Scattering Amplitudes and Non-Invertible Symmetry

Shota Komatsu

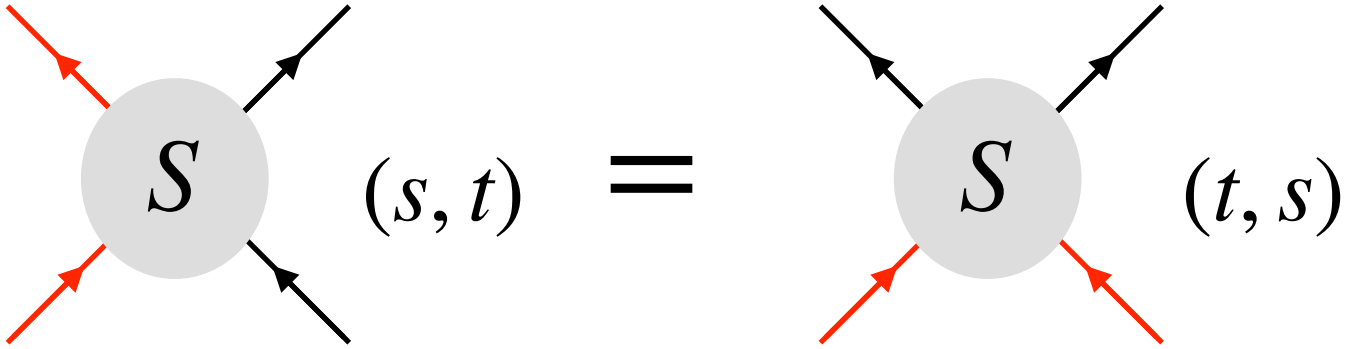
Based on works with Christian Copetti (Oxford) Lucía Córdova (CERN)

2403.04835 : “Non-Invertible Symmetries, Anomalies and Scattering Amplitudes”

2408.13132: “S-Matrix Bootstrap and Non-Invertible Symmetries”

Crossing symmetry

- **Crossing symmetry**: One of important properties of S-matrix



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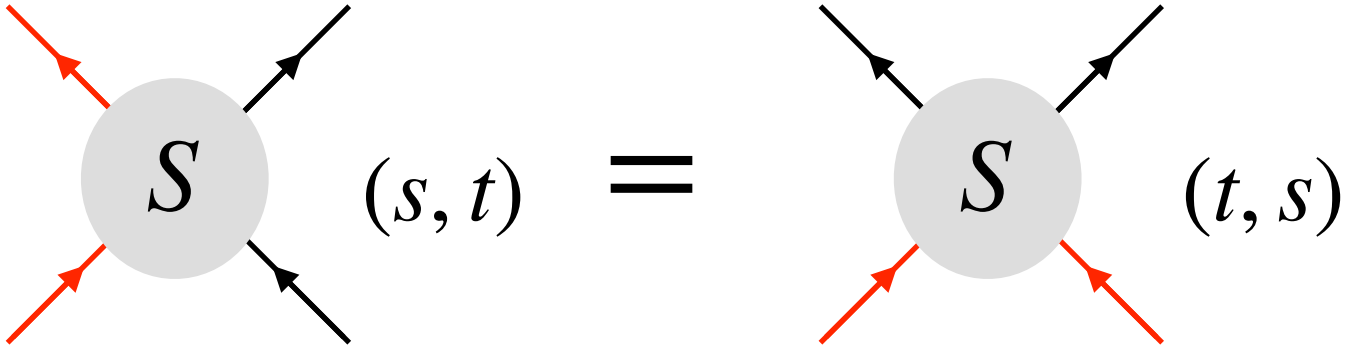

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- Useful in practice. No need to repeat the computation!
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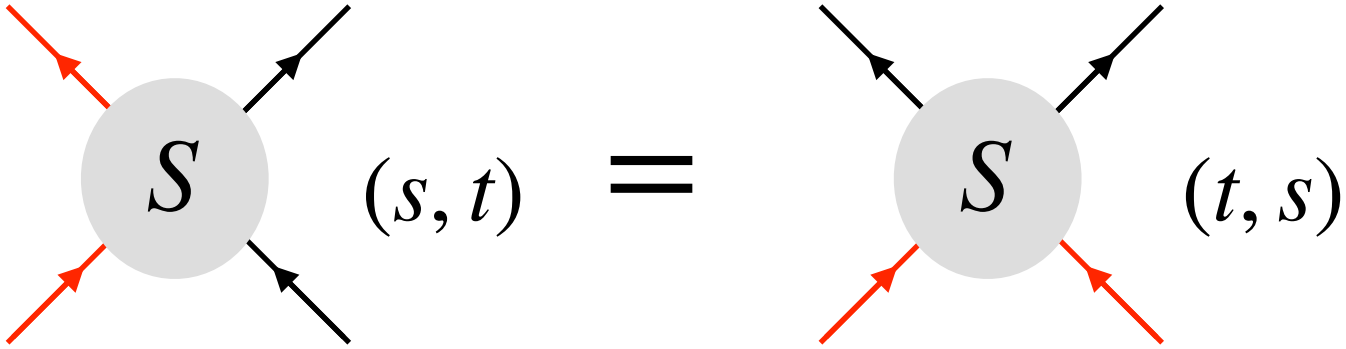

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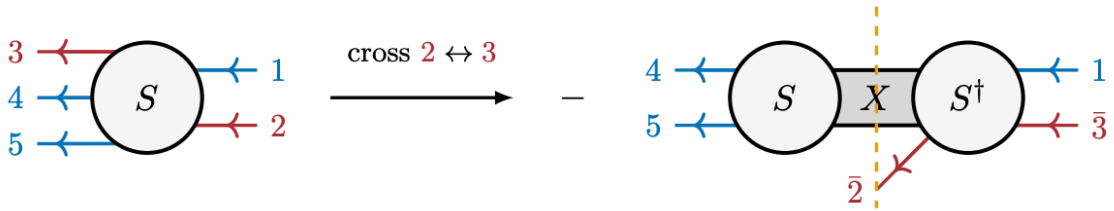
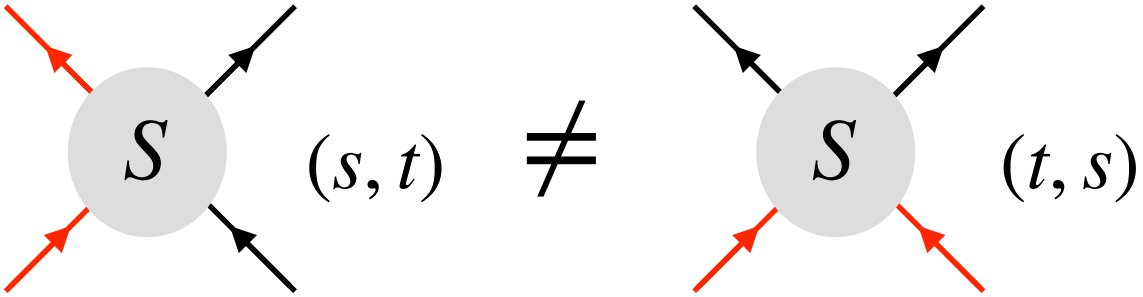
- Useful in practice. No need to repeat the computation!
- Rigorous proof in some cases. [Bros, Epstein, Glaser], ….., [Mizera], …
- Relates higher-pt S-matrix to in-in obs. Relevant for gravitational waveforms. [Caron-Huot, Giroux, Hannesdottir, Mizera]



Takeaway of this talk

- In the presence of long-range modes (soft / topological), crossing symmetry of S-matrix is sometimes modified.



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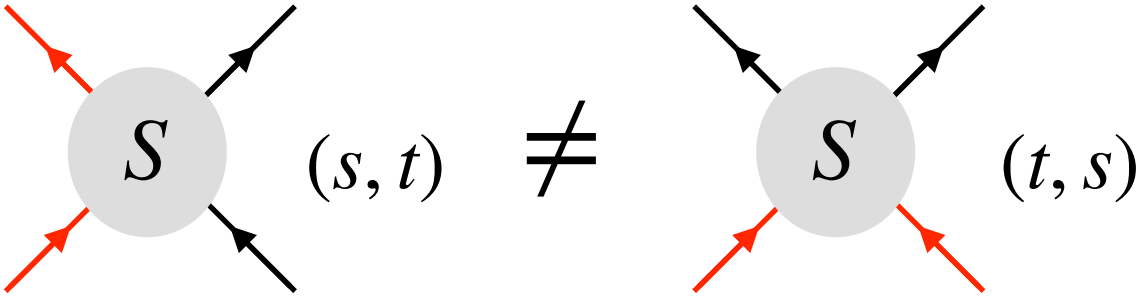

- Modification comes from (non-local) corrections to norms of in- and out- states due to long-range modes.

$$\langle \{p_i, Q_i\} \,|\, \{p_i, Q_i\} \rangle_{\text{full}} = f_{\text{long}}(\{p_i, Q_i\})) \times \langle \{p_i, Q_i\} \,|\, \{p_i, Q_i\} \rangle_{\text{short}}$$

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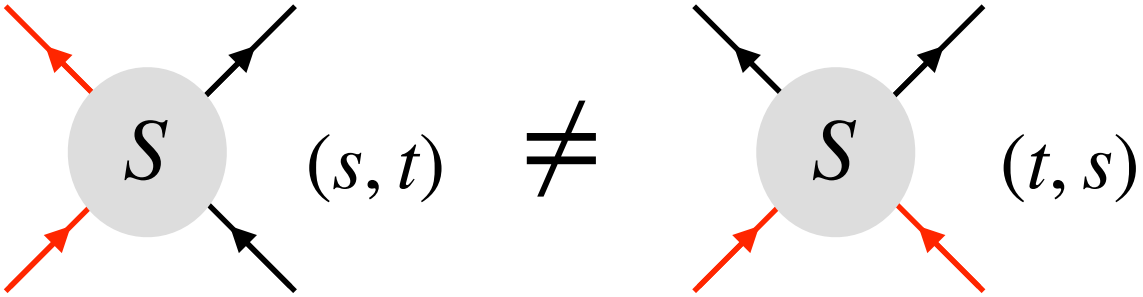

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- Today I'll focus on simplest case:
  Theories with non-invertible sym & topological modes in 1+1 d.
  But we expect similar modifications in higher d & soft modes
  (Relevant for QCD & gravity waves / charge-monopole….?)

Plan

1. Non-invertible symmetries in 1+1 dim
2. Integrable flow from tricritical Ising and S-matrix
3. Derivation of modified crossing rules
4. Conclusion

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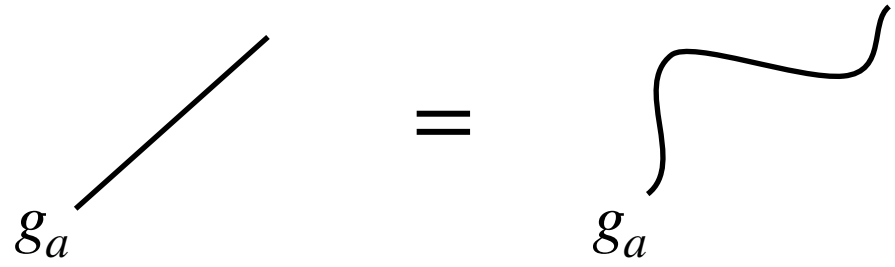
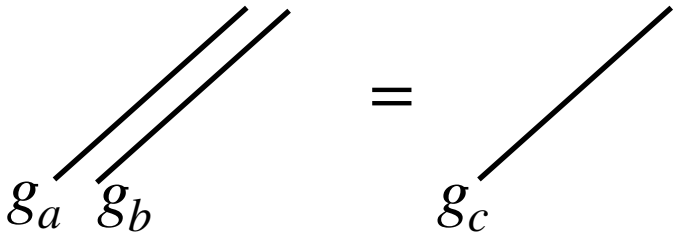
Non-Invertible Symmetries in 1+1 dim

- Symmetries in QFT $\subset$ (d-1)-dim topological operators



- Usual symmetries (group): $g_a g_b = g_c$ $\qquad g_{a,b,c} \in G$



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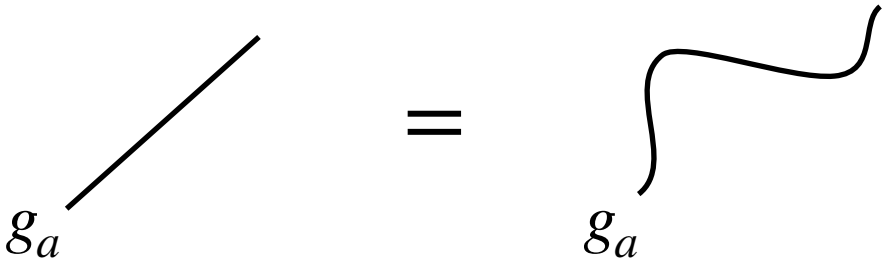

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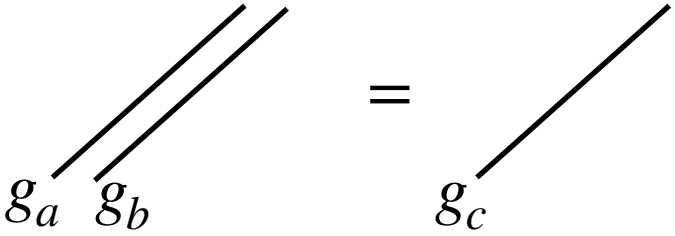
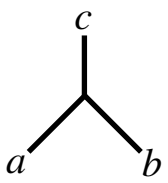


- Non-invertible symmetries (in 1+1 dim):

Fusion category $\qquad \mathscr{L}_a \mathscr{L}_b = \sum_{\mathscr{L}_c} N_{ab}{}^c \mathscr{L}_c \qquad (N_{ab}{}^c \in \mathbb{Z}_{\geq 0})$

Data characterizing fusion categories

$$\mathscr{L}_a \mathscr{L}_b = \sum_{\mathscr{L}_c} N_{ab}{}^c \mathscr{L}_c$$

- Fusion coefficient $N_{ab}{}^c$: 

If $N_{ab}{}^c \neq 0$

- Quantum dimension : $\langle \mathcal{L}_a \rangle =$ 

$= \mathrm{d}_a$ 1 group-like, $\sqrt{2}$ etc. for non-inv.

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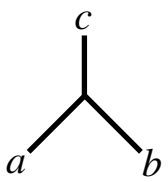
- Fusion coefficient $N_{ab}{}^c$: 

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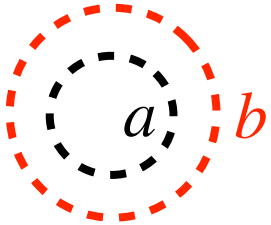
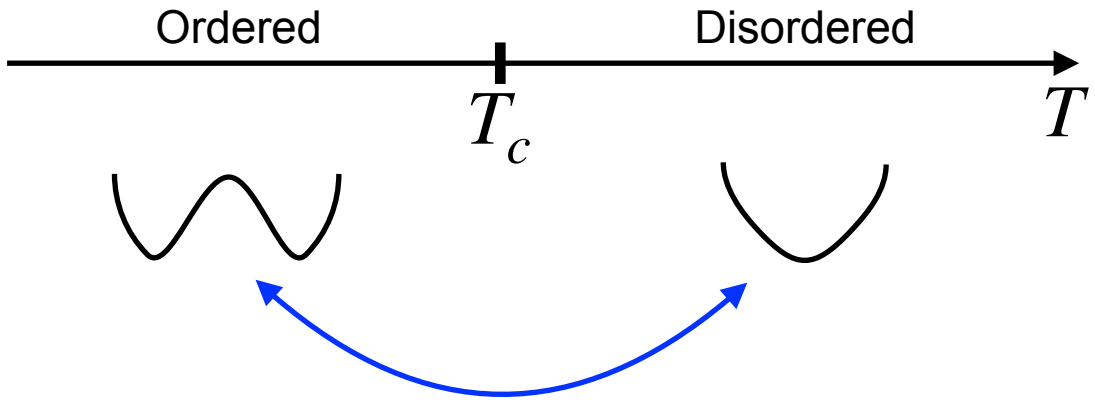
They satisfy consistency conditions.

e.g. 

$\mapsto \quad \mathrm{d}_a \mathrm{d}_b = \sum_c N_{ab}{}^c \mathrm{d}_c$

Non-Invertible Kramers-Wannier

- 2d Ising model: $Z = \sum_{\{\sigma\}} \exp\left[\beta \sum_{<ij>} \sigma_i \sigma_j\right]$



Kramers-Wannier duality $e^{-2\beta_{\text{low}}} \leftrightarrow \tanh \beta_{\text{high}}$

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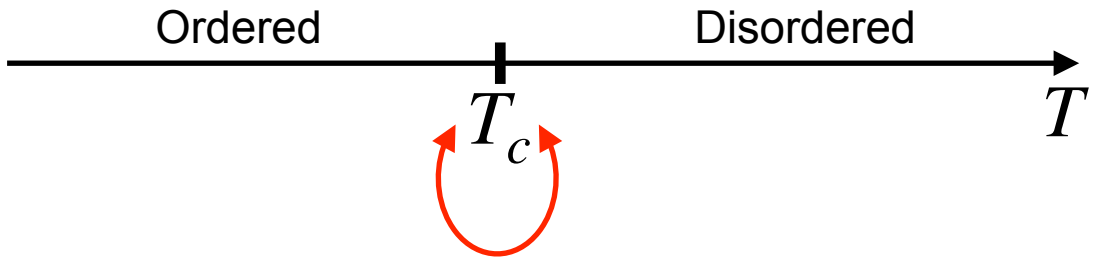

Kramers-Wannier **symmetry**

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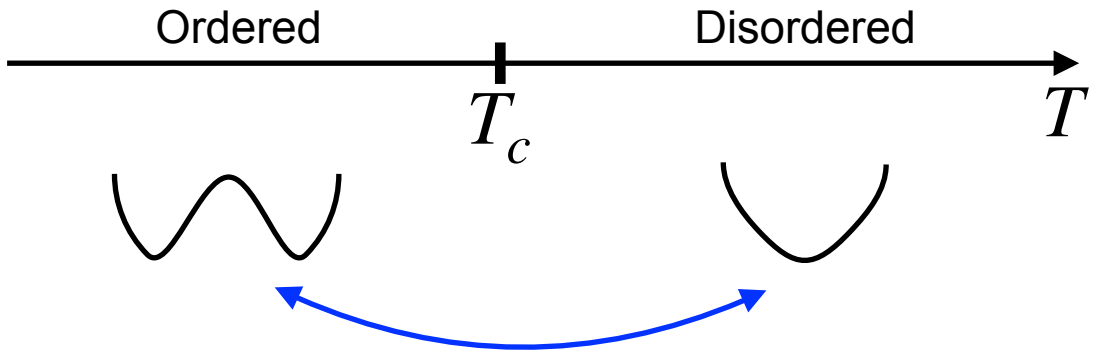

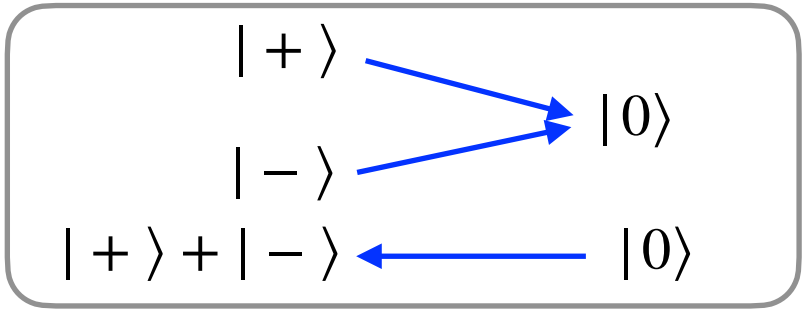

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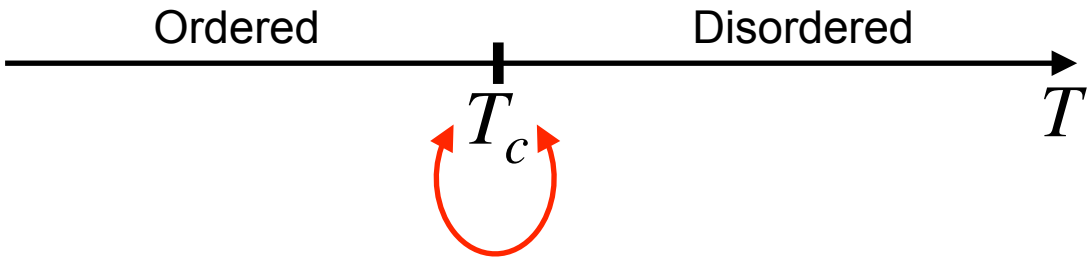
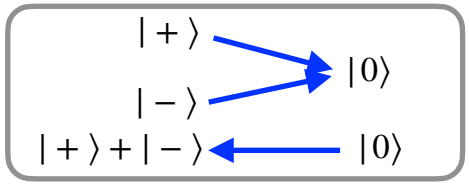


Kramers-Wannier **symmetry** $\mathcal{N}$

- Symmetry of 2d Ising CFT

$$\eta^2 = 1\,, \qquad \mathcal{N}^2 = 1 + \eta\,, \qquad \mathcal{N}\eta = \eta\mathcal{N} = \mathcal{N}$$

$$(\mathrm{d}_1 = \mathrm{d}_\eta = 1, \quad \mathrm{d}_\mathcal{N} = \sqrt{2})$$ topological modes!



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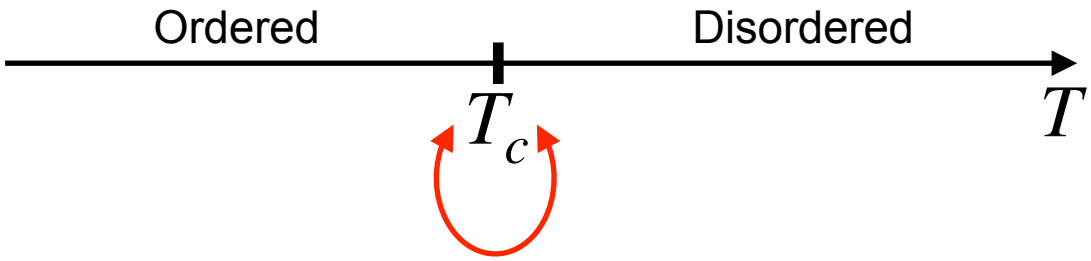

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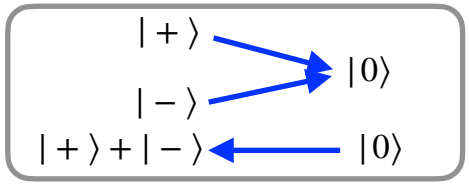
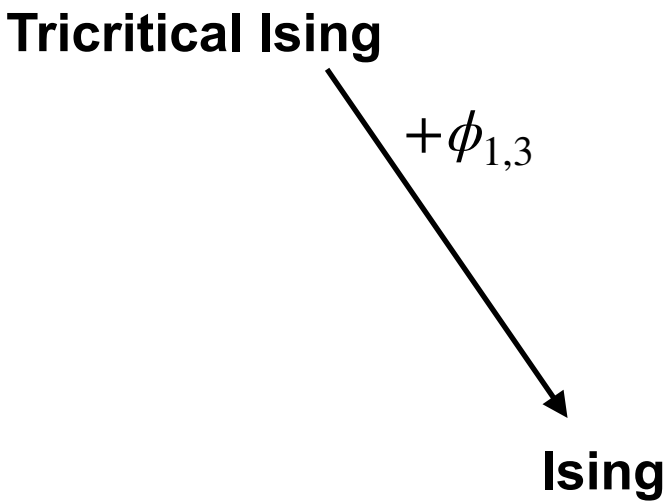


- In Ising CFT, KW symmetry broken by relevant perturbations

- RG flow preserving KW symmetry…?

Flow from tricritical Ising

- Ising CFT is the simplest unitary minimal model ($\mathscr{M}_{3,4}$).
- Next simplest is tricritical Ising CFT ($\mathscr{M}_{4,5}$).
- Non-invertible symmetric RG flow from tricritical: [Chang, Lin, Shao, Wang, Yin]



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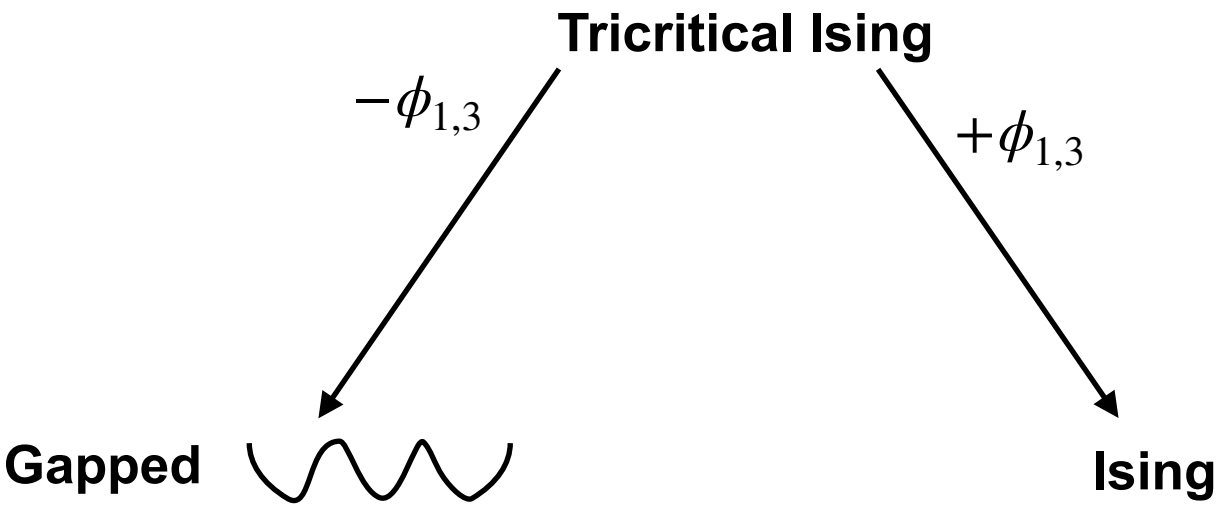
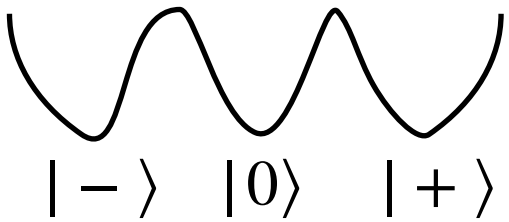


Action on vacua



- $\mathbb{Z}_2$-defect exchanges $|+\rangle$ and $|-\rangle$

$$\eta : |+\rangle \leftrightarrow |-\rangle$$

- $\mathcal{N}$-defect:

$$\mathcal{N}|0\rangle = |+\rangle + |-\rangle$$

$$\mathcal{N}|+\rangle = \mathcal{N}|-\rangle = |0\rangle$$

- "Superposition" of disordered and ordered vacua in Ising

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Integrability along the flow

Tricritical Ising $\xrightarrow{-\phi_{1,3}}$ 

- At UV CFT fixed point, there exist ∞ many higher spin charges.

  because of Virasoro

- Perturbation by $\phi_{1,3}$ preserve higher spin charges.

  [Zamolodchikov 1989]

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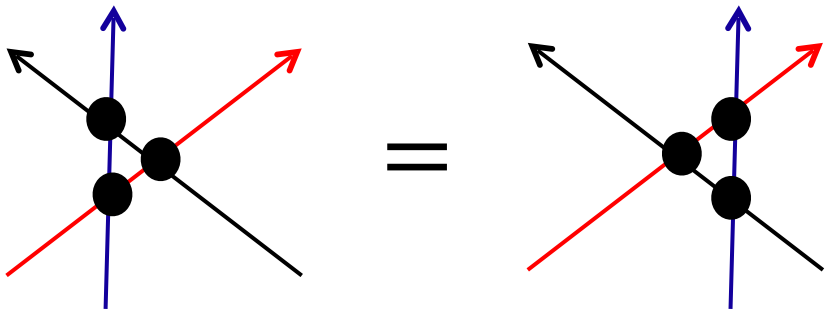
[Shankar, Witten] [Parke]

- 2→2 S-matrices satisfy the Yang-Baxter equation.



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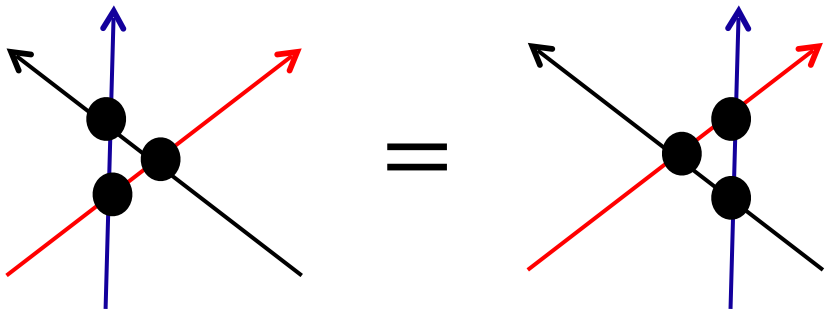
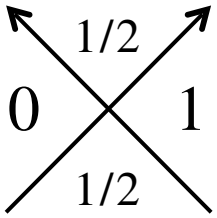


- Imposing unitarity, crossing & YB, the S-matrix can be almost uniquely "**bootstrapped**".

  [Zamolodchikov 1991]

S-matrix by Zamolodchikov

- Particles in IR = kinks interpolating between adjacent vacua.

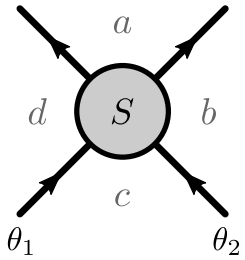


etc .

*Warning: Change of notation!

- It also depends on total energy: $s = (p_1 + p_2)^2 = 4m^2 \cosh^2(\theta/2)$

$$S_{dc}^{ab}(\theta) =$$

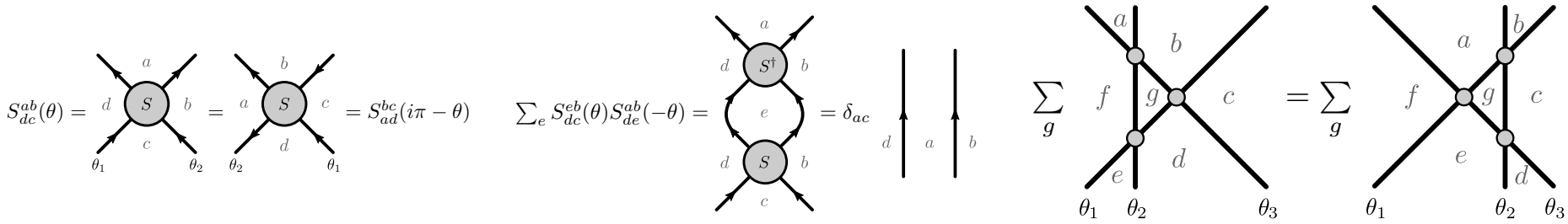


$$\theta = \theta_1 - \theta_2$$

$$s + t = 4m^2 \qquad t = -4m^2 \sinh^2(\theta/2)$$

S-matrix by Zamolodchikov

- Imposing crossing & unitarity & YB,



we can fix S-matrix uniquely as

$$\widehat{S}^{ab}_{dc}(\theta) = Z(\theta) \left(\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}\right)^{\frac{i\theta}{2\pi}} \left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\, \sinh\left(\frac{\theta}{n}\right)\, \delta_{bd} + \sinh\left(\frac{i\pi - \theta}{n}\right)\, \delta_{ac}\right]$$

$$(n = 3, \quad \mathrm{d}_0 = \mathrm{d}_1 = 1, \quad \mathrm{d}_{1/2} = \sqrt{2})$$

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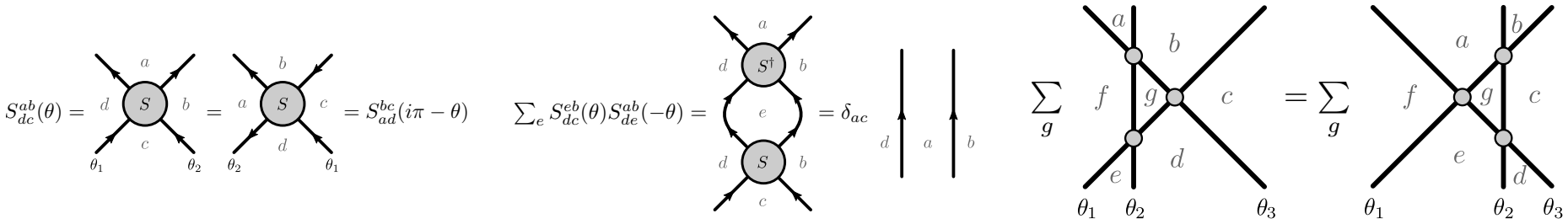

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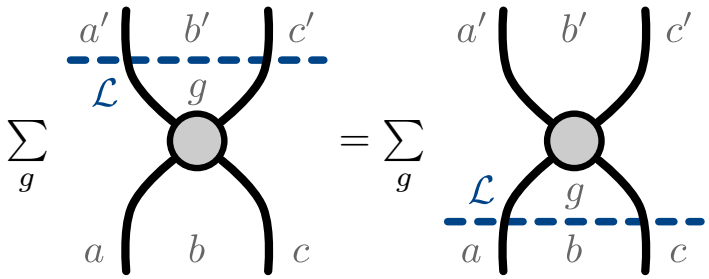
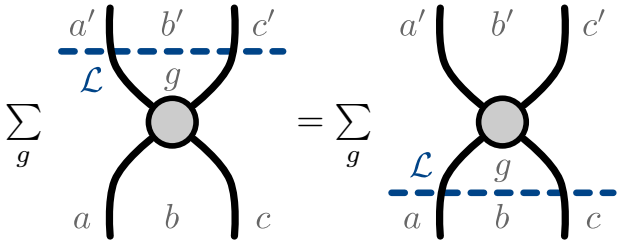
- We expect that it also preserves noninvertible symmetries:



?

Paradox



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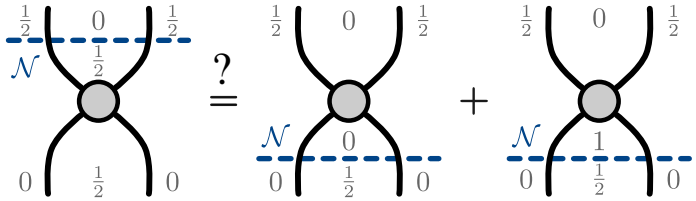
- We found that $\widehat{S}$ commutes with η ($\mathbb{Z}_2$), but not with $\mathcal{N}$.



$$\mathcal{N}: \quad S^{\frac{1}{2}0}_{0\frac{1}{2}}(\theta) \stackrel{?}{=} S^{0\frac{1}{2}}_{\frac{1}{2}0}(\theta) + S^{0\frac{1}{2}}_{\frac{1}{2}1}(\theta)$$

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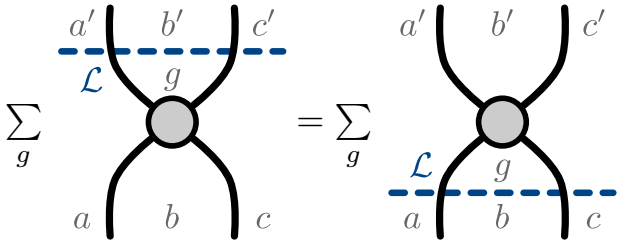

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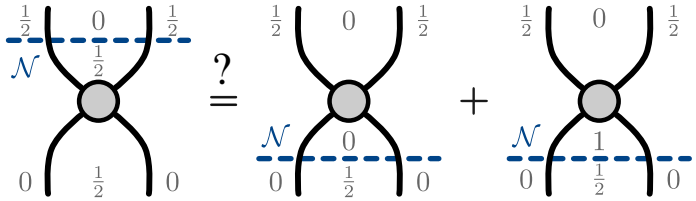
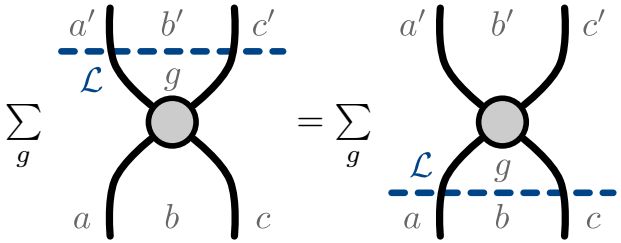


$$\mathcal{N}: \quad S^{\frac{1}{2}0}_{0\frac{1}{2}}(\theta) \stackrel{?}{=} S^{0\frac{1}{2}}_{\frac{1}{2}0}(\theta) + S^{0\frac{1}{2}}_{\frac{1}{2}1}(\theta)$$

- The following 4 properties are mutually incompatible.
  - Unitarity
  - Crossing
  - Integrability (YB)
  - Noninvertible symmetry

Paradox



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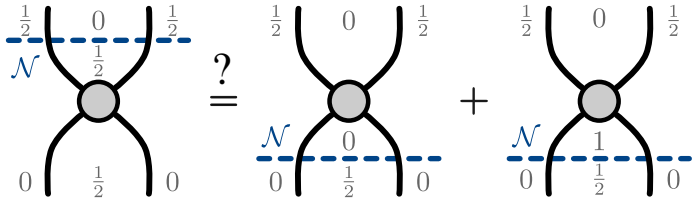

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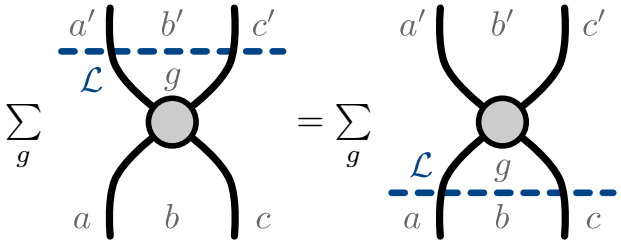

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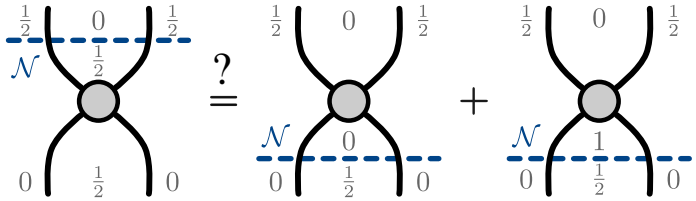

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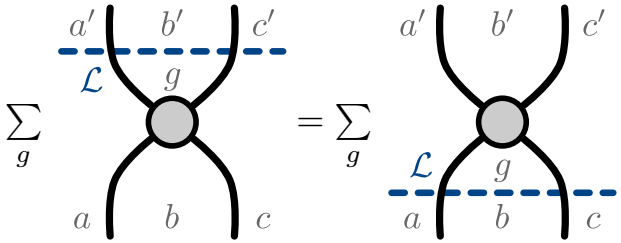

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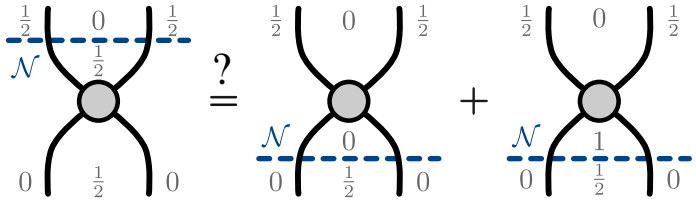

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- The following 4 properties are mutually incompatible.
  - Unitarity ✓ basic principle
  - Crossing
  - Integrability (YB) ✓ deformation of CFT
  - Noninvertible symmetry ✓ deformation of CFT
- The only viable option is to give up crossing.

New proposal

$$S_{dc}^{ab}(\theta) = Z(\theta)\left[\sqrt{\frac{\mathrm{d}_a\mathrm{d}_c}{\mathrm{d}_b\mathrm{d}_d}}\,\sinh\left(\frac{\theta}{n}\right)\,\delta_{bd} + \sinh\left(\frac{i\pi-\theta}{n}\right)\,\delta_{ac}\right]$$

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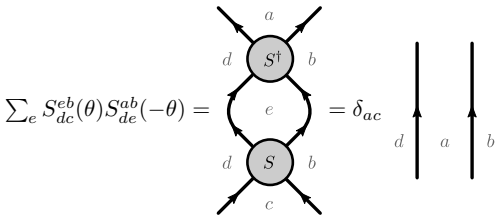
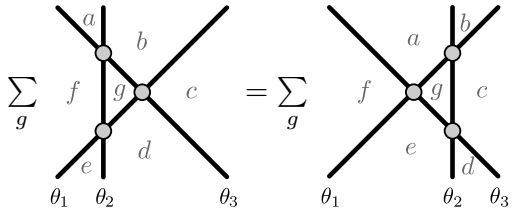
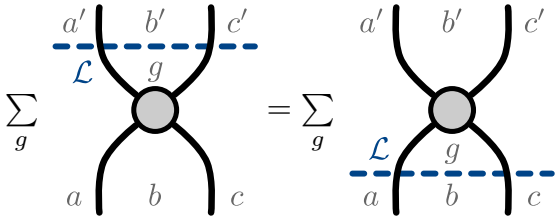


Unitarity



Yang-Baxter



Non-invertible

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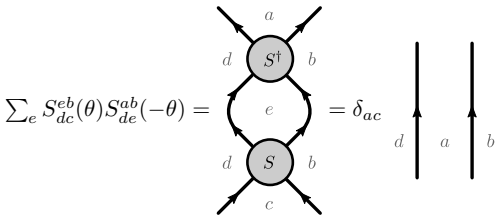

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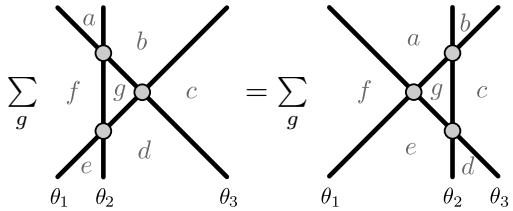

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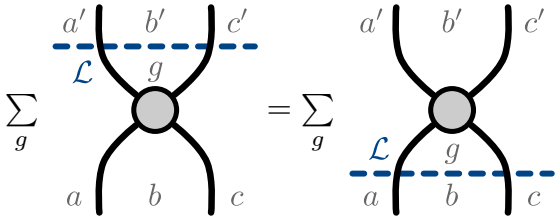


Non-invertible

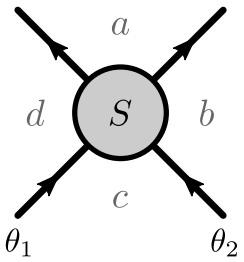
- Crossing symmetry is modified:

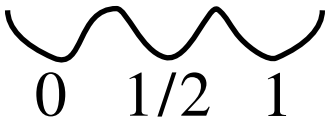
$$S_{dc}^{ab}(\theta) = \sqrt{\frac{\mathrm{d}_a\mathrm{d}_c}{\mathrm{d}_b\mathrm{d}_d}}\,S_{ad}^{bc}(i\pi-\theta)$$

- Physical origin? (The topic of the rest of the talk)

1. Non-invertible symmetries in 1+1 dim
2. Integrable flow from tricritical Ising and S-matrix
3. Derivation of modified crossing rules
4. Conclusion

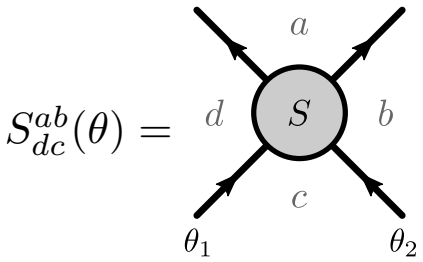
Key physical input

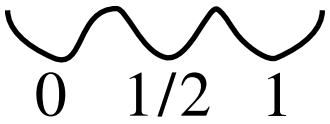
$S^{ab}_{dc}(\theta) =$ 



- The IR dynamics is described by a nontrivial TQFT.

Key physical input





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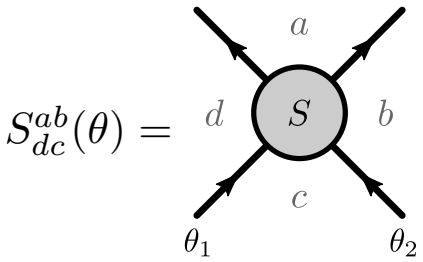
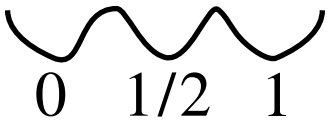
1. In the IR, the worldline of a kink = symmetry line $\mathcal{N}\,(=:\nu)$

$$\mathcal{N}\,|0\rangle = |1/2\rangle, \qquad \mathcal{N}\,|1/2\rangle = |0\rangle + |1\rangle, \qquad \mathcal{N}\,|1\rangle = |1/2\rangle\,.$$

Key physical input





- The IR dynamics is described by a nontrivial TQFT.

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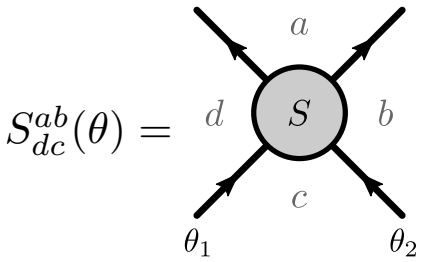
2. The vacua are in 1-to-1 correspondence with symmetry lines.

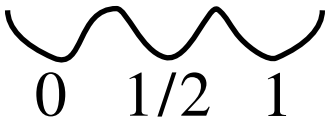
$$|0\rangle \leftrightarrow \mathbf{1} \qquad |1/2\rangle \leftrightarrow \mathcal{N} \qquad |1\rangle \leftrightarrow \eta$$

("Adjoint" representation of fusion category)

$$\mathcal{N}\,|1/2\rangle = |0\rangle + |1\rangle \quad \leftrightarrow \quad \mathcal{N}^2 = \mathbf{1} + \eta$$

Key physical input





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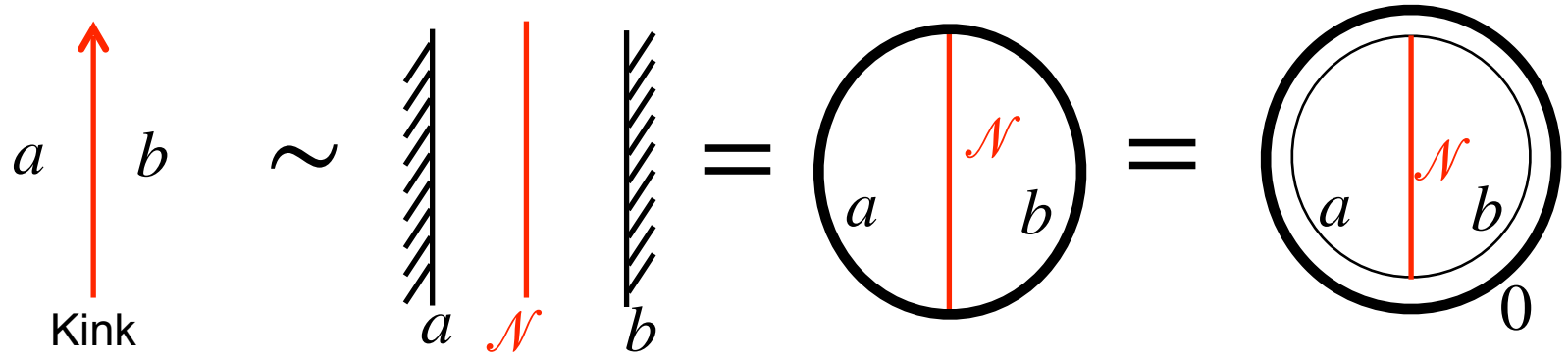

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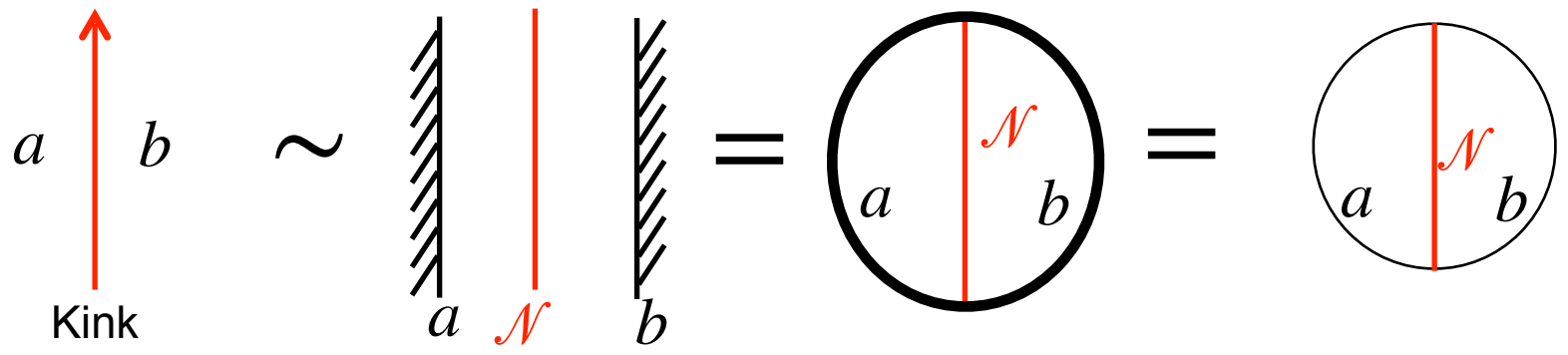


kink states $\leftrightarrow$ network of lines

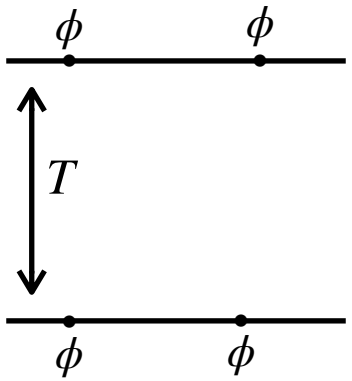
Modified crossing for S-matrix

- Kink creation operators are non-local (boundary cond. changing op)

  Unclear how to get S-matrix from LSZ reduction.

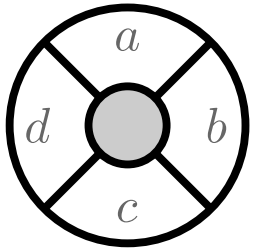
- Use alternative (discussed in Itzykson-Zuber Ch.5)

$$S(\{p_i\}) = \lim_{T\to\infty} \int \prod_j dv_j e^{ip_j x(v_j)} \prod_k (n_k \cdot \overleftrightarrow{\partial}_{x_k}) G(\{x(v_k)\})$$



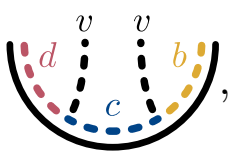
- In our case,

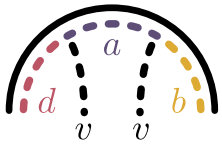
$$S^{ab}_{dc}(\theta) \propto$$

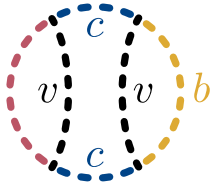


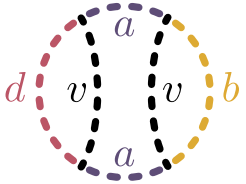
$$\Big|_{\text{analyt. cont.}}$$

- But we need to take into account normalization.

$|\text{in}\rangle\rangle =$ 

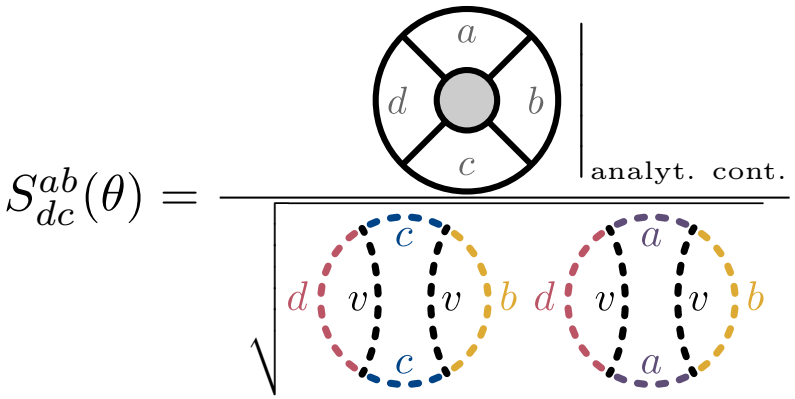
$|\text{out}\rangle\rangle =$ 

$\langle\langle \text{in}|\text{in}\rangle\rangle =$ 

$\langle\langle \text{out}|\text{out}\rangle\rangle =$ 

Modified crossing for S-matrix

- Thus the correct expression is



- Numerator : disk correlation function, crossing symmetric.
- Denominator : depend on the channel we consider.

Modified crossing for S-matrix

- Thus the correct expression is

$$S_{dc}^{ab}(\theta) = \frac{\left.\begin{array}{c} a \\ d \quad b \\ c \end{array}\right|_{\text{analyt. cont.}}}{\sqrt{\begin{array}{c} c \\ d\ v\ v\ b \\ c \end{array} \quad \begin{array}{c} a \\ d\ v\ v\ b \\ a \end{array}}}$$

- Numerator : disk correlation function, crossing symmetric.
- Denominator : depend on the channel we consider.

- Modified crossing:

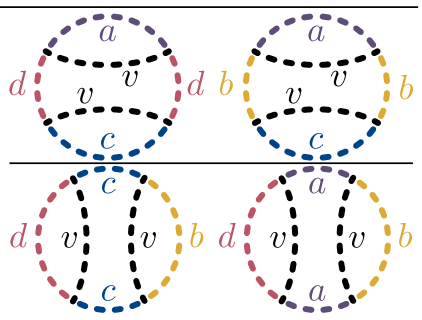
$$S_{dc}^{ab}(\theta) = \sqrt{\frac{\begin{array}{c} a \\ d\ v\ d \\ c \end{array} \quad \begin{array}{c} a \\ b\ v\ b \\ c \end{array}}{\begin{array}{c} c \\ d\ v\ v\ b \\ c \end{array} \quad \begin{array}{c} a \\ d\ v\ v\ b \\ a \end{array}}}\, S_{ad}^{bc}(i\pi - \theta)$$

$$S_{dc}^{ab}(\theta) = \sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\, S_{ad}^{bc}(i\pi - \theta)$$

1. Non-invertible symmetries in 1+1 dim
2. Integrable flow from tricritical Ising and S-matrix
3. Derivation of modified crossing rules
4. Conclusion

Conclusion & Future

$$S^{ab}_{dc}(\theta) = \sqrt{\frac{}{}} \; S^{bc}_{ad}(i\pi - \theta)$$



$$S^{ab}_{dc}(\theta) = \sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}} \, S^{bc}_{ad}(i\pi - \theta)$$

- Topological modes in 1+1 d leads to modified crossing symmetry.
- Physically, it comes from corrections to norms of in- and out-states due to TQFT dynamics.
- Similar modified crossing observed in Chern-Simons matter in 2+1 d.

[Mehta, Patel, Prakash, Minwalla, Sharma],…

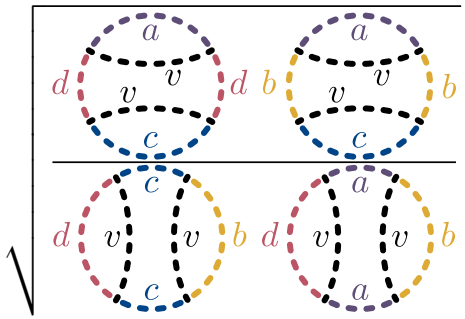
In our examples, importance of norms is clearer & TQFT d.o.f is more hidden.

- Soft modes in gauge theories and gravity? Charge-monopole scattering? Gravity waves?

Backups

Future

$$S_{dc}^{ab}(\theta) = \sqrt{\quad} \; S_{ad}^{bc}(i\pi - \theta)$$

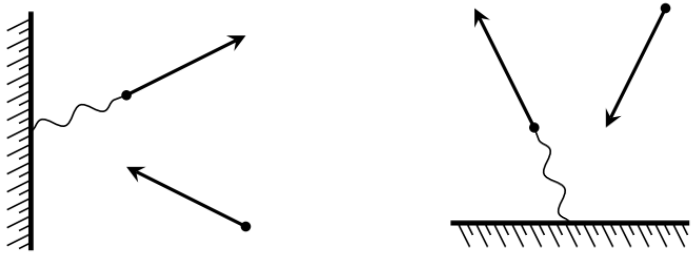


$$S_{dc}^{ab}(\theta) = \sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}} \, S_{ad}^{bc}(i\pi - \theta)$$

- S-matrix bootstrap with categorical symmetry (Haagerup fusion category)

  [WIP]

- Modified crossing in monopole scattering?



[Csaki, Hong, Shirman, Telem, Terning, Waterbury]

[van Beest, Boyle Smith, Delmastro, Komargodski, Tong]

[van Beest, Boyle Smith, Delmastro, Mouland, Tong]

- Toy model for IR effects (Faddeev-Kulish) in gravity and gauge theory?