A New Twist on Spinning (A)dS Correlators

Facundo Rost

UNIVERSITY OF AMSTERDAM

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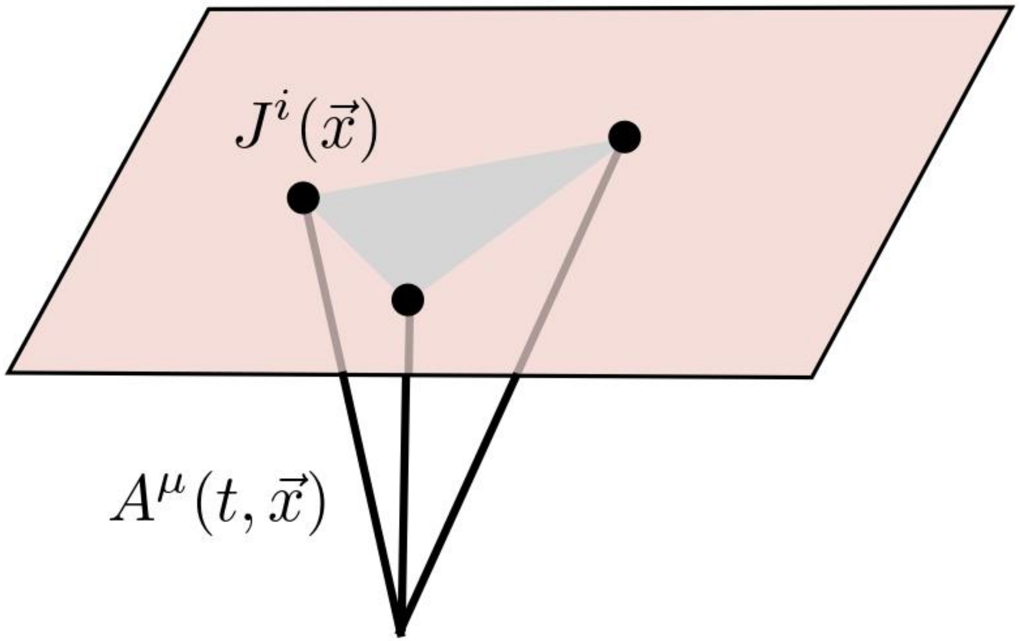
A Challenge

We study correlators on the 3d boundary of a 4d bulk de Sitter spacetime.

These are CFT correlators.



$$\langle J^i(\vec{x}_1) J^j(\vec{x}_2) J^k(\vec{x}_3) \rangle$$

Correlators of massless, spinning fields (conserved currents) are **complicated**.

Example: Graviton Four-Point Function

$$\psi^{(s)}_{\gamma^4}(\{\mathbf{k}\}) = M_{\rm Pl}^2 H^2 \psi^{(s)}_{\phi^4}(\epsilon_1\cdot\epsilon_2)^2(\epsilon_3\cdot\epsilon_4)^2 + 2f^{(s)}_{(2,2)}\left[s^2\Pi^{(s)}_{1,1}(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)W_s + 2W_s^2\right]$$
$$- 2f^{(s)}_{2,0}\Pi^{(s)}_{1,0}(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)W_s + g^{(s)}_{\rm cont.}(\epsilon_1\cdot\epsilon_3)(\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4) - 2f_{\rm cont.}W_sW_s^{(c)},$$

where

$$\psi^{(s)}_{\phi^4}(\{\mathbf{k}\}) = \frac{1}{6H^2M_{\rm Pl}^2}\left[f^{(s)}_{(2,2)}s^4\Pi^{(s)}_{2,2} + f^{(s)}_{(2,1)}s^2\Pi^{(s)}_{2,1} + f^{(s)}_{(2,0)}(E_LE_R - sk_T)\Pi^{(s)}_{2,0} + f_c\right].$$

$$f^{(s)}_{(2,2)} = \frac{1}{k_T} - \frac{s^2}{k_TE_LE_R} + \frac{sk_1k_2}{k_TE_L^2E_R} + \frac{sk_3k_4}{k_TE_R^2E_L} + \frac{2sk_1k_2k_3k_4}{k_T^2E_L^2E_R^2} - \frac{s(k_1k_2+k_3k_4)}{k_T^2E_LE_R}$$
$$+ \frac{k_1k_2}{k_T^2E_L} + \frac{k_3k_4}{k_T^2E_R} + \frac{2k_1k_2k_3k_4}{k_T^3E_LE_R},$$

$$f^{(s)}_{(2,1)} = -\frac{2k_1k_2k_3k_4}{k_T^3} - \frac{k_{12}k_3k_4 + k_{34}k_1k_2}{k_T^2} - \frac{k_{12}k_{34}}{k_T},$$

$$f^{(s)}_{(2,0)} = -f^{(s)}_{(2,1)},$$

$$f_c = -\frac{2k_1k_2k_3k_4(k_{12}k_{34}+s^2)}{k_T^3} - \frac{1}{k_T^2}(k_1k_2k_3 + k_1k_2k_4 + k_1k_3k_4 + k_2k_3k_4)(k_{12}k_{34}+s^2)$$
$$- \frac{1}{4k_T}\Big[24k_1k_2k_3k_4 + 8(k_1k_3+k_2k_3+k_1k_4+k_2k_4)^2$$
$$+ (k_1k_2+k_3k_4)\left[9(k_1k_3+k_2k_3+k_1k_4+k_2k_4) - 12(k_1k_2+k_3k_4)\right]$$
$$+ \frac{1}{2}(k_{12}^2+k_{34}^2+2s^2)\Big(3(k_1k_2+k_3k_4) + 4(k_1k_3+k_2k_3+k_1k_4+k_2k_4)\Big)\Big]$$
$$- \frac{3(k_1-k_2)^2(k_3-k_4)^2(k_1k_2+k_3k_4)}{4s^2k_T} - 3(k_1k_2k_3 + k_1k_2k_4 + k_3k_4k_1 + k_3k_4k_2)$$
$$+ \frac{k_T}{8}\Big[21(k_1k_2+k_3k_4) + 34(k_1k_3+k_2k_3+k_1k_4+k_2k_4)$$
$$+ 3(k_{12}^2+k_{34}^2+2s^2) - \frac{6}{s^2}(k_1-k_2)^2(k_3-k_4)^2\Big] - \frac{9}{8}k_T^3,$$

$$W_s = (\epsilon_1\cdot\epsilon_2)\left[(\epsilon_3\cdot k_1)(\epsilon_4\cdot k_2) - (\epsilon_3\cdot k_2)(\epsilon_4\cdot k_1)\right] + (\epsilon_3\cdot\epsilon_4)\left[(\epsilon_1\cdot k_3)(\epsilon_2\cdot k_4) - (\epsilon_1\cdot k_4)(\epsilon_2\cdot k_3)\right]$$
$$+ \left[(\epsilon_1\cdot k_2)\epsilon_2 - (\epsilon_2\cdot k_1)\epsilon_1\right]\cdot\left[(\epsilon_3\cdot k_4)\epsilon_4 - (\epsilon_4\cdot k_3)\epsilon_3\right],$$

$$W_s^{(c)} = (\epsilon_1\cdot\epsilon_4)(\epsilon_2\cdot\epsilon_3) - (\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4),$$

$$f_{\rm cont.} = \prod_{i=1}^{4}(1 - k_i\partial_{k_i})k_T(\log k_T - 1) = \frac{2\mathbb{e}_4}{k_T^3} + \frac{\mathbb{e}_3}{k_T^2} + \frac{\mathbb{e}_2}{k_T} - k_T,$$

$$g^{(s)}_{\rm cont} = -(k_{12}k_{34}+s^2)\left[\frac{4\mathbb{e}_4}{k_T^3} - \frac{(k_{12}-k_{34})(k_{12}^2+2k_1k_2-k_{34}^2-2k_3k_4)}{2k_T^2} + 3\frac{k_1k_2+k_3k_4}{k_T} - \frac{3k_T}{2}\right]$$
$$- \frac{4\mathbb{e}_4}{k_T} + \frac{8k_{12}^2k_{34}^2 - 4k_{12}^4 - 4k_{34}^4 - 2k_1k_2k_{34}^2 - 2k_3k_4k_{12}^2 + 8k_1k_2k_{12}^2 + 8k_3k_4k_{34}^2}{3k_T}$$
$$+ \frac{14}{3}k_T(k_1k_2+k_3k_4) - \frac{10k_T^3}{9} + \frac{16}{9}\sum_{a=1}^{4}k_a^3.$$

[Bonifacio et al, ‘22]

Spinning Amplitudes

In contrast, amplitudes in flat space are **very simple**:

$$A(1^-2^-3^+\cdots n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle (n-1)n\rangle\langle n1\rangle}$$

Why? Spinor-helicity variables trivialize all kinematic constraints:

1. Lorentz symmetry

2. No gauge redundancy

What Are The Right Variables?

We may be using inconvenient variables for cosmological correlators.

The right variables must make all kinematic constraints manifest:

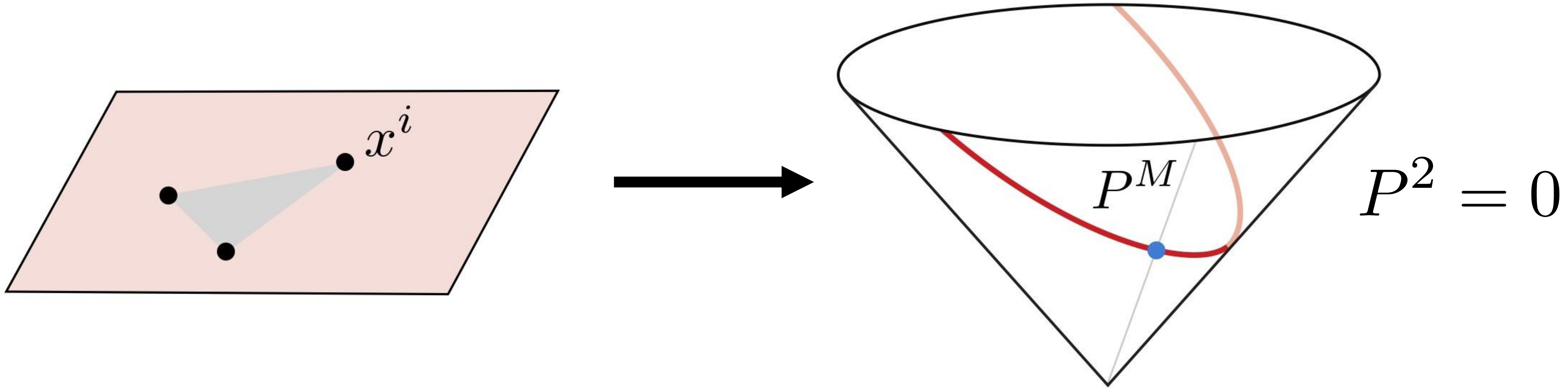
1. Conformal symmetry

2. Current conservation $\longrightarrow$ $\partial_i J^i = 0$

Goal: Find variables that expose the hidden simplicity of correlators.

Embedding Space

We map 3d positions to the projective null cone of 5d Minkowski:



- Conformal symmetry is trivialized $\longrightarrow$ $\langle J_1 J_2 \cdots J_n \rangle = F(P_i \cdot P_j)$
- Conservation is not $\longrightarrow$ $\partial_M \langle J_1^M J_2 \cdots J_n \rangle = 0$

A Lesson from 2d CFT

Embedding space is four-dimensional.
Hence, we can use ordinary SHV for the null cone position:

$$P_{\alpha\dot{\alpha}} = \begin{pmatrix} P_0 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & P_0 - P_3 \end{pmatrix} = \Lambda_\alpha \tilde{\Lambda}_{\dot{\alpha}}$$

- Works well for all Δ. What about conserved tensors?

$$J(\Lambda) \quad T(\Lambda)$$

- Conservation is trivialized through **holomorphicity**.
- How does this generalize to 3d CFTs?

Spinors in 3d CFTs

Embedding space is five-dimensional.
Hence, we must use SHV in 5d flat space:

[Binder et al, '20;
Chiodaroli et al, '22]

$$P^{AB} \equiv P^M (\Gamma_M)^{AB} = \epsilon^{ab} \Lambda_a^A \Lambda_b^B$$

$A, B = 1, 2, 3, 4$

$a, b = 1, 2$

- A current can be written as $J^{a_1 a_2 \cdots a_{2S}}(\Lambda_a)$
- The conservation condition is

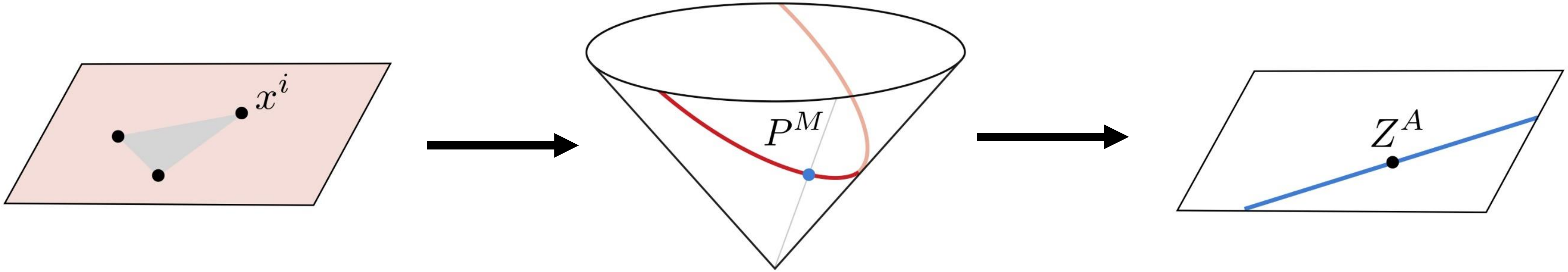
$$\partial_M J^M = 0 \longrightarrow \partial_{a_1 a_2} J^{a_1 a_2 \cdots a_{2S}}(\Lambda_a) = 0$$

How to trivialize it?

Discovering Twistors

Conservation ⟶ Holomorphic functions that only depend on Λ_a^A through

$$Z^A = \pi^a \Lambda_a^A$$ ⟵ This is a twistor.



Currents in Twistor Space

Integrate over the arbitrary π^a:

$$J^{a_1 a_2 \cdots a_{2S}}(\Lambda_a) = \int D\pi \, \pi^{a_1} \pi^{a_2} \cdots \pi^{a_{2S}} F(\pi^a \Lambda_a)$$

- Automatically conserved. Note that $\epsilon_{ab}\pi^a\pi^b = 0$
- Projective invariance:

$$F(rZ) = r^{-2(S+1)} F(Z) \longrightarrow \Delta = S + 1$$

Currents in Twistor Space

Index-free form:

$$J \equiv \zeta_{a_1} \cdots \zeta_{a_{2S}} J^{a_1 \cdots a_{2S}}(\Lambda_a) = \int D\pi \, (\zeta_c \pi^c)^{2S} \, F(\pi^a \Lambda_a)$$
$$= \int DZ \, (\Upsilon \cdot Z)^{2S} \, F(Z) \, ,$$

where $\Upsilon \cdot Z = \Upsilon^A \Omega_{AB} Z^B$ with $\Omega_{AB} = \begin{pmatrix} 0 & 1_{2\times2} \\ -1_{2\times2} & 0 \end{pmatrix}$

Alternative representation in dual twistor space: [Neiman, '17]

$$\tilde{J} = \int DW \left(\Upsilon \cdot \frac{\partial}{\partial W} \right)^{2S} \tilde{F}(W) \quad \text{where} \quad W_A = \pi^a (\Omega_{AB} \Lambda_a^B)$$

Correlators in Twistor Space

Apply this to three-point functions of spin-S currents:

$$\langle J_1 J_2 J_3 \rangle = \left[\prod_{k=1}^{3} \int DZ_k \, (\Upsilon_k \cdot Z_k)^{2S} \right] F(Z_i \cdot Z_j)$$

$$\langle \tilde{J}_1 \tilde{J}_2 \tilde{J}_3 \rangle = \left[\prod_{j=1}^{3} \int DW_k \left(\Upsilon_k \cdot \frac{\partial}{\partial W_k} \right)^{2S} \right] \tilde{F}(W_i \cdot W_j)$$

- Conservation and conformal symmetry manifest.
- Scaling: $F(r_i Z_i) = (r_1 r_2 r_3)^{-2S-2} F(Z_i)$,

  $\tilde{F}(r_i W_i) = (r_1 r_2 r_3)^{2S-2} \tilde{F}(W_i)$

Correlators in Twistor Space

Bootstrap three-point functions in twistor space via scaling:

$$F(Z_i \cdot Z_j) = \int \frac{d^3 c_{ij}}{(2\pi)^3} \exp\left(-i c_{12} Z_1 \cdot Z_2 + \text{cyclic}\right) A(c_{ij}) \,,$$

$$\tilde{F}(W_i \cdot W_j) = \int \frac{d^3 c_{ij}}{(2\pi)^3} \exp\left(-i c_{12} W_1 \cdot W_2 + \text{cyclic}\right) \tilde{A}(c_{ij}) \,,$$

where

$$A(c_{ij}) = (c_{12} c_{23} c_{31})^S \quad \text{and} \quad \tilde{A}(c_{ij}) = \frac{1}{(c_{12} c_{23} c_{31})^S} \,.$$

Correlators in Twistor Space

This gives the known three-point correlators in embedding space:

$$A(c_{ij}) = (c_{12}c_{23}c_{31})^S \mapsto \langle J_1 J_2 J_3 \rangle = \{\ F^3\ ,\ R^3\ ,\ \ldots\ \}$$

$$\tilde{A}(c_{ij}) = \frac{1}{(c_{12}c_{23}c_{31})^S} \mapsto \langle \tilde{J}_1 \tilde{J}_2 \tilde{J}_3 \rangle = \{\ \mathrm{YM}\ ,\ \mathrm{GR}\ ,\ \ldots\ \}$$

- Remarkably simple!
- Double copy at 3 points!
- They can also be mapped to known correlators in momentum space.

Conclusions and Outlook

In this work:

- Found variables that make all kinematic constraints manifest.
- Exposed a hidden simplicity of three-point functions.

Next steps:

- Extend to higher points.
- Cosmological Parke-Taylor.
- Positive geometries.

Extra Slide: Back to Momentum Space

- Half-Fourier transform:

$$Z_i^A \equiv \begin{pmatrix} \lambda_i^\alpha \\ \mu_{i,\dot\alpha} \end{pmatrix} \quad \Rightarrow \quad g(\lambda_i, \tilde\lambda_i) \equiv \int d^2\mu_i \exp(i\tilde\lambda_i \cdot \mu_i)\, F(Z_i)\,,$$

$$W_{i,A} \equiv \begin{pmatrix} \tilde\mu_{i,\alpha} \\ \tilde\lambda_i^{\dot\alpha} \end{pmatrix} \quad \Rightarrow \quad \tilde g(\lambda_i, \tilde\lambda_i) \equiv \int d^2\tilde\mu_i \exp(-i\lambda_i \cdot \tilde\mu_i)\, \tilde F(W_i)\,.$$

- They give known results in Fourier space:

$$g(\lambda_i, \tilde\lambda_i) = \{\ F^3\ ,\ R^3\ ,\ \ldots\ \}$$

$$\tilde g(\lambda_i, \tilde\lambda_i) = \mathrm{Disc}\{\ \mathrm{YM}\ ,\ \mathrm{GR}\ ,\ \ldots\ \}$$

Spinning (A)dS Correlators