The q_T spectrum beyond leading power

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*work in collab. with Giancarlo Ferrera and Wan-Li Ju, based on arXiv:2312.14911

1. Introduction
2. Power expansion of the q_T spectrum
3. Results
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Introduction

Expansion in powers of q_T^2 for $q_T \to 0$ in $pp \to B + X$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} = \underbrace{\sum_{m,n} \frac{\alpha_s^m \, c_{-1}^{(m,n)} \, L^n}{q_T^2}}_{\text{LP}} + \underbrace{\sum_{m,n} \alpha_s^m \, c_0^{(m,n)} \, L^n}_{\text{NLP}} + \underbrace{\sum_{m,n} q_T^2 \, \alpha_s^m \, c_1^{(m,n)} \, L^n}_{\text{N}^2\text{LP}} + \ldots$$

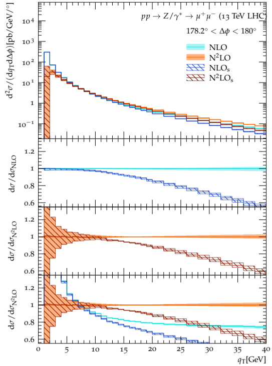
with $L = \log[q_T^2/m^2]$ and coefficients $c_i^{(m,n)}$.

Although a few NLP results are known, the main focus of research has been on LP which dominates the $q_T \to 0$ behaviour.

Leading power resummation

At LP there is a long history resumming the q_T spectrum in singlet production. **q_T resummation**

- CSS framework
- momentum space resummation
- SCET-based resummation
- ...



LP resummation, however, misses some important aspects of the physics of the process in question, both at small and medium q_T.

This talk: systematically construct subleading power coefficients.

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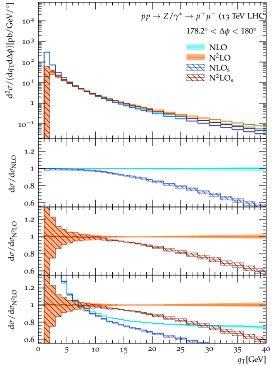

LP resummation, however, misses some important aspects of the physics of the process in question, both at small and medium q_T.

This talk: systematically construct subleading power coefficients.

Power expansion of the q_T spectrum

Consider the process the q_T spectrum of the process $pp \to B + X$, where B is any massive colour neutral final state.

We restrict ourselves to an analysis at NLO, ie. X is made up of one single parton.

Momentum labels

$$i + j \to B + k$$

Use light-cone decomposition

$$k^\mu \equiv [k_+, k_-, k_\perp] \qquad \text{and} \qquad q_T^2 = k_T^2 = k_+ k_-$$

Power expansion of $pp \to B + X$

Consider the NLO q_T spectrum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\,\mathrm{d}q_\mathrm{T}^2} = \frac{1}{16\pi\, s^2} \sum_{i,j} \int_0^{k_+^\mathrm{max}} \mathrm{d}k_+ \int_0^{k_-^\mathrm{max}} \mathrm{d}k_-\, \delta(k_+ k_- - q_\mathrm{T}^2)\, \frac{f_{i/n}(\xi_n)}{\xi_n}\, \frac{f_{j/\bar{n}}(\xi_{\bar{n}})}{\xi_{\bar{n}}} \times \overline{\sum_{\mathrm{col,pol}}} \left|\mathcal{M}(i+j \to B+k)\right|^2$$

$$\equiv \frac{1}{16\pi\, s^2} \sum_{i,j} \int_{k_+^\mathrm{min}}^{k_+^\mathrm{max}} \frac{\mathrm{d}k_+}{k_+} \frac{f_{i/n}(\xi_n)}{\xi_n} \frac{f_{j/\bar{n}}(\xi_{\bar{n}})}{\xi_{\bar{n}}} \overline{\left|\mathcal{M}_{[\kappa]}\right|^2}$$

with $\kappa \in \{gg, gq, q\bar{q}\}$.

Assuming a propagator structure in $\overline{\left|\mathcal{M}_{[\kappa]}\right|^2}$, we now need to determine the coefficients of $q_\mathrm{T}^{2\omega}\, \alpha_s^m\, c_\omega^{(m,n)}\, L^n$ for $\omega \geq -1$.

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$$\times\overline{\sum_{\mathrm{col,pol}}}\left|\mathcal{M}(i+j\to B+k)\right|^2$$

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Kinematics for $i + j \to B + k$

Invariants

$$s_{ik} = -\,2p_i \cdot p_k = -\left(k_+ + m_{\mathrm{T}} e^{-Y}\right) k_-$$
$$s_{jk} = -\,2p_j \cdot p_k = -\left(k_- + m_{\mathrm{T}} e^{+Y}\right) k_+$$

Transverse momentum and mass, expand in limit $q_{\mathrm{T}} \to 0$

$$q_{\mathrm{T}}^2 = k_+ k_- \qquad \text{and} \qquad m_{\mathrm{T}} = m \cdot \sum_{h=0}^{\infty} \frac{1}{h!} \frac{\Gamma[\frac{3}{2}]}{\Gamma[\frac{3}{2} - h]} \left(\frac{q_{\mathrm{T}}^2}{m^2}\right)^h .$$

Momentum fractions

$$\xi_n = \frac{k_+ + m_{\mathrm{T}} e^{-Y}}{\sqrt{s}} \qquad \text{and} \qquad \xi_{\bar{n}} = \frac{k_- + m_{\mathrm{T}} e^{+Y}}{\sqrt{s}}$$

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Hard function

Extract powers of $k_\pm$

$$\overline{\left|\mathcal{M}_{[\kappa]}\right|^2} \propto \sum_{\{\beta\}} \sum_{\rho,\sigma} \frac{(k_+)^\sigma}{(k_+ + m_T e^{-Y})^{\beta_n - 1}} \frac{(k_-)^\rho}{(k_- + m_T e^{Y})^{\beta_{\bar{n}} - 1}} \mathcal{H}^{\rho,\sigma}_{[\kappa],\{\beta\}}(m_T)$$

Procedure not unique, one can always multiply numerator and denominator with common factor, eg. $(k_- + m_T e^Y)^2$, and then expand numerator again.

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Expand in q_{T} for $q_{\mathrm{T}} \to 0$ or $m_{\mathrm{T}} \to m$

$$\mathcal{H}^{\rho,\sigma}_{[\kappa],\{\beta\}}(m_{\mathrm{T}}) = \sum_{h=0}^{\infty} \frac{(m_{\mathrm{T}}-m)^h}{h!} \left\{ \frac{\partial^h}{\partial m_{\mathrm{T}}^h} \mathcal{H}^{\rho,\sigma}_{[\kappa],\{\beta\}}(m_{\mathrm{T}})\bigg|_{m_{\mathrm{T}}\to m} \right\}$$

$$= \sum_{h,l=0}^{\infty} \sum_{g=0}^{h} \frac{(-1)^{h-g}}{g!\,l!\,(h-g)!} \frac{\Gamma[\frac{g}{2}+1]}{\Gamma[\frac{g}{2}-l+1]} \left(\frac{q_{\mathrm{T}}^2}{m^2}\right)^l m^h \mathcal{H}^{(h),\rho,\sigma}_{[\kappa],\{\beta\}}(m)\,,$$

We have extracted powers of q_{T} from $\mathcal{H}$, the corresponding coefficient contains derivatives of $\mathcal{H}$.

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PDFs

Introduce derivatives of scaled PDFs $f_{i/n}$

$$F^{(\alpha_n)}_{i/n,\beta_n}\left(Q_n\right) \equiv \frac{\partial^{\alpha_n}}{\partial Q_n^{\alpha_n}}\left[\frac{f_{i/n}\left(Q_n/\sqrt{s}\right)}{Q_n^{\beta_n}}\right]$$

The full result only contains the zero-rank scaled PDFs, $F^{(0)}_{i/n,\beta_n} \propto f_{i/n}$, but the expansion in q_{T} will contain all $\alpha_n \geq 0$.

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&F^{(0)}_{i/n,\beta_n}\left(k_+ + m_{\mathrm{T}} e^{-Y}\right) \\
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Problem: Need derivatives of PDFs. LHAPDF uses bi-cubic interpolator in $\{\log\xi, \log Q\}$.

Solution: Either upgrade degree of interpolation function or construct suitably differentiable approximant.

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Hard function and PDF

Taken together,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\,\mathrm{d}q_T^2} = \sum_{\omega=-1}^{\infty} \left(q_T^2\right)^{\omega} \sum_{[\kappa]} \sum_{\{\alpha,\beta\}} \sum_{\rho,\sigma} \widetilde{\mathcal{H}}^{(\omega),\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} \left\{ \widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} + \Delta\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} \right\}.$$

For simplicity, $\widetilde{\mathcal{H}}^{(\omega),\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}$ absorbs all factorials, Γ-functions, m^{α_n}, etc.
All explicit q_T dependences of $\mathcal{H}$ and $f_{i/n}$ have been extracted.

However, we still have to extract the q_T dependences from the k_+ integration and its boundaries.

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k_+ integration boundaries

Divide into two regions

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} = \int_{\tilde{k}_+^{\min}}^{\tilde{k}_+^{\max}} \frac{\mathrm{d}k_+}{k_+} (k_-)^\rho \, (k_+)^\sigma \, F^{(\alpha_n)}_{i/n,\beta_n}\left(k_+ + m e^{-Y}\right) \, F^{(\alpha_{\bar{n}})}_{j/\bar{n},\beta_{\bar{n}}}\left(k_- + m e^{Y}\right)$$

$$\Delta\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} = \left(\int_{\tilde{k}_+^{\max}}^{k_+^{\max}} + \int_{k_+^{\min}}^{\tilde{k}_+^{\min}}\right) \frac{\mathrm{d}k_+}{k_+} (k_-)^\rho \, (k_+)^\sigma \, F^{(\alpha_n)}_{i/n,\beta_n}\left(k_+ + m e^{-Y}\right) \, F^{(\alpha_{\bar{n}})}_{j/\bar{n},\beta_{\bar{n}}}\left(k_- + m e^{Y}\right)$$

with the integration boundaries

$$k_+^{\max} = \sqrt{s} - m_T e^{-Y} \qquad k_+^{\min} = \frac{q_T^2}{\sqrt{s} - m_T e^{+Y}}$$

$$\tilde{k}_+^{\max} = \sqrt{s} - m e^{-Y} \qquad \tilde{k}_+^{\min} = \frac{q_T^2}{\sqrt{s} - m e^{+Y}} \, .$$

Expand in q_T.

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Expand in $q_{\rm T}$.

Boundary correction

$\Delta\widetilde{\mathcal{I}}$ contains all m_T dependences and unamb. scaling for $e^{\pm Y} \sim \mathcal{O}(1)$.

$$k_+ \in [\tilde{k}_+^{\max}, k_+^{\max}] \;:\quad k_+ \sim \mathcal{O}(m)\,,\quad k_- \sim \mathcal{O}(q_\mathrm{T}^2/m)\,,\quad \vec{k}_\mathrm{T} = -\vec{q}_\mathrm{T}$$
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$k_\pm$ are hardest momenta kinematically allowed $\to$ ultra-collinear modes.

On one side

$$\Delta\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\bigg|_{uc} = \sum_{\tilde{\alpha}_n,\tilde{\alpha}_{\bar{n}}=0}^{\infty} \frac{(q_\mathrm{T}^2)^{\rho+\tilde{\alpha}_{\bar{n}}}}{\tilde{\alpha}_n!\,\tilde{\alpha}_{\bar{n}}!}\, F^{(\alpha_n+\tilde{\alpha}_n)}_{i/n,\beta_n}\left(\sqrt{s}\right)\, F^{(\alpha_{\bar{n}}+\tilde{\alpha}_{\bar{n}})}_{j/\bar{n},\beta_{\bar{n}}}\left(m e^Y\right)\, \mathcal{B}^{\rho,\sigma}_{+,\{\alpha,\beta\}}$$

associated with PDF at $\xi = 1$, will play small role for finite q_T.

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Interior domain

$\widetilde{\mathcal{I}}$ contains a number of different scales, assuming $e^{\pm Y} \sim \mathcal{O}(1)$

- ultrasoft $k_+^{\min} \sim \mathcal{O}(q_\mathrm{T}^2/m)$
- soft $k_\pm \sim \mathcal{O}(q_\mathrm{T})$
- hard $k_+^{\max} \sim \mathcal{O}(m)$

Expand by regions

$$
\begin{aligned}
n\text{-collinear mode}: &\quad \nu_n < k_+ \le \tilde{k}_+^{\max} && k^\mu \sim \mathcal{O}(m, q_\mathrm{T}^2/m, q_\mathrm{T}) \\
\text{transitional range}: &\quad \frac{q_\mathrm{T}^2}{\nu_{\bar{n}}} < k_+ \le \nu_n && \text{mult. modes for } \nu_n \lesssim m e^{-Y} \\
\bar{n}\text{-collinear mode}: &\quad \tilde{k}_+^{\min} \le k_+ \le \frac{q_\mathrm{T}^2}{\nu_{\bar{n}}} && k^\mu \sim \mathcal{O}(q_\mathrm{T}^2/m, m, q_\mathrm{T})
\end{aligned}
$$

Scales $\{\nu_n, \nu_{\bar{n}}\}$ separate n- and $\bar{n}$-collinear regions.

Interior domain – n-, $\bar{n}$-collinear region

In n-collinear region, $k_+ \sim \mathcal{O}(m)$, $k_- \sim \mathcal{O}(q_T^2/m)$, expand

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Big|_c + \widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Big|_{\bar{c}} = \sum_{\omega=\rho}^{\infty} (q_T^2)^\omega \left[\widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}(\nu_n)\Big|_c + \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}(\nu_{\bar{n}})\Big|_{\bar{c}} \right]$$

q_T^2 dependence extracted, $\widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}$ collects all hard scales, is $\mathcal{O}(1)$.

Interior domain – transitional region

Expand F around $me^{\pm Y}$

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Big|_t = \sum_{\lambda,\eta=0}^{\infty}\int_{\frac{q_\mathrm{T}^2}{\nu_{\bar n}}}^{\nu_n}\frac{\mathrm{d}k_+}{k_+}\,\frac{(k_-)^{\rho+\lambda}\,(k_+)^{\sigma+\eta}}{\lambda!\,\eta!}\,F^{(\alpha_n+\eta)}_{i/n,\beta_n}\left(me^{-Y}\right)F^{(\alpha_{\bar n}+\lambda)}_{j/\bar n,\beta_{\bar n}}\left(me^{Y}\right)$$

$$= \sum_{\lambda,\eta=0}^{\infty}\frac{F^{(\alpha_n+\eta)}_{i/n,\beta_n}\left(me^{-Y}\right)F^{(\alpha_{\bar n}+\lambda)}_{j/\bar n,\beta_{\bar n}}\left(me^{+Y}\right)}{\lambda!\,\eta!}$$

$$\times\left\{\underbrace{\frac{(q_\mathrm{T}^2)^{\rho+\lambda}\,\nu_n^{\sigma+\eta-\rho-\lambda}}{\sigma+\eta-\rho-\lambda}\,\bar\delta^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col}}+\underbrace{\frac{(q_\mathrm{T}^2)^{\sigma+\eta}\,\nu_{\bar n}^{\rho+\lambda-\sigma-\eta}}{\rho+\lambda-\sigma-\eta}\,\bar\delta^{\sigma+\eta}_{\rho+\lambda}}_{\bar n\text{-col}}\right.$$

$$\left.+\underbrace{(q_\mathrm{T}^2)^{\sigma+\eta}\ln\left[\frac{\nu_n\nu_{\bar n}}{q_\mathrm{T}^2}\right]\delta^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col},\ \bar n\text{-col},\ s}\right\}$$

Even for fixed λ, η multiple powers of q_T.

Interior domain – transitional region

Expand F around $me^{\pm Y}$

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Bigg|_t = \sum_{\lambda,\eta=0}^{\infty} \int_{\frac{q_T^2}{\nu_{\bar n}}}^{\nu_n} \frac{\mathrm{d}k_+}{k_+} \frac{(k_-)^{\rho+\lambda}\,(k_+)^{\sigma+\eta}}{\lambda!\,\eta!} F^{(\alpha_n+\eta)}_{i/n,\beta_n}\left(me^{-Y}\right) F^{(\alpha_{\bar n}+\lambda)}_{j/\bar n,\beta_{\bar n}}\left(me^{Y}\right)$$

$$= \sum_{\lambda,\eta=0}^{\infty} \frac{F^{(\alpha_n+\eta)}_{i/n,\beta_n}\left(me^{-Y}\right)\, F^{(\alpha_{\bar n}+\lambda)}_{j/\bar n,\beta_{\bar n}}\left(me^{+Y}\right)}{\lambda!\,\eta!}$$

$$\times \Bigg\{ \underbrace{\frac{(q_T^2)^{\rho+\lambda}\,\nu_n^{\sigma+\eta-\rho-\lambda}}{\sigma+\eta-\rho-\lambda}\,\bar\delta^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col}} + \underbrace{\frac{(q_T^2)^{\sigma+\eta}\,\nu_{\bar n}^{\rho+\lambda-\sigma-\eta}}{\rho+\lambda-\sigma-\eta}\,\bar\delta^{\sigma+\eta}_{\rho+\lambda}}_{\bar n\text{-col}}$$

$$+ \underbrace{(q_T^2)^{\sigma+\eta}\ln\left[\frac{\nu_n\nu_{\bar n}}{q_T^2}\right]\delta^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col},\ \bar n\text{-col},\ s} \Bigg\}$$

Even for fixed λ, η multiple powers of q_T.

Interior domain – transitional region

Expand F around $me^{\pm Y}$

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Big|_t = \sum_{\lambda,\eta=0}^{\infty} \int_{\frac{q_\mathrm{T}^2}{\nu_{\bar{n}}}}^{\nu_n} \frac{\mathrm{d}k_+}{k_+} \frac{(k_-)^{\rho+\lambda}\,(k_+)^{\sigma+\eta}}{\lambda!\,\eta!} F^{(\alpha_n+\eta)}_{i/n,\beta_n}\left(me^{-Y}\right) F^{(\alpha_{\bar{n}}+\lambda)}_{j/\bar{n},\beta_{\bar{n}}}\left(me^{Y}\right)$$

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$$\times \left\{ \underbrace{\frac{(q_\mathrm{T}^2)^{\rho+\lambda}\,\nu_n^{\sigma+\eta-\rho-\lambda}}{\sigma+\eta-\rho-\lambda}\,\bar{\delta}^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col}} + \underbrace{\frac{(q_\mathrm{T}^2)^{\sigma+\eta}\,\nu_{\bar{n}}^{\rho+\lambda-\sigma-\eta}}{\rho+\lambda-\sigma-\eta}\,\bar{\delta}^{\sigma+\eta}_{\rho+\lambda}}_{\bar{n}\text{-col}} \right.$$

$$\left. + \underbrace{(q_\mathrm{T}^2)^{\sigma+\eta}\ln\left[\frac{\nu_n\nu_{\bar{n}}}{q_\mathrm{T}^2}\right]\delta^{\sigma+\eta}_{\rho+\lambda}}_{n\text{-col}\,,\ \bar{n}\text{-col}\,,\ \mathrm{s}} \right\}$$

Even for fixed λ, η multiple powers of q_T.

Interior domain – transitional region

Collect and refactor

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}}\Big|_t = \sum_{\omega=\rho}^{\infty} (q_{\mathrm{T}}^2)^{\omega} \left[\widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}(\nu_n)\Big|_{cs} + \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}(\nu_{\bar{n}})\Big|_{\bar{c}s}\right]$$

Defined n- and $\bar{n}$-collinear-soft sectors based on ν_n, $\nu_{\bar{n}}$ dependence.
Besides hard scale $k_{\pm}$ now also $\log[\nu_n/q_{\mathrm{T}}]$ and $\log[\nu_{\bar{n}}/q_{\mathrm{T}}]$.

Interior domain

Combine all regions

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} = \sum_{\omega} (q_T^2)^{\omega} \left\{ \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{c} + \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{cs} + \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{\bar{c}s} + \widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{\bar{c}} \right\}.$$

One can show that the sum is independent of ν_n and $\nu_{\bar{n}}$.

Interior domain

This can be generalised to various other rapidity regulators, instead of scales ν_n and $\nu_{\bar{n}}$,

$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} = \sum_{\omega=\rho}^{\infty} \left(q_T^2\right)^{\omega} \left[\widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\Big|_{c} - \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\Big|_{c0} + \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\Big|_{\bar{c}} - \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\Big|_{\bar{c}0} \right]$$

where the $\widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}$ are modifications of the $\widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}$ ($|_c$ collects the collinear contributions) with the integration boundaries extended to zero, regulated by an ad-hoc rapidity regulator.

$|_{\bar{c}0}$ is the zero-bin subtrahend.

We have shown that the result for pure and exponential rapidity regulator is the same as with momentum cut-offs, the Δ and η regulators can produce residual divergences beyond LP but can be suitably modified.

Interior domain

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$$\widetilde{\mathcal{I}}^{\rho,\sigma}_{[\kappa],\{\alpha,\beta\}} = \sum_{\omega=\rho}^{\infty} \left(q_T^2\right)^{\omega} \left[\widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{c} - \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{c0} + \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{\bar{c}} - \widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}\bigg|_{\bar{c}0} \right]$$

where the $\widetilde{\mathcal{G}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}$ are modifications of the $\widetilde{\mathcal{I}}^{\rho,\sigma,(\omega)}_{[\kappa],\{\alpha,\beta\}}$ ($|_c$ collects the collinear contributions) with the integration boundaries extended to zero, regulated by an ad-hoc rapidity regulator.

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We have shown that the result for pure and exponential rapidity regulator is the same as with momentum cut-offs, the Δ and η regulators can produce residual divergences beyond LP but can be suitably modified.

Power expansion

We now have the following q_T-expanded q_T spectrum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{c} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{cs} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{\bar{c}s} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{\bar{c}} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{b.c.}$$

with

Individual terms depend on rapidity regulator, but sum does not.

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with

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\right|_{c} \equiv \sum_{i,j=\{g,q,\bar{q}\}} \sum_{\omega=-1}^{\infty} \left(\frac{q_\mathrm{T}^2}{m^2}\right)^{\omega} \int_{x_n}^{\tilde{z}_n} \mathrm{d}z_n \left[\mathbf{F}_{i/n}\left(\frac{x_n}{z_n}\right)\right]^\mathbf{T}$$
$$\times \left\{\mathbf{R}_{cs}^{(\omega),ij}(z_n) + \frac{\mathbf{P}_{cs}^{(\omega),ij}}{1-z_n} + \mathbf{B}_{cs}^{(\omega),ij}(z_n)\left[\delta\left(x_n - z_n\right) - \delta\left(\tilde{z}_n - z_n\right)\right]\right\}$$
$$\times \mathbf{F}_{j/\bar{n}}\left(x_{\bar{n}}\right)$$

Individual terms depend on rapidity regulator, but sum does not.

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with

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_{\mathrm{T}}^2}\right|_{cs} \equiv \sum_{i,j=\{g,q,\bar{q}\}} \sum_{\omega=-1}^{\infty} \left(\frac{q_{\mathrm{T}}^2}{m^2}\right)^{\omega} \int_{\tilde{z}_n}^{1} \mathrm{d}z_n \left[\mathbf{F}_{i/n}\left(\frac{x_n}{z_n}\right)\right]^{\mathbf{T}} \times \left\{\mathbf{R}_{cs}^{(\omega),ij}(z_n) + \mathbf{P}_{cs}^{(\omega),ij}\left[\frac{1}{1-z_n}\right]_+ + \mathbf{D}_{cs}^{(\omega),ij}\,\delta\left(1-z_n\right) + \mathbf{B}_{cs}^{(\omega),ij}(\tilde{z}_n)\,\delta\left(\tilde{z}_n - z_n\right)\right\} \mathbf{F}_{j/\bar{n}}(x_{\bar{n}})\,.$$

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with

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_\mathrm{T}^2}\bigg|_{b.c.} \equiv \sum_{i,j=\{g,q,\bar{q}\}} \sum_{\omega=-1}^{\infty} \left(\frac{q_\mathrm{T}^2}{m^2}\right)^{\omega} \Bigg\{ \left[\mathbf{F}_{i/n}(1)\right]^\mathbf{T} \cdot \mathbf{B}_{uc}^{(\omega),ij}(x_n) \cdot \mathbf{F}_{j/\bar{n}}(x_{\bar{n}}) + \left[\mathbf{F}_{i/n}(x_n)\right]^\mathbf{T} \cdot \mathbf{B}_{u\bar{c}}^{(\omega),ij}(x_{\bar{n}}) \cdot \mathbf{F}_{j/\bar{n}}(1) \Bigg\}$$

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2}\right|_c + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2}\right|_{cs} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2}\right|_{\bar{c}s} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2}\right|_{\bar{c}} + \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2}\right|_{b.c.}$$

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Individual terms depend on rapidity regulator, but sum does not.

Results – Higgs q_T spectrum at NLO

Input parameters:

- $m_H = 125\,\text{GeV}$
- $v = 246.22\,\text{GeV}$
- $\mu_R = \mu_F = m_H$
- PDF: `MSHT20nlo_as118`
  (fitted with Chebyshev polynomials),
  α_s accordingly

$$\left.\frac{d\sigma_H^{\langle\text{asy}\rangle}}{dq_T^2}\right| = \sum_m \frac{\Delta_{\text{LP}}^{(m)}}{q_T^2}(L_H)^m$$

$L_H = \log[q_T/m_H]$

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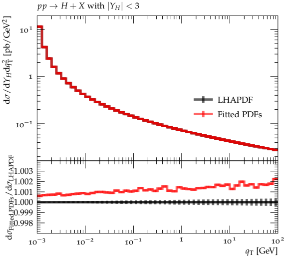

$$\left.\frac{\mathrm{d}\sigma_H^{\langle\mathrm{asy}\rangle}}{\mathrm{d}q_{\mathrm{T}}^2}\right| = \sum_m \frac{\Delta_{\mathrm{LP}}^{(m)}}{q_{\mathrm{T}}^2}(L_H)^m + \sum_m \Delta_{\mathrm{NLP}}^{(m)}(L_H)^m$$

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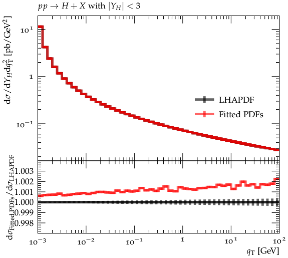

$$\left.\frac{\mathrm{d}\sigma_H^{\langle\text{asy}\rangle}}{\mathrm{d}q_T^2}\right|_{\text{LP}} = \sum_m \frac{\Delta^{(m)}_{\text{LP}}}{q_T^2}\,(L_H)^m + \sum_m \Delta^{(m)}_{\text{NLP}}\,(L_H)^m$$

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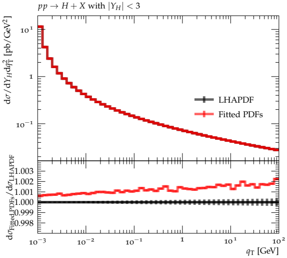

$$\left.\frac{\mathrm{d}\sigma_H^{\langle\mathrm{asy}\rangle}}{\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{NLP}} = \sum_m \frac{\Delta_{\mathrm{LP}}^{(m)}}{q_{\mathrm{T}}^2}\,(L_H)^m + \sum_m \Delta_{\mathrm{NLP}}^{(m)}\,(L_H)^m + \sum_m q_{\mathrm{T}}^2\,\Delta_{\mathrm{N^2LP}}^{(m)}\,(L_H)^m$$

$$L_H = \log[q_{\mathrm{T}}/m_H]$$

Results – Higgs q_T spectrum at NLO

Input parameters:

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- $v = 246.22\,\text{GeV}$
- $\mu_R = \mu_F = m_H$
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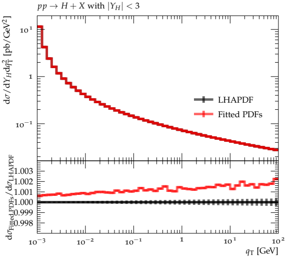
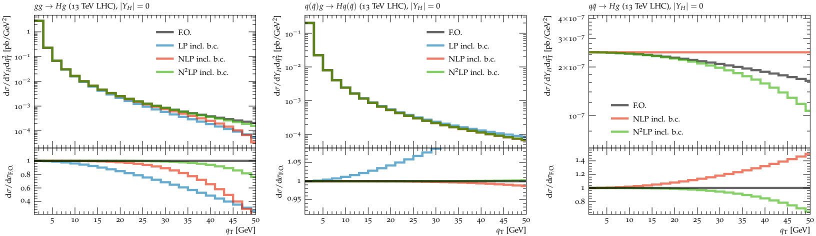


$$\left.\frac{d\sigma_H^{\langle\text{asy}\rangle}}{dq_T^2}\right|_{\text{N}^2\text{LP}} = \sum_m \frac{\Delta_{\text{LP}}^{(m)}}{q_T^2}\,(L_H)^m + \sum_m \Delta_{\text{NLP}}^{(m)}\,(L_H)^m + \sum_m q_T^2\,\Delta_{\text{N}^2\text{LP}}^{(m)}\,(L_H)^m$$

$$L_H = \log[q_T/m_H]$$

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

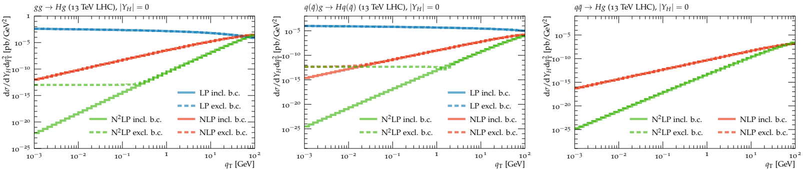
Ratio to exact spectra



- LP reproduces leading singularity
- NLP & N^{2}LP improve finite-q_T behaviour
  eg. $< 1\%$ deviation at $q_T = 30$ GeV in $gg \to Hg$ and $gq \to Hq$
  $\Rightarrow$ important for matching systematics
- no LP in $q\bar{q} \to Hg$, NLP const., N^{2}LP first non-trivial

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact spectra

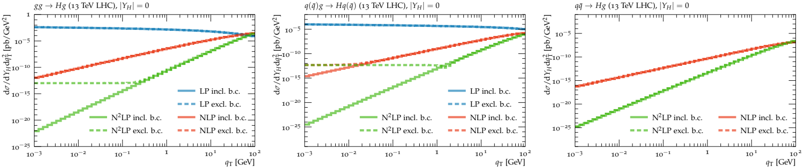


$$\frac{\mathrm{d}\sigma_H^{\langle \mathrm{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma_H^{\langle \mathrm{asy}\rangle}}{\mathrm{d}q_T^2}\Bigg|_{\mathrm{LP}} = \sum_m \quad \Delta_{\mathrm{NLP}}^{(m)} \quad (L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$, power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact spectra

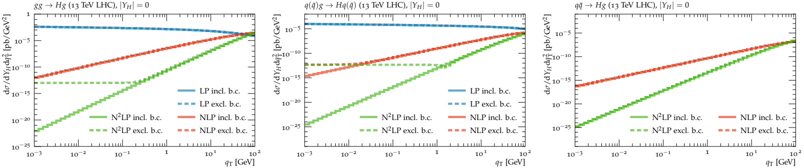


$$\frac{\mathrm{d}\sigma_H^{\langle\text{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma_H^{\langle\text{asy}\rangle}}{\mathrm{d}q_T^2}\Bigg|_{\text{NLP}} = \sum_m q_T^2\,\Delta^{(m)}_{\text{N}^2\text{LP}}\,(L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$, power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact spectra

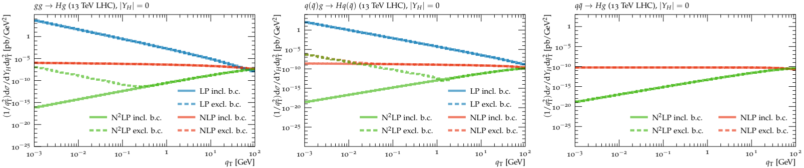


$$\left.\frac{\mathrm{d}\sigma_H^{\langle \mathrm{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma_H^{\langle \mathrm{asy}\rangle}}{\mathrm{d}q_T^2}\right|_{\mathrm{N^2LP}} = \sum_m q_T^4\, \Delta^{(m)}_{\mathrm{N^3LP}}\, (L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$,
  power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact weighted spectra

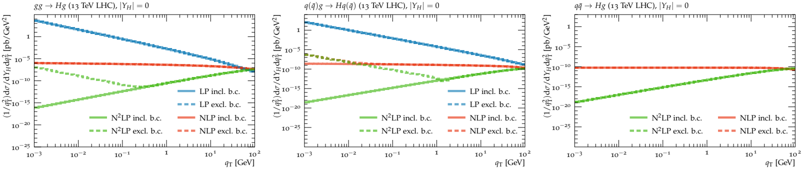


$$\frac{1}{q_T^2}\frac{\mathrm{d}\sigma_H^{\langle\mathrm{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{1}{q_T^2}\frac{\mathrm{d}\sigma_H^{\langle\mathrm{asy}\rangle}}{\mathrm{d}q_T^2}\Bigg|_{\mathrm{LP}} = \sum_m \frac{1}{q_T^2}\,\Delta_{\mathrm{NLP}}^{(m)}\;(L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$,
  power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact weighted spectra

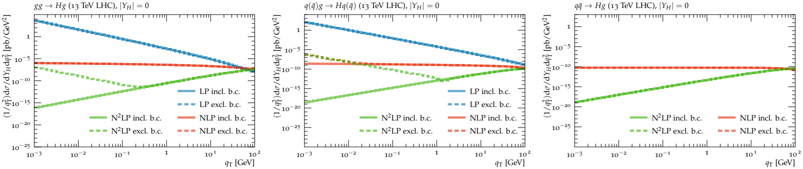


$$\frac{1}{q_T^2}\,\frac{\mathrm{d}\sigma_H^{\langle \mathrm{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{1}{q_T^2}\,\frac{\mathrm{d}\sigma_H^{\langle \mathrm{asy}\rangle}}{\mathrm{d}q_T^2}\Bigg|_{\mathrm{NLP}} = \sum_m \quad \Delta^{(m)}_{\mathrm{N^2LP}}\,(L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$, power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $Y_H = 0$

Difference to exact weighted spectra

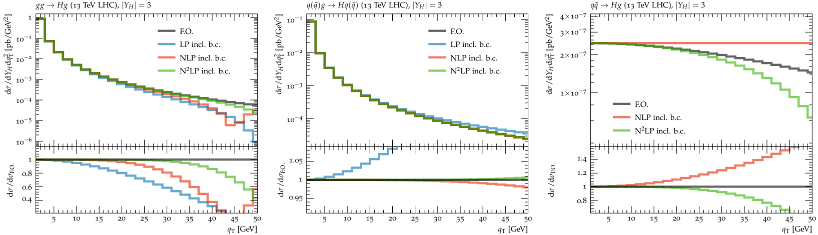


$$\frac{1}{q_T^2}\,\frac{\mathrm{d}\sigma_H^{\langle \mathrm{F.O.}\rangle}}{\mathrm{d}q_T^2} - \frac{1}{q_T^2}\,\frac{\mathrm{d}\sigma_H^{\langle \mathrm{asy}\rangle}}{\mathrm{d}q_T^2}\Bigg|_{\mathrm{N^2LP}} = \sum_m q_T^2\;\Delta^{(m)}_{\mathrm{N^3LP}}\;(L_H)^m + \ldots$$

- LP still integrably divergent as $q_T \to 0$,
  power corrections essential to control difference
- without boundary corrections, revert to LP accuracy at small q_T

Results – Higgs q_T spectrum at NLO – $|Y_H| = 3$

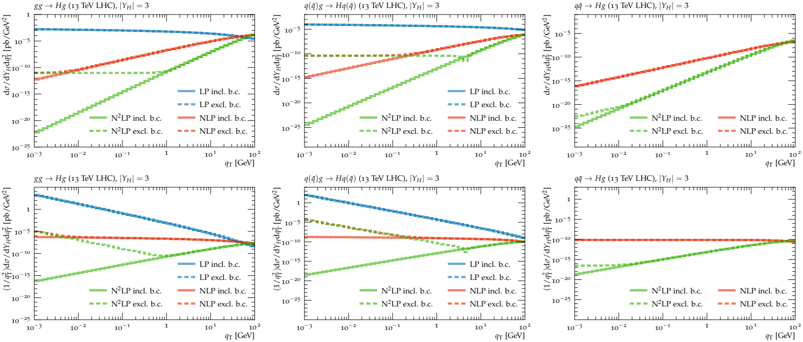
Ratio to exact spectra



- $e^{\pm Y_H}$ now $\mathcal{O}(10)$, impacts scaling of $k_\pm$, $k_\pm^{\max}$, and x_n, $x_{\bar{n}}$
  $\to$ impacts assumptions of scaling in the individual dynamic regions
- increased sensitivity to subleading power corrections

Results – Higgs q_{T} spectrum at NLO – $Y_H = 0$

Difference to exact (weighted) spectra



- increased sensitivity to boundary corrections, driven by asymmetry of x_n and $x_{\bar{n}}$

Conclusions

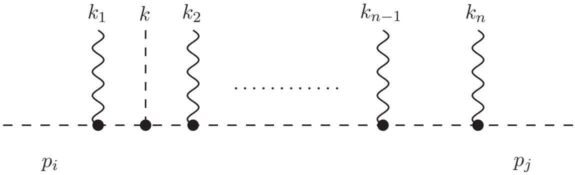
- presented a systematic framework to construct the small-q_T expansion at NLO up to arbitrary power accuracy
  - applicable to all conservative rapidity regulators
  - included contributions from integration boundaries, which were found to be dominating subleading power corrections for $q_T \to 0$
- applied to $pp \to H + X$ up to N^{2}LP as an example
  $\to$ exactly reproduce the exact result at a given power accuracy
- equally applicable to any process with similar propagator structure, eg. Drell-Yan

Thank you!

Backup

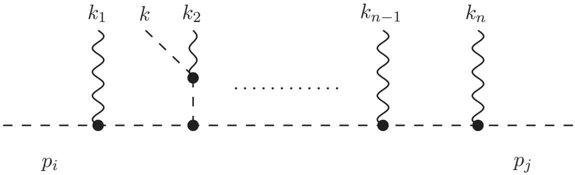
Multi-boson production

A) **no bosons attached to coloured final state**



⇒ straight forward

B) **at least one boson attached to coloured final state**



⇒ needs more care to derive factorised form