



The Sound of Black Holes

Vítor Cardoso

Niels Bohr Institute
& Técnico Lisbon

Tory Island, County Donegal,
Ireland, 1995.

Photograph by Martine Franck

Standard Model not the final word



Death and the Miser, by Frans Francken the Young

Galaxy interactions

Gravitational lensing

Rotation curves

Velocities of galaxies in clusters

Structure formation

...

General Relativity not final word

Theorem (Penrose 1965; 1969):

For “reasonable” matter, trapped surface formation results in “singularity,” where at least one of the following holds:

- a. Negative local energy occurs.
- b. Einstein's equations are violated.
- c. The space-time manifold is incomplete.
- d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)

General Relativity is deterministic (strong CCC)

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel, Costa & Heusler 2012):
A stationary, asymptotically flat, *vacuum* BH solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi$$
$$- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$
$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass M and angular momentum $J=aM$, iff $a < M$

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

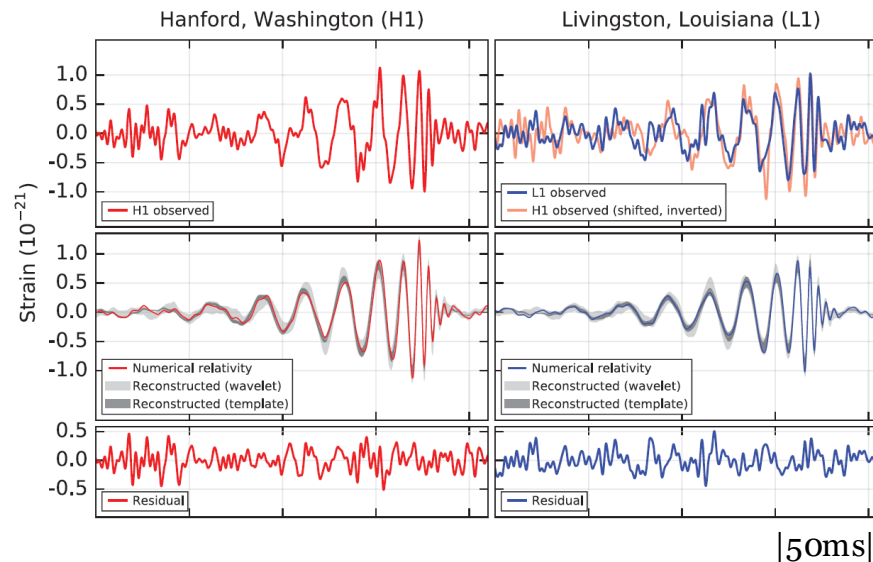
S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975



“But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future.”

S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 September 1986

Final state is compact!





How do we know it's a black hole?

Dark

Massive

Why is this enough?

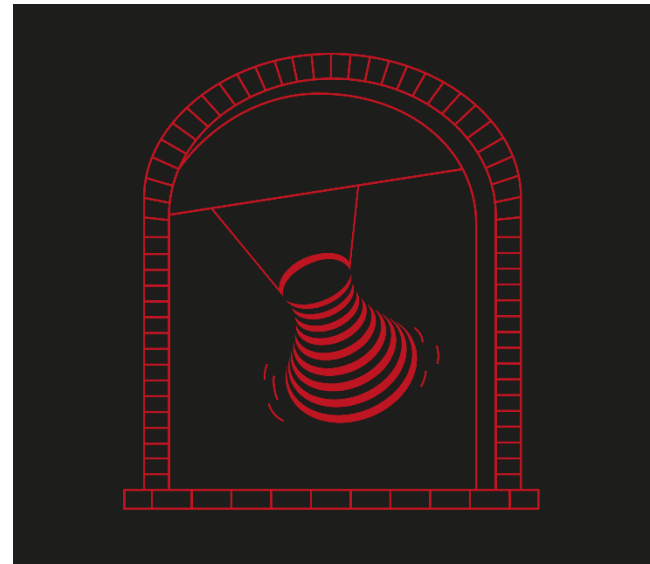
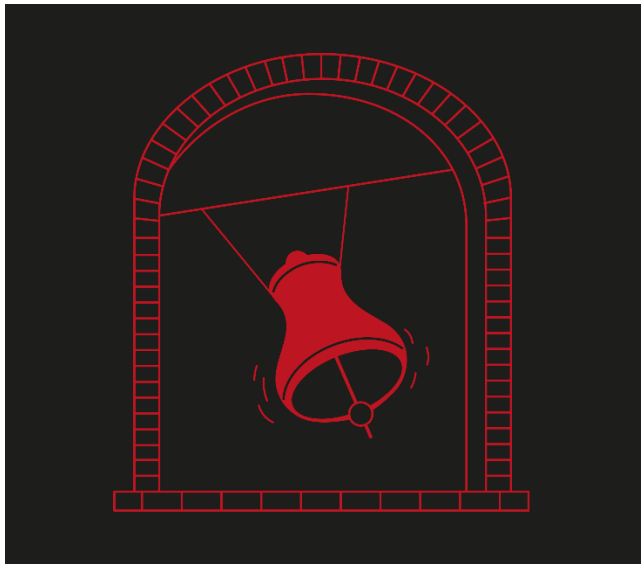
Black holes are end-point of gravitational collapse, using the equation of state that is thought to prevail in known matter.

No other massive, dark object has been seen to arise from generic collapse of known matter.

Fundamental questions

Are we really looking at black holes?

Is it a Kerr black hole? How can we constrain alternatives?

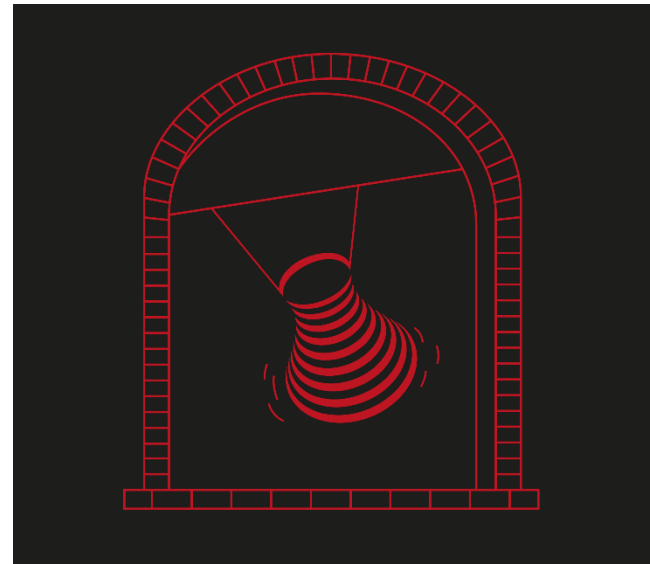


Answer requires understanding of theoretical framework, PDE analysis, precise modelling, observations, challenging simulations & data analysis techniques

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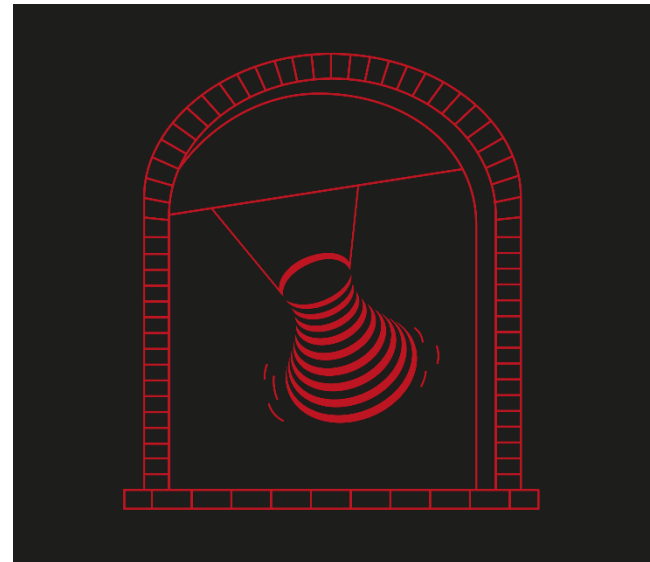


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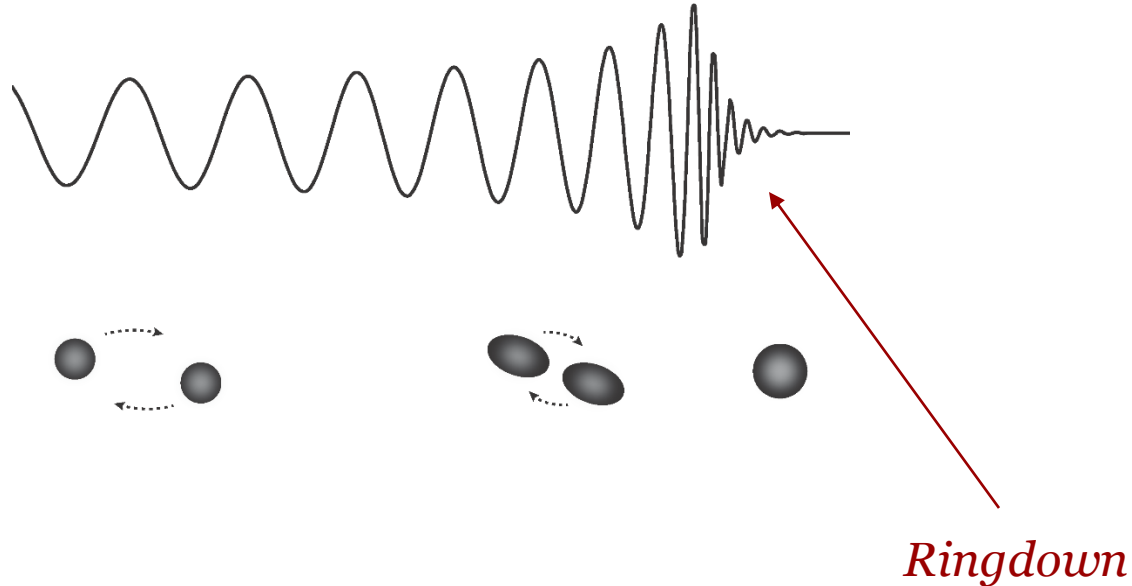
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Black hole spectroscopy



$$\frac{\partial^2 \Psi}{\partial r_*^2} - \frac{\partial^2 \Psi}{\partial t^2} - V(r_*) \Psi = S \quad \sim e^{-i\omega t}$$

$$(\omega^2 - \mathcal{L}) \psi = s \rightarrow \psi = (\omega^2 - \mathcal{L})^{-1} s$$

poles = QNMs

$$h = \sum_{nlm} A_{nlm} e^{-t/\tau_{nlm}} \sin(2\pi f_{nlm} t) Y_{lm}(\theta, \phi)$$

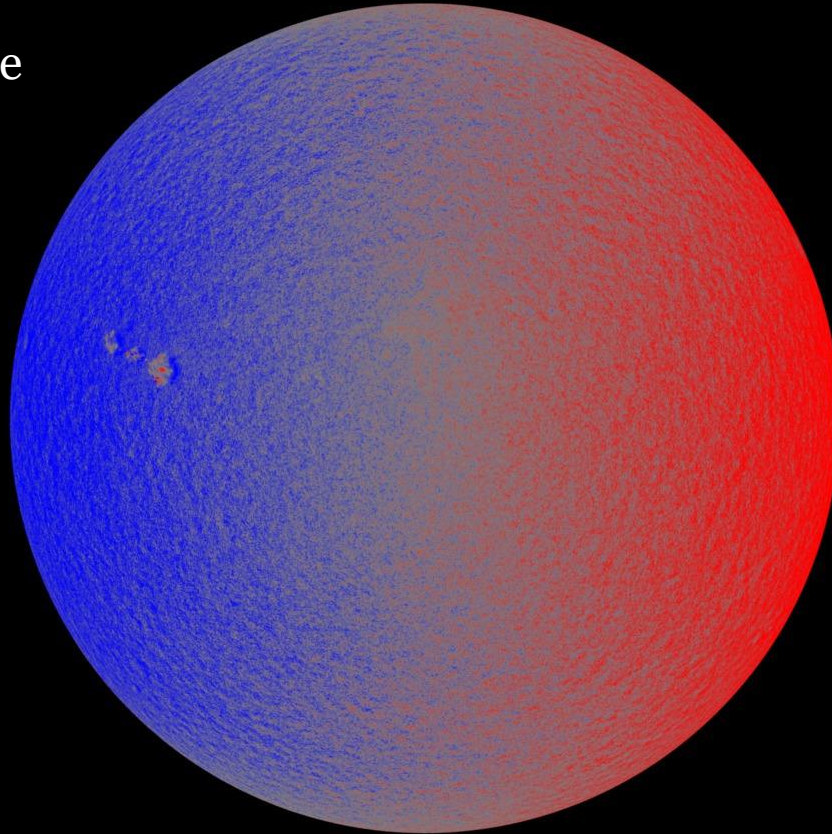
Helioseismology

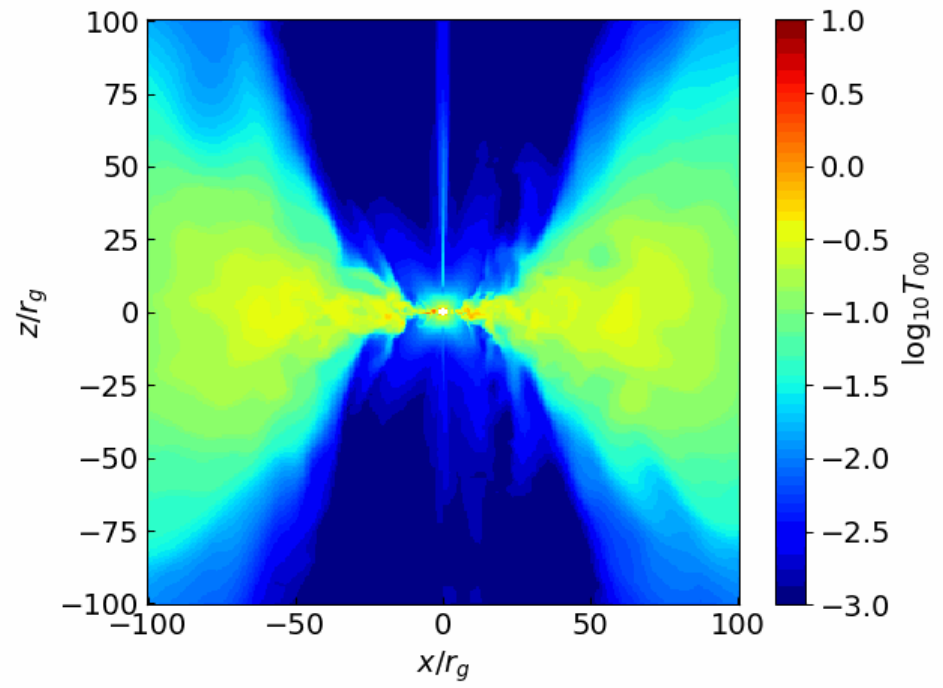
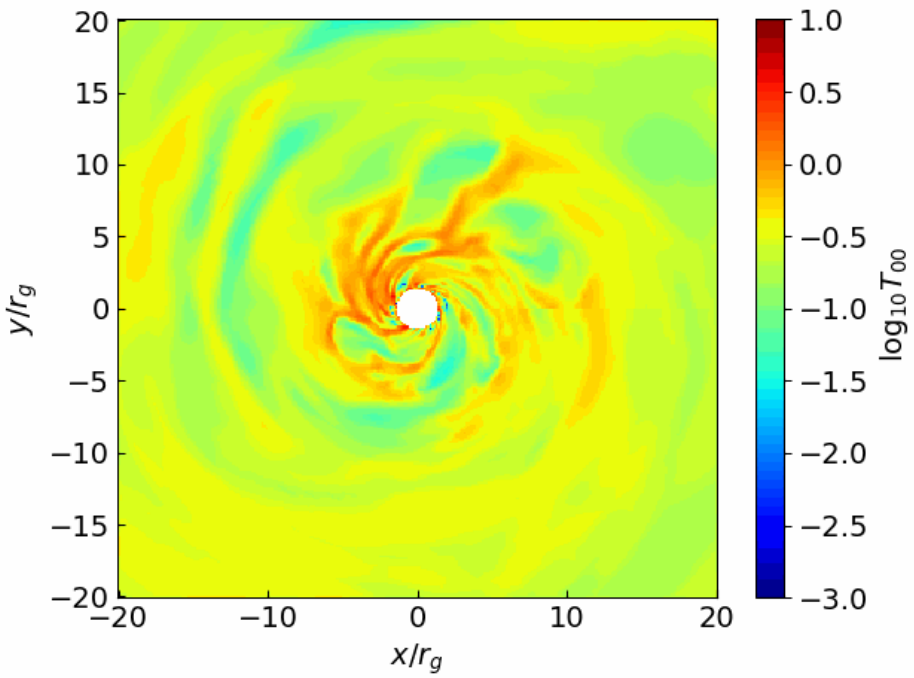
Solar spectrum sensitive to radial structure, opacity, temperature...

More than 10^6 modes with periods between 4 and 6 minutes

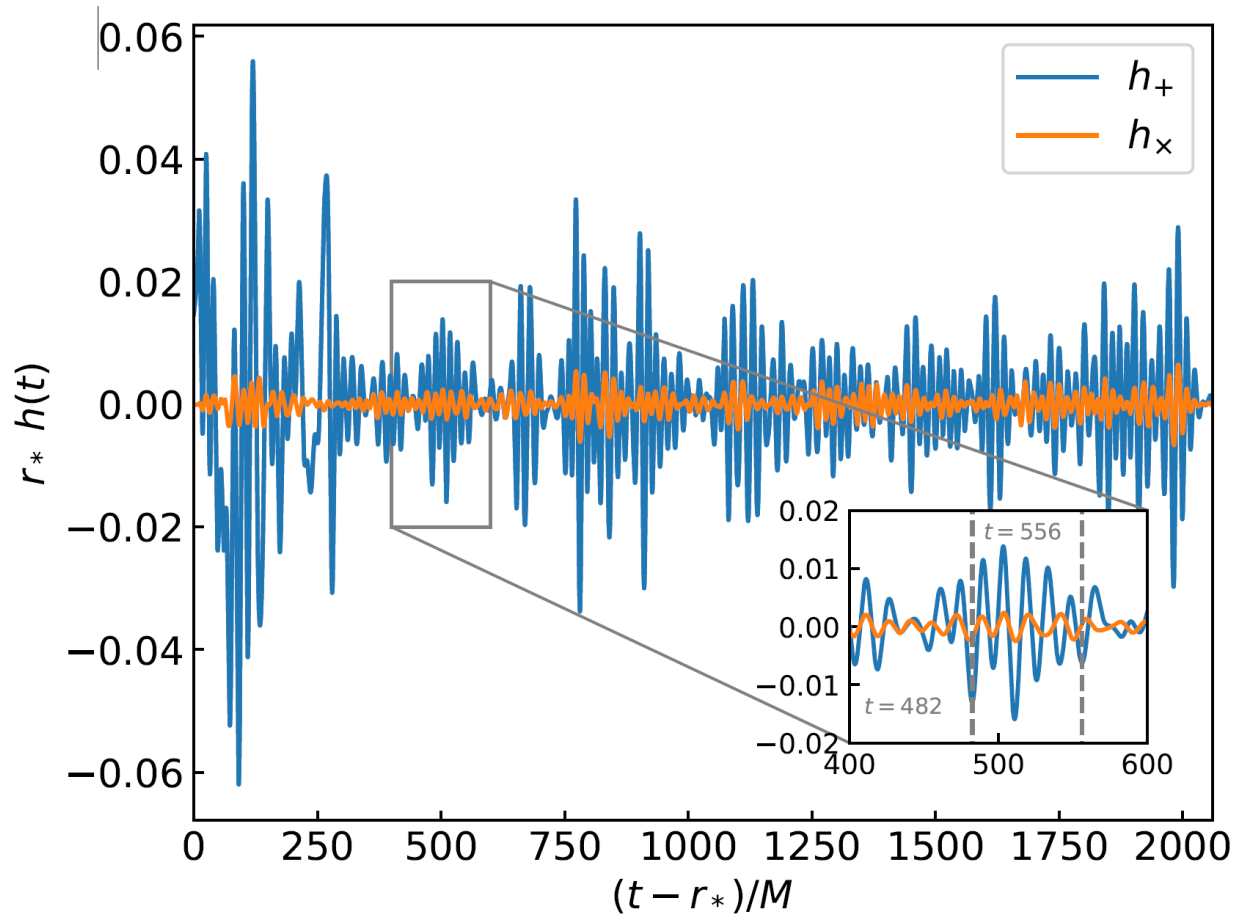
Quality factors of order $10^6 - 10^9$

Driven by turbulence

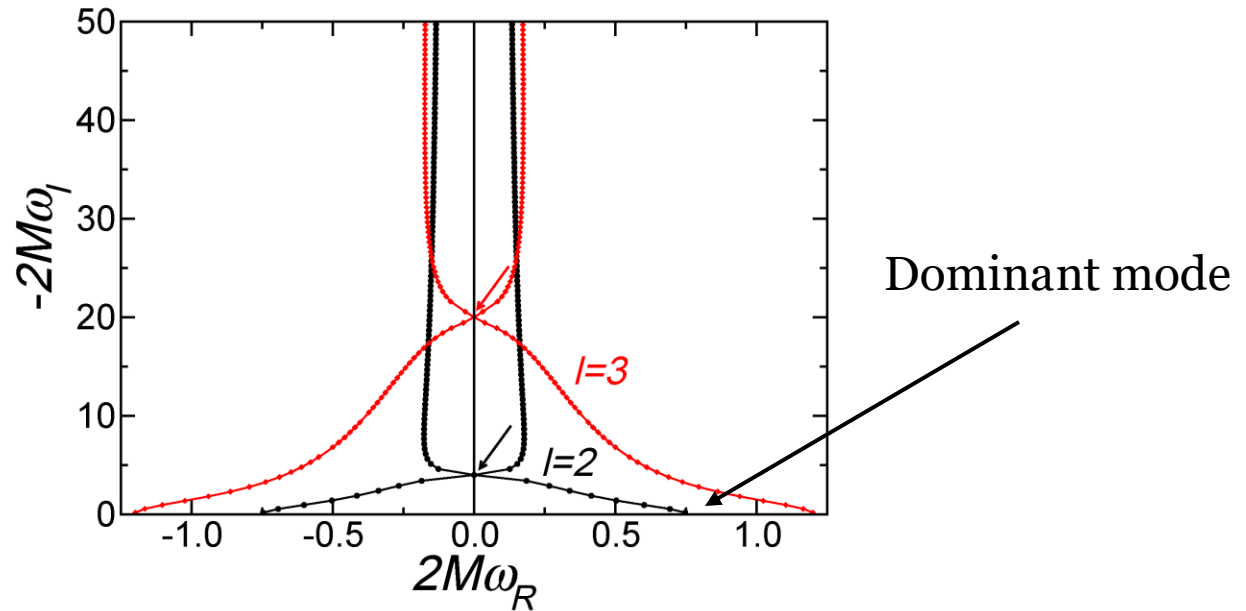




Yuan +, to appear



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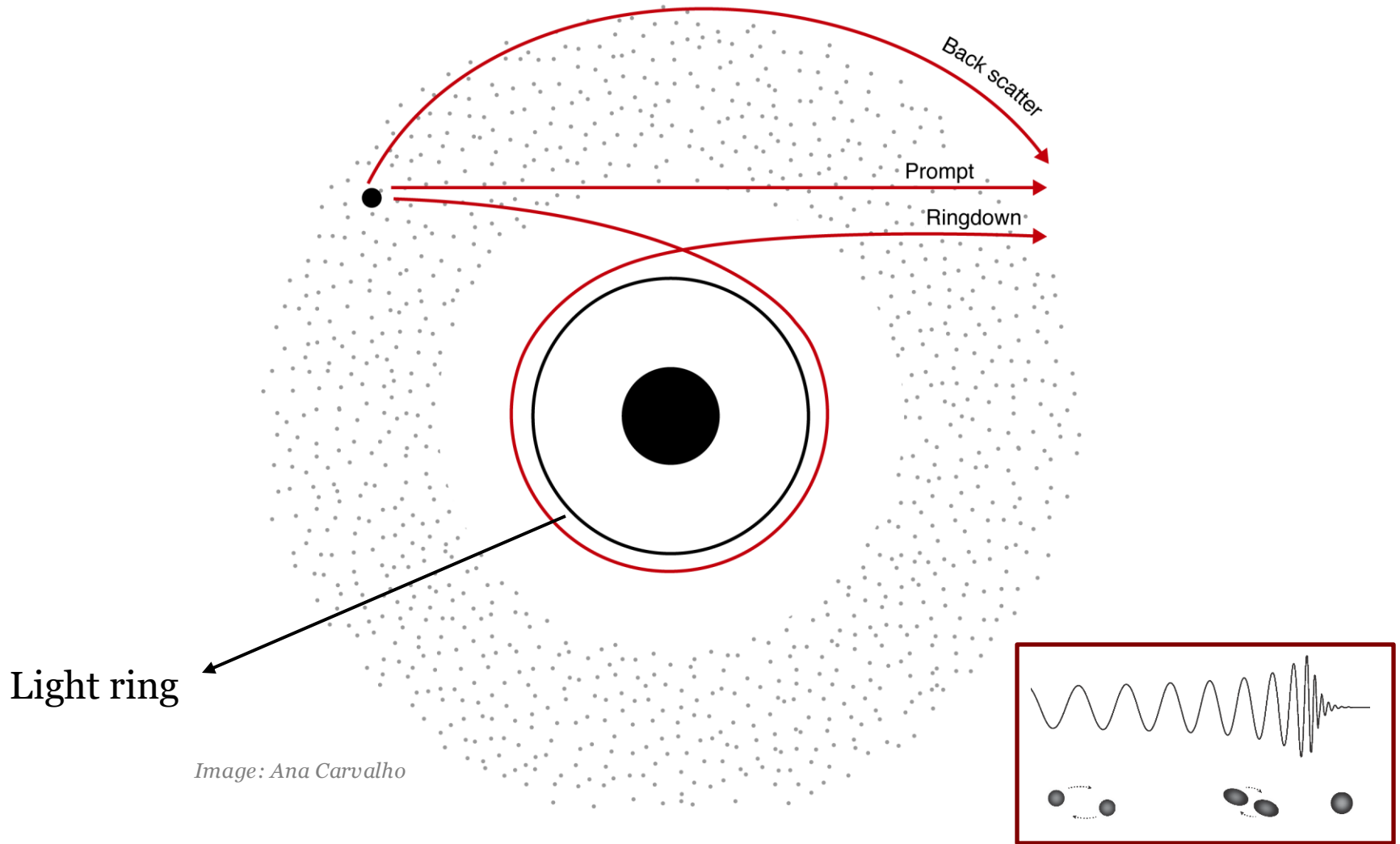


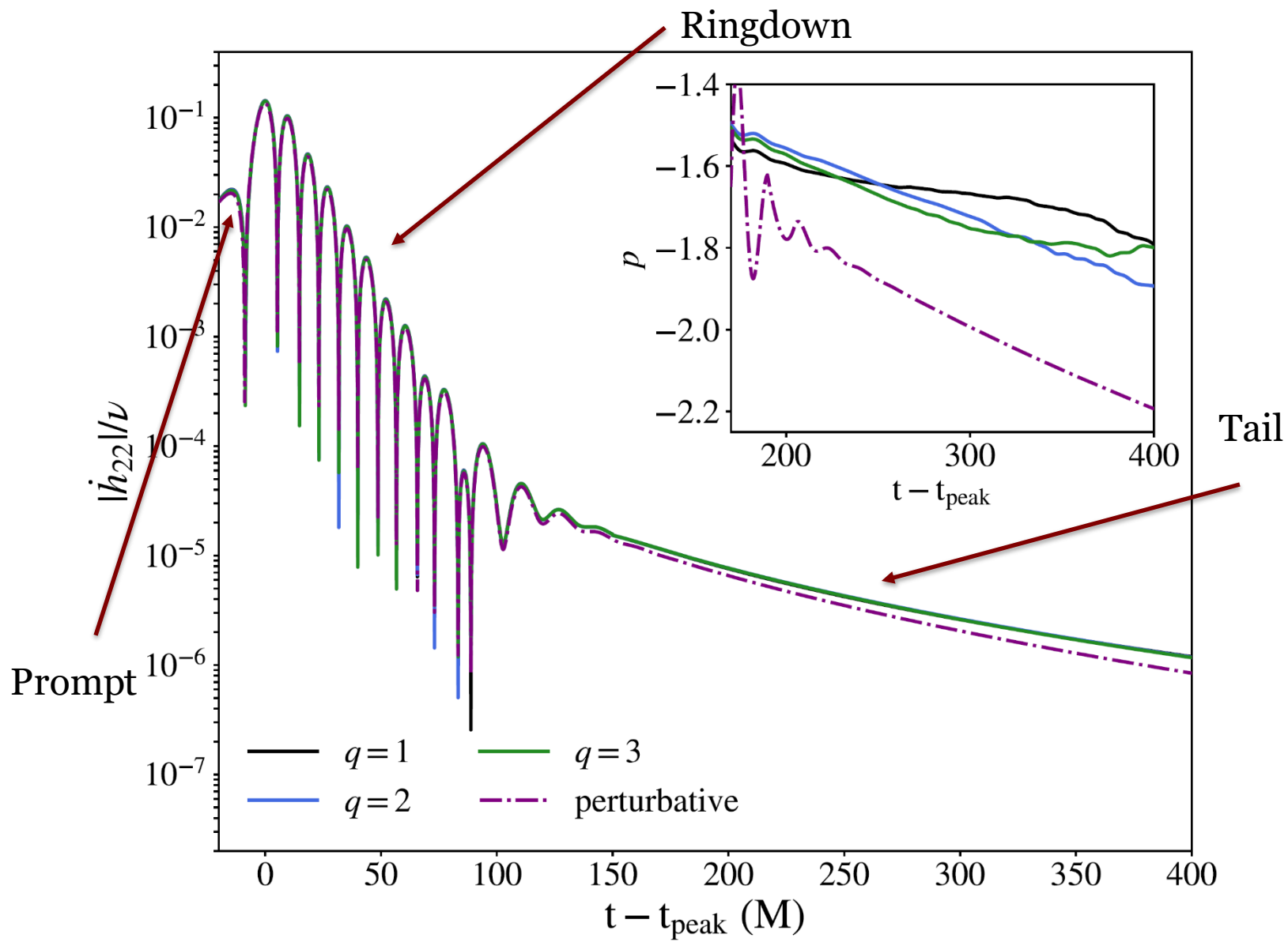
$$f = \omega_R/2\pi = 1.207 \left(\frac{10 M_\odot}{M} \right) \text{ kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10 M_\odot} \right) \text{ ms.}$$

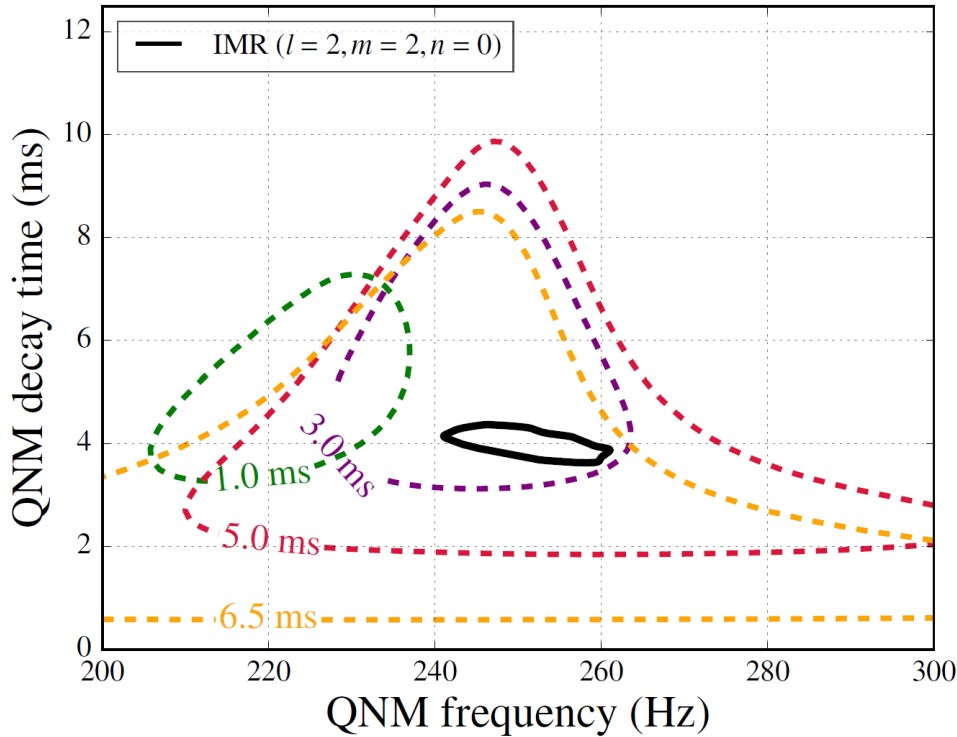
Quality factors of ~ 3 . Black holes don't ring, they relax with a thud.

When is a *linear* ringdown description valid?



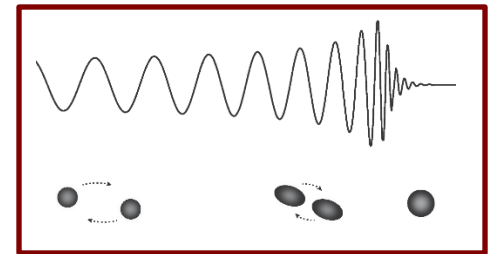


One and two-mode estimates: the start of spectroscopy



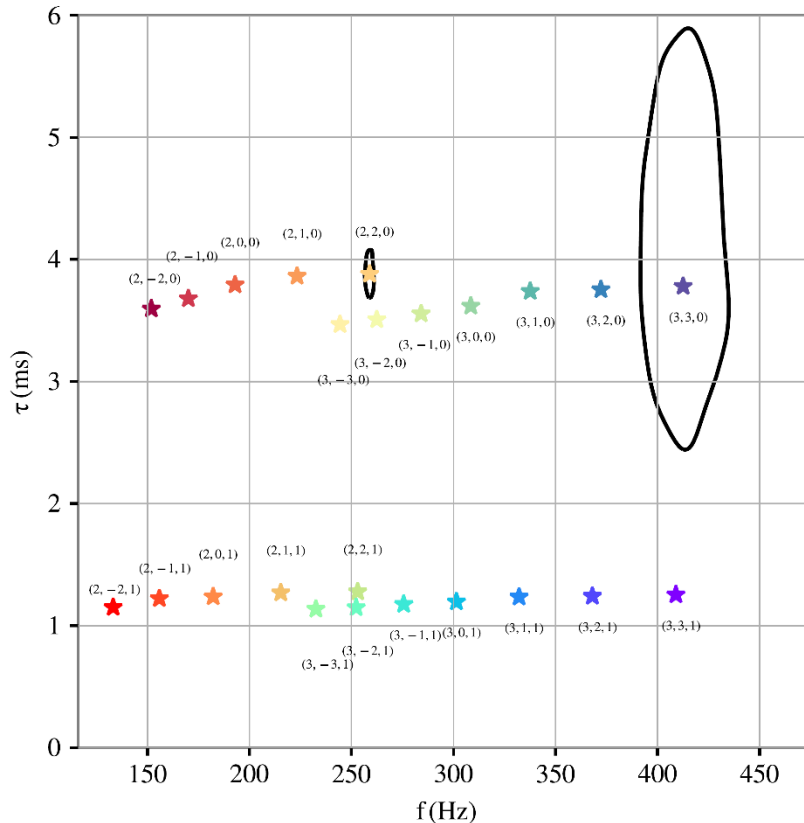
90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant



LSC PRL116:221101 (2016); arXiv:2010.14529;
For future detectors: Berti+ PRL117:10102 (2016)

One and two-mode estimates: the start of spectroscopy



90% posterior distributions.

Black solid is 90% posterior of QNM from a future event with SNR=40 in ringdown.

LISA will see SNRs of thousands...

Courtesy of Gregorio Carullo

See also Berti+ PRL117:10102 (2016); Bhagwat+ arXiv:2304.02283

Challenges

Strength of the poles (“excitation factors”)

Inner products and waveform reconstruction with QNMs
 (“excitation coefficients”)

Norm of resolvent in entire complex plane (pseudospectrum)

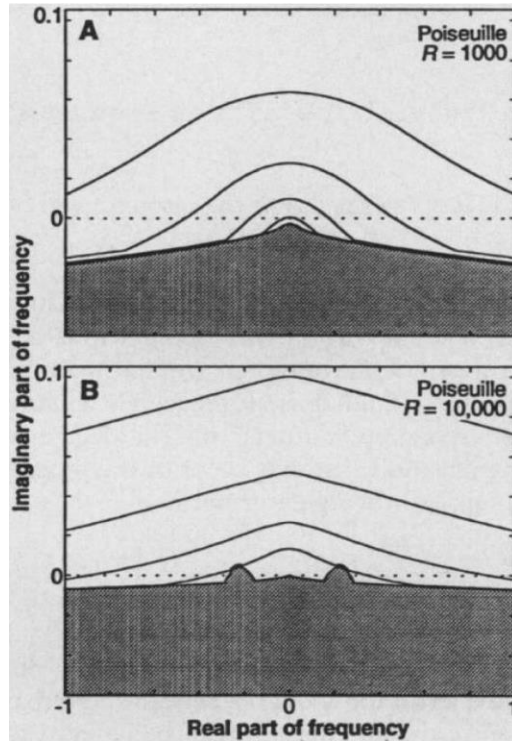
Environments (more soon)

Nonlinearities

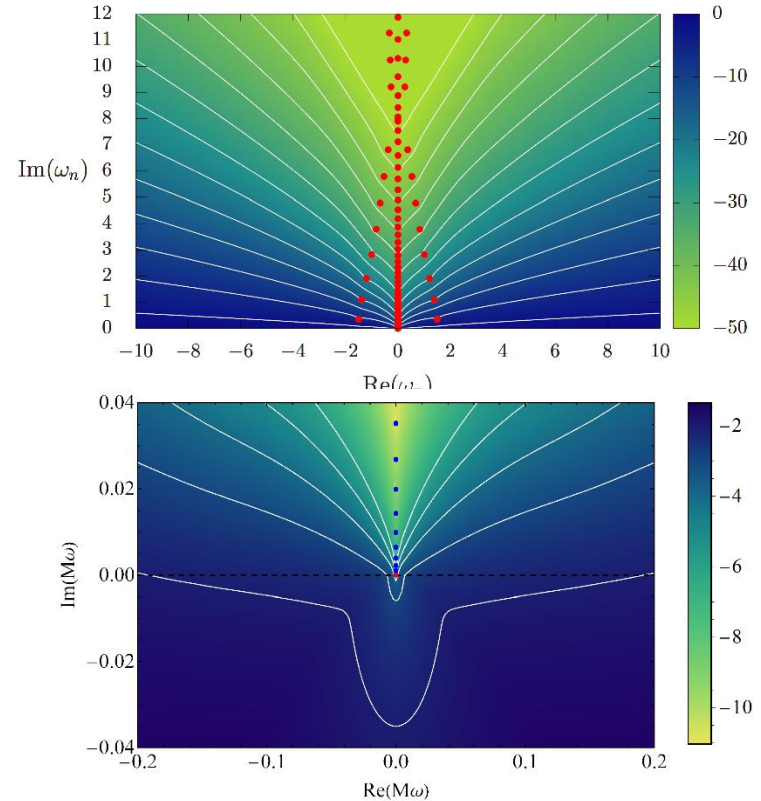
Interlude: pseudospectra

Trefethen+ Science 261: 578 (1993); Jaramillo+ PRX11:031003 (2021)

$$\left\| (\omega^2 - \mathcal{L})^{-1} \right\| \geq \epsilon^{-1}$$



(plane) Poiseuille: parabolic velocity profile between static infinite plates



Pseudospectrum of extremal BHs

K. Destounis+ PRD104:084091 (2021)
Also V. Boyanov+ PRD107:064012 (2022)

On the maximum luminosity and nonlinearities

Event	Peak Luminosity
Three Gorges dam	$3 \times 10^{-43} \mathcal{L}_P$
Most powerful laser	$3 \times 10^{-38} \mathcal{L}_P$
Tsar Bomba	$3 \times 10^{-27} \mathcal{L}_P$
Solar luminosity	$1 \times 10^{-30} \mathcal{L}_P$
γ -ray bursts	$1 \times 10^{-5} \mathcal{L}_P$
Inspiralling BHs	$2 \times 10^{-3} \mathcal{L}_P$
High-energy BH collision	$2 \times 10^{-2} \mathcal{L}_P$
Critical collapse	$2 \times 10^{-1} \mathcal{L}_P$
End point of BH evaporation	\mathcal{L}_P

$$\mathcal{L}_P = \frac{c^5}{G} = 3.6 \times 10^{52} \text{ W}$$

Conjecture:

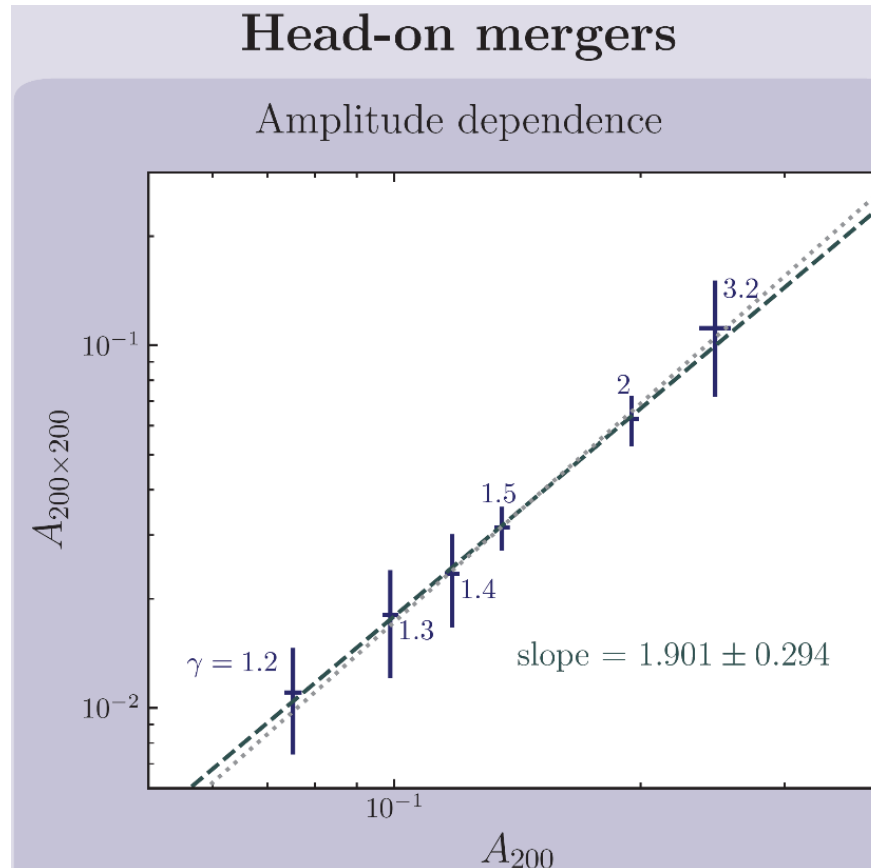
Maximum possible value of luminosity in a past-regular spacetime is the Planck luminosity \mathcal{L}_P

Cardoso + PRD97:084013 (2018)

also Gibbons & Barrow MNRAS 446:3874 (2015); Schiller PRD104:124079 (2021)

Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V \Psi^{(2)} \sim \left(\Psi^{(1)} \right)^2$$



Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V \Psi^{(2)} \sim \left(\Psi^{(1)} \right)^2$$

$$R_{220 \times 220} = \frac{A_{220 \times 220}}{A_{220}^2}$$

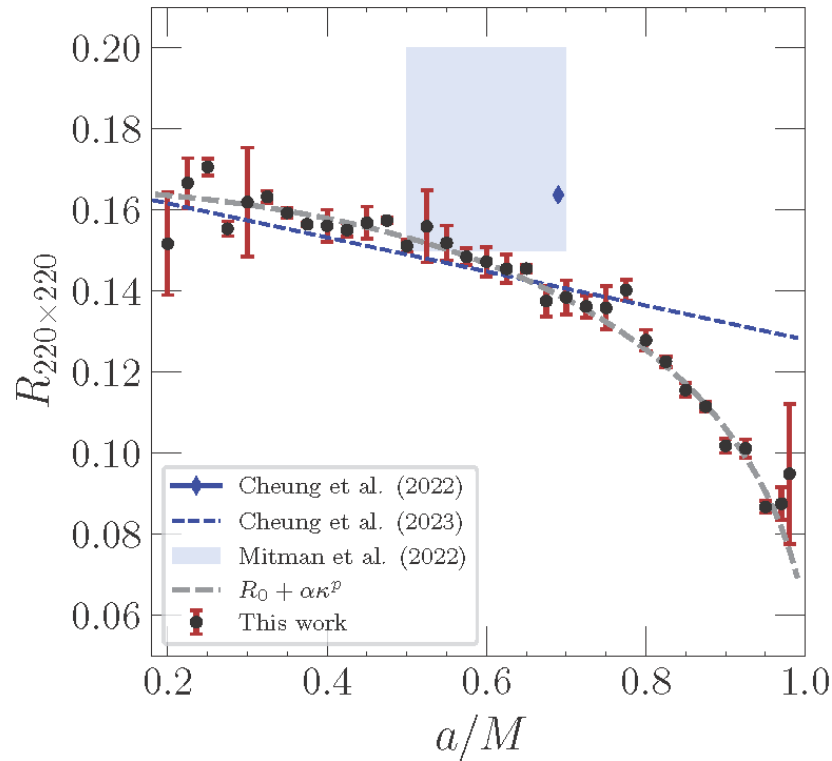
For quasi-circular inspirals of non spinning binaries, from NR:

$$R_{220 \times 220} = 0.16$$

It is not hopelessly small!

Cheung + PRL130:081401 (2023); Mitman + PRL130:081402 (2023)
Plenty of work, see Bucciotti + (2024), Bourg + (2024), Kehagias + (2024) etc

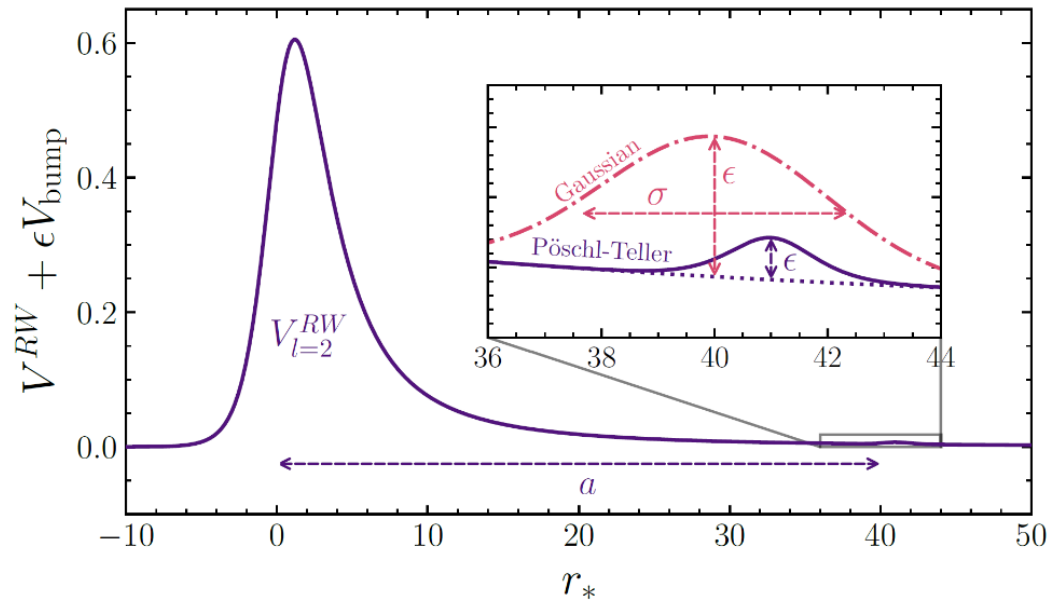
Nonlinearities in ringdown: second-order



Mild dependence on spin

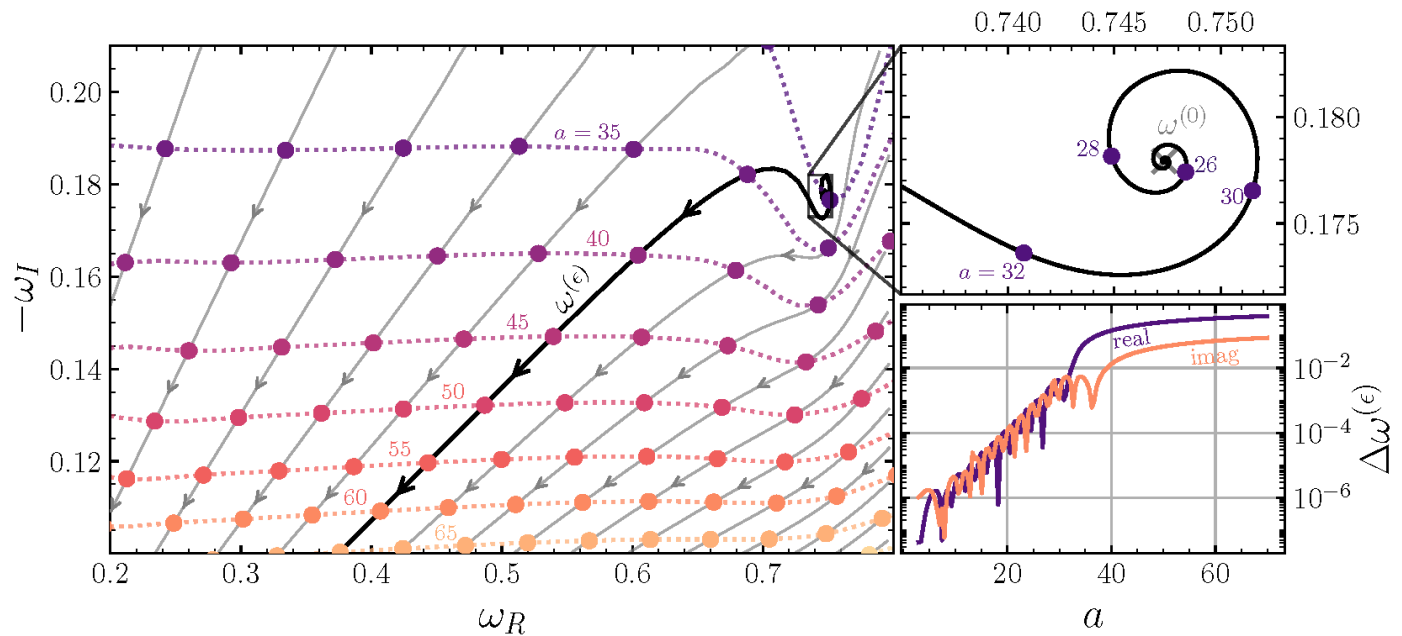
Spectral stability: the elephant and the flea

Spectrum is unstable: Nollert gr-qc/9602032; Barausse + PRD89:104059 (2014); Jaramillo+ PRX 11: 031003 (2021); Cheung+ PRL128:111103 (2022); PRD106:084011 (2022); Iannicari + PRL133:211401 (2024); Oshita + PRD110:084070 (2024); Yang + (2024)



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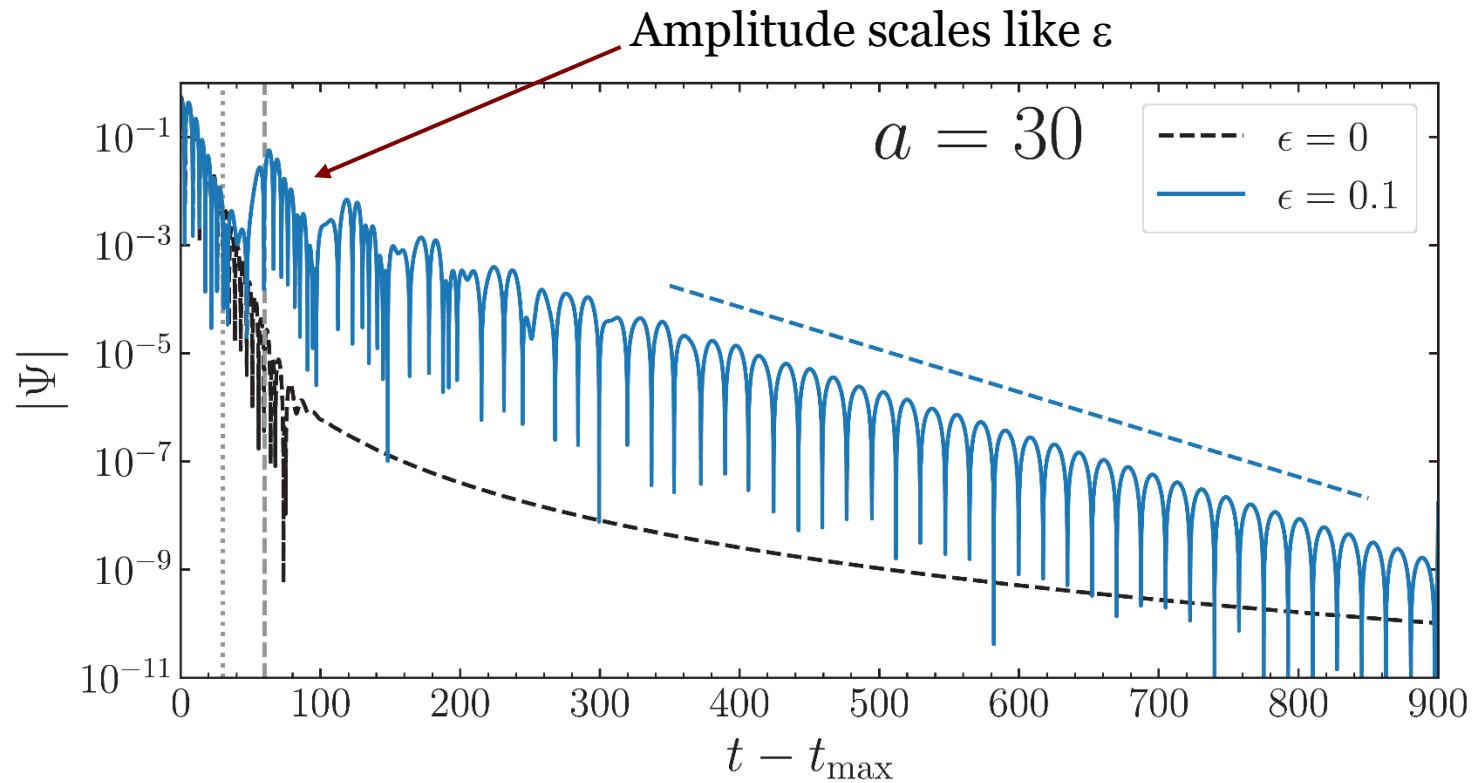
Spectrum is unstable: Cheung+ PRL128:111103 (2022); Berti +PRD106:084011 (2022)

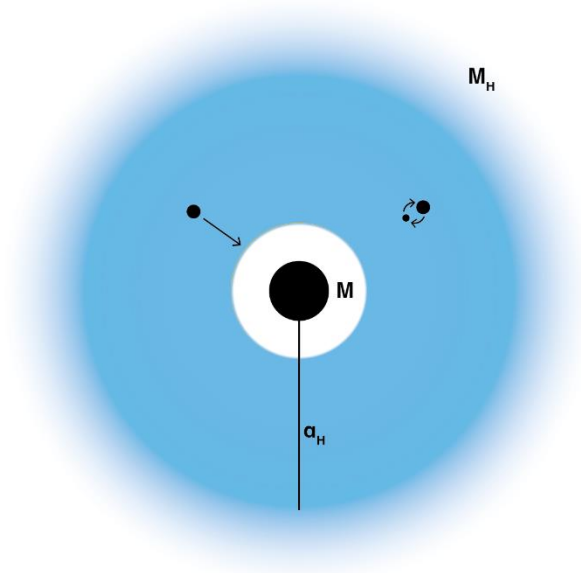


$$\epsilon = 10^{-6}$$

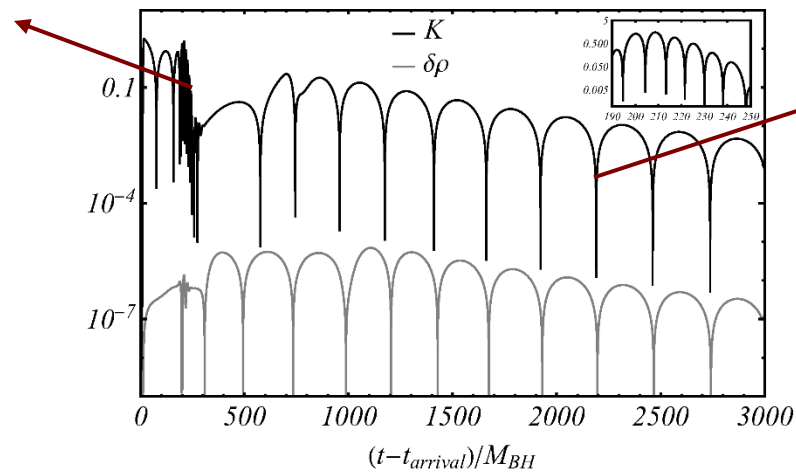
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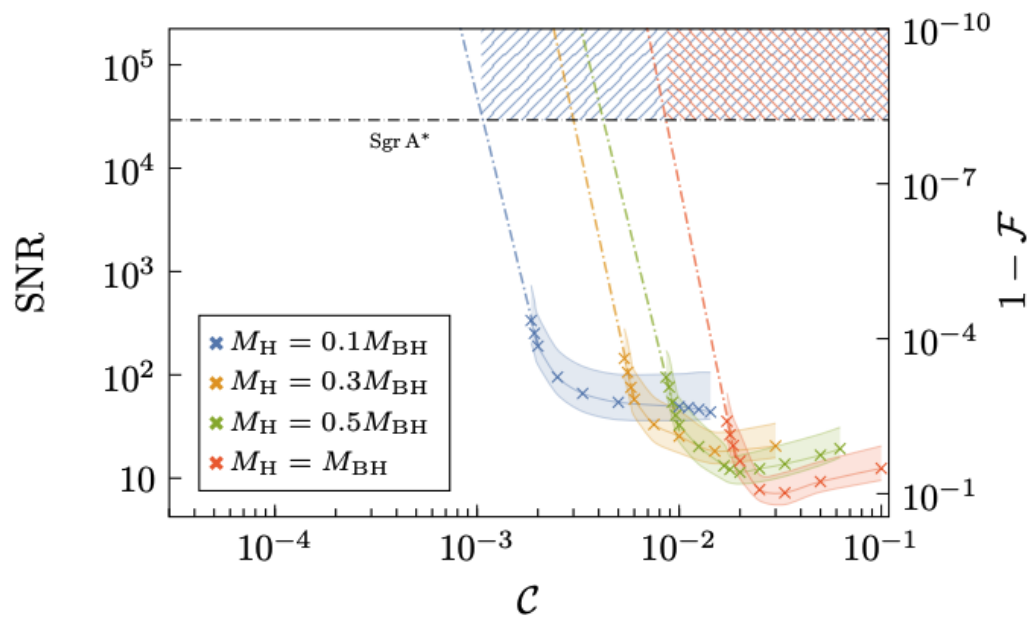
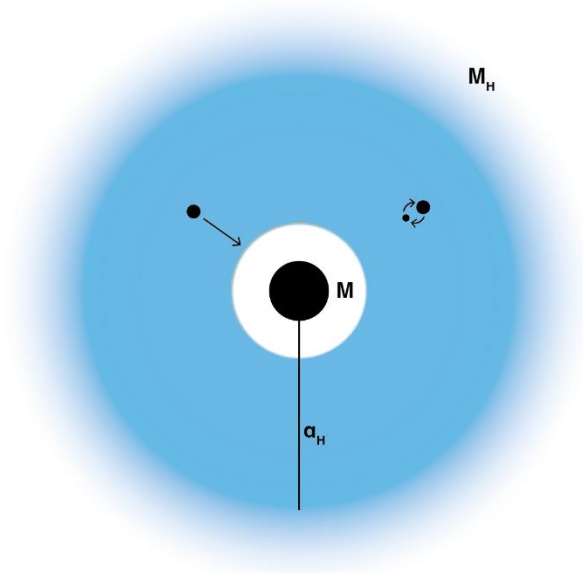




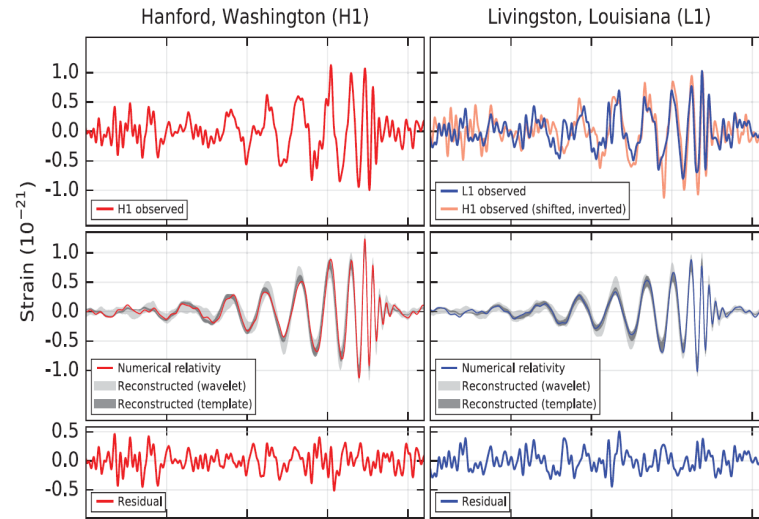
Light ring
modes (BH
ringdown)



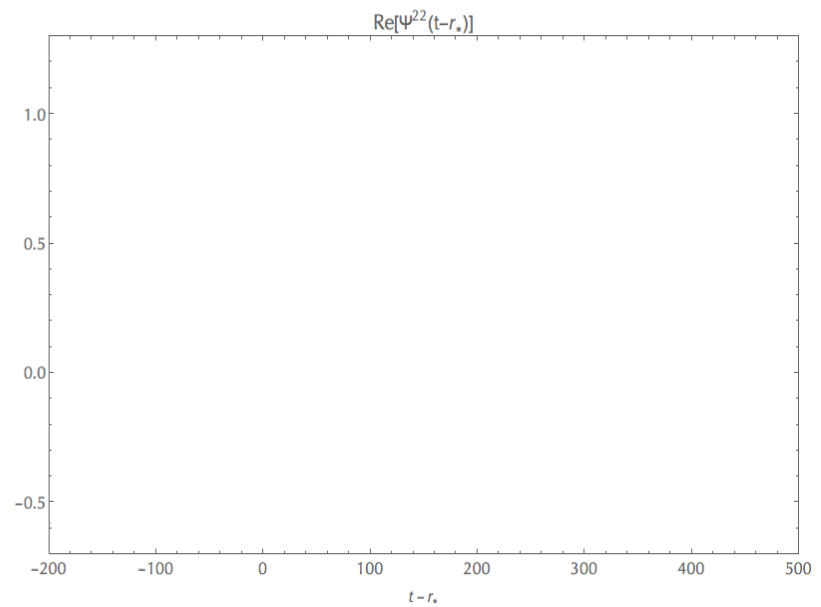
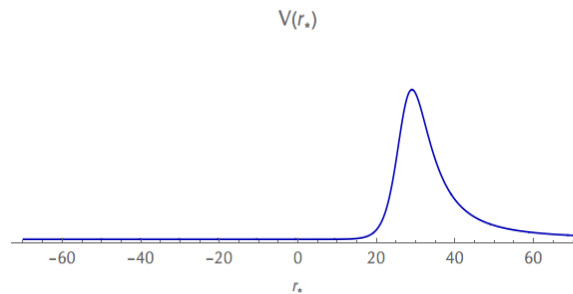
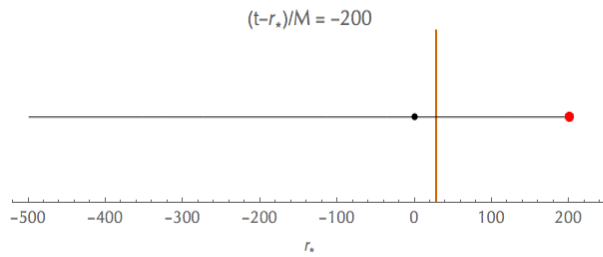
Fluid modes



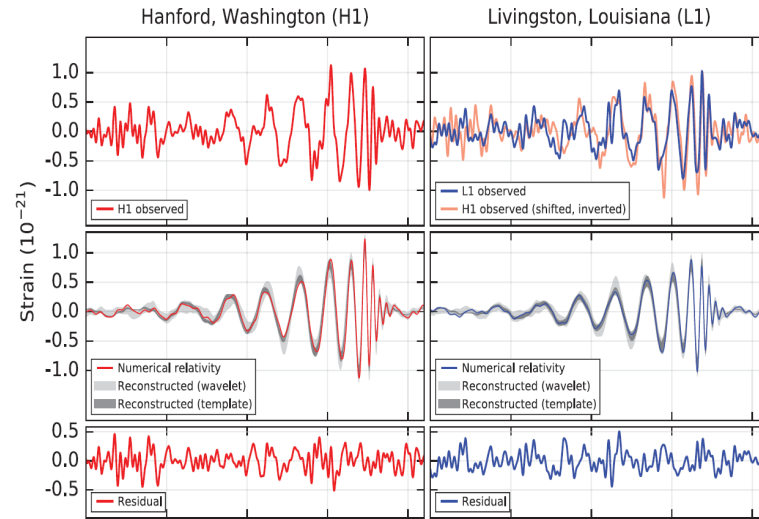
Post-merger



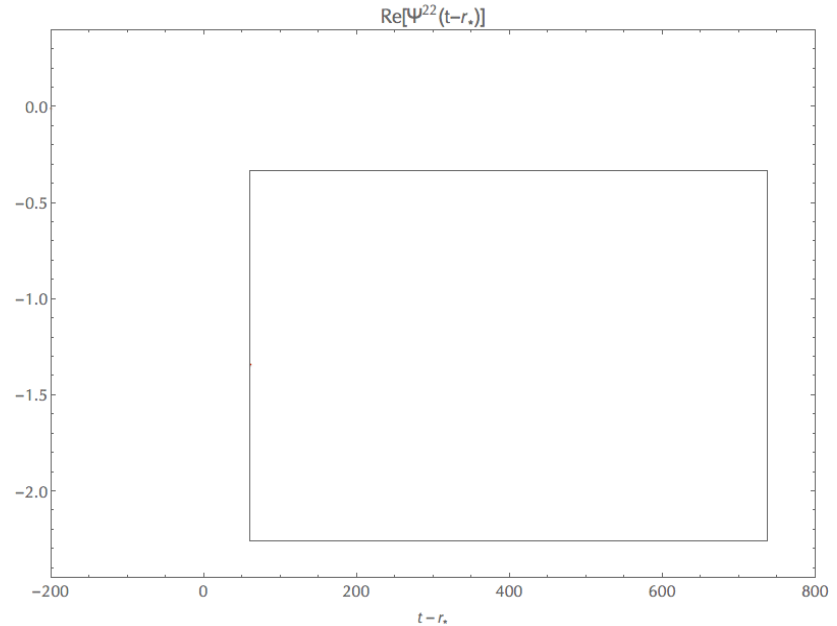
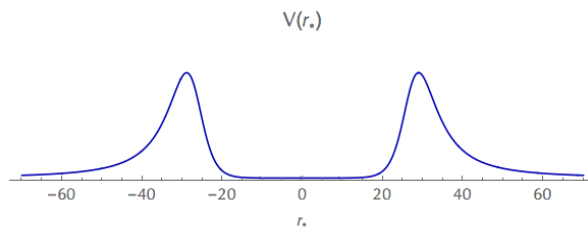
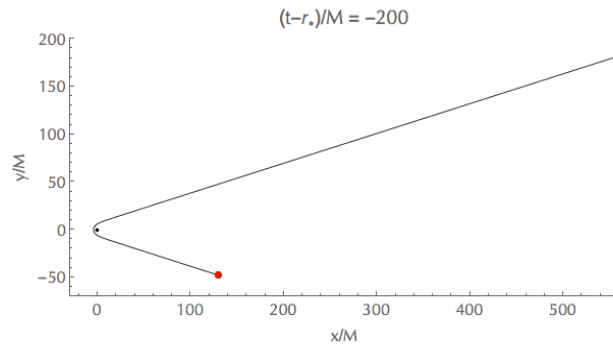
$$\mathcal{E} = 1.5, \mathcal{J} = 0$$

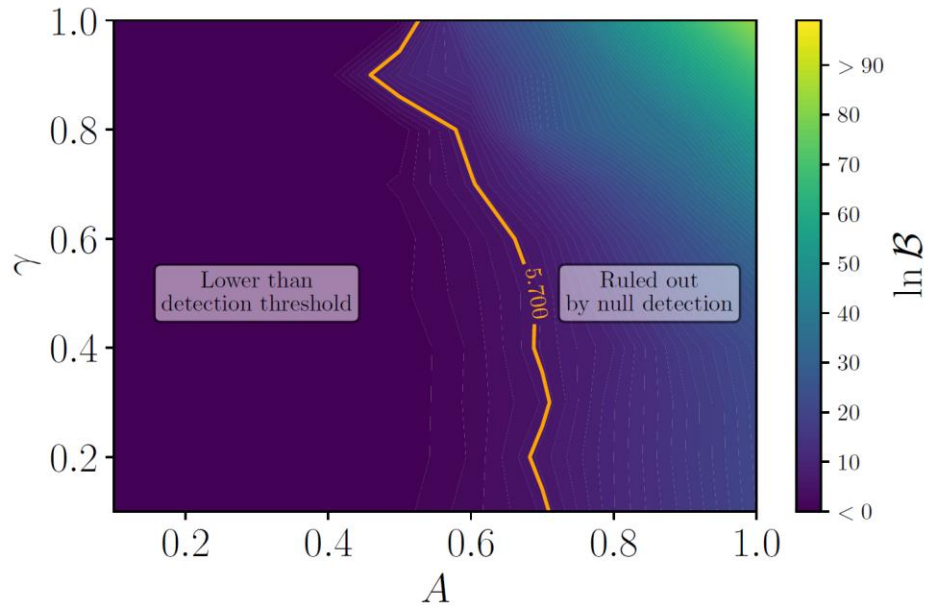
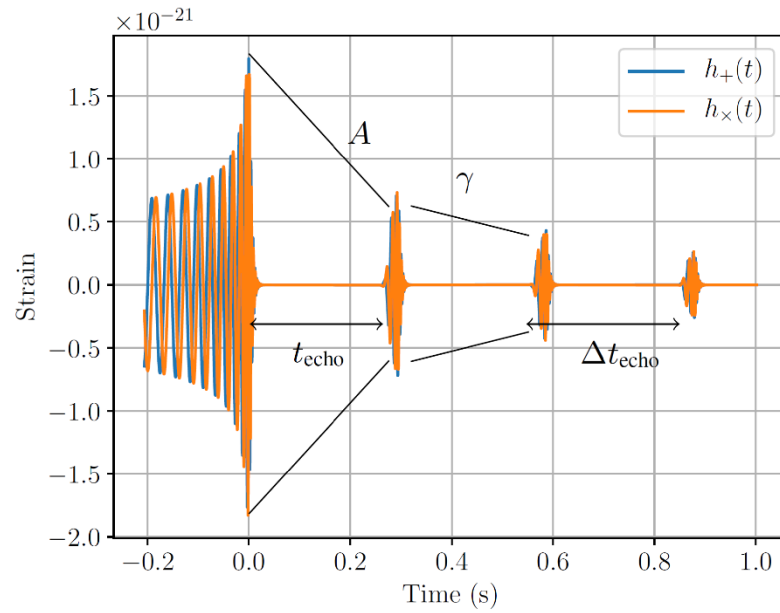


Post-merger



$$\mathcal{E} = 1.5, r_{\min} = 4.3M, r_0 - 2M = 10^{-6}M$$





Lessons learned?

Black hole spectroscopy has begun. Relaxation stage informs on final state conjecture, near-horizon physics and environments, and nonlinear effects. Dissecting every part of signal.

Environments are important to understand, current lore is that they are outside precision of detectors or redshift away for large scale galactic potentials. Much more work is necessary to understand extreme environments.

Echoes general feature of near-horizon physics, no fine-tuning

Spectroscopy can be used to test General Relativity or extensions thereof (e.g. Maenaut + 2024 for higher curvature corrections)

HUGE progress in last 10 years, community review ongoing (to appear, 2025)

Thank you



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Quantum Surprises?

Bekenstein & Mukhanov 1995
Kleban+2019; Cardoso+ 2019; Agullo+ arXiv:2007.03700


i. Postulate some area quantization

$$A = \alpha l_P^2 N = \alpha \frac{\hbar G}{c^2} N$$

$$\Delta A = \alpha \frac{\hbar G}{c^3} \Delta N = 32\pi \frac{G^2}{c^4} M \Delta M$$

ii. Compute absorbed energy of graviton

$$\Delta M = \alpha \frac{c\hbar}{32\pi G} \frac{\Delta N}{M}$$

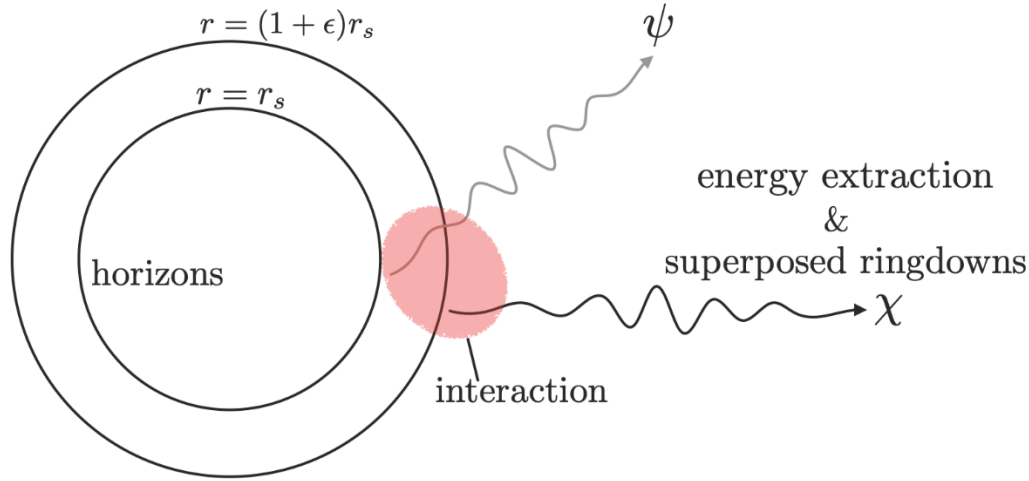
$$\omega_n = \frac{\Delta M c^2}{\hbar} = \frac{n\alpha}{32\pi} \frac{c^3}{MG}$$


Classical! Consequences for ringdown, TLNs, tidal heating

Agullo + PRL126:041302 (2021)
Deppe + arXiv:2411.056045

Double horizons

Cardoso, Oshita, Mukohyama, Takahashi, arXiv:2404.05790



$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{\epsilon}{2}(\phi^\mu\partial_\mu\chi)^2 - \frac{1}{2}(\nabla_\mu\phi^\mu)^2(\epsilon_{11}\chi^2 + 2\epsilon_{12}\chi\psi + \epsilon_{22}\psi^2)$$

$\phi^\mu = g^{\mu\nu}\partial_\nu\phi$ with scalar $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$. Re-write

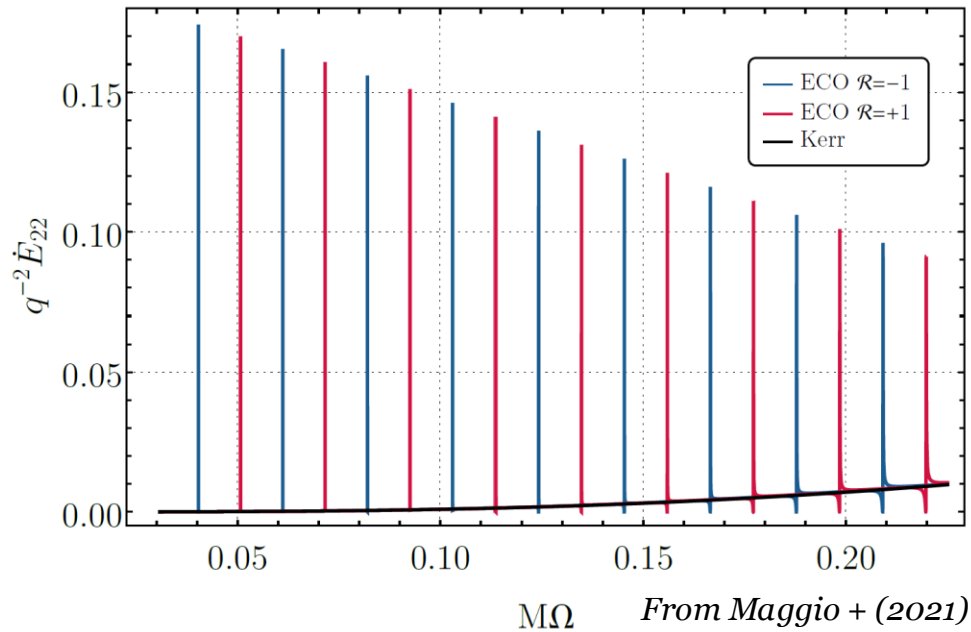
$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}(\nabla_\mu\phi^\mu)^2(\epsilon_{11}\chi^2 + 2\epsilon_{12}\chi\psi + \epsilon_{22}\psi^2)$$

Inverse disformal metric $\tilde{g}^{\mu\nu} := g^{\mu\nu} - \epsilon\phi^\mu\phi^\nu$:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{\epsilon}{1 + \epsilon}\phi_\mu\phi_\nu.$$

Early inspiral: resonances

$$r_{\text{surface}} = r_+ (1 + 10^{-10})$$



Cardoso, del Rio & Kimura PRD101:069902 (2020)

Maggio, van de Meent & Pani PRD104:104026 (2021)

Sago & Tanaka PRD104:064009 (2021); Sago & Tanaka arXiv:2202.04249

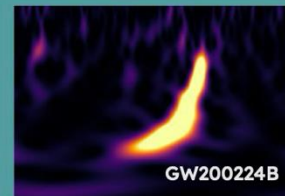
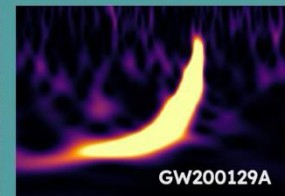
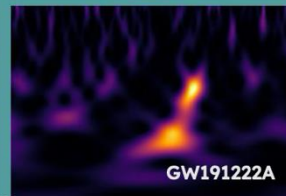
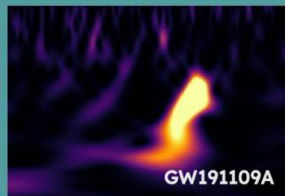
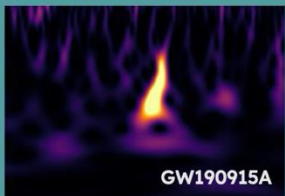
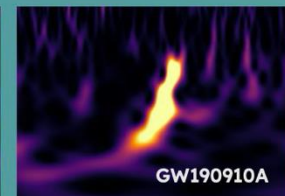
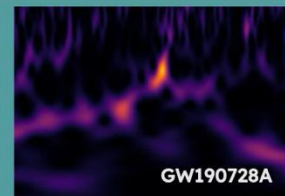
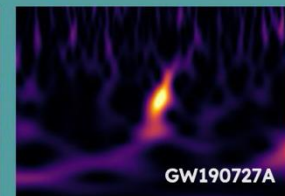
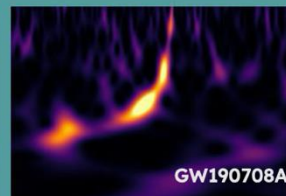
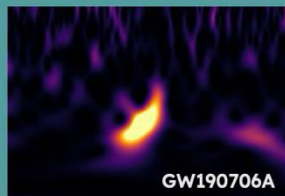
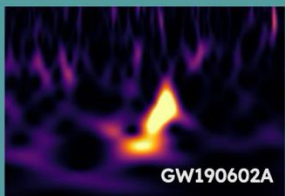
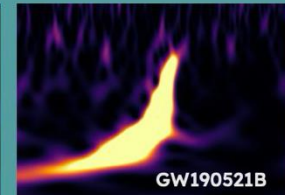
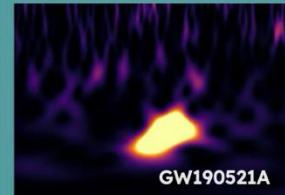
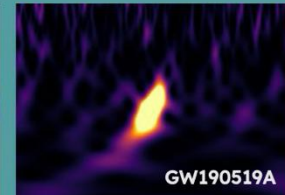
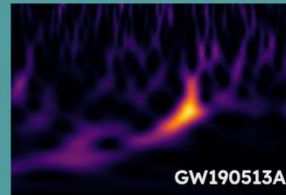
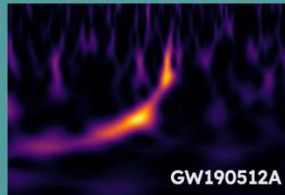
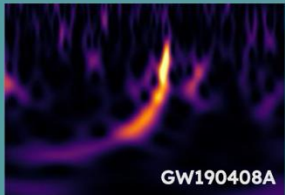
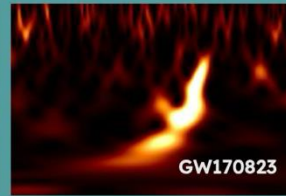
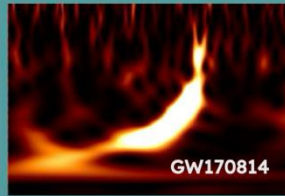
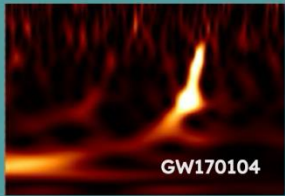
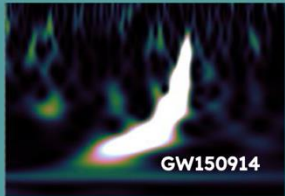
Caution with frequency-domain calculations: see Cardoso & Duque PRD105:104023 (2022)

Ringdown Gravitational Wave Catalog

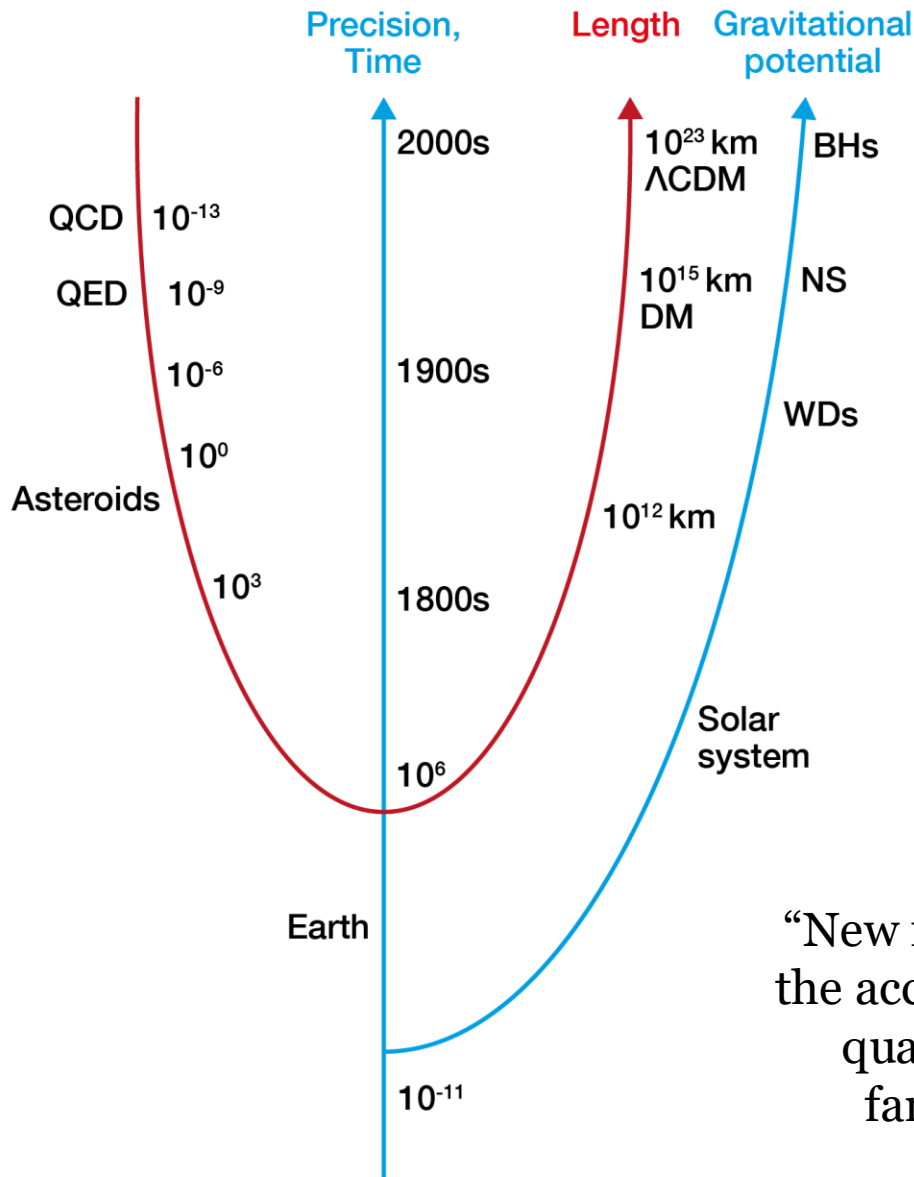


Detections from 2015-2020
passing the ringdown selection criteria

LVK, 2021

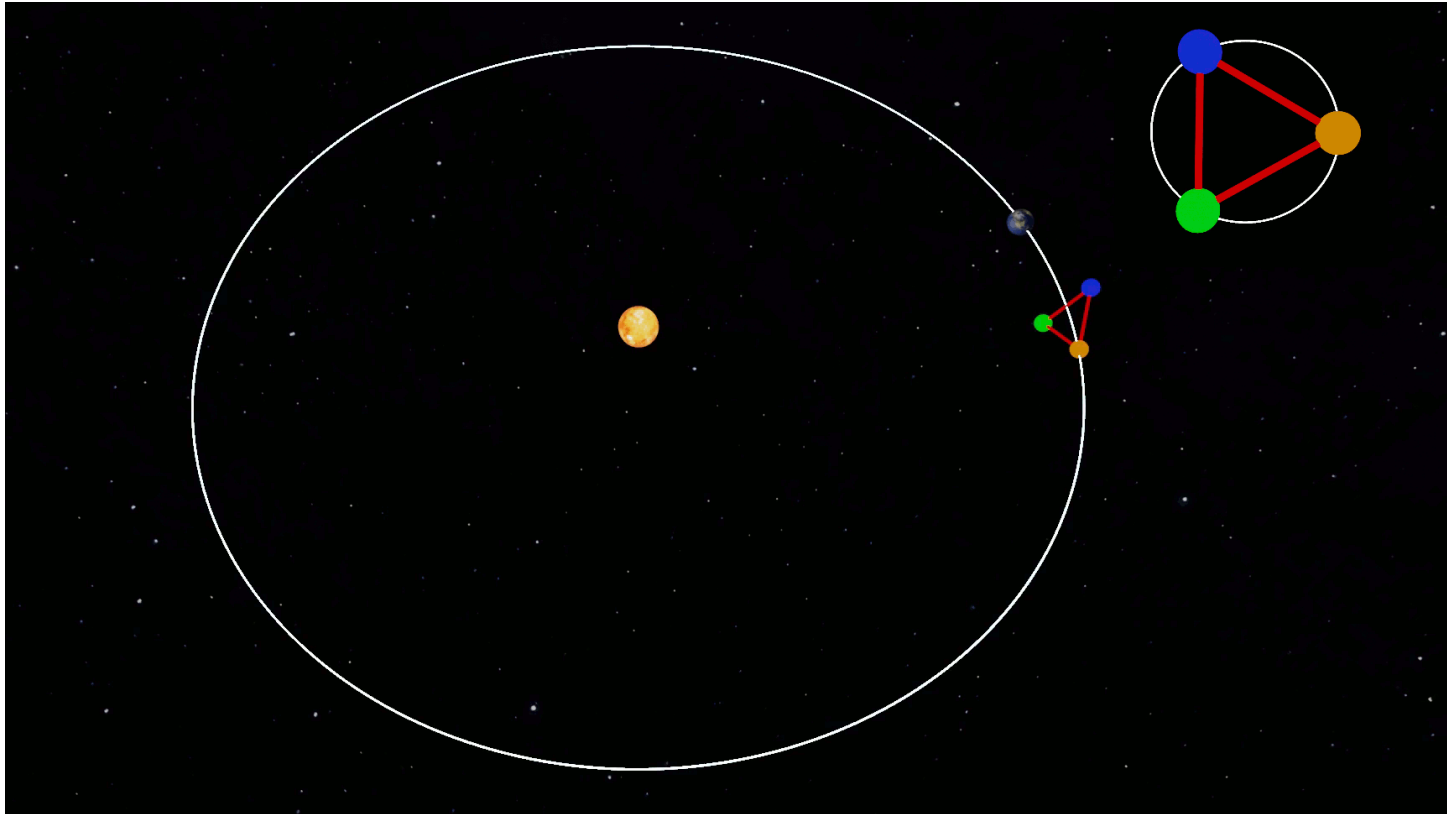


The dark universe



“New realms will become visible by improving the accuracy of the numerical measurement of quantities with which one has long been familiar.” [Maxwell, Scientific papers]

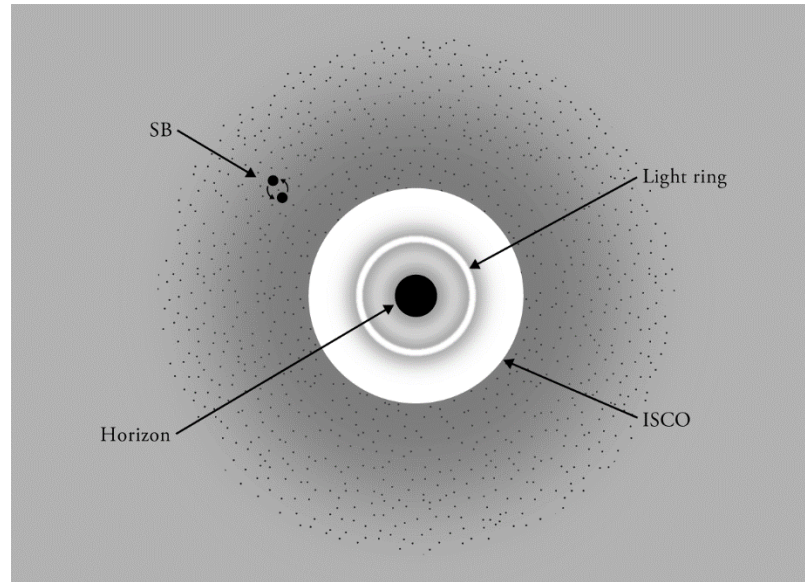
Observing from space with LISA (Laser Interferometer Space Antenna)



The most amazing space mission ever conceived!

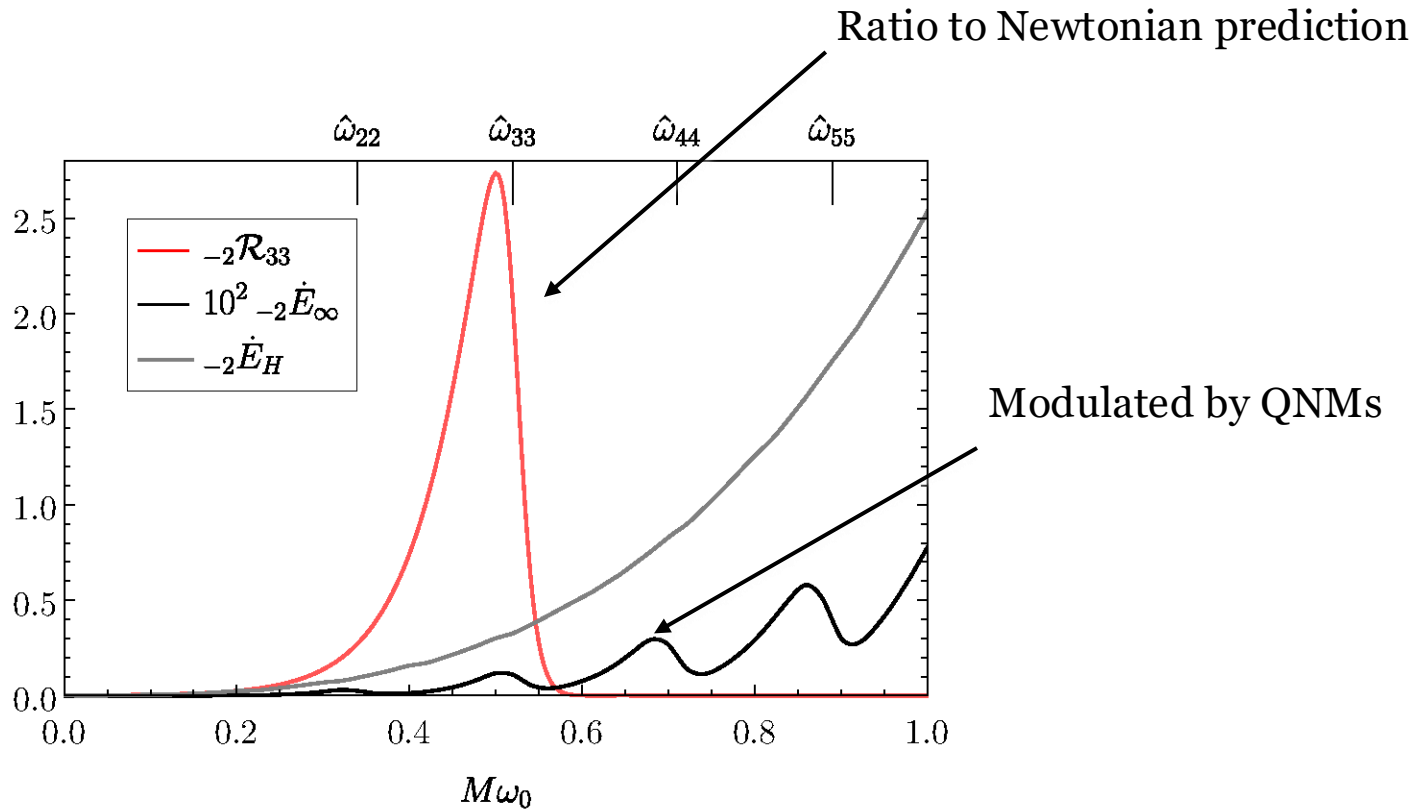
Spectroscopy: resonant excitation

Cardoso and Duque PRD103:Lo81501 (2021)



Spectroscopy: resonant excitation

Cardoso and Duque PRD103:Lo81501 (2021)



Energy output when tuning fork stands at ISCO of SMBH of spin $a = 0.9M$, as function of tuning fork frequency. Modal energy output ratio peaks at the lowest QNM. Also shown is flux integrated over modes: substantial component going down the SMBH horizon, total flux is modulated by QNM contributions.