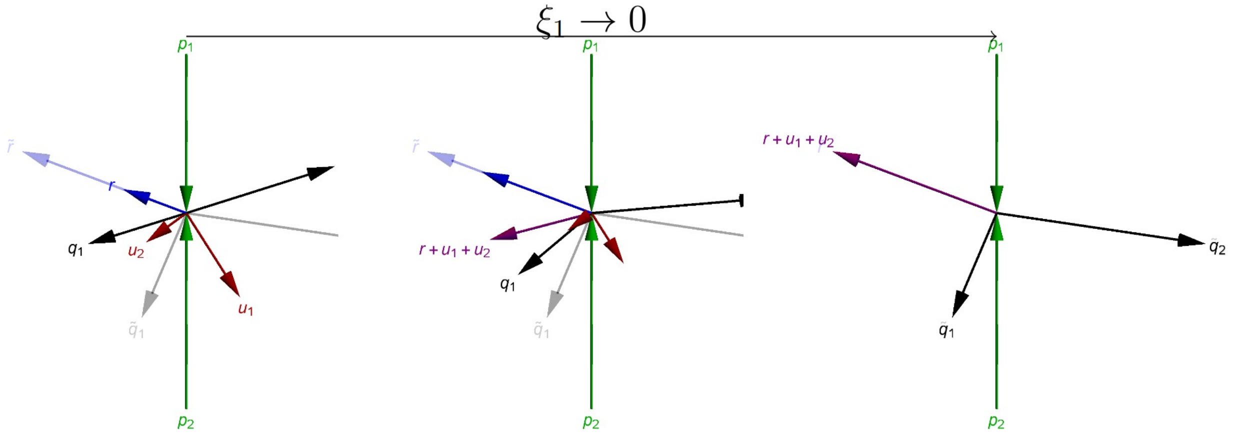
Double unresolved limits of kinematics

Example: Sector 1

Triple-collinear limit:



Double-soft limit:



Numerical pole cancellation

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$$pp \to e^+ e^- \mu^+ \mu^- + X$$

Contribution	ε^{-1}	ε^{-2}	ε^{-3}	ε^{-4}
$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}}$	0.000430986 ± 2.0e-06	0.000181251 ± 2.7e-07	3.96055e-05 ± 2.1e-08	1.79091e-05 ± 9.6e-10
$\hat{\sigma}^{\mathrm{RV}}_{\mathrm{DU}}$	-0.00056284 ± 5.1e-07	-0.00023461 ± 1.2e-07	-9.86939e-05 ± 1.4e-08	-2.93685e-05 ± 1.1e-09
$\hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}}$	4.60138e-07 ± 1.5e-11	4.30794e-05 ± 1.4e-09	5.90985e-05 ± 1.9e-09	1.14615e-05 ± 3.7e-10
$\hat{\sigma}^{\mathrm{C1}}_{\mathrm{DU}}$	0.000118897 ± 1.4e-07	6.25903e-05 ± 2.5e-08	1.43081e-05 ± 2.1e-09	0
$\hat{\sigma}^{\mathrm{C2}}_{\mathrm{DU}}$	1.31055e-05 ± 4.1e-09	-5.22942e-05 ± 8.1e-09	-1.43028e-05 ± 1.7e-09	0
Sum	6.08638e-07 ± 2.1e-06	1.65e-08 ± 3.0e-07	1.54e-08 ± 2.6e-08	2.1e-09 ± 1.5e-09
$\hat{\sigma}^{\mathrm{RV}}_{\mathrm{FR}}$	6.9779e-05 ± 6.2e-08	7.95598e-05 ± 6.1e-09	0	0
$\hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}}$	-0.000119352 ± 9.0e-09	-7.95678e-05 ± 6.0e-09	0	0
$\hat{\sigma}^{\mathrm{C2}}_{\mathrm{FR}}$	4.95713e-05 ± 1.3e-08	0	0	0
Sum	-1.7e-09 ± 6.4-08	-8e-09 ± 8.5e-09	0	0

Sector counting

Process	Number of independent (triple/double) sectors
$e^+e^- \to u\bar{u}gg$	(8/2)
$e^+e^- \to u\bar{u}u\bar{u}$	(16/0)
$e^+e^- \to u\bar{u}d\bar{d}$	(32/0)
$e^+e^- \to u\bar{u}ggg$	(12/6)
$e^+e^- \to u\bar{u}u\bar{u}g$	(32/4)
$e^+e^- \to u\bar{u}d\bar{d}g$	(64/8)
...	...
$e^+e^- \to u\bar{u} + (2+n)g$	(12/6)
$e^+e^- \to u\bar{u}d\bar{d}s\bar{s}c\bar{c}b\bar{b} + (2+n)g$	(444/220)
$gg \to ggggg$	(12/7)
$u\bar{u} \to d\bar{d}s\bar{s}g$	(96/18)
$u\bar{u} \to d\bar{d}s\bar{s}c\bar{c}b\bar{b}$	(256/12)
...	...

Table 8: Number of triple and double collinear sectors (counting each sub-sector separately) for some example proceses.