Tidal Love numbers from Gravitational Raman Scattering

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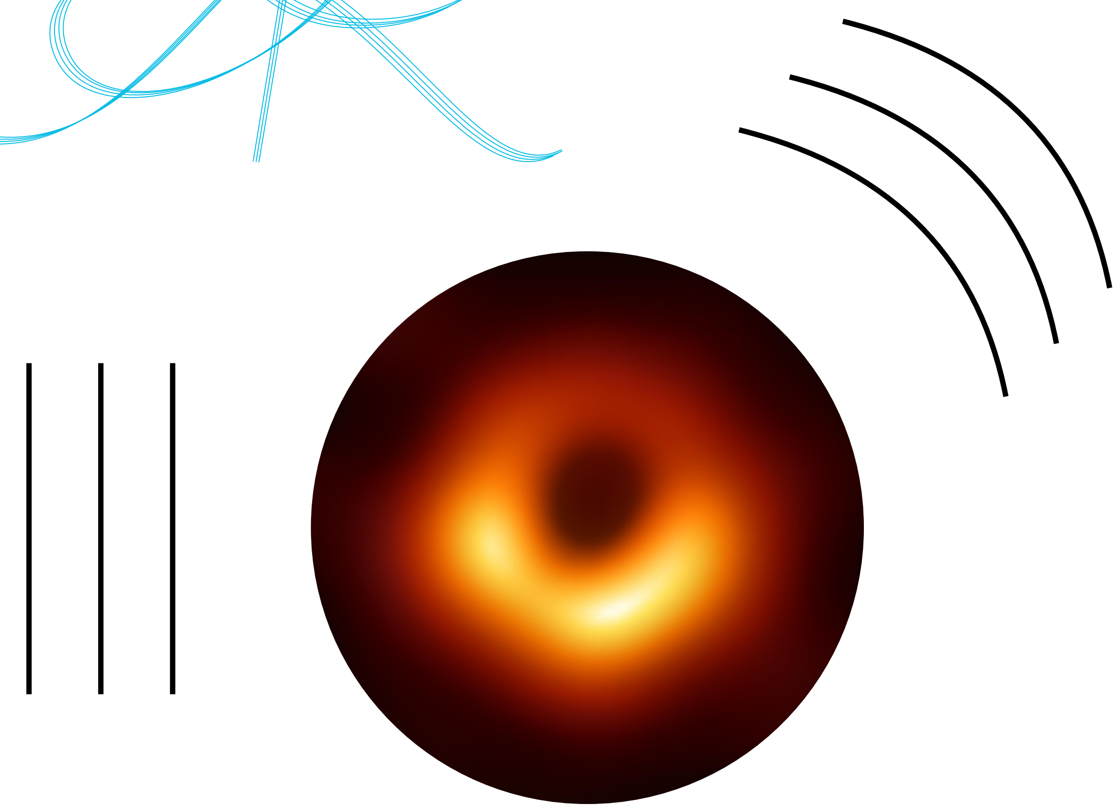
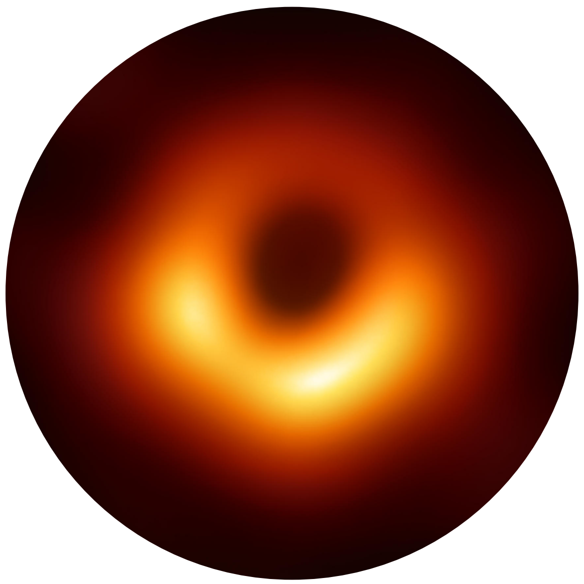


QCD Meets Gravity 2024 @ NTU, Taipei

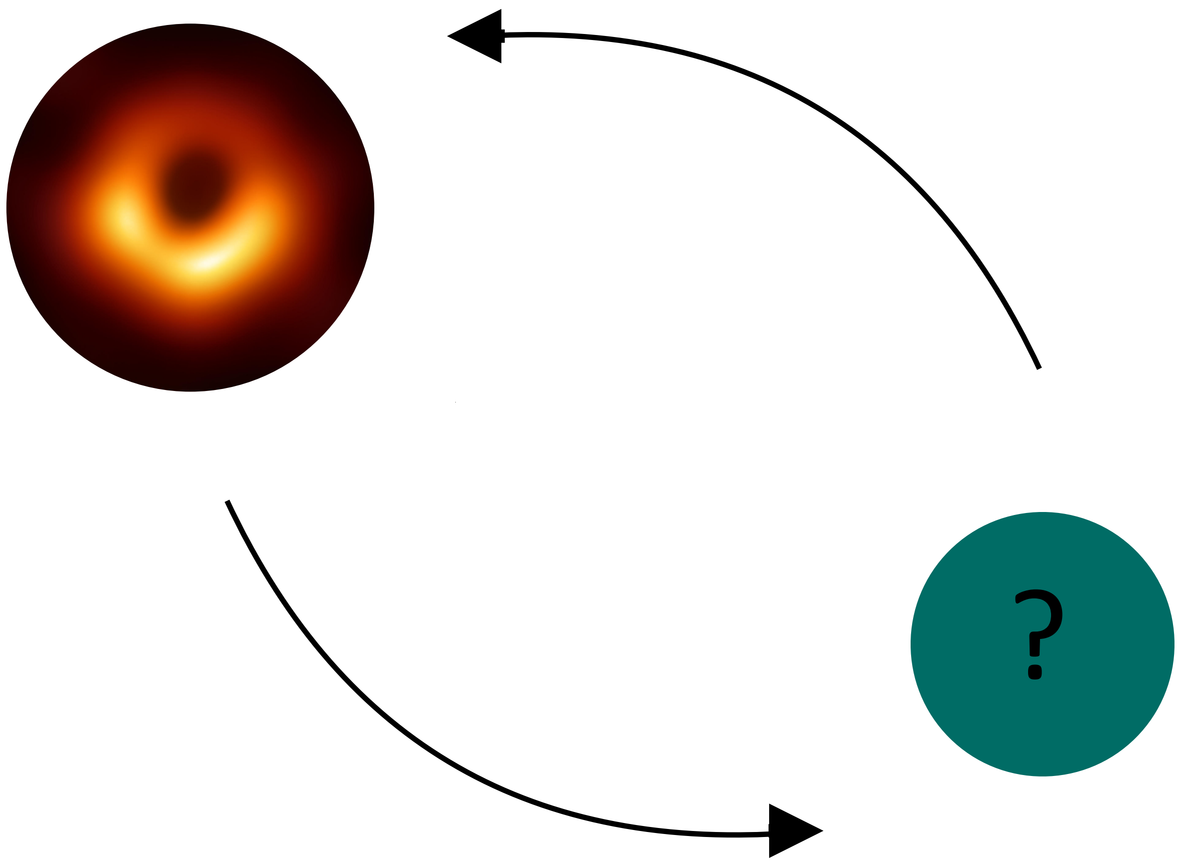
How do we tell apart…

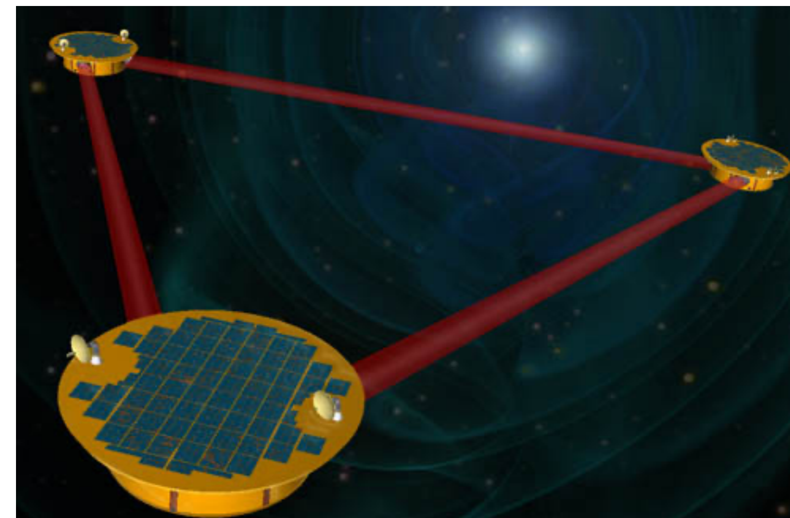


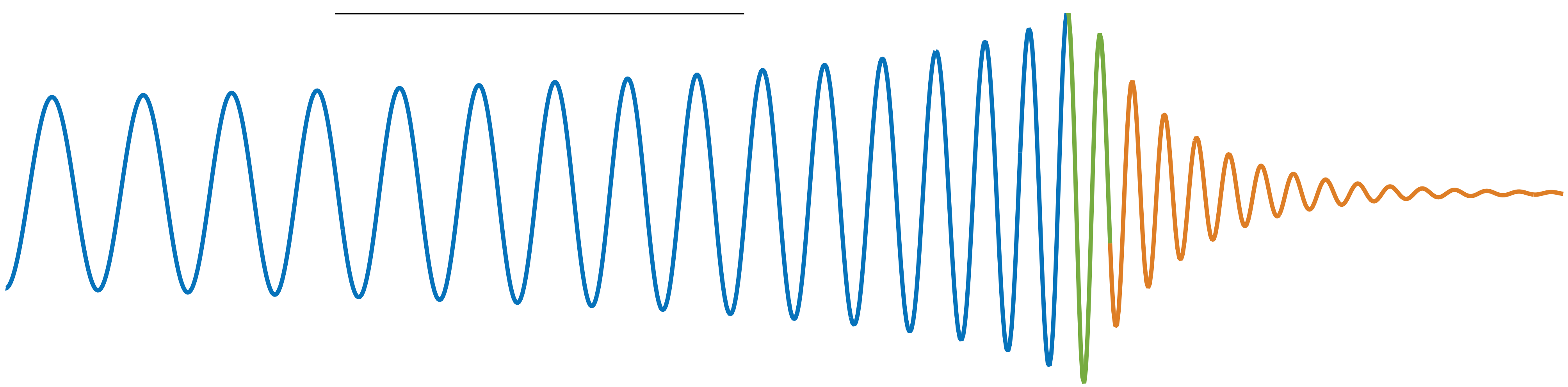
vs.

if we can only probe $\omega \ll 1/R$?

What are we really going to learn?

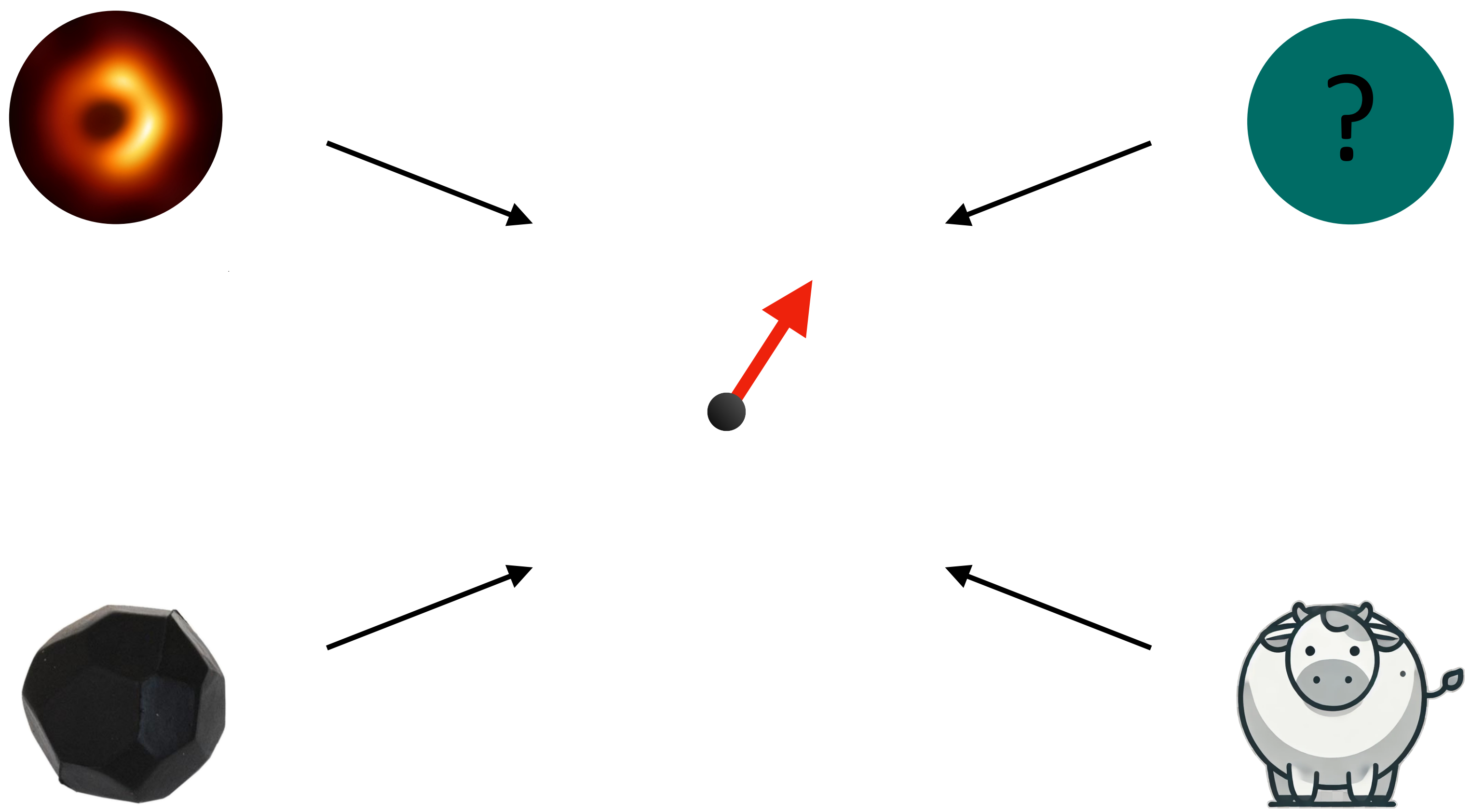






The simplest EFT

Black holes, Neutron stars, coal… $\longrightarrow$ point particle with given mass, spin



$$S = m\int d\tau\sqrt{g_{\mu\nu}(x)\dot{x}^{\mu}x^{\nu}} \quad + \quad \text{spin} \quad + \quad \mathcal{O}(R\omega)^{\#}$$

Power counting

The EFT for a point particle coupled to gravity has in principle two small parameters

"EFT expansion" $R\omega \ll 1$ (c.f. E/Λ in NLSM)

"Loop expansion" $R_s\omega \ll 1$ (c.f. $E/4\pi f_\pi$ in NLSM)

The EFT for "stars" is weakly coupled $R \gg R_s$ (c.f. $m_\sigma \gg f_\pi$ in LSM)

The EFT for BH and NS is strongly coupled $R \sim R_s$ (c.f. $\Lambda \sim 4\pi f_\pi$ in QCD)

This means in order to understand contact terms for BH and NS we will need to calculate "loops" in EFT!

The simplest EFT

In the language of EFT, point particle only first approximation must be augmented by dynamical multipoles [Goldberger, Rothstein; Porto]

$$S = \int d\tau \left[m(\tau) + Q_{ij}(\tau) E^{ij} + Q_{ijk}(\tau) \nabla^{(i} E^{jk)} + \text{magnetic} + \cdots \right]$$

mass

quadrupole

octupole

$$E_{ij} = C_{i0j0}$$
$$B_{ij} = {}^*C_{i0j0}$$

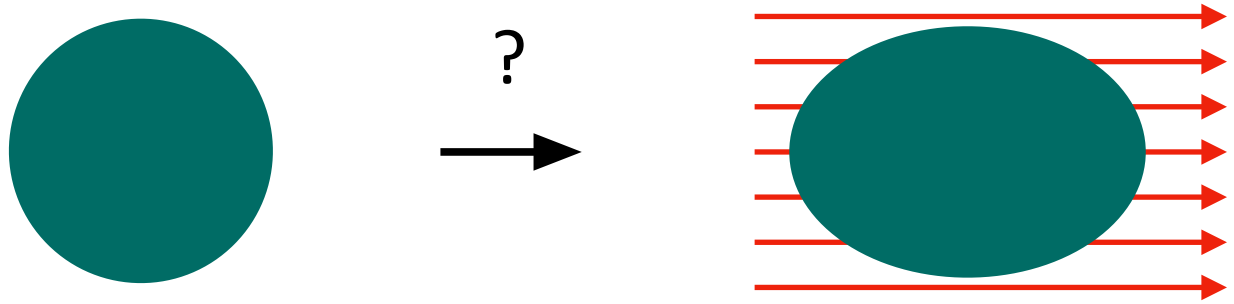
Multipoles encode microscopic/UV degrees of freedom

dim. analysis: $Q_\ell \sim m R^\ell q_\ell$

Can be gapless, so generically they can describe dissipation

Tidal Love numbers

Linear response to applied external gravitational field.



"gravitational polarizability"

$$Q_{ij}^{\text{ind}} = \lambda_2 E_{ij}^{\text{ext.}}$$

Generically is a function of frequency

$$\lambda_\ell(\omega) = \int d\tau\, e^{i\omega\tau}\, \theta(\tau)\, \langle [Q_\ell(\tau), Q_\ell(0)] \rangle$$

$$= \lambda_\ell + \lambda_{\ell\omega}\,\omega + \lambda_{\ell\omega^2}\,\omega^2 + \cdots$$

Static tide

Dynamical tides

$$\lambda_{\ell\omega^n} \sim mR^{2\ell+n}$$

n even: conservative

n odd: dissipative

Tidal effects in the EFT

Non-minimal couplings by integrating out multipoles

$$\Delta S^{\text{con.}} = \lambda_2 \int d\tau E_{\mu\nu}^2 + \lambda_{2\omega^2} \int d\tau (\dot{E}_{\mu\nu})^2 + \text{magnetic} + \cdots$$

and dissipation (in-in effective action)

$$\Delta S^{\text{dis.}} = \lambda_{2\omega} \int d\tau\, E_{-\mu\nu} \dot{E}_{+}^{\mu\nu} + \text{magnetic} + \cdots$$

Leading Love number enters as $\lambda_2 \sim R^5$, so EFT predicts that cannot tell apart point particle from BH until at least $O(G^5)$

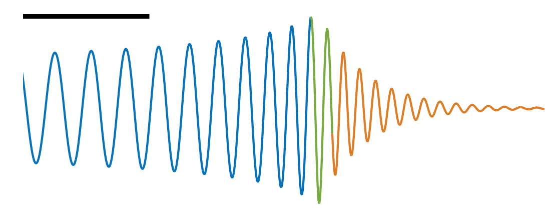
Why are these interesting?

The modify the post-Newtonian potential

[Damour; Cheung, Solon; Bern, **JPM**, Roiban, Sawyer, Shen; Porto, many others]

$$V(r) \sim \lambda_2 \frac{R^4}{r^4}\frac{R_s^2}{r^2}$$

"5PN"

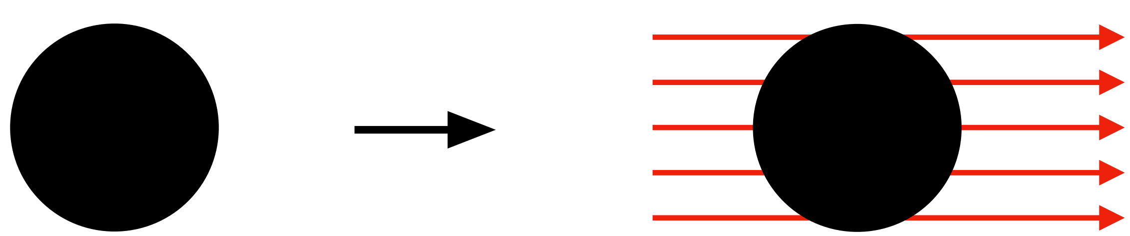


So we should be able to measure with gravitational wave detectors. Equation of state of NS?

[Flanagan, Hinderer; Chia, Ivanov, Zhou]

Static responses are zero for BH in D=4 GR!

[Damour, Nagar; Binnington, Poisson; …]



Excellent possible window into new physics!

[Cardoso, Franzin, …]

Static linear response

Static Regge-Wheeler/Teukolsky equation $\nabla^2_{BH} h_{\mu\nu}|_{\omega=0} = 0$

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d}{dr}{}_sU_{\ell m}(r)\right) + V_{eff}(r,\omega=0)\,{}_sU_{\ell m}(r) = 0$$

incoming b.c.

$$U_\ell(r) \to A_\ell^{(\mathrm{src})} r^\ell + A_\ell^{(\mathrm{resp})}\frac{1}{r^{\ell+1}}$$

Static Love numbers are measured as linear response:

$$\lambda_\ell \sim \frac{A_\ell^{(\mathrm{resp.})}}{A_\ell^{(\mathrm{src.})}}$$

Expert in the audience: Vitor

Dynamical tides run!

Classical GR coupled to a point particle is non-renormalizable! We should expect classical UV divergences

$$\mathbb{O}(r_c \sim 1/\mu, \omega) = \frac{1}{\epsilon} + \lambda_\ell(\omega) + 2\gamma_\ell(\omega)\,\lambda_\ell(\omega)\,\log(r_c\omega) + \beta_\ell(\omega)\,\log(r_c\omega) + \cdots$$

These will be absorbed by tidal coefficients

$$\lambda_\ell(r_c, \omega) = \bar{\lambda}_\ell(\omega) + 2\gamma_\ell(\omega)\,\bar{\lambda}_\ell(\omega)\,\log(R/r_c) + \beta_\ell(\omega)\,\log(R/r_c) + \cdots$$

"Bare coupling" "Anomalous dimension" "Beta function"

UV info.

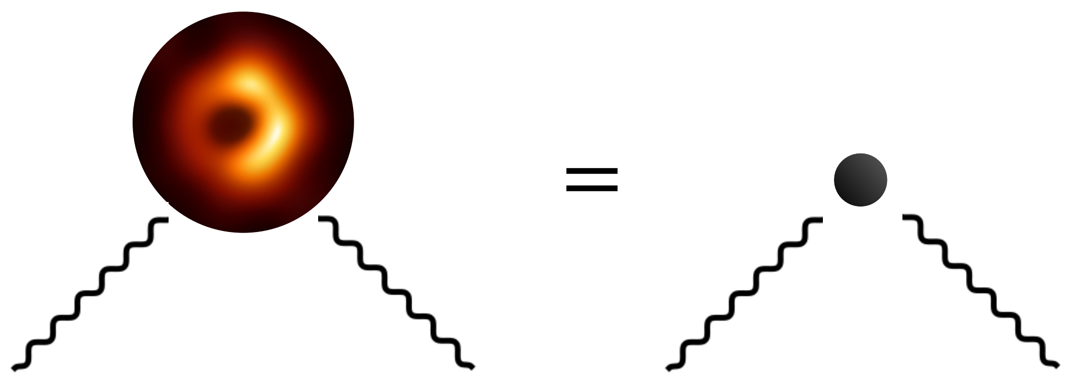
$$\mu\frac{dQ_\ell(\omega)}{d\mu} = \gamma_\ell(\omega)Q_\ell(\omega) \qquad \mu\frac{d}{d\mu}[(\mu^2)^{\gamma_\ell}\lambda_\ell(\omega)] = \beta_\ell(\omega)$$

Running complicates response calculation and introduces ambiguities

For more about these anomalous dimensions see Zihan's talk on Thursday!

 👍 $\gamma_\ell(\omega)$

Idea: Matching using scattering amplitudes



$$\mathscr{M}^{BHPT} = \mathscr{M}^{EFT}$$

Benefits: observable = gauge and field redef. inv.
+ we can use the methods of scattering amplitudes :)

(see also [Bautista, Guevara, Kavanagh, Vines; Ivanov, Zhou; + Saketh])

The “Gravitational Compton Amplitude”

There has been some discussion in the literature, in particular in the context of Kerr BH [too many people to cite, sorry]

A tree amplitude with consistent 4-pt factorization was built, and then there is disagreement about the “correct” contact terms.

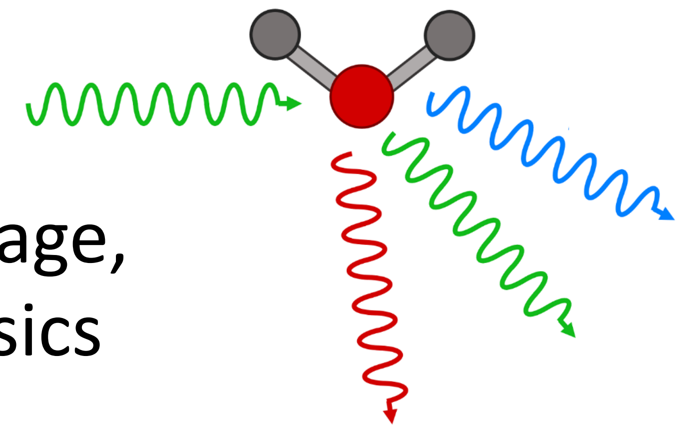
The EFT for BH is strongly coupled, so it is <u>not possible</u> to match BHPT to a tree amplitude.*

E.g., for Schwarzschild, a meaningful matching of the leading contact term to BHPT in principle requires computing Compton to 4 loops!

*Even worse, most contact terms run, and hence are scheme dependent.

Compton vs. Raman

[Ivanov, Li, **JPM**, Zhou]



"Gravitational Compton Amplitude" carries baggage, we propose a rebranding inspired by atomic physics

Raman = Compton + Loops + Dissipation

$$i\mathscr{M} = \eta_\ell e^{i\delta_\ell} - 1$$

$$\delta_\ell = (R_s\omega)\,\delta_\ell^{1\mathrm{PM}} + (R_s\omega)^2\,\delta_\ell^{2\mathrm{PM}} + (R_s\omega)^3\,\delta_\ell^{3\mathrm{PM}} + \cdots + (R_s\omega)^5\,\delta_\ell^{5\mathrm{PM}} + \cdots$$

$$\eta_\ell = (R_s\omega)^6\,\eta_\ell^{7\mathrm{PM}} + \cdots$$

Leading tides λ_2 + loops

Scalar toy model

Gravitational tides start at 5PM, consider a simpler model

$$\Delta S^{\mathrm{con.}} = \lambda_{1\omega^0}\int d\tau(\nabla\phi)^2 + \lambda_{0\omega^2}\int d\tau\dot{\phi}^2 + \cdots$$

$$\Delta S^{\mathrm{dis.}} = \lambda_{0\omega}\int d\tau\,\phi_-\dot{\phi}_+ + \lambda_{0\omega^3}\int d\tau\,\phi_-\dddot{\phi}_+ + \lambda_{1\omega}\int d\tau\,\nabla\phi_-\nabla\dot{\phi}_+ + \cdots$$

Power-counting

$\lambda_{0\omega} \sim R_s^2$ $\lambda_{1\omega^0} \sim \lambda_{0\omega^2} \sim R_s^3$ $\lambda_{0\omega^3} \sim \lambda_{1\omega} \sim R_s^4$

Leading "Relative 2-loop order"

Note, no monopole static Love number due to symmetry $\phi \to \phi + a$

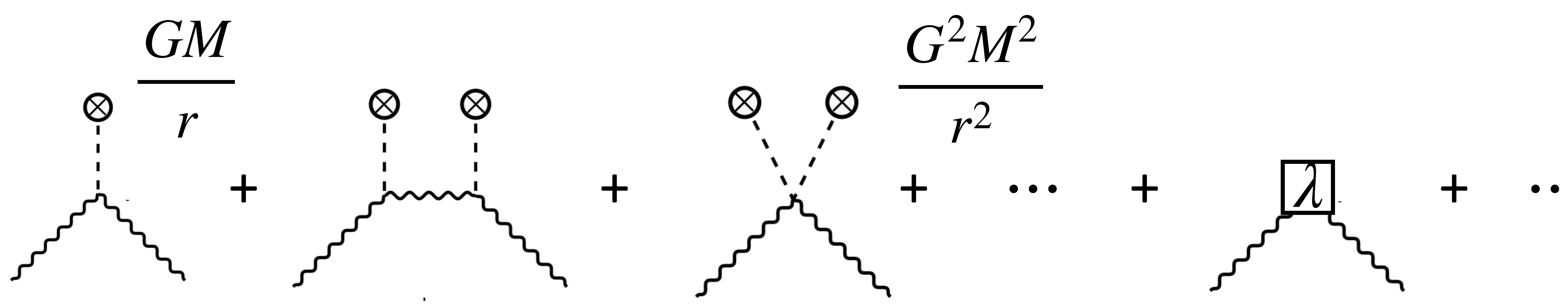
Scattering amplitude in Effective Field Theory

Integrand from background field method

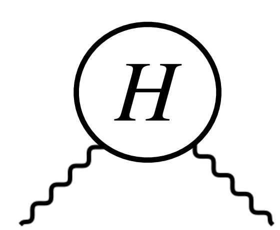
Instead of computing all the Feynman diagrams, we can expand around background solution

$$S = \int d^D x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu}(x) \partial_\mu \phi \partial_\nu \phi + \lambda_{\omega^2} \int d\tau \dot{\phi}^2 + \cdots$$

$i\mathscr{M} =$



Same for photons, for gravitons we also need to include "recoil vertex"



$$\sim M \int d\tau \delta\Gamma^{\mu}_{00} \frac{1}{\partial_\tau^2} \delta\Gamma_{\mu 00}$$

Integrals from angular diff. Eqs.

After PM expansion amplitude only has non-trivial dependence on angle

$$x = \frac{k_1 \cdot k_2}{2\omega^2} = \sin^2 \frac{\theta}{2}$$

$$\mathscr{M}(\omega, x = \sin^2 \tfrac{\theta}{2}) = R_s \left(\mathscr{M}(x)^{1\text{PM}} + (R_s\omega)\mathscr{M}(x)^{2\text{PM}} + (R_s\omega)^2 \mathscr{M}(x)^{3\text{PM}} + \cdots \right)$$

Can use IBP and derive simple canonical differential equations in the angle

$$\frac{d\vec{f}}{dx} = \epsilon A(x)\vec{f} = \epsilon \left[\frac{A_0}{x} + \frac{A_{+1}}{x-1} + \frac{A_{-1}}{x+1} \right] \vec{f}$$

Landau analysis shows that they are likely HPL to all orders.

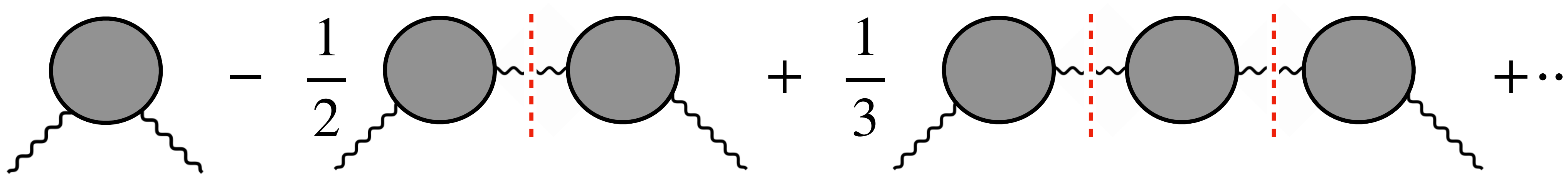
Phase from exponential representation

Amplitude is IR divergent at all orders, but phase shift only at leading

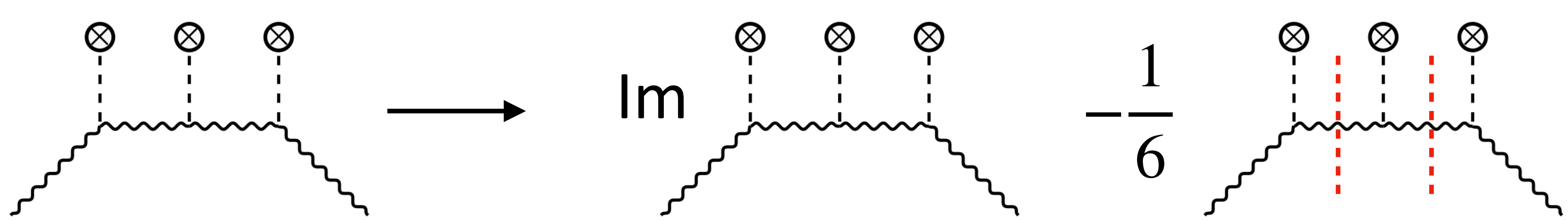
We can cancel infrared divergences by taking the logarithm at operator level

$$i\mathcal{M} = e^{i\Delta} - 1$$ c.f. exponential representation

$i\Delta =$



Can be conveniently done at the level of masters $f_i \to \bar{f}_i = f_i - f_i|_{\text{cuts}}$



Result is manifestly infrared finite

Example

For example, this is the result for the momentum-space phase for the helicity-preserving photon amplitude

$$\Delta^{\mathrm{3PM}}_{(1)} = (Gm)^3\omega^2\pi\Bigg[\left(-\frac{15}{4x}+\frac{3x}{x^2-1}-\frac{2x}{(x^2-1)^2}\right)J_1(x)-\left(\frac{4}{3x^2}+\frac{3}{x^2-1}+\frac{x^2+1}{(x^2-1)^2}\right)J_2(x)$$
$$+\left(\frac{20}{3(x^2-1)}-\frac{7(x^2+1)}{3(x^2-1)^2}\right)\log(x)+\left(\frac{1}{x^2-1}-\frac{x^2+1}{3(x^2-1)^2}\right)\pi^2+\frac{7}{3(x^2-1)}\Bigg]$$

$$J_1(x) \equiv 2\mathrm{Li}_2(-x)-2\mathrm{Li}_2(x)+\log(x^2)\log\left(\tfrac{1+x}{1-x}\right) \qquad J_2(x) \equiv \mathrm{Li}_2\left(x^2\right)+\log(x^2)\log(1-x^2)$$

Helicity violating vanishes.

One can easily perform the partial wave transform of this.

Match BHPT on the nose!

The scalar phase shift in the EFT

Result matches BHPT in the far zone for $\ell \neq 0$

$$\delta_\ell^{G^3}\Big|_{\text{EFT}}^{FZ} = (R_s\omega)^3 \left[\frac{1}{6}\psi^{(2)}(1+\ell) + \frac{-11+15\ell+15\ell^2}{2(-1+2\ell)(1+2\ell)(3+2\ell)}\psi^{(1)}(1+\ell) + \frac{3\left(3\ell^2+3\ell-2\right)}{2\ell(\ell+1)(2\ell-1)(2\ell+3)}\right]$$

Result for $\ell = 0$ has UV divergence that is renormalized by dynamical love number

$$\delta_{\ell=0}^{G^3}\Big|_{\text{EFT}} = (R_s\omega)^3 \left[\frac{1}{4\epsilon_{\text{UV}}} + \frac{25}{12} - \ln\left(\frac{2\omega}{\bar{\mu}}\right) + \frac{11}{36}\pi^2 - \frac{1}{3}\zeta_3\right] + \frac{\lambda_{0\omega^2}}{4\pi}\omega^3$$

Warning! Scheme dependent. Also, refrain from using general-ℓ PW.

Scattering amplitude in BHPT

The calculation in BHPT

Regge-Wheeler equation with wave boundary conditions $\Box_{BH}\, h_{\mu\nu} = 0$

$$f_s(\theta) = \frac{2\pi}{i\omega} \sum_{\ell=s}^{\infty} {}_{-s}S_{\ell}^{s}(1, a\omega)\, {}_{-s}S_{\ell}^{s}(\cos\theta, a\omega)(\eta_{\ell s} e^{2i\delta_{\ell s}} - 1)$$

Phase shifts receive contributions from "near zone" ($r \sim R_s$) and "far zone" $r \gg R_s$

$$\delta_{\ell s} = \underbrace{\delta_{\ell s}\,|^{NZ}}_{\text{Contains information about Love numbers}} + \underbrace{\delta_{\ell s}\,|^{FZ}}_{\text{Computable in EFT, mod counterterms}}$$

Wave scattering off BH

Regge-Wheeler/Teukolsky equation $\qquad \Box_{BH}\, h_{\mu\nu} = 0$

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d}{dr}\,{}_sU_{\ell m}(r)\right) + V_{eff}(r)\,{}_sU_{\ell m}(r) = 0$$

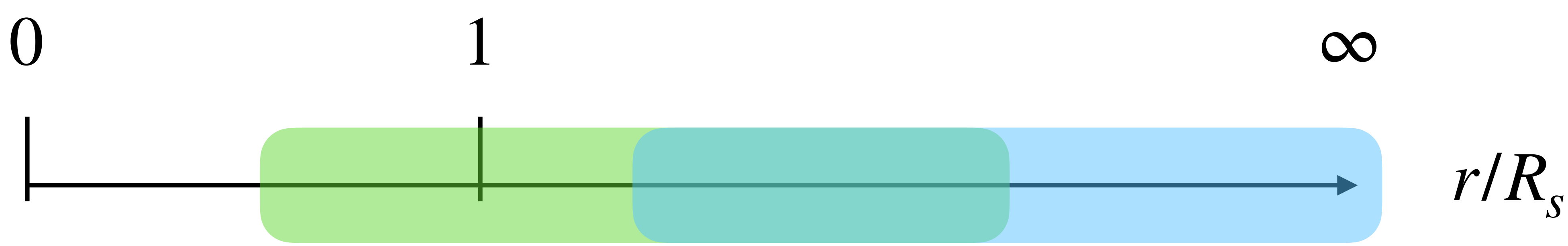
incoming b.c. $\qquad {}_{-s}U_{\ell m}(r) \to B^{(\text{inc})}_{-s\ell m} r^{-1} e^{-i\omega r} + B^{(\text{refl})}_{-s\ell m} r^{-1+2s} e^{i\omega r}$

"Connection coefficients for the confluent Heun equation" $\qquad \eta_{\ell s} e^{2i\delta_{\ell s}} \sim \dfrac{B^{(\text{refl})}_{-s\ell m}}{B^{(\text{inc})}_{-s\ell m}}$

Two methods

1. Method of matched asymptotic expansions [Mano, Suzuki, Takasugi]

Sum of Hypergeoms. Sum of Coulomb wfs.



Match both asymptotic series to determine coefficients.

2. Relation to Seiberg-Witten theory: [Aminov, Grassi, Hatsuda; …]

Radial Teukolsky equation $\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d}{dr}{}_sU_{\ell m}(r)\right) + V_{eff}(r)\,{}_sU_{\ell m}(r) = 0$

=

Quantum Curve for $SU(2)$ $\mathcal{N}=2$ SYM with $N_f = 3$ hypers

The phase shift in BHPT

Result for scalar wave toy model in the far zone of Schwarzschild

$$\delta_\ell^{G^3}\Big|_{\rm BHPT}^{FZ} = (R_s\omega)^3\left[\frac{1}{6}\psi^{(2)}(1+\ell) + \frac{-11+15\ell+15\ell^2}{2(-1+2\ell)(1+2\ell)(3+2\ell)}\psi^{(1)}(1+\ell) + \frac{3\left(3\ell^2+3\ell-2\right)}{2\ell(\ell+1)(2\ell-1)(2\ell+3)}\right]$$

Note pole at $\ell = 0$, related to a counter-term ambiguity in EFT, $\lambda_2\dot{\phi}^2$.
Near and far zone individually not well defined.

Result after adding the "near zone"

$$\delta_{\ell=0}^{G^3}\Big|_{\rm BHPT} = (R_s\omega)^3\left[\frac{6}{12} - \gamma_E - \ln(2R_s\omega) + \frac{11}{36}\pi^2 - \frac{1}{3}\zeta_3\right]$$

Matching

Finally we are ready to match:

$$\mathscr{M}^{BHPT} = \mathscr{M}^{EFT} \quad \longleftrightarrow \quad \delta_\ell^{BHPT} = \delta_\ell^{EFT}\big|_{\bar{\mu}=1/R_s}$$

Comparing to EFT to BHPT result we can extract dynamical Love number!

$$\lambda_{0\omega^2}(\bar{\mu}) = -4\pi R_s^3 \left[\frac{1}{4\epsilon_{\mathrm{UV}}} + \ln(\bar{\mu}R_s) + \frac{19}{12} + \gamma_E\right]$$

Note: initially presented incorrect matching coefficient, corrected after recent communication with Caron-Huot, Correia, Isabella, Solon, who also computed

See Giulia's talk!

Our results

PM order	Scalar	Photon	Graviton
$R_s\omega$	✓	✓	✓
$(R_s\omega)^2$	✓	✓	✓
$(R_s\omega)^3$	$\lambda_1 = 0,\ \lambda_{0\omega^2}(\mu)$	$\lambda_1^E = \lambda_1^B = 0$	✓
$(R_s\omega)^4$	–	–	–
$(R_s\omega)^5$	$\lambda_2,\ \lambda_{1\omega^2},\ \lambda_{0\omega^4}$	$\lambda^E_{1\omega^2},\ \lambda^B_{1\omega^2}$	$\lambda_2^E,\ \lambda_2^B$
$(R_s\omega)^6$	–	–	–
$(R_s\omega)^7$	$\cdots$	$\cdots$	$\lambda^E_{2,\omega^2},\ \lambda^B_{2,\omega^2}$

We also matched dissipative tides and their running to relative two-loop order, which I do not have time to discuss.

Conclusions & Future

- We can compute scattering of waves off compact objects using effective field theory and tools from scattering amplitudes.
- These can be used to extract the tidal properties of compact objects. Demonstrated by matching dynamical black holes tides.
- Much simpler problem than massive scattering. Confident we can push to very high orders and match relevant tides. Bootstrap?
- Generalization to Kerr seems straightforward
- Dynamical tides renormalize subdivergences in massive scattering starting at 7PN.
- See Zihan's talk for application of scattering phase shift to waveforms!